Chapter 3
Division of labour and social co-ordination modes: a simple simulation model

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> I still believe that, by what is implicit in its reasoning, economics has come nearer than any other social science to an answer to that central question of all social sciences: How can the combinations of fragments of knowledge existing in different minds bring about results which, if they were to be brought about deliberately, would require a knowledge on the part of the directing mind which no single person can possess? To show that in this sense the spontaneous actions of individuals will, under conditions which we can define, bring about a distribution of resources which can be understood as if it were made according to a single plan, although nobody has planned it, seems to me an answer to the problems which has sometimes been metaphorically described as that of the "social mind". (von Hayek 1948, p. 54)

It is customary to consider economic organizations as social systems which make the co-ordination of individual plans and decisions possible. However, little attention has been given to the connection between forms of coordination and the process which characterizes industrial societies: the process of increasing division of labour.

The fact that co-ordination and the division of labour have been somewhat connected in their historical development is hardly possible to doubt: as economic organizations increased their degree of division of labour and knowledge, the problems of co-ordination among a growing number of increasingly inter-related producers and decision-makers became more and more complex. Co-ordination of distributed decisions by markets, firms and other economic institutions appears as the other side of the process of the increasing division of labour.

The historical evidence does not show clearly whether the two proc-esses of co-ordination and the division of labour have co-evolved as two aspects of the same phenomenon or have in the main proceeded independently, in terms of both temporal and causal relationships, but some evidence at the microeconomic level—such as analyses of the processes of design and managerial planning in modern corporations—seem to support the hypothesis of co-evolution. In economic organizations in which planning and design are highly purposeful activities, division of labour and co-ordination are the joint result of these very activities. But can the hypothesis of co-evolution of co-ordination and division of labour be extended also to the cases in which—both in markets and in organizations—they are not the outcome of a purposeful planning and design process, but

are emergent and partly unintended properties of the interaction between distributed decision-making activities (cf. von Hayek 1948)?

Behind this question lie two different ways of conceiving the division of labour:

1. As the outcome of a single mind, which designs, divides and coordinates: this is the view behind the Marxian capitalist and the Schumpeterian entrepreneur.
2. As the emergent, unplanned and unconscious outcome of the interaction of local and incomplete decisions.

We believe that an intermediate form can be found in large business organizations, where both forms, top-down conscious design and bottom-up partially unintended adaptation, co-exist. To understand why this is so we have first to take into consideration the role played by routines and contracts; both routines and contracts are necessarily incomplete and partly tacit, and this implies that some discretion must be left to the actors. Thus incompleteness and tacitness are likely to bring about cognitive conflicts.

To clarify this point, suppose that the top management puts into place a redesign of the organization in order to react to some kind of environmental change. The implementation of the new division of labour that is required by such a change gives rise to a complex process of adaptation which is far from that anticipated by the traditional theory of planning. On the one hand, the implementation of a new organizational design requires managers and employees to rethink their jobs and revise their competencies, but on the other hand—to be effective—any new design requires local checks and adjustments, i.e. the resolution of cognitive conflicts arising from a possible mismatch between the general requirements of the project and the specific, idiosyncratic knowledge of any single agent.

These considerations lead us to a brief remark on the role of managers: for what has been said, we cannot clearly distinguish between managing and doing:

> the blue collar worker on the most routinized assembly line must repeatedly make decisions about how to handle non-standard situations, and in particular when to call one to the attention of the supervisor. On the other hand, sales managers, in addition to managing their salespersons, often spend considerable amounts of time with clients, engaged in selling, and thus in 'doing'.
>
> (Radner 1992)

"Doing", i.e. the application of a given routine, is never a mere execution of given rules but always involves some discretionary behaviour, if only because a fully specified routine, which does not leave any space for autonomous decision-making, would require an endless list of contingent behaviours. The essence of managerial activities seems rather to lie in the capability to fill knowledge and information gaps and in the capability to design the organization of labour and "imagine how to do".

Therefore top-down plans—which are necessarily incomplete for reasons we will discuss below—have to match pieces of knowledge and skills which are largely local and tacit. Inevitably, they therefore have a largely uncertain and unintended outcome.

