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ScienceDirect
Physica A 387 (2008) 277–280
PHYSICA A
www.elsevier.com/locate/physa

On the proper behavior of atoms: A comment on a critique

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Received 2 May 2007
Available online 12 September 2007

Abstract

A recent article [S. Keen, R. Standish, Profit maximization, industry structure, and competition: a critique of neoclassical theory, Physica A 370 (2006) 81–85] suggests that the Cournot model, which is widely used in Economics, is inappropriate and the article proposes an alternative. I argue that the supporting arguments compute derivatives incorrectly which, amongst other things, have the effect of confusing the relationship between an individual actor in a market and the total mass of firms. I also indicate that the proposed alternative was considered about 20 years ago and found to be unsatisfactory. Thus, it should not be surprising when the authors fail to use their own proposal in the second half of the paper.

Keywords: Equilibrium; Oligopoly; Cournot; Nash; First order condition; Maximization; Simulation

1. Introduction

Keen and Standish (KS, [1]) sought to show that the theory of firm behavior most often asserted by economists is internally inconsistent both in the most famous case of atomistic firms and in its updated form of a Cournot oligopoly. KS's paper is interesting because it offers two styles of proof, one based on calculus and first order conditions and one based on numerical simulation, to conclude that the standard solution is wrong. This conclusion could be serious since the Cournot model is one of the oldest problems in economics to use mathematics and many researchers have applied it to study many kinds of problems [2].

I argue the proofs should not be considered as convincing for a couple of reasons. First, I argue that the words being used to describe the problem are not connected to the mathematics being used in the proofs. More precisely, the mathematical analysis assumes an important conclusion about how to combine the actions of independent actors. If the conditions change slightly then the proposed method of analysis reveals an inconsistency that is hidden in the presentation used by KS. Second, I argue that the analysis focuses on interpreting a particular first order condition even though a more careful consideration of the maximization problem would reveal other types of solutions. Third, I argue that the new parameter proposed by the authors was studied by economists and was rejected for most purposes. This earlier rejection may explain why the authors choose to ignore their own proposal in the second half of their paper.

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0378-4371/$ - see front matter
doi:10.1016/j.physa.2007.08.053

2. Concerning a mathematical "Proof"

The paper starts with an equality: that $\mathrm{d}P/\mathrm{d}q_i = \mathrm{d}P/\mathrm{d}Q$ where P is the price paid by consumers of a good, q_i is the production of a single firm i and Q is the total production by all firms in an industry. KS conclude that this equality reveals a logical flaw because it implies that each atom in an industry has the same effect on the price as the entire mass of firms. I suggest that a flawed premise is being used since it is also true that the effect on P would be about 100 times larger if the change in output by a single firm increased from $\mathrm{d}q_i = 1$ to 100. So, before analyzing the effect of a change by a mass of firm, a more relevant question is: is $\mathrm{d}q_i = 1$ or 100 (or -1 or -100)? An equilibrium is special because the most preferred change in output for each firm is 0. And, if there are 100 firms in that equilibrium, then the total effect of these most preferred changes could be the same as the effect of the change preferred by a single firm.

How is $\mathrm{d}q_i$ determined? The authors use a standard expression for a firm's profit. In this setting, a firm's profits can be calculated as total revenue minus total cost. The total cost for a firm depends on the firm's output. Total revenue depends on consumer behavior, as represented by a market demand curve, where consumers are willing to buy as much as firms in the industry are willing to sell at the market price. If there are lots of firms, e.g. n is infinitely large, then a single firm has an infinitesimal market share. And, if it increases its output then it bears an infinitesimal share of the reduction in price regardless of $\mathrm{d}P/\mathrm{d}Q$; the effect on total revenue of a change in *its* output is infinitely larger, in relative terms, than the effect of any resulting change in the market price. Thus, and in contrast to the claim made on the first page of KS, it is not necessary to assume that $\mathrm{d}P/\mathrm{d}q_i$ is close to 0 in order to conclude that the marginal revenue of a *single firm* is close to the market price in the case of atomistic firms and in Cournot's model.

