① ⑧

* Questão não considerada

Nota

1.* —	7 - 1,0
2 - 1,0	8 - 1,0
3 - 1,0	9 - 1,0
4 - 1,0	10 - 1,0
5 - 0,9	11 - 1,0
6 - 1,0	12 - 0,7

9,7

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LISTA 2 - ANÁLISE

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1a) $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

X e Y são independentes ~ Normal trivariada, com:

$\mu_1 = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ $\Sigma_1 = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $\mu_2 = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$ $\Sigma_2 = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

Pelo Resultado 3:

$$\Rightarrow \begin{bmatrix} X \\ Y \end{bmatrix} \sim N_6\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}\right)$$

$$Z = [X - Y] = \begin{bmatrix} x_1 - y_1 \\ x_2 - y_2 \\ x_3 - y_3 \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{\begin{bmatrix} X \\ Y \end{bmatrix}} = \underbrace{\begin{bmatrix} x_1 - y_1 \\ x_2 - y_2 \\ x_3 - y_3 \end{bmatrix}}_{Z}$$

1 - 1,0

Pelo Resultado 1

$\Rightarrow Z \sim N_3(\mu_3, \Sigma_3)$

$$\mu_3 = A.\begin{bmatrix}\mu_1\\ \mu_2\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & -1 & 0\\ 0 & 0 & 1 & 0 & 0 & -1\end{bmatrix} \cdot \begin{bmatrix}2\\2\\2\\3\\4\\2\end{bmatrix} = \begin{bmatrix}-1\\-2\\0\end{bmatrix} = \mu_3$$

$$\Sigma_3 = A.\begin{bmatrix}\Sigma_1 & 0\\ 0 & \Sigma_2\end{bmatrix}.A'$$

$$= \begin{bmatrix}1 & 0 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & -1 & 0\\ 0 & 0 & 1 & 0 & 0 & -1\end{bmatrix} \cdot \begin{bmatrix}3 & 2 & 1 & 0 & 0 & 0\\ 2 & 4 & 1 & 0 & 0 & 0\\ 1 & 1 & 2 & 0 & 0 & 0\\ 0 & 0 & 0 & 4 & 2 & 0\\ 0 & 0 & 0 & 2 & 4 & 2\\ 0 & 0 & 0 & 0 & 2 & 4\end{bmatrix} \cdot \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ -1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix}$$

$$= \begin{bmatrix}3 & 2 & 1 & -4 & -2 & 0\\ 2 & 4 & 1 & -2 & -4 & -2\\ 1 & 1 & 2 & 0 & -2 & -4\end{bmatrix} \cdot \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ -1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{bmatrix} = \begin{bmatrix}7 & 4 & 1\\ 4 & 8 & 3\\ 1 & 3 & 6\end{bmatrix} = \Sigma_3$$

b) $$Z = \begin{bmatrix}X - Y\\ X + Y\end{bmatrix} = \begin{bmatrix}X_1 - Y_1\\ X_2 - Y_2\\ X_3 - Y_3\\ X_1 + Y_1\\ X_2 + Y_2\\ X_3 + Y_3\end{bmatrix}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}}_{\begin{bmatrix} X \\ Y \end{bmatrix}} = \underbrace{\begin{bmatrix} x_1 - y_1 \\ x_2 - y_2 \\ x_3 - y_3 \\ x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{bmatrix}}_{Z}$$

Pelo Resultado 1

$\Rightarrow Z \sim N_6(\mu_3, \Sigma_3)$

$$\mu_3 = A \cdot \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$$

$$= A \cdot \begin{bmatrix} 2 \\ 2 \\ 2 \\ 3 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 5 \\ 6 \\ 4 \end{bmatrix} = \mu_3$$

$$\Sigma_3 = A \cdot \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \cdot A^T$$

$$= \begin{bmatrix} 3 & 2 & 1 & -4 & -2 & 0 \\ 2 & 4 & 1 & -2 & -4 & -2 \\ 1 & 1 & 2 & 0 & -2 & -4 \\ 3 & 2 & 1 & 4 & 2 & 0 \\ 2 & 4 & 1 & 2 & 4 & 2 \\ 1 & 1 & 2 & 0 & 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 4 & 1 & -1 & 0 & 1 \\ 4 & 8 & 3 & 0 & 0 & -1 \\ 1 & 3 & 6 & 1 & -1 & -2 \\ -1 & 0 & 1 & 7 & 4 & 1 \\ 0 & 0 & -1 & 4 & 8 & 3 \\ 1 & -1 & -2 & 1 & 3 & 6 \end{bmatrix} = \Sigma_3$$

c) $Z = \begin{bmatrix} X \\ Y \end{bmatrix}$

$$Z \sim N_6\left(\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}, \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \right)$$