Before discussing the reasons for the incompleteness of planning, it is useful to note that, when considered in connection with the process of division of labour, the very notion of co-ordination takes on a meaning different from the one implicit in neo-

classical economics. For the latter, coordination means making individual and independent decisions compatible; here the problem of co-ordination concerns the relationship between the top-down activity of designing new organizational set-ups and the adaptive, intelligent, bottom-up reactions by managers and employees, to adapt the organization to the external environment.

Division of labour, distributed knowledge and specialization

The analysis of the process of division of labour as a primary determinant in economic development goes back to Adam Smith and lies at the heart of classical economics. But few of these early analytical efforts have been continued in modern neo-classical economics, and they have been brought back to their central role only within recent non-neo-classical studies of the process of technological evolution (e.g. Dosi et al. 1988).

A possibly fruitful way of interpreting the process of division of labour is to consider it within the general framework of the theory of problem-solving. From this point of view, the division of labour derives from the decomposition of problems into subproblems to be solved independently. Direct observation of the behaviour of organizations and individuals as problem-solving activity—within the tradition initiated by March and Simon (1958)—suggests that such behaviour is a peculiar and unstable balance between two opposite situations: on the one side purely routinized behaviour, in which a series of operations are repeated again and again, and on the other an active and conscious search for solutions to new problems (or new solutions to old problems) faced by the organization.

Without entering the debate on how to represent problem-solving activities formally, some general points are worth mentioning for their relevance to the subject of this chapter.

First, problem-solving activities are characterized by a search in a problem space which generally leads to the decomposition of the original problems into at least partially independent subproblems. If such a decomposition is feasible, subproblems can be solved in parallel and subsequently co-ordinated: the original problem is therefore decomposed into a set of connected subproblems. In the language of economics and organization science, this corresponds to the expectation that the decomposition of a production process *(lato sensu)* into independent subsystems, which can be dealt with in relative isolation and in parallel, will lead to increasing returns, in spite of the co-ordination costs which the decomposition generates.

Second, the working hypothesis we propose—along the lines of Simon and March—is that the search in the problem space, based on the division of the given problem into subproblems, is a model for the division of knowledge; even if division of knowledge and division of labour are not the same process (two employees co-operating in the same organization can have the same competencies and largely overlapping knowledge bases but different jobs), the former is necessary to the latter.

Third, it must be emphasized that the search in the space of sub-problems is a highly uncertain and conjectural process. In fact, when a problem has been decomposed into a set of subproblems, in general not all these subproblems will be solvable immediately and some will have to be decomposed in turn into simpler ones. The decomposition then

recursively proceeds until all the relevant subproblems have been solved. Problem-solving by subproblem decomposition is therefore an uncertain activity, for two reasons:

1. There is no set of decomposition rules which a priori allows agents to achieve a certain result.
2. The solvability of the original problem can only be verified when all the relevant subproblems have been solved (Egidi 1992).

This implies that plans and projects within firms are not merely executed by different actors: they also necessarily require a multi-agent process of learning and adaptation that can generate cognitive conflicts among the different actors.

These features allow us to consider co-ordination within organizations as the complement to the division of knowledge and labour which follows the realization of a new project. The further the division of labour proceeds, the more the different parts require co-ordination and the more information becomes dispersed. A crucial problem then arises: can we assume that the co-evolution of division of labour and co-ordination also takes place within markets?

A first approach would claim that markets are mere co-ordinating devices (by means of price mechanisms), whereas the division of labour is performed entirely within organizations (Coase 1937). A more careful examination would suggest that the two processes take place in a complementary way in both markets and business organizations, the main difference being the degree of awareness which characterizes the actors.

The context of Schumpeterian competition is a good starting point to clarify how division of labour and co-ordination can both take place in markets: suppose that a cluster of product innovations is generated in the economic system. If the new products are substitutes for old intermediate goods, they will activate the modification of the existing productive and organizational routines and a "second generation" of adaptive innovators will come along: they in turn, by using the new products will produce either new goods or the old ones in a new manner. In the first case, we have a dynamic innovation process, in which new projects lead to the modification of other projects. This is not merely a matter of the diffusion of an innovation. On the contrary, what we are describing is an "avalanche" of innovations that are activated recursively by the original cluster of innovations. Two different "responses" could occur as a consequence of the original innovative actions. In the first case, they only give rise to local dynamic processes, which do not alter the basic structure of the division of labour. In the second case, the reaction of the system causes a new definition of the division of labour, knowledge and competencies within the economic system (often going beyond the intentions and expectations of the innovators).

