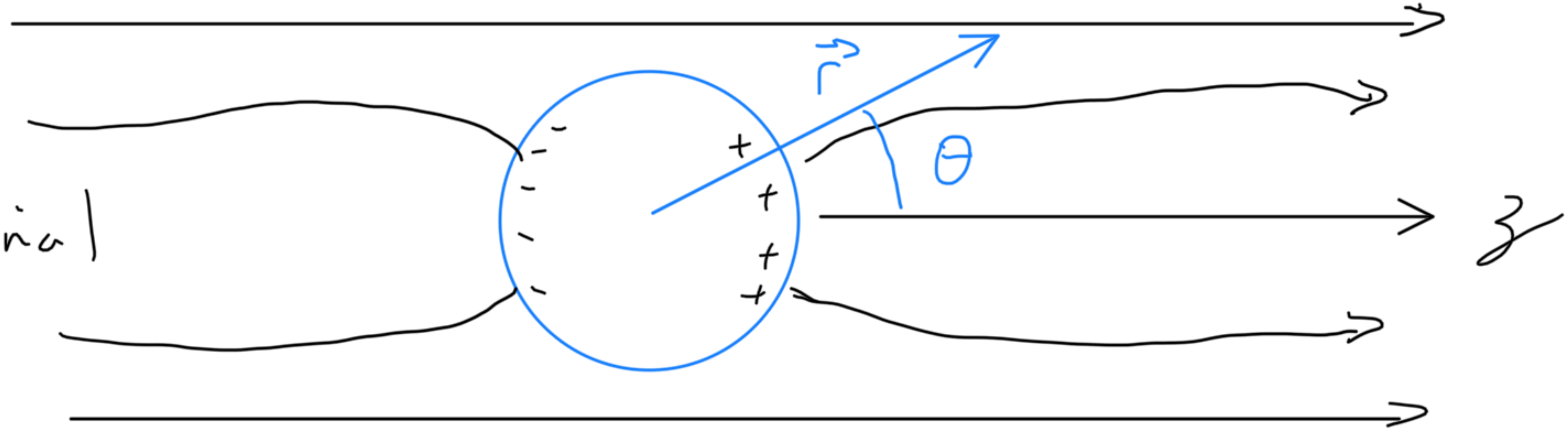
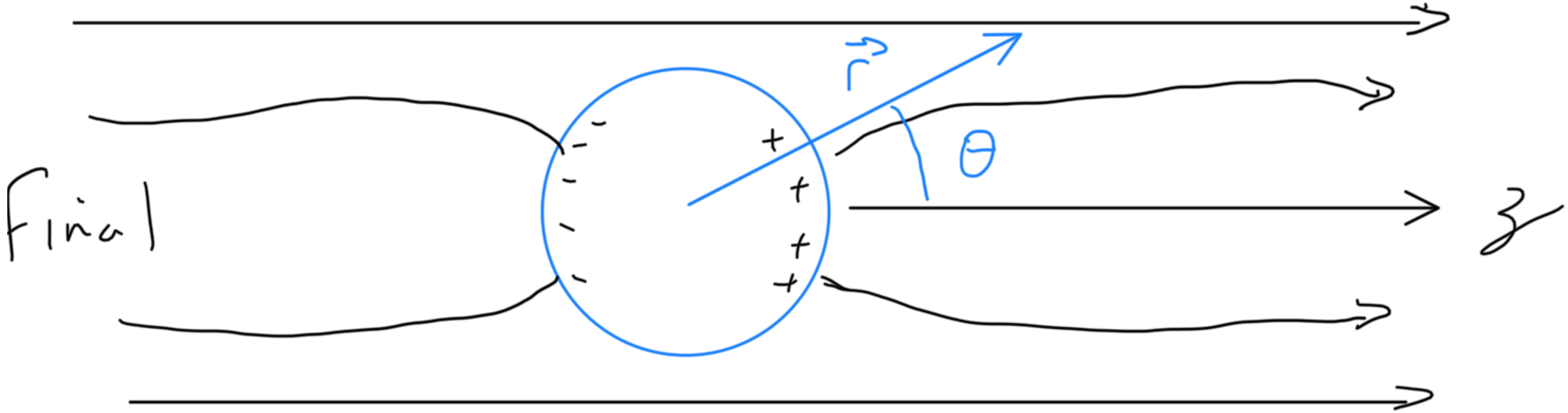
Aplicação: Esfera dielétrica em campo uniforme