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \\ 3 \\ 4 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 & 0 & 0 & 0 \\ 2 & 4 & 1 & 0 & 0 & 0 \\ 1 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 2 & 0 \\ 0 & 0 & 0 & 2 & 4 & 2 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{bmatrix}$$

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2a) $$\rho(X_1, X_3) = \frac{cov(X_1, X_3)}{\sqrt{\sigma_{11}} \cdot \sqrt{\sigma_{33}}} = \frac{4}{\sqrt{25} \cdot \sqrt{3}} = \frac{4}{5 \cdot \sqrt{3}}$$

b) $$\rho\left(X_1, \frac{1}{2} X_2 + \frac{1}{2} X_3\right) = \frac{cov\left(X_1, \frac{1}{2}(X_2 + X_3)\right)}{\sqrt{\sigma_{11}} \cdot \sqrt{var\left(\frac{1}{2}(X_2 + X_3)\right)}}$$

$$= \frac{\frac{1}{2} cov(X_1, (X_2 + X_3))}{\sqrt{25} \cdot \sqrt{\frac{1}{4}\left[var(X_2) + var(X_3) + 2 \cdot cov(X_2, X_3)\right]}}$$

$$= \frac{\frac{1}{\cancel{2}}\left(2\,cov(x_1, x_2) + cov(x_1, x_3)\right)}{5 \cdot 1 \cdot \sqrt{9}}$$

$$1 \times \cancel{2}$$

$$= \frac{-2+4}{5 \cdot 3} = \frac{2}{15}$$

3)

$$\begin{bmatrix} x_{11} & x_{12} & \dots & x_{1p} \\ x_{21} & & & \\ \vdots & & & \\ x_{n1} & \dots & & x_{np} \end{bmatrix} \qquad R = \begin{bmatrix} \underbrace{\frac{s_{11}}{\sqrt{s_{11}} \cdot \sqrt{s_{11}}}}_{r_{11}} & \dots & \frac{s_{1p}}{\sqrt{s_{11}} \cdot \sqrt{s_{1p}}} \\ \vdots & & \\ \frac{s_{p1}}{\sqrt{s_{pp}} \cdot \sqrt{s_{11}}} & \dots & \underbrace{\frac{s_{pp}}{\sqrt{s_{pp}} \cdot \sqrt{s_{pp}}}}_{r_{pp}} \end{bmatrix}$$

com $s_{jk} = \frac{1}{n} \sum_{i=1}^{n} (x_{ij} - \bar{x}_j) \cdot (x_{ik} - \bar{x}_k) \quad k, j = 1, 2, \dots, p$

$$\begin{array}{c} x_{11}' \\ \\ \\ x_{1p}' \end{array} \begin{bmatrix} \frac{x_{11} - \bar{x}_1}{\sqrt{s_{11}}} & \frac{x_{12} - \bar{x}_2}{\sqrt{s_{22}}} & \dots & \frac{x_{1p} - \bar{x}_p}{\sqrt{s_{pp}}} \\ \vdots & & & \\ \frac{x_{p1} - \bar{x}_1}{\sqrt{s_{11}}} & \dots & & \frac{x_{pp} - \bar{x}_p}{\sqrt{s_{pp}}} \end{bmatrix} \qquad R = \begin{bmatrix} s_{11}' & \dots & s_{1p}' \\ s_{21}' & & \\ \vdots & & \\ s_{p1}' & \dots & s_{pp}' \end{bmatrix}$$