This clarifies how a new division of labour and knowledge can be gen-erated within a market. It should be noted that co-ordination within markets also involves the transmission of knowledge and competencies: prices do not convey sufficient information to support the co-ordination processes that arise from the division of labour. The importance of this point and the provisional conclusions we have reached are reinforced if we consider an intermediate organizational set-up, as does Williamson in his "fundamental transformation" (Williamson 1975). When two or more business enterprises engage in a new common project, an improvement in human and physical assets internal to each enterprise and a transfer of knowledge among the enterprises will

normally be required. The emergence of sunk costs in physical and human capital guarantees the strength and stability of the links among the businesses. Even in this case, the relationship among firms which takes place in the market will involve not only price signals but also (and fundamentally!) the transmission of pieces of knowledge and information. Conflicts and bargaining between the parties will most probably be solved by protest and dispute instead of by leaving the market (Hirschman 1970). Markets are only one of a number of devices for communication and co-ordination between organizations.

To summarize, the transfer of knowledge and competence is a fundamental aspect of co-ordination which takes place not only within organizations but also among organizations in markets. Even if we believe that a fully developed model of the properties of the division of labour and co-ordination and their co-evolution should be at the top of the research agenda, to model this kind of process is far beyond our present abilities and intentions. More modestly, we intend to pick up some of the properties discussed above and try to compare the performances of the different co-ordination forms that arise from the division of knowledge and labour in a context of boundedly rational behaviour. In the next two sections the relevant assumptions will be discussed.

Division of labour and returns to scale

A fundamental question that has been already hinted at is whether and under what conditions subproblem decomposition—which we suggested was a useful representation of the process of division of labour—can generate more efficient forms of the organization of production. This question can be answered in many different ways: one of the first analyses was provided by Simon (1962) with his parable of the two watchmakers, called Tempus and Hora. Watches are complex objects made up of a very large number of small pieces. Tempus assembles his watches sequentially and every time he is disturbed—for instance, by his clients' phone calls—he has to start the assembly all over again. Hora instead proceeds by first assembling subunits consisting of only a few components and then putting the sub-units together. When he makes mistakes or is disturbed by external events he only has to restart assembling the current subunit. His average time to complete a watch will therefore be much shorter than Tempus's average.

In this example there is a clear advantage in the division of labour: when a perturbation occurs in a sequential system, it affects the whole system, when instead it occurs in a system made up of small independent and parallel subunits it will not propagate outside the affected subunit. But if we rule out the possibility of perturbations and mistakes, this advantage disappears and the parallel system seems rather less efficient because of its higher co-ordination costs. Only if we consider the positive, long-term effects that the division of labour might have on individuals' working capability, because of learning-by-doing and specialization, can we again find advantages in an increase in the degree of task decomposition. Learning therefore seems to be a key factor in explaining the division of labour. But how do individual learning processes co-ordinate in a framework in which the very definition of tasks is an emergent property?[1] In the rest of this chapter we shall investigate this question by means of a very simple model of division of labour and some simulations of different social co-ordination mechanisms.

Division of labour in the accountants' model

We shall elaborate on some of the points outlined in the previous section by means of a simple model of organization, which will be then made a little richer. The model is the accountants' model suggested by Sobel (1992) which addresses directly the problem raised by Simon's parable of the two watchmakers.

Suppose that the problem to be solved is counting an unknown and possibly very large number of items, for instance, dollar bills, and delivering a correct answer. Each accountant is characterized by a certain productive capacity, i.e. the number of bills he can count successfully in a given time interval, or, alternatively, by a certain probability of making mistakes, which plausibly we can assume to increase with the time he spends counting, because he gets more and more tired and error-prone, and/or more and more bored and likely to have his attention distracted. In the first case the accountant simply cannot work over his productive capacity; in the second he has no given limit, but the likelihood that he will make a mistake will increase with the size of his workload. If an accountant makes a mistake and does not realize it, he will jeopardize the result of the entire counting process; if he is aware of the mistake he will instead be forced to start counting again from the beginning. In all these cases splitting the task into subtasks is necessary to ensure the efficacy and efficiency of the production process.