Final



Aplicação: Esfera dielétrica em campo uniforme



Esfera metálica:

$$V(\vec{r}) = -E_0 r \cos\theta + E_0 \frac{R^3}{r^2} \cos\theta \quad , \quad r > R$$

$$\sigma(\theta) = 3 \varepsilon_0 E_0 \cos\theta$$

$$\vec{E}(\vec{r}) = -\nabla \cdot V$$

Em coord. cartesianas $V(\vec{r}) = -E_0 z + E_0 R^3 \dfrac{z}{\sqrt{x^2+y^2+z^2}^{\,3/2}}$

$$\frac{\partial}{\partial x}\frac{z}{r^3} = z\frac{\partial}{\partial x}\left(\frac{1}{(x^2+y^2+z^2)^{3/2}}\right) = -\frac{3}{2}z\frac{1}{(x^2+y^2+z^2)^{5/2}} \cdot 2x = -\frac{3x}{r^5}\cdot z$$

$$\frac{\partial}{\partial z}\left(\frac{z}{r^3}\right) = z\frac{\partial}{\partial z}\left(\frac{1}{r^3}\right) + \frac{1}{r^3} = z\cdot\left(-3\frac{z}{r^5}\right) + \frac{1}{r^3}$$

$$\vec{E}(\vec{r}) = 3\frac{\vec{r}}{r^5} z E_0 R^3 + E_0\left(1 - \frac{R^3}{r^3}\right)\hat{k}$$

Vemos a comparação com o campo de dipolo

$$V_d(\vec{r}) = \vec{p} \cdot \hat{r} \cdot \frac{1}{4\pi\varepsilon_0 r^2} \Rightarrow \vec{p} = 4\pi\varepsilon_0 \sigma_0 R^3 \hat{k}$$

$$\vec{E}_d(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^3} \left[3 (\hat{r} \cdot \vec{p}) \hat{r} - \vec{p} \right]$$

Campo fora da esfera: $\vec{E} = \vec{E}_0 + \vec{E}_d$

Campo dentro da esfera: $\vec{E} = 0 = \vec{E}_0 + \vec{E}'$

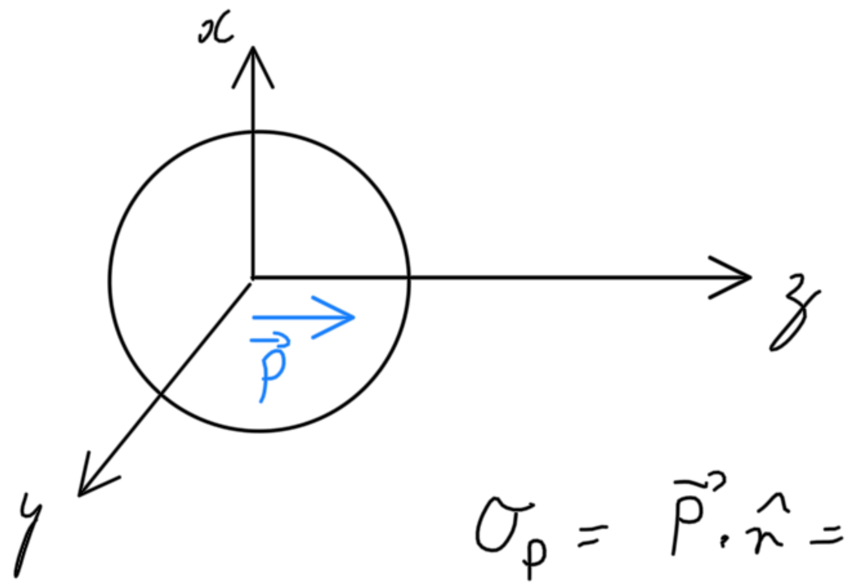
↳ originado na distri. de carga

$$\therefore \vec{E}' = -\vec{E}_0$$

E para um dielétrico?

Campo $\vec{E}_0$ induz um dipolo $\vec{P}$ na esfera.

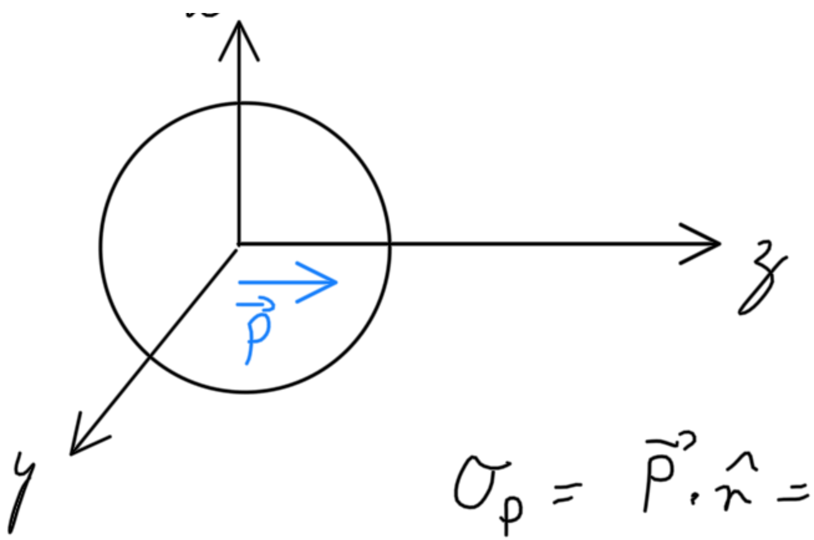
Qual o campo gerado por uma esfera uniformemente polarizada?