$$\bar{x}_j' = \frac{1}{n} \times \sum_{i=1}^{n} \frac{x_{ij} - \bar{x}_j}{\sqrt{s_{jj}}} = \frac{1}{n\sqrt{s_{jj}}} \cdot \sum_{i=1}^{n} x_{ij} - n\bar{x}_j = \frac{n\bar{x}_j - n\bar{x}_j}{n\sqrt{s_{jj}}} = 0$$

$$s_{11}' = \frac{1}{n} \sum_{i=1}^{n} (x_{i1}' - \bar{x}_1') \cdot (x_{i1}' - \bar{x}_1')$$

$$= \frac{1}{n} \sum_{i=1}^{n} \left(\frac{x_{i1} - \bar{x}_1}{\sqrt{s_{11}}}\right) \cdot \left(\frac{x_{i1} - \bar{x}_1}{\sqrt{s_{11}}}\right) = \frac{1}{\sqrt{s_{11}} \cdot \sqrt{s_{11}}} \cdot \frac{1}{n} \sum_{i=1}^{n} (x_{i1} - \bar{x}_1) \cdot (x_{i1} - \bar{x}_1)$$

$$= \frac{s_{11}}{\sqrt{s_{11}} \cdot \sqrt{s_{11}}} = r_{11}$$

$$s_{jk}^{*} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_{ij}-\bar{x}_j}{\sqrt{s_{jj}}}\right)\left(\frac{x_{ik}-\bar{x}_k}{\sqrt{s_{kk}}}\right) \quad k,j=1,2,\dots,p$$

$$= \frac{1}{\sqrt{s_{jj}}\cdot\sqrt{s_{kk}}}\cdot\frac{1}{n}\sum_{i=1}^{n}(x_{ij}-\bar{x}_j).(x_{ik}-\bar{x}_k)$$

$$= \frac{s_{jk}}{\sqrt{s_{jj}}.\sqrt{s_{kk}}} = r_{jk}$$ => matriz R = matriz covariância amostral da variável padronizada.

4a) $X = \begin{bmatrix} 1 & 4 & 3 \\ 6 & 2 & 6 \\ 8 & 3 & 3 \end{bmatrix}$ $\quad \underset{\sim}{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

↳ amostra $n=3$

$b'\underset{\sim}{x} = [1 \;\; 1 \;\; 1].\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $\quad c'\underset{\sim}{x} = [1 \;\; 2 \;\; -3].\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$b'\underset{\sim}{x} = [x_1 + x_2 + x_3]$ => $b'\underset{\sim}{x}_1 = 15$ $\quad \underset{\sim}{x}_i \equiv$ amostra i

$b'\underset{\sim}{x}_2 = 9$

$b'\underset{\sim}{x}_3 = 12$ => $b'\underset{\sim}{x} = [15 \;\; 9 \;\; 12]$

$c'\underset{\sim}{x} = [x_1 + 2x_2 - 3x_3]$ => $c'\underset{\sim}{x}_1 = -11$

$c'\underset{\sim}{x}_2 = -1$

$c'\underset{\sim}{x}_3 = 6$ => $c'\underset{\sim}{x} = [-11 \;\; -1 \;\; 6]$

$$Y = \begin{bmatrix} b'\underset{\sim}{x} \\ c'\underset{\sim}{x} \end{bmatrix} = \begin{bmatrix} 15 & 9 & 12 \\ -11 & -1 & 6 \end{bmatrix}$$

$$\underset{\text{Amostral}}{E[Y]} = \begin{bmatrix} E[b'\underset{\sim}{x}] \\ E[c'\underset{\sim}{x}] \end{bmatrix} = \begin{bmatrix} \frac{15+9+12}{3} \\ \frac{-11-1+6}{3} \end{bmatrix} = \begin{bmatrix} 12 \\ -2 \end{bmatrix}$$

kajoma

$$Var(Y)_{\text{Amostral}} = \begin{bmatrix} Var(b'\underset{\sim}{x}) & cov(b'\underset{\sim}{x}, c'\underset{\sim}{x}) \\ cov(c'\underset{\sim}{x}, b'\underset{\sim}{x}) & var(c'\underset{\sim}{x}) \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\sum_{i=1}^{3}(b'\underset{\sim}{x}_i - 12)^2}{3} & \dfrac{\sum_{i=1}^{3}(b'x_i - 12).(c'\underset{\sim}{x}_i + 2)}{3} \\ \sum_{i=1}^{3}(b'\underset{\sim}{x}_i - 12).(c'\underset{\sim}{x}_i + 2) & \sum_{i=1}^{3}(c'\underset{\sim}{x}_i + 2)^2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -10 \\ -10 & \dfrac{146}{3} \end{bmatrix}$$