Of course, the decomposition of the global task into subtasks must be technically possible, which is usually the case only to a certain extent, because there are often technical indivisibilities. The example of counting bills is in this sense very simple because the problem can be decomposed into any number of subtasks, limited only by the fact that the minimum subtask is counting one bill.

Suppose that the global task is to count a stack of N bills of one dollar. Suppose also that each accountant can successfully count at most k bills in a given time interval, at the end of which he issues a new bill whose face-value corresponds to the number of one-dollar bills he has counted (at most, k). Such bills are put in a new, smaller stack and can be counted by other accountants who will issue new, higher-valued bills and so on, until a stack of "fictitious" bills is obtained whose size is no bigger than k and which can be therefore counted by a single accountant who will be able to announce the total number.

Let us illustrate the problem with a trivial example: suppose that N= 10,000 and all accountants have a counting capacity of k=10 bills. We will need therefore L_1=1000 accountants at the first level, who will count the 10,000 bills, issue 1000 bills of face value $10 and put them in a new stack. To count this new stack, L_2=100 accountants are needed, who will issue 100 bills valued $100 and put them in a new stack that needs L_3=10 accountants to count them. Finally, only one accountant (L_4=1) is needed to count these 10 bills of $1,000 and announce the result of the entire counting process. All in all, we need 1111 accountants to complete the task in 4 time units, and we obtain a perfect pyramidal subdivision of subtasks.

In general, if $N=K^n$, i.e. the size of the task is an integer power of the individual productive capacity, the optimal hierarchical structure is formed by

n=log N/log k

levels, each consisting of 1, k, k^2,…, k^{n-1} accountants.

When the ratio log N/log k is not an integer, the number n of levels of the pyramid will be the first integer greater than or equal to this ratio. This implies that the pyramid will necessarily contain some slack resources which cannot fully be exploited.[2] Let us consider a general task of size N bills. If every accountant can count up to k bills, the lowest level of the organization will produce N/k bills, the second N/k^2 and so on up to level w such that $N/k^{\omega} \leq k$. If we have exactly $N/K^{\omega} = k$ all resources in the pyramid will be fully exploited, otherwise the productive capacity of the accountant at the top of the hierarchy will not be entirely exploited. Moreover the numbers $N/k, N/k^2, \ldots, N/k\omega$ might not all be integers: some idle resources will appear at each level for which the ratio is not an integer number.

The ratio between size of the task N and total amount M of labour required for its completion is given by

$$R(k)=N/M=k^n/1+k+k^2+\ldots+k^n=(1-k)k^n/1-K^n+^1.$$

Thus $R(k)$ tends to 1 as n tends to infinity and the process exhibits asymptotically constant returns to scale.

If the number N of bills to be counted increases, the organization can respond by increasing the level of employment and/or the productive capacity k. The latter can increase also with N constant (equivalent to an increase of productivity, or Marxian exploitation).

Division of labour and co-ordination

If dividing a complex task into subtasks to be handled separately is necessary and/or more efficient than carrying it out as a unit, this process of division of labour implies a growing and non-trivial co-ordination problem. The previous section analyzed some formal properties of a toy model of the division of labour and showed under what conditions a pyramidal structure can implement an optimal subdivision of a given task. In this section we examine how such a structure could emerge in different institutional set-ups and adapt to random environmental disturbances.

Generally speaking, we can imagine at least three different ways of at- taining co-ordination:

1. A central planner can use the model outlined in the previous section to design the optimal organization and adapt it to changes in the size of the task and/or changes in the workers' capabilities. To perform this task, he or she needs to know at every moment in time the exact value of every worker's productive capacity and the size of the global task.[3]
2. A boundedly rational central co-ordinator, instead of attempting an exhaustive plan, can adjust the organizational structure adaptively by moving workers where needed. Such a co-ordinator can operate as a sort of Walrasian auctioneer: he or she receives messages about all the flows between the different levels of the organization and about unused resources, and moves workers between adjacent levels of the hierarchy in such a way as to fill the gaps between demand and supply.

The information requirements of this boundedly rational co-ordinator are quite different from those of a fully rational central planner: while the latter needs precise information about the size of the overall task and the characteristics of each accountant, the former needs information on all the flows between different hierarchical levels. Whereas the central planner needs precise information, as even small amounts of noise will make the entire organization ineffective, the boundedly rational co-ordinator will generally need some signals (even if they are only qualitative ones) about supply-demand disequilibria. Moreover this boundedly rational co-ordinator can be replaced completely by inter-level co-ordinators who each take care only of the relationship between demand and supply at one interface between levels, regardless of what happens in other parts of the organization.