$$\vec{P} = P\,\hat{k}$$

$$\sigma_p = \vec{P} \cdot \hat{n} = P \cdot \cos\theta$$

$$\rho_p = -\nabla \cdot \vec{P} = 0$$



$\vec{P} = P\,\hat{k}$

$\sigma_p = \vec{P} \cdot \hat{n} = P \cdot \cos\theta$

$\rho_p = -\nabla \cdot \vec{P} = 0$

Potencial dentro da esfera: $r < R$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(\vec{r}\,')}{|\vec{r}-\vec{r}\,'|}\, da$$

Solução? Eq. de Poisson em coord. esféricos

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l\, r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Solução? Eq. de Poisson em coord. esféricos

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$$

Dentro da esfera: $B_l = 0$ (elimina divergência na origem)

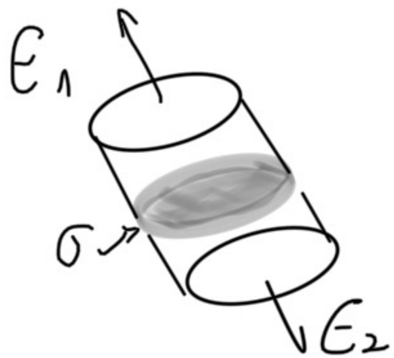
Fora da esfera: $A_l = 0$ (elimina divergência para $r \to \infty$)

Continuidade: $\lim_{r \to R^+} V(r,\theta) = \lim_{r \to R^-} V(r,\theta)$

$$\sum_{l=0}^{\infty} \frac{B_l}{R^{l+1}} P_l(\cos\theta) = \sum_{l=0}^{\infty} A_l R^l P_l(\cos\theta)$$

$$\Rightarrow \quad \frac{B_l}{R^{l+1}} = A_l R^l \Rightarrow \quad B_l = A_l R^{2l+1}$$

Carga na superfície => descontinuidade



$$(\vec{E}_1 - \vec{E}_2)\,\hat{n} = \frac{\sigma}{\varepsilon_0}$$

$$\frac{\partial V_1}{\partial n} - \frac{\partial V_2}{\partial n} = -\frac{\sigma}{\varepsilon_0}$$

$$V_1 = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta) \Rightarrow \frac{\partial}{\partial r} V_1 = \sum_{l=0}^{\infty} -(l+1)\frac{B_l}{r^{l+2}} P_l(\cos\theta)$$

$$V_2 = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) \Rightarrow \frac{\partial}{\partial r} V_2 = \sum_{l=0}^{\infty} l A_l r^{l-1} P_l(\cos\theta)$$

$$\sigma = P\cos\theta$$

$$\left(\frac{\partial}{\partial r} V_1 - \frac{\partial}{\partial r} V_2\right)\Big|_{r=R} = \sum_{l=0}^{\infty} -\left[(l+1)\frac{B_l}{R^{l+2}} + l A_l R^{l-1}\right] P_l(\cos\theta)$$

$$\sigma = P \cos\theta$$

$$\left(\frac{\partial}{\partial r} V_1 - \frac{\partial}{\partial r} V_2\right)\Bigg/_{r=R} = \sum_{l=0}^{\infty} -\left[(l+1)\frac{B_l}{R^{l+2}} + l A_l R^{l-1}\right] P_l(\cos\theta)$$

$$B_l = A_l R^{2l+1} \Rightarrow$$

$$\left(\frac{\partial}{\partial r} V_1 - \frac{\partial}{\partial r} V_2\right)\Bigg/_{r=R} = \sum_{l=0}^{\infty} -A_l\left[(l+1)\frac{R^{2l+1}}{R^{l+2}} + l R^{l-1}\right] P_l(\cos\theta)$$

$$= \sum_{l=0}^{\infty} -A_l R^{l-1}(2l+1) P_l(\cos\theta) = -\frac{P_0}{\varepsilon_0}\cos\theta$$

Como $P_1(x) = x$, $A_l = A_1 \delta_{l1}$ (todos nulos, exceto $l=1$)

$$A_1 \cdot R^{1-1}(2\cdot 1+1)\cdot \cos\theta = \frac{P}{\varepsilon_0}\cos\theta$$

$$A_1 = \frac{P}{3\varepsilon_0} \qquad ; \; B_1 = A_1 \cdot R^{2l+1} = A_1 \cdot R^3 = \frac{P}{3\varepsilon_0} R^3$$