b) $\bar{X} = \begin{bmatrix} 8/3 \\ 14/3 \\ 14/3 \end{bmatrix}$

$Cov(x_i, x_j) = E(x_i . x_j) - E[X_i].E[X_j]$

$$cov(x_1, x_1) = \frac{1+16+9}{3} - \frac{8}{3}.\frac{8}{3}$$

$$= \frac{26}{3} - \frac{64}{9} = \frac{78}{9} - \frac{64}{9} = \frac{14}{9}$$

$$cov(X_1, X_2) = E(X_1 . X_2) - E[X_1].E[X_2]$$

$$= \frac{6+8+18}{3} - \frac{8}{3}.\frac{14}{3} = \frac{32}{3} - \frac{112}{9} = \frac{96}{9} - \frac{112}{9} = \frac{-16}{9}$$

$$cov(x_1, X_3) = \frac{8+12+9}{3} - \frac{8}{3}\frac{14}{3} = \frac{29}{3} - \frac{112}{9} = \frac{87}{9} - \frac{112}{9} = \frac{-25}{9}$$

$$cov(X_2, X_2) = \frac{36+4+36}{3} - \frac{14}{3}.\frac{14}{3} = \frac{76}{3} - \frac{196}{9} = \frac{228}{9} - \frac{196}{9} = \frac{32}{9}$$

$$cov(X_2, X_3) = \frac{48+6+18}{3} - \frac{14}{3}.\frac{14}{3} = \frac{72}{3} - \frac{196}{9} = \frac{216}{9} - \frac{196}{9} = \frac{20}{9}$$

$$cov(X_3, X_3) = \frac{64+9+9}{3} - \frac{14}{3}.\frac{14}{3} = \frac{82}{3} - \frac{196}{9} = \frac{246}{9} - \frac{196}{9} = \frac{50}{9}$$

$$\Rightarrow S = \begin{bmatrix} 14/9 & -16/9 & -25/9 \\ -16/9 & 32/9 & 20/9 \\ -25/9 & 20/9 & 50/9 \end{bmatrix}$$

$b'\bar{x} = [1\ 1\ 1] \cdot \begin{bmatrix} 8/3 \\ 14/3 \\ 14/3 \end{bmatrix} = \frac{36}{3} = 12 \Rightarrow E[b'\underset{\sim}{x}] = b\bar{x}$

$b'Sb = [1\ 1\ 1] \cdot S \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{27}{9} & \frac{36}{9} & \frac{45}{9} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 6 \Rightarrow Var(b'\underset{\sim}{x}) = b' \cdot Sb$

$c'\bar{x} = [1\ 2\ -3] \cdot \bar{x} = -\frac{6}{3} = -2 \Rightarrow E[c'\underset{\sim}{x}] = c'\bar{x}$

$c'Sc = [1\ 2\ -3] \cdot S \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} \frac{57}{9} & -\frac{12}{9} & -\frac{135}{9} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \frac{146}{3} \Rightarrow Var(c'\underset{\sim}{x}) = c' \cdot Sc$

$b'Sc = [1\ 1\ 1] \cdot S \cdot \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} -\frac{27}{9} & \frac{36}{9} & \frac{45}{9} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} = -10 \Rightarrow cov(b'\underset{\sim}{x}, c'\underset{\sim}{x}) = b'Sc$

$\frac{0{,}9}{1{,}0}$

5 a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \sim N_3(0, \Sigma) \qquad \Sigma = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$