As to the kind of cognitive capabilities that are required by the central planner and the boundedly rational co-ordinator, the former has to develop a highly abstract and general decision rule. This requires that the problem has been understood in its general features and decomposed into subproblems. The boundedly rational co-ordinator, on the other hand, can implement the organizational structure adaptively, using a process of trial and error which can proceed without a general understanding of the problem (Dosi & Egidi 1991, Dosi et al. 1993).

But the process of adaptive co-ordination involves a cost, given by the loss of efficiency incurred during the process of adaptation. While the perfectly rational central planner computes the optimal organization in his mind, the boundedly rational co-ordinator carries out the design process in real time and corrects mistakes only after experiencing the loss of efficiency they cause. On the other hand, if the central planner makes mistakes, these are likely to damage the organization more persistently, because the planner is unable to process signals which detect the presence of inefficiencies and make consequential adjustments, and a thorough rede-sign is always needed even to cope with small perturbations.

3. Co-ordination can also be achieved with a completely decentralized mechanism, a quasi-market in which each accountant adjusts his production and/or his position according to quantity and/or price signals, which are processed independently by each individual. Each interface between levels of the organization constitutes a market where the accountants of the lower level sell the “fictitious” bills they produce and the accountants of the higher level buy them. Demand and supply in each of these markets depend on the number and productive capacity of the accountants at the two levels. Suppose, for instance, that the overall productive capacity at the lower level is not sufficient to supply the accountants at the higher level with enough bills. Some of the higher-level accountants will not be able to produce enough because of insufficient input and will therefore tend to move to other parts of the organization. These adjustments could take place through simple quantity signals or through more complex price mechanisms of Marshallian type (for instance, excess demand generates a price increase which raises the profits of sellers; and this attracts new sellers who make the supply increase and balance the initial excess demand).

A decentralized co-ordination mechanism of this kind requires only local information processing: each accountant can process disequilibrium signals autonomously according to his own local knowledge about his own characteristics and without any need to know

about other accountants' characteristics. However, such a mechanism requires a powerful system of incentives in order to fuel the agents' search for maximum profits. Finally, as in the case of a central co-ordinator, decentralized co-ordination takes place through costly adjustments in real time.

To summarize, in the first case, all the decision-making capability is concentrated in the central planner, who has developed a highly context-independent co-ordinating rule (n=log$N$/log$k$). The accountants do not even have the possibility of sending signals about inter-level disequilibria to the central planner. In the second case, the central co-ordinator follows a behavioural routine based on disequilibrium signals sent by the accountants. In the third case, there is no central co-ordination.

In the next section of the chapter we will study how co-ordination could emerge as an adaptive property in the different environments defined by centralized and decentralized co-ordination mechanisms and explore some of their properties by means of a simulation model.

Division of labour and the emergence of co-ordination: a simulation model

In this section some of the properties of the co-ordination modes outlined in the previous section will be investigated further by means of a simulation model of adaptive learning based on genetic algorithms and classifier systems (Holland 1975 and 1986, Goldberg 1989).

Let us consider a population of h accountants. Each of them is characterized by two variables: his position in the hierarchy (i.e. the integer number which identifies the hierarchical level where the agent is placed) and his productive capacity (i.e. the maximum number of banknotes he can count in a given time interval). Both variables can be encoded by their binary representations. In our examples we have used a string of six bits to represent each accountant: the first three bits for his position in the hierarchy (levels 0 to 7) and three bits for his counting capacity (from 0 to 7 banknotes per unit of time). The ith accountant is represented by the string:

A_i: $p_1p_2p_3k_1k_2k_3$ $p, k \varepsilon \{0, 1\}$

A set of h such strings provides, at each moment in time, a complete representation of the organization in which the position in the hierarchy (level) and the task of each individual (productive capacity) are specified.

Accountants can move (or be moved) across hierarchical levels and can modify their productive capacity according to signals of input-output disequilibrium either at the individual (in the case of decentralized co-ordination) or at the inter-level interface (in the case of a boundedly rational central co-ordinator). Such signals, appropriately interpreted, constitute the condition part of the adaptive learning system.