Dentro da esfera: $V(r) = \frac{P}{3\varepsilon_0} \cdot r \cos\theta = \frac{P}{3\varepsilon_0} z$

$$\Rightarrow \vec{E}_p = -\frac{P}{3\varepsilon_0} \hat{k}$$

Fora da esfera: $V(r) = \frac{P}{3\varepsilon_0} \frac{1}{r^2} R^3 \cos\theta = \vec{p} \cdot \hat{r} \frac{1}{4\pi\varepsilon_0 r^2}$

$\Rightarrow$ campo de dipolo $\vec{p} = \frac{4\pi}{3} P R^3 \hat{k}$

Voltemos assim ao começo:

Campo $\vec{E}_0 \rightarrow$ Polarização $\vec{P} = \chi \varepsilon_0 \vec{E}_0$

$\vec{P} \rightarrow$ soma um campo $\vec{E}_1 = -\dfrac{P}{3\varepsilon_0} \hat{k} = -\dfrac{\chi}{3} \vec{E}_0$

Por superposição $\rightarrow \vec{E}_1$ corrige a polarização, somando $\vec{P}_1$

$$\vec{P}_1 = \chi \varepsilon_0 \vec{E}_1 = (\varepsilon_0 \chi) \left(-\frac{\chi}{3}\right) \vec{E}_0 \Rightarrow$$

$$\Rightarrow \vec{E}_2 = -\frac{\vec{P}_1}{3\varepsilon_0} = \left(-\frac{\chi}{3}\right)^2 \vec{E}_0$$

Sucessivamente: $\vec{E}_n = \left(-\dfrac{\chi}{3}\right)^n \vec{E}_0$

Campo resultante $\vec{E} = \sum_{n=0}^{\infty} \vec{E_n} = \vec{E_0} \sum_{n=0}^{\infty} \left(-\frac{\chi}{3}\right)^n$

Serie $\sum_{n=0}^{\infty} (-\alpha)^n = 1 + \sum_{n=1}^{\infty} (-\alpha)^n = 1 + \sum_{n=1}^{\infty} (-\alpha)^{n-1} \cdot (-\alpha)$

$m = n-1$

$$= 1 - \alpha \sum_{m=0}^{\infty} (-\alpha)^m \Rightarrow \sum_{n=0}^{\infty} (-\alpha)^n = \frac{1}{1+\alpha}$$

$$\vec{E} = \left(\frac{1}{1+\chi/3}\right) \vec{E_0} = \left(\frac{3}{3+\chi}\right) \vec{E_0} = \left(\frac{3}{2+\epsilon/\epsilon_0}\right) \vec{E_0}$$

$$\epsilon = \epsilon_0 (1+\chi)$$

$$\vec{P} = \chi \epsilon_0 \vec{E} \Rightarrow \vec{P} = \frac{3\epsilon_0 \chi}{3+\chi} \vec{E_0} =$$

$$\sigma = \vec{P} \cdot \hat{n} = 3 \epsilon_0 \left(\frac{\chi}{3+\chi}\right) E_0 \cos\theta$$

$$\vec{E} = E_0 \hat{k} + \frac{1}{4\pi\varepsilon_0} \frac{1}{r^3} \left[3 (\hat{r} \cdot \vec{p}) \hat{r} - \vec{p} \right] \quad (r > R)$$

Metalli

$$\vec{E} = 0 \quad (r < R)$$

$$\sigma(\theta) = 3 \varepsilon_0 E_0 \cos\theta$$

$$\vec{p} = 4\pi \varepsilon_0 E_0 R^3 \hat{k}$$

Dielettrico

$$\vec{E} = \left(\frac{3}{3+\chi} \right) \vec{E}_0$$

$$\sigma(\theta) = 3 \varepsilon_0 E_0 \cos\theta \cdot \left(\frac{\chi}{3+\chi} \right)$$

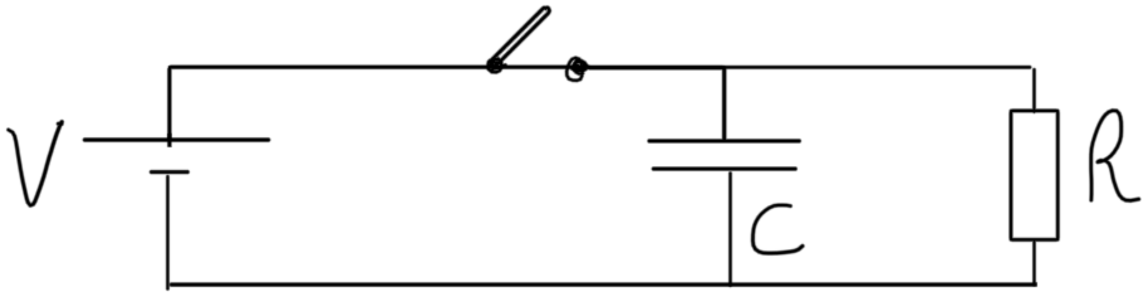
$$\vec{p} = 4\pi \varepsilon_0 E_0 R^3 \left(\frac{\chi}{3+\chi} \right) \hat{k}$$

Eletromagnetismo

Complementos

- Construindo um capacitor.
- Comparando uma esfera dielétrica e uma esfera metálica em um campo eletrostático.