When analyzing the Cournot model in greater depth, the mathematical problems start when the authors maximize an expression for profits without distinguishing the things which a firm can control and the things which it cannot control. Eq. (4) in KS reports the derivative of profits of firm i with respect to industry output, Q. It contains no mathematical error but it is irrelevant to the question being discussed because it assumes the conclusion that $Q = nq_i$. In other words, the authors confuse the choices and preferences of the actor who chooses $\mathrm{d}q_i$, i.e. firm i, with the choices and preferences of an unidentified actor who chooses $\mathrm{d}Q$.[1]

More careful attention to the difference between a partial derivative and a total derivative might have prevented this error at either of two steps in the argument. The point of failure in the first step might be revealed more clearly if the model were changed slightly and in a way that would be relevant to anybody who wanted to combine the analysis with some data: suppose that each of the n firms has a slightly different marginal cost function. If so, then $Q = nq_i$ for all i is unlikely to be true of an equilibrium. But any attempting to solve an analog of Eq. (4) would require solving n different first order conditions *simultaneously* with a scalar Q.[2] So, what was asserted to be a convenient assumption should be seen as necessary. The link between Q and q_i needs to be clarified in terms of the logic and in mathematical terms.

A second point of failure might be revealed by incorporating their proposal into the analysis more thoroughly. The authors introduce a variable, θ_i, which shows how a firm *reacts* to the actions of competitors. To *add* θ_i as part of an operationally valid experiment, and to show that some part of each firm's choice is independent of other firms, it must be admitted that the managers of firm i choose *two* variables, (α_i, θ_i) where

$$q_i = \alpha_i + \theta_i q_j. \tag{1}$$

[1] In response to my request for a proof, Professor Keen sent me a working paper [3]. Eq. (1.10) of the paper is critical and asserts that $\mathrm{d}q_i/\mathrm{d}Q = \Sigma_j\, \partial q_i/\partial q_j$. Though not stated explicitly, this equality seems to be a consequence of the fact that $Q = \Sigma_j\, q_j$. In words, the critical assertion is true if and only if the effect of a change in total output (i.e. a change in q_1 or q_2 or … or q_n which in combination adds up to one unit) is equal to the combined effect of a unit change in q_1 *and* a unit change in q_2 *and* … and a unit change in q_n. Notice also that this assertion implies that a unit increase in q_i may cause an increase in q_i since $\partial q_i/\partial q_i$ is unconstrained.

[2] To use a simpler version of the analysis that the authors introduce later, suppose that the total cost of producing a firm's output has the following form $tc_i\,(q_i) = A_i + C_i\, q_i$ where C_i is marginal cost and differs between firms. Replacing q_i by Q/n and computing a system of n first order conditions to replace KS Eq. (4) produces an inconsistent system. If we did not replace q_i by Q/n, and computed the partial derivative of each firm's profit with respect to its own output only, q_i, then the resulting system of n equations would produce a solution for $(q_1,\dots,q_n)$. If the symmetry assumption is re-introduced after a general solution has been found, then it can be proven that $Q = nq_i$ for any i in the particular equilibrium.

The role of the intercept in a firm's output function is vaguely defined in KS even though, if relevant, Eq. (6) should be replaced by two first order conditions:

$$\partial \text{Profit}_i / \partial \alpha_i = 0 \tag{2}$$

and

$$\partial \text{Profit}_i / \partial \theta_i = 0. \tag{3}$$

Computing these first order conditions would have shown that this system of equations does not produce a unique solution for θ_i. In fact, given the output of other firms, the characteristic which all of the solutions for (α_i, θ_i) share is a common level of output.

θ_i represents what was called a "conjectural variation" (e.g. Ref. [4]). The concept fell out of use in the leading journals of Economics at about the time that the concept of Nash Equilibrium entered widespread use and years before John Nash became a Nobel Laureate. The reasons are explained more fully in some popular textbooks [5]. The essential conceptual barrier to implementing a conjectural variation is that it is hard to build an internally consistent model which asks a person to *react* to a choice made by somebody else *before* that person has made their choice.[3] If KS had proposed a dynamic model then it might have represented the concept of a reaction but proposing a longer horizon would also add other dimensions to the optimization problem that KS do not consider. Thus, analysis based on the concept of conjectural variation is a poor cousin of true strategic analysis.

3. Concerning a "Proof" by simulation

One way for a firm to anticipate the actions of a competitor is to learn from experience. Using a simulation where behavior can evolve may reveal this experience in a controlled environment and may produce a better understanding of an optimal reaction. Using a numerical simulation also forces a model builder to propose behaviors which are implementable. Thus, it may be noteworthy that KS's simulation omits their parameter θ_i. This second attempt to defend their proposal is suspect for two more reasons.