The case of the boundedly rational co-ordinator

The boundedly rational central co-ordinator is able to adapt the entire organization according to signals of disequilibrium/equilibrium at the interface between different

levels of the hierarchy. It can thus be represented by a set of condition-action rules where the condition classifies such signals and the action defines a complete organizational structure. In detail, we have:

(a) Environmental messages (equilibrium/disequilibrium signals) are given by the concatenation of eight binary strings (one for each interlevel interface, including the interface between the last level and the "final demand"). Each one of these eight substrings is composed of two digits:

$$s_1 s_2 \; s \quad \{0, 1\}$$

where the first digit is set to 1 when there is an excess supply at that interface and is set to 0 otherwise; the second digit is set to 1 when there is an excess demand at that interface and is set to 0 otherwise.

(b) Conditions are strings of the same length (16 bits) as the environmental messages which they classify:

$$c_{11} c_{12} c_{21} c_{22} \ldots c_{81} c_{82} \; c \quad \{0, 1, \#\}$$

Each bit position may be set to 0, 1, or a "wild card" marker, #.

(c) Action parts are binary strings of length $6h$ that define the whole organizational structure:

$$p_{11} p_{12} p_{13} k_{11} k_{12} k_{13} \ldots p_{h1} p_{h2} p_{h3} k_{h1} k_{h2} k_{h3} \; p, k \quad \{0, 1\}$$

In this way an adaptive and boundedly rational central co-ordination can be represented by a classifier system. Here we briefly review its basic features (more details can be found in, for instance, Holland 1986, Holland et al. 1986, Goldberg 1989).

An adaptive learning system is a system of condition-action rules such as those so far described. In addition, each rule is attributed a strength coefficient which, as a first approximation, measures how successful that rule has been in the past, and a specificity coefficient (the number of bits in the condition part that are not set to the wild card #), which measures the strictness of the condition. The smaller the cardinality of the set of environmental messages that satisfy that condition, the higher is its specificity, the highest specificity belonging to rules whose conditions are satisfied by only one environmental message.

This set of rules is processed according to the following cycle throughout the simulation process:

1. Condition matching: a message is received from the environment which informs the system about the disequilibrium/equilibrium conditions at the inter-level interface. This message is compared with the conditions of all the rules, and those rules which match, i.e. those which apply to such a state of the world, enter the following step.
2. Competition among matched rules: all the rules whose condition is satisfied compete in order to select the one which is allowed to execute its action, i.e. to implement the organizational structure specified by its action part. To enter this competition each rule

makes a bid based on its strength and on its specificity. In other words, the bid of each matched rule is proportional to its past usefulness (strength) and its relevance to the present situation (specificity):

$$\text{Bid}(R_j, t) = k_1(k_2 + k_3 \text{Specificity}(R_j))\ \text{Strength}(R_j, t)$$

where k_1, k_2 and k_3 are constant coefficients. The winning rule is chosen randomly, with probability proportional to the bids.

3. Action and strength updating: the winning rule executes the action indicated by its action part. It has its own strength reduced by the amount of the bid and increased by the payoff that the action receives, given the occurrence of the "real" state of the world. If the jth rule is the winner of the competition, we have:

$$\text{Strength}(R_j, t+1) = \text{Strength}(R_j, t) + \text{Payoff}(t) - \text{Bid}(R_j, t)$$

4. Generation of new rules: the system must be able not only to select the most successful rules, but also to discover new ones. This is ensured by applying "genetic operators" which, by recombining and mutating elements of the most successful existing rules, introduce new ones which could improve the performance of the system. In this way new rules are injected constantly into the system and scope for new opportunities is always made available. The genetic operators used in our simulations (as in standard classifier systems) are crossover and mutation.

The case of decentralized co-ordination

In this case each accountant can move autonomously across the hierarchical levels of the organization and change productive capacity according to individual disequilibrium signals. Some kind of market mechanism is thus necessary to generate local signals to reflect global disequilibria. One of the simplest ways in which such a quasi-market could operate[4] is by delivering quantity signals through rationing: if at an interface between levels there is an excess demand, some (possibly all) of the accountants of the higher level will not be able to get all the banknotes they would like and will be subject to rationing on the demand side. If instead there is excess supply, some (possibly all) of the accountants of the lower level will not be able to sell all the banknotes they would like and will be subject to rationing on the supply side.