Aula 9: Descarga



Carga inicial $\quad Q = C \cdot V$

Descarga: $\quad I(t) = V/R = -\frac{d}{dt} Q(t)$

$$\Rightarrow \frac{d}{dt} Q(t) = -\frac{Q}{CR} \Rightarrow \quad Q(t) = A e^{-t/\tau}$$

$$V(t) = \frac{Q}{C} = \frac{A}{C} e^{-t/\tau} = V_0 e^{-t/\tau} \qquad \tau = R \cdot C$$

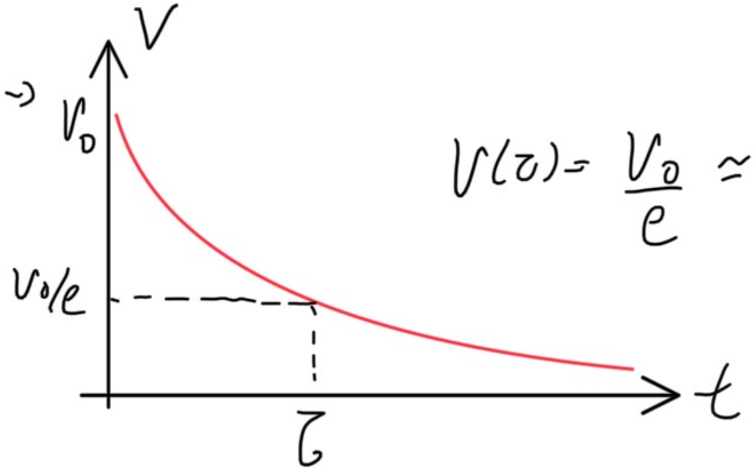
$$V(t) = \frac{Q}{C} = \frac{A}{C} e^{-t/\tau} = V_0 e^{-t/\tau} \qquad \tau = R \cdot C$$

Energia dissipada: $P = \frac{d}{dt}\mathcal{E} = V \cdot I = \frac{V^2}{R} = \frac{V_0^2}{R} e^{-2t/\tau}$