$x_j \sim N(\mu_j, \sigma_{jj})$

$\Rightarrow x_1 \sim N(0, 4)$

b) $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim N_2\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix} \right)$

c) $X_1 / X_2 = x_2, X_3 = x_3 \sim N_1\left(\mu_1 + \Sigma_{12} \Sigma_{22}^{-1} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix}, \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21} \right)$

$$N_1\left([1 \;\; 0] \cdot \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \cdot \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} , \; 4 - [1 \;\; 0] \cdot \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

$$N_1\left([3 \; -1] \cdot \begin{pmatrix} x_2 \\ x_3 \end{pmatrix} , \; 4 - [3 \; -1] \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$$

$$N_1\left(3x_2 - x_3 , \; 4 - 3 \right) = N_1\left(3x_2 - x_3 , 1 \right)$$ erro de contas

$$N_1\left(\frac{3}{5} x_2 - \frac{x_3}{5} , \frac{17}{5} \right)$$

d) $\rho_{ik} = \dfrac{cov(X_i, X_k)}{\sqrt{var X_i} \cdot \sqrt{var X_k}}$

$\Rightarrow \rho_{12} = \dfrac{1}{2\sqrt{2}}$

$\Rightarrow \rho_{13} = 0$

$\Rightarrow \rho_{23} = \dfrac{1}{\sqrt{6}}$

e) $X_2 / X_3 = x_3 \sim N_1\left(\mu_2 + \Sigma_{12} \Sigma_{22}^{-1} (x_3) , \; \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21} \right)$

$\sim N_1\left(1 \cdot \frac{1}{3} \cdot x_3 , \; 2 - 1 \cdot \frac{1}{3} \cdot 1 \right)$

$\sim N_1\left(\frac{x_3}{3} , \frac{5}{3} \right)$

f) $Z = 4X_1 - 6X_2 + X_3$

$Z \sim N(\mu_Z, \Sigma_Z)$

$A = [4 \; -6 \; 1] \Rightarrow Z = A \cdot X$

$\mu_Z = A \cdot \mu = 0$

$$\Sigma_Z = A \cdot \Sigma \cdot A' = [4 \; -6 \; 1] \begin{bmatrix} 4 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -6 \\ 1 \end{bmatrix} = [10 \; -7 \; -3] \cdot \begin{bmatrix} 4 \\ -6 \\ 1 \end{bmatrix} = 79$$

1,1

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6) a) X_1, X_2, X_3, X_4 ind $\sim N_p(\mu, \Sigma)$

$$V_1 = \frac{1}{4} X_1 - \frac{1}{4} X_2 + \frac{1}{4} X_3 - \frac{1}{4} X_4$$

$$V_1 \sim N_p\left(\sum_{j=1}^{n} c_j \mu, \left(\sum_{j=1}^{n} c_j^2\right). \Sigma\right)$$

$$V_1 \sim N_p\left(0_{p\times1}, \frac{1}{4}\Sigma_{p\times p}\right)$$

$$b = \left(\frac{1}{4}, \frac{-1}{4}, \frac{+1}{4}, \frac{-1}{4}\right)$$

$$c = \left(\frac{+1}{4}, \frac{1}{2}, \frac{-1}{4}, \frac{-1}{4}\right)$$

$$V_2 = \frac{1}{4} X_1 + \frac{1}{2} X_2 - \frac{1}{4} X_3 - \frac{1}{4} X_4$$

$$b'c = \frac{1}{16} - \frac{1}{8} - \frac{1}{16} + \frac{1}{16} = \frac{-1}{16}$$

$$V_2 \sim N_p\left(\frac{1}{4}\mu_{p\times1}, \frac{7}{16}\Sigma_{p\times p}\right)$$

b) $$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \sim N_{2p}\left(\begin{bmatrix} 0_{p\times1} \\ \frac{1}{4}\mu_{p\times1} \end{bmatrix}, \begin{bmatrix} \frac{1}{4}\Sigma_{p\times p} & \frac{-1}{16}\Sigma_{p\times p} \\ \frac{-1}{16}\Sigma_{p\times p} & \frac{7}{16}\Sigma_{p\times p} \end{bmatrix}\right)$$