First, the authors' simulation proposes that firms maximize profits as a result of random choices. It would have been better if they had let the firms accumulate relevant experience and then computed a firm's (expected) profits for many different possible levels of output, including deviations from the simulated solution. If the output indicated by simulated solution produced the maximum possible profit for each individual firm, conditional on its experience, then they would have proven their conclusion. Since the analysis does not show what the profit might be if the firm produced at any other level of output, the simulations present a solution without revealing whether that solution represents a profit maximum.

Second, the problem being simulated may differ from the problem as originally stated. Though numerical simulations have some advantages, their disadvantage is that the coding used in a simulation may embed other assumptions. These embedded assumptions may be added intentionally or not, and may be declared or not. In any case, assumptions used to encode the evolutionary process are important because, in many cases, one can prove that an Evolutionary Stable Solution is also a Cournot (Nash) solution [6]. The fact that these authors find a different solution, combined with the authors' decision to not use their parameter θ_i, indicates that the difference between the traditional solution and their proposal may be sensitive to changes in the embedded assumptions. Previous work on evolutionary models certainly raises doubts about whether their simulation is sufficient to establish the generality of their conclusion.[4]

[3]It is possible to ask how a firm would react to different possible conjectures of what the other firms might do just as it is possible to ask how consumers would react to different possible prices. But, if the strategies of different firms are not "in equilibrium" then each firm can anticipate that its conjecture about the actions of the other independent firms will not be confirmed by those actions. In equilibrium, the conjectures are confirmed by the actions. Proving that an equilibrium does exist shows when it is possible for conjectures to be consistent with the actions that are a consequence of those conjectures.

[4]Dixon and Somma [7] offered an evolutionary model of conjectural variation using bounded rationality.

4. Other empirically relevant strategies that were not considered

The authors could have used their simulation to explore aspects of behavior that are hard to represent algebraically. For example, some economists argue that a firm must maximize its profits not because it wants to but because the market process punishes non-maximizing behavior: e.g. a firm may be forced to leave or may be forced to lower costs if it is threatened with new entrants. Exploring whether their proposed behavior is compatible with a more general market process would represent a natural application of one of their comments (KS, fn. 3). Unfortunately, a more complete consideration of their comment would also indicate more weaknesses.

As noted in KS and as noted above, economists often assume that all firms in an industry use the same cost function and that it displays constant returns to scale. Even when known to be descriptively inaccurate, using a simplifying assumption can make a logically correct proof more convincing by removing parameters whose presence complicate the proofs unnecessarily.

This particular simplifying assumption is widely used because the implications of the alternatives are discussed implicitly or explicitly in many textbooks. Consider the additional ways in which a firm could respond if the cost function does not fit the standard economic model. First, as noted above, different firms might produce different amounts when facing the same price and this fact would invalidate the assertion that $Q = nq_i$. Second, if a cost function displays decreasing returns to scale, and if a simple condition is satisfied,[5] then a firm could add another plant and increase its profits. Or a new firm could enter on a smaller scale and be more profitable than an existing firm. If a cost function displays increasing returns to scale, then the well-known concept of "natural monopoly" becomes relevant. If so then a solution which satisfies a first order necessary condition, such as KS Eq. (3), may fail to satisfy a second order sufficient condition. None of these considerations are discussed even though, to establish whether the conclusion is robust and widely applicable, they should have been considered.

Acknowledgment

I thank Steve Keen for several discussions that helped to refine the content of this paper. It should be apparent that any remaining errors and omissions are my responsibility.

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[5]The condition establishes a critical level of the fixed cost A. Suppose the firm divides the original level of output between the plants equally. Doing so would decrease total variable cost and, if the fixed cost of building a second production facility is not too high then this action would increase profits. To use the example proposed in the paper, suppose that the original total cost of producing q_i is $tc(q_i) = A + Cq_i + Dq_i^2 + Eq_i^3$ where $A, C, D, E > 0$ and q_i may be the solution given by KS. Then there exists a positive upper bound on A such that, for all A less than this bound, $A + Cq_i + Dq_i^2 + Eq_i^3 > 2(A + C(q_i/2) + D(q_i/2)^2 + E(q_i/2)^3)$.

This condition is a sufficient condition since, if dividing production between the plants also decreases marginal cost then the firm can increases its profit even more by increasing total output at least a little.