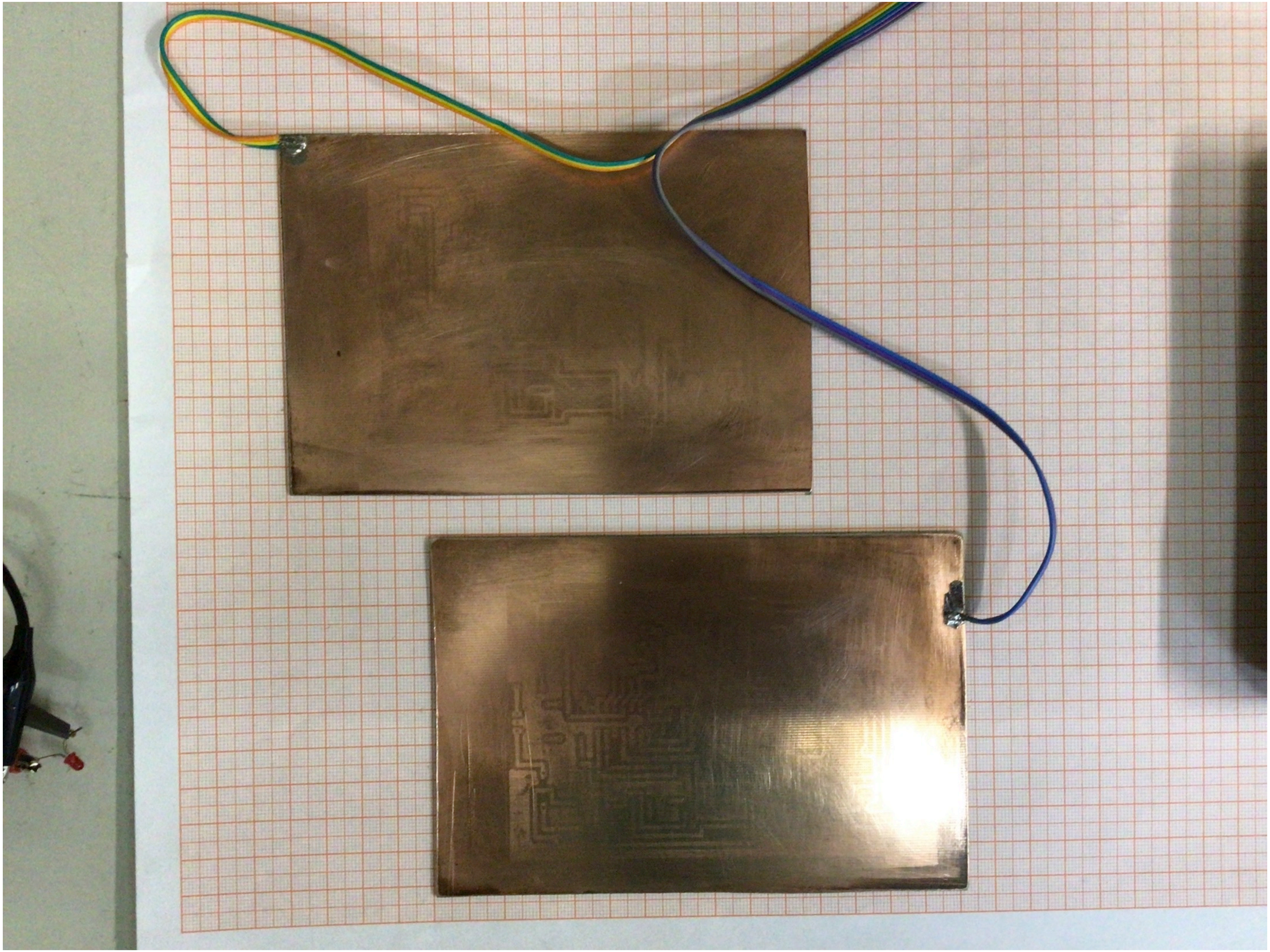
$$\mathcal{E}(t) = \int_0^t P\, dt = \frac{V_0^2}{R}\left[e^{-2t/\tau}\right]_0^t \cdot -\frac{RC}{2}$$