Thus, in this case, each accountant is an independent decision-maker who can be modelled by an autonomous classifier system, and the links be-tween the different classifier systems are given by rationing signals. Each accountant is represented by a set of condition-action rules, where the condition classifies such signals and the action defines his own position/ capacity pair. In more detail we have:

(a) environmental messages (equilibrium/disequilibrium signals) are in general different for each accountant and given by a two-digit binary string:

$$s_1 s_2\ s \quad \{0, 1\}$$

where the first digit is set to 1 when the accountant has been rationed on the demand side (i.e. there is an excess demand at the interface between that accountant's level and the lower one) and is set to 0 otherwise; the second digit is set to 1 when the accountant has been rationed on the supply side (i.e. there is an excess supply at the interface between that accountant's level and the higher one) and is set to 0 otherwise.

(b) Conditions are therefore strings of two bits which classify such environmental messages:

$$c_1 c_2 \; c \quad \{0, 1, \#\}$$

(c) Action parts are binary strings, each six bits in length which define the accountant's position in the hierarchy and his productive capacity:

$$p_1 p_2 p_3 k_1 k_2 k_3 \; p, k \quad \{0, 1\}$$

Each classifier is then processed by exactly the same execution cycle as that already described for the centralized co-ordination case.

The two co-ordination systems have been tested on a simple problem. The task is to count 25 bills and there are 6 accountants, whose capacity can vary between 0 and 7 banknotes and whose position in the hierarchy can vary from level 0 to level 7. Level 0 is a stand-by position: accountants in this position do not enter the production process[5]. Let us examine in more detail the nature of the environmental signals which characterize the two institutional set-ups:

1. In the case of the boundedly rational central co-ordinator, the winning rule's action part is decoded in order to obtain the corresponding organizational design. The productive capacity of all the accountants who are at hierarchical level 1 is then summed to obtain the total demand for banknotes at this level. If the total demand is less than 25, the environmental message will signal an excess supply in the follow-ing iteration (the first digit will be set to 1 and the second to 0). If the total demand is greater than 25, the environmental message will signal an excess demand (the first digit will be set to 0 and the second to 1). Only if the total demand is equal to 25 will the environmental message signal an equilibrium situation (both digits set to 0). The total supply of level 1 can now be computed in this way:

   (i) if the total demand is less than or equal to 25, all accountants at level 1 will be able to exploit their productive capacity to the full. Thus the total supply will be given by a number of banknotes equal to the number of accountants, and each note will have a face-value equal to the productive capacity of the accountant who produced it; and

   (ii) if instead the total demand is greater than 25, some accountants (randomly chosen) will be unable to use their own productive capacity fully. Total supply will be given by a number of banknotes equal to the number of accountants at the first level who received at least one banknote and their face-values will be given by the production of the accountants who produced them.

Once the total supply at the interface between levels 1 and 2 has been so computed, we can determine the total demand as the sum of the productive capacities of all the accountants who are placed at level 2 of the hierarchy. We can then set the third and fourth digits of the environmental message according to the disequilibrium and equilibrium situations which are thus realized. The same procedure can be repeated for all the organizational levels. At the last level we will suppose the existence of a "final" demand of one banknote of face value 25. If more banknotes are offered, an excess supply will be signalled and the face-value of the only purchased banknote will determine the overall payoff to the organization: if its value is 25, the organization (i.e. the winning rule) will receive a positive payoff, otherwise it will receive a negative payoff proportional to the absolute difference between 25 and the face-value itself.

2. In the case of decentralized co-ordination, inter-level supplies and demands are computed in exactly the same way, but environmental messages are determined separately for each accountant, by means of random rationing. For instance, if at a given interface between levels, demand is higher than supply, banknotes are assigned, one by one, to a randomly chosen accountant who still has some unused productive capacity. All accountants on the supply side will therefore have sold all the banknotes they produced, and at the following iteration will receive a message whose second digit is set to 0. As to the accountants on the demand side, some (possibly none) of them will have been given all the banknotes they required and at the following iteration will receive a message whose first digit is set to 0. The others (possibly all of them) will find themselves with at least a part of their demand unmet and at the following iteration will receive a message whose second digit is set to 1. The organizational payoff is in this case distributed to all the accountants' winning rules through a bucket-brigade mechanism (Holland 1986).