1/1

7 a) $$\sigma_{12} = \rho_{12} . \sqrt{\sigma_{11}}\sqrt{\sigma_{22}} = \frac{\sqrt{2}}{2}$$

$$X \sim N_2\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 & \sqrt{2}/2 \\ \sqrt{2}/2 & 1 \end{bmatrix}\right)$$

$$f(\underset{\sim}{x}) = \frac{1}{(2\pi)^{\frac{2}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left\{\frac{-1}{2}(\underset{\sim}{x} - \mu_x)'\Sigma^{-1}(x - \mu_x)\right\}$$

b) $$(x-\mu)'\Sigma^{-1}(x-\mu) = [x_1 \quad x_2-2] . \begin{bmatrix} 2/3 & -\sqrt{2}/3 \\ -\sqrt{2}/3 & 4/3 \end{bmatrix} \circ \begin{bmatrix} x_1 \\ x_2-2 \end{bmatrix}$$

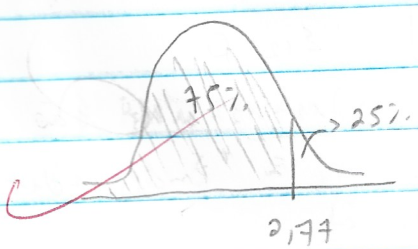
$$= \frac{2}{3}x_1^2 - \frac{\sqrt{2}}{3}(x_2-2).x_1 - \frac{\sqrt{2}}{3}(x_1).(x_2-2) + \frac{4}{3}(x_2-2)^2$$

c) $P((x-\mu)'\Sigma^{-1}(x-\mu) \le 2{,}77)$

$x \sim N_2(\mu, \Sigma)$

$(x-\mu)'\Sigma^{-1}(x-\mu) \sim \chi^2_2$

$P((x-\mu)'\Sigma^{-1}(x-\mu) \le \chi^2_2(\alpha)) = 1-\alpha$



$\Rightarrow P((x-\mu)'\Sigma^{-1}(x-\mu) \le 2{,}77) = 0{,}75$

8) a) $X \sim N_3(\mu, \Sigma)$ $\quad \mu = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Pelo Resultado 2: $X^{(1)}_q$ e $X^{(2)}_{p-q}$ são independentes se e somente se $\Sigma_{12} = 0_{q \times p-q}$

$Y = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \quad \Sigma_Y = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} \quad \Sigma_{12} = -2 \Rightarrow X_1$ e X_2 não são independentes

b) $Y = \begin{pmatrix} X_2 \\ X_3 \end{pmatrix} \quad \Sigma_Y = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix} \quad \Sigma_{12} = 0 \Rightarrow X_2$ e X_3 são independentes

c) $Y = \begin{pmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \\ X_3 \end{pmatrix} \quad \Sigma_Y = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad \Sigma_{12} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow (X_1, X_2)$ e X_3 são independentes.

d) $Y = \begin{pmatrix} \frac{X_1 + X_2}{2} \\ X_3 \end{pmatrix} \quad \underbrace{\begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}}_{X} = Y$

$\Rightarrow Y \sim N_2(\mu_Y, \Sigma_Y)$

$$\Sigma_Y = A . \Sigma . A' = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} . \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/2 & 3/2 & 0 \\ 0 & 0 & 2 \end{bmatrix} . \begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$

$\Sigma_{12} = 0$

$\Rightarrow \frac{X_1 + X_2}{2}$ e X_3 são independentes

e) $Y = \begin{pmatrix} X_2 \\ X_2 - \frac{5}{2} X_1 - X_3 \end{pmatrix}$

$$\underbrace{\begin{bmatrix} 0 & 1 & 0 \\ -\frac{5}{2} & 1 & -1 \end{bmatrix}}_{A} . \underbrace{\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}}_{X} = Y$$

$\Rightarrow Y \sim N_2(\mu_Y, \Sigma_Y)$

$$\Sigma_Y = A . \Sigma . A' = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{5}{2} & 1 & -1 \end{bmatrix} . \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix} . \begin{bmatrix} 0 & -5/2 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 5 & 0 \\ -\frac{9}{2} & 10 & -2 \end{bmatrix} . \begin{bmatrix} 0 & -5/2 \\ 1 & 1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 10 & \frac{101}{4} \end{bmatrix}$$