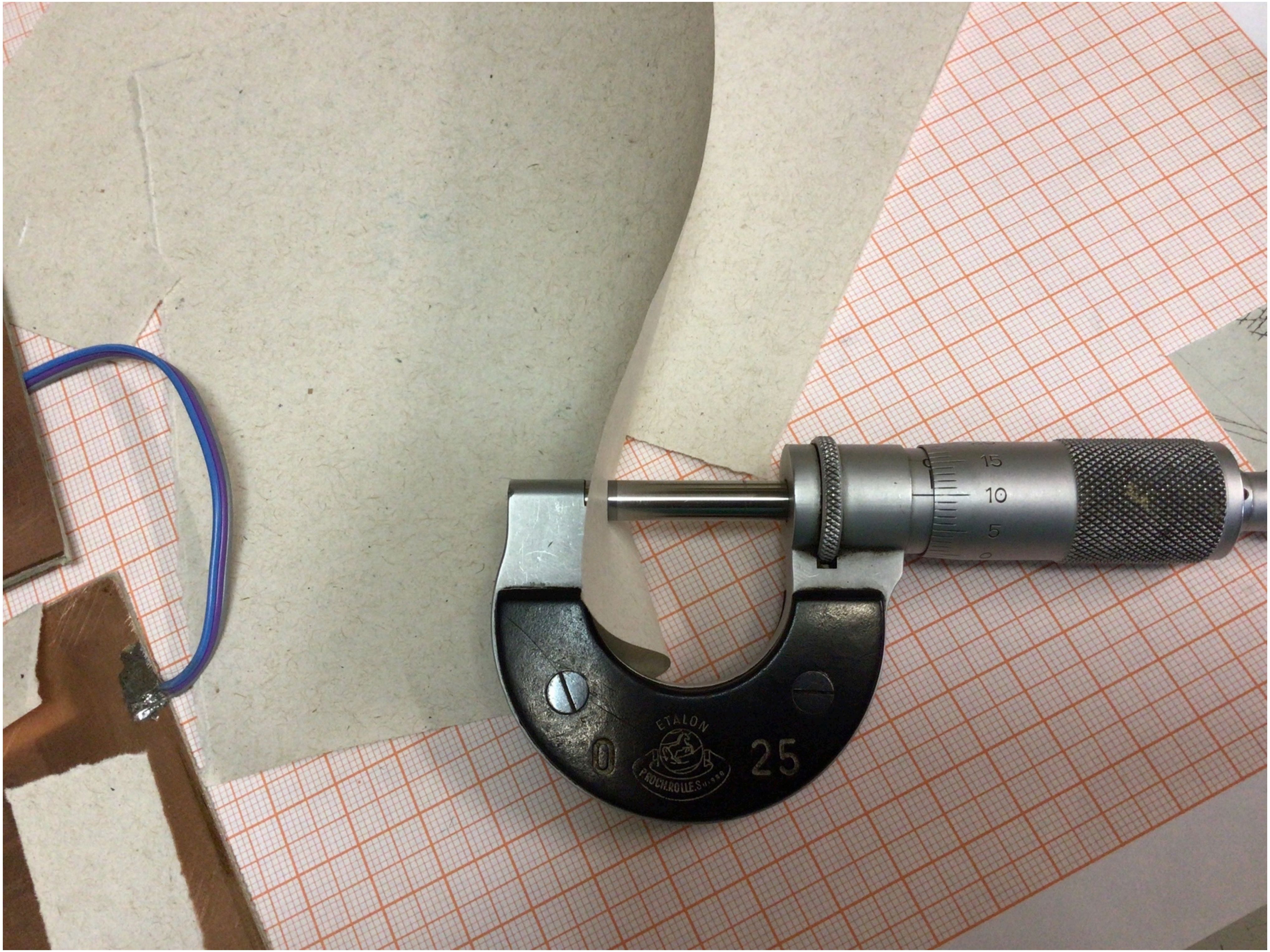
$$= -\frac{V_0^2 C}{2} \lim_{t \to \infty}\left[e^{-2t/\tau} - 1\right] = \frac{V_0^2 C}{2}$$