Simulations have been carried out in order to test the adaptive performance of the systems in different environmental conditions. Let us ex- amine briefly the main results:

1. The first set of results concerns the simplest situation: an error-free and stationary world. The stack of banknotes to be counted always contains 25 notes, and no mistakes can occur in the counting process, i.e. the counting can always be performed without any disturbance and the face-value of the output always equals the amount of used productive capacity. In this situation the boundedly rational centralized co-ordination mechanism is considerably more efficient in finding the optimal structure as far as both the speed of convergence to an effective organizational design and its stability are concerned.

   The structure of the payoff function was found to play a crucial role in determining the speed of convergence and the type of organization which emerges. In particular, punishments for unused productive capacities are essential for reducing slack resources, and a payoff function that takes into account the total task-completion time is necessary to obtain an equal distribution of capacities across agents.

2. In error-prone environments, accountants can make mistakes. Two different kinds of mistake are possible. In the first case accountants are aware of the mistakes they make and correct them by restarting the counting process. The payoff function contains a

penalty for the time (steps of the counting process) taken to perform the overall task. A second type of mistake is represented by a random variable (with mean 0) which might cause a deviation between the amount of productive capacity used by an accountant and the face-value of the banknote he produces. In this case accountants are not aware of the mistakes they make and they deliver a result which is not correct. The decentralized mechanism is more efficient in dealing with both types of mistake. Local adjustments therefore seem to increase efficiency in coping with local disturbances.

Conclusions and directions for further research

We have pointed out that the division of labour determines a hierarchical structure of tasks of pyramidal form. Does this structure directly relate to power relations within organizations, i.e. is the hierarchical system of task division connected to the hierarchical system of power and control? The answer is, generally speaking, no. The pyramidal structure of our example emerges only from functional relations (each agent has to use a multiplicity of inputs coming from different agents) and is independent of control and power relations. But, on the other hand, the shape of the pyramid can have some important consequences for the relationship between agents. The number of agents in fact decreases as we climb the hierarchy and thus, if the relationships between adjacent levels are managed by a market, such a market cannot be a competitive one because of its strong asymmetry. Market power will be higher for agents placed in higher hierarchical positions.

In our simple model we have supposed that movement of workers across hierarchical levels can take place instantaneously and at no cost: this is clearly a most unrealistic hypothesis. Different positions in the hierarchy require different capabilities and a knowledge of different routines. There is likely to be a trade-off in the organization between the time a worker takes to learn the routines associated with a different hierarchical level, and the increase of production time and number of mistakes resulting from an increase in individual workloads.

Moreover, in real organizations the types of knowledge required normally differ between hierarchical levels: higher levels in the hierarchy demand a broader but less precise kind of knowledge (regardless of the intellectual capabilities of workers). The division of labour usually involves a strong cognitive asymmetry between different levels of the task as a part of the general decomposition of knowledge, but this element does not appear in our accountants' model.

Finally, real economic organizations always possess some mechanism of control to spot elements which do not work properly, either because of real problems or because of opportunistic behaviour. They also include a parallel and connected system of incentives that aims to avoid the appearance of such problems. All these elements should be embodied in our model (which could become a sort of "beehive" rather than an accountants' model) to make it more realistic.

Notes

1. The problem of the co-ordination of learning processes within an economic organization has also been studied, in a framework in which the division of labour is given, in Marengo (1992).
2. The presence of such idle resources is a possible reason for the existence of economies of scale in this technology: a multiplicity of tasks can be internalized in a single organization, which can therefore reduce the amount of idle resources. In the above example, two separate tasks of $5,000 would be handled more efficiently by a single organization rather than separately. But this issue lies outside the scope of this chapter.
3. This requirement seems rather paradoxical. If the planner knew exactly the number of bills to be counted there would be no need to carry out the counting process. This paradox also arises in less caricatured production processes, although less strikingly. Fully centralized and exhaustive planning would require perfect knowledge of every aspect of the production process: only in this case could the co-ordination problem be solved in the planner's mind without the need for local adjustments.
4. Markets with prices are much more complex and to model them correctly we should introduce a series of hypotheses on individual utility functions, which would take us far from the core issue of the chapter.
5. The possibility of leaving the organization must be allowed for if we want the system to be able to adjust when there are more accountants than the optimum number.