$\Sigma_{12} = 10$

$\Rightarrow X_2$ e $X_2 - \frac{5}{2} X_1 - X_3$ não são independentes

9a) $\underset{\sim}{X}_1, \underset{\sim}{X}_2, \dots, \underset{\sim}{X}_{20}$ amostra aleatória $\sim N_6(\mu, \Sigma) \Rightarrow \underset{\sim}{X}_1 \sim N_6(\mu, \Sigma)$

$(X \sim N_p(\mu, \Sigma)$

$\Rightarrow (X - \mu)' \Sigma^{-1} (X - \mu) \sim \chi^2_p$

Então:

$(\underset{\sim}{X}_1 - \mu)' \Sigma^{-1} (\underset{\sim}{X}_1 - \mu) \sim \chi^2_6$

9b) Resultado (máxima verossimilhança):

$\bar{x} \sim N_6\left(\mu, \frac{1}{20}\Sigma\right)$

1,0

Resultado (Teorema central do Limite multivariado):

$\sqrt{n}\,(\bar{x}-\mu) \xrightarrow{d} N_6(0, \Sigma)$

c) Resultado (máxima verossimilhança):

$(n-1).S^* \sim W(\Sigma, 5)$

↳ whishart $\quad S^* = \frac{1}{20-1} \sum_{i=1}^{20} (\underset{\sim}{x}_i - \bar{x}).(\underset{\sim}{x}_i - \bar{x})'$

1,0

10) a) $\rho_{ij} = \frac{s_{ij}}{\sqrt{s_{ii}}.\sqrt{s_{jj}}}$

1,0

$\Rightarrow \rho_{11} = \rho_{22} = \rho_{33} = 1$

$\Rightarrow \rho_{12} = \frac{600,51}{\sqrt{5691,34}.\sqrt{126,05}} = 0,704$

$\rho_{13} = \frac{217,25}{\sqrt{5691,34}.\sqrt{23,11}} = 0,547$

$\rho_{23} = \frac{23,37}{\sqrt{126,05}.\sqrt{23,11}} = 0,433$

$$\rho = \begin{bmatrix} 1 & 0,704 & 0,547 \\ 0,704 & 1 & 0,433 \\ 0,547 & 0,433 & 1 \end{bmatrix}$$

b) Coeficiente de variação $= CV_i = \frac{s_{ii}}{\bar{x}_i}$

$\Rightarrow CV_1 = \frac{\sqrt{5691,34}}{527,74} = 0,143 \qquad CV_2 = \frac{\sqrt{126,05}}{54,64} = 0,205$

$CV_3 = \frac{\sqrt{23,11}}{25,13} = 0,191$

11) $\begin{pmatrix} X \\ Y \end{pmatrix} \sim N_2(\mu, \Sigma)$

9/10

$X \sim N(8,4)$

$Y \sim N(3,2)$

$\sigma_{XX} = 4, \quad \sigma_{YY} = 2 \qquad E(Y/X=x) = -1 + \frac{1}{2}x$

$$\begin{pmatrix} X \\ Y \end{pmatrix} \sim N_2\left(\begin{bmatrix} 8 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 & \sigma_{12}(X,Y) \\ \sigma_{12}(X,Y) & 2 \end{bmatrix}\right)$$

$$E[X_1/X_2=x_2] = \mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(x_2 - \mu_2)$$

$$\Rightarrow E[Y/X=x] = 3 + \frac{\sigma_{12}}{4}(x-8) = 3 + \frac{\sigma_{12} \cdot x}{4} - \sigma_{12} \cdot 2$$

$$= -1 + \frac{1}{2} \cdot x$$

$$\Rightarrow \frac{1}{2} \cdot x = \frac{\sigma_{12} \cdot x}{4} \Rightarrow \sigma_{12} = 2$$

$$\Rightarrow \rho_{XY} = \frac{cov(X,Y)}{\sqrt{\sigma_{11}} \cdot \sqrt{\sigma_{22}}} = \frac{2}{\sqrt{2} \cdot 2} = \frac{1}{\sqrt{2}}$$

$$Var(X_1/X_2=x_2) = \sigma_{11} \cdot (1 - \rho_{12}^2)$$

$$\Rightarrow Var(Y/X=x) = 2 \cdot \left(1 - \left(\frac{1}{\sqrt{2}}\right)^2\right) = 2 \cdot \left(1 - \frac{1}{2}\right) = 2 \cdot \frac{1}{2} = \cancel{\frac{1}{2}} \; 1$$

8

0,7 / 1,0

Lista 2, Exercício 13

Lucas Bortolucci

17 de abril de 2018

b) sem responder o item c)

a)

Vetor de médias:

```
# Importando os dados
library(readxl)
BigMac <- read_excel("~/Downloads/BigMac.xlsx")

# Fazendo a Matriz que será usada
matriz = matrix(nrow = 45, ncol = 5)

# Populando a Matriz com a coluna do BigMac
for(i in 1:45)
{
  matriz[i, 1] = BigMac$BigMac[i]
}

# Populando a Matriz com a coluna do Bread
for(i in 1:45)
{
  matriz[i, 2] = BigMac$Bread[i]
}

# Populando a Matriz com a coluna do EngSal
for(i in 1:45)
{
  matriz[i, 3] = BigMac$EngSal[i]
}

# Populando a Matriz com a coluna do TeachSal
for(i in 1:45)
{
  matriz[i, 4] = BigMac$TeachSal[i]
}

# Populando a Matriz com a coluna do Service
for(i in 1:45)
{
  matriz[i, 5] = BigMac$Service[i]
}

# Vetor de Médias
vetor_medias = c(mean(matriz[, 1]), mean(matriz[, 2]), mean(matriz[, 3]), mean(matriz[, 4]), mean(matr
vetor_medias
```

```
## [1] 53.28889 25.35556 31.01556 19.92889 19.92889
```

Matriz de covariância:

```r
# Matriz de Variância e Covariância
matriz_cov = var(matriz)
matriz_cov
```

```
##             [,1]       [,2]      [,3]      [,4]       [,5]
## [1,]  2032.3465   837.6677 -608.0228 -446.0744 -2152.3232
## [2,]   837.6677  1129.3253 -314.0488 -213.8901 -1135.6566
## [3,]  -608.0228  -314.0488  389.6445  263.9261  1116.3889
## [4,]  -446.0744  -213.8901  263.9261  217.4894   932.6313
## [5,] -2152.3232 -1135.6566 1116.3889  932.6313  8279.7980
```

b) Matriz de correlação:

```r
# Matriz de Correlação
matriz_corr = matrix(nrow = 5, ncol = 5)
for(i in 1:5){

  for(j in 1:5){

    matriz_corr[i, j] = matriz_cov[i, j]/(sqrt(matriz_cov[i, i])*sqrt(matriz_cov[j, j]))
  }
}
matriz_corr
```

```
##            [,1]       [,2]       [,3]       [,4]       [,5]
## [1,]  1.0000000  0.5529214 -0.6832611 -0.6709489 -0.5246851
## [2,]  0.5529214  1.0000000 -0.4734274 -0.4315807 -0.3713878
## [3,] -0.6832611 -0.4734274  1.0000000  0.9066279  0.6215432
## [4,] -0.6709489 -0.4315807  0.9066279  1.0000000  0.6949941
## [5,] -0.5246851 -0.3713878  0.6215432  0.6949941  1.0000000
```

c) Variáveis com maior coefiente de correlação linear

```r
# Par de Variáveis com maior correlação

# Matriz com as Correlações entre as variáveis diferentes em valor absoluto e entre as variáveis iguais
matriz_corr_posit = matriz_corr

for(i in 1:5){

  for(j in 1:5){

    if(i == j){
      matriz_corr_posit[i, j] = 0
    }

    if(matriz_corr_posit[i, j] < 0){
      matriz_corr_posit[i, j] = matriz_corr_posit[i, j]*(-1)
    }
  }
}

max_corr = max(matriz_corr_posit)

i_max = 1
j_max = 1
```

```
for(i in 1:5){

  for(j in 1:5){

    if(matriz_corr_posit[i, j] == max_corr){
      i_max = i
      j_max = j
    }
  }
}

variaveis = c("BigMac", "Bread", "EngSal", "TeachSal", "Service")

# Maior Correlação
variaveis[i_max]
```

```
## [1] "TeachSal"
```

```
variaveis[j_max]
```

```
## [1] "EngSal"
```

:ri.