Determinar τ →

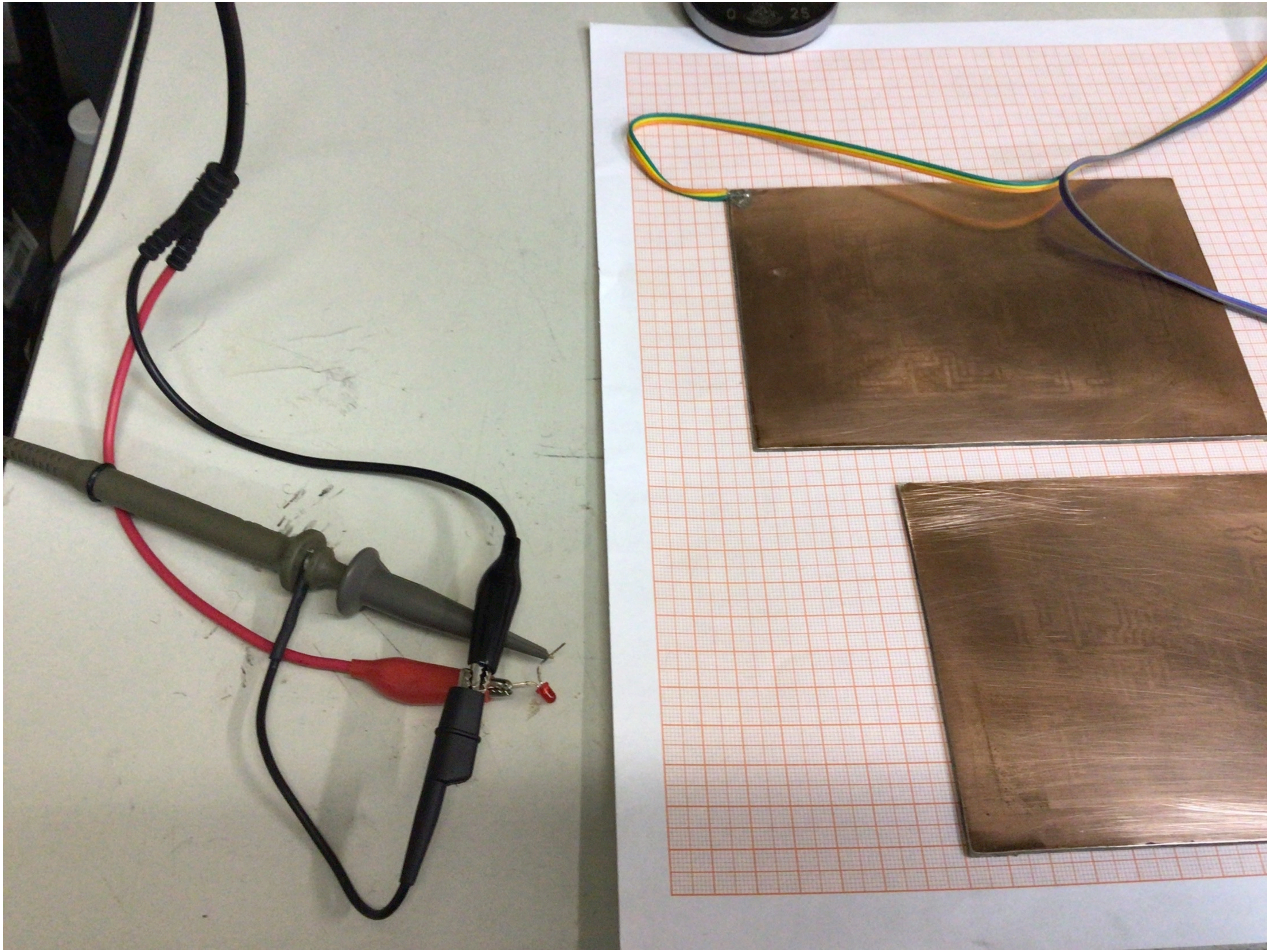


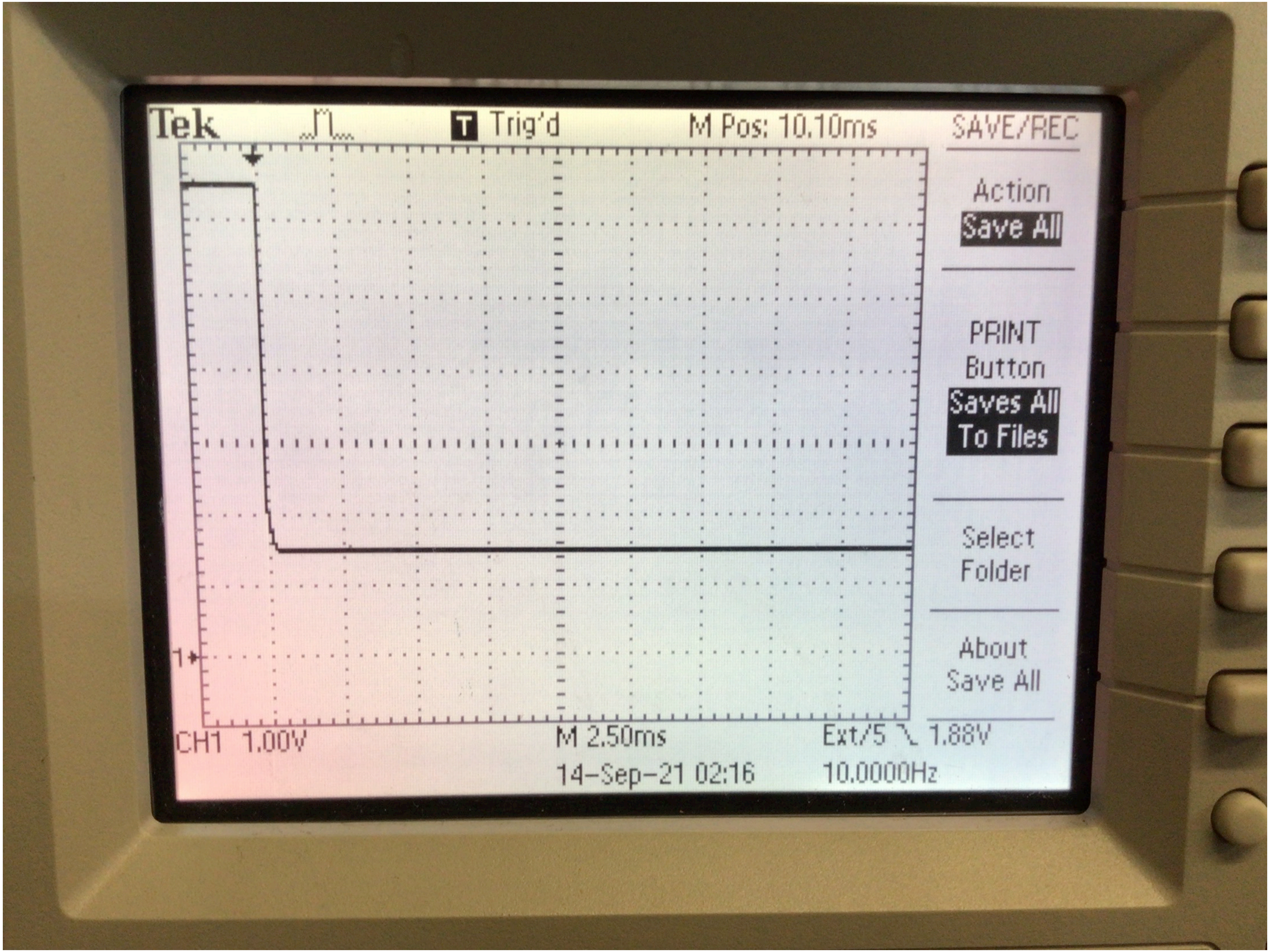
$$V(\tau) = \frac{V_0}{e} \simeq \frac{V_0}{2,7} \simeq \frac{V_0}{3}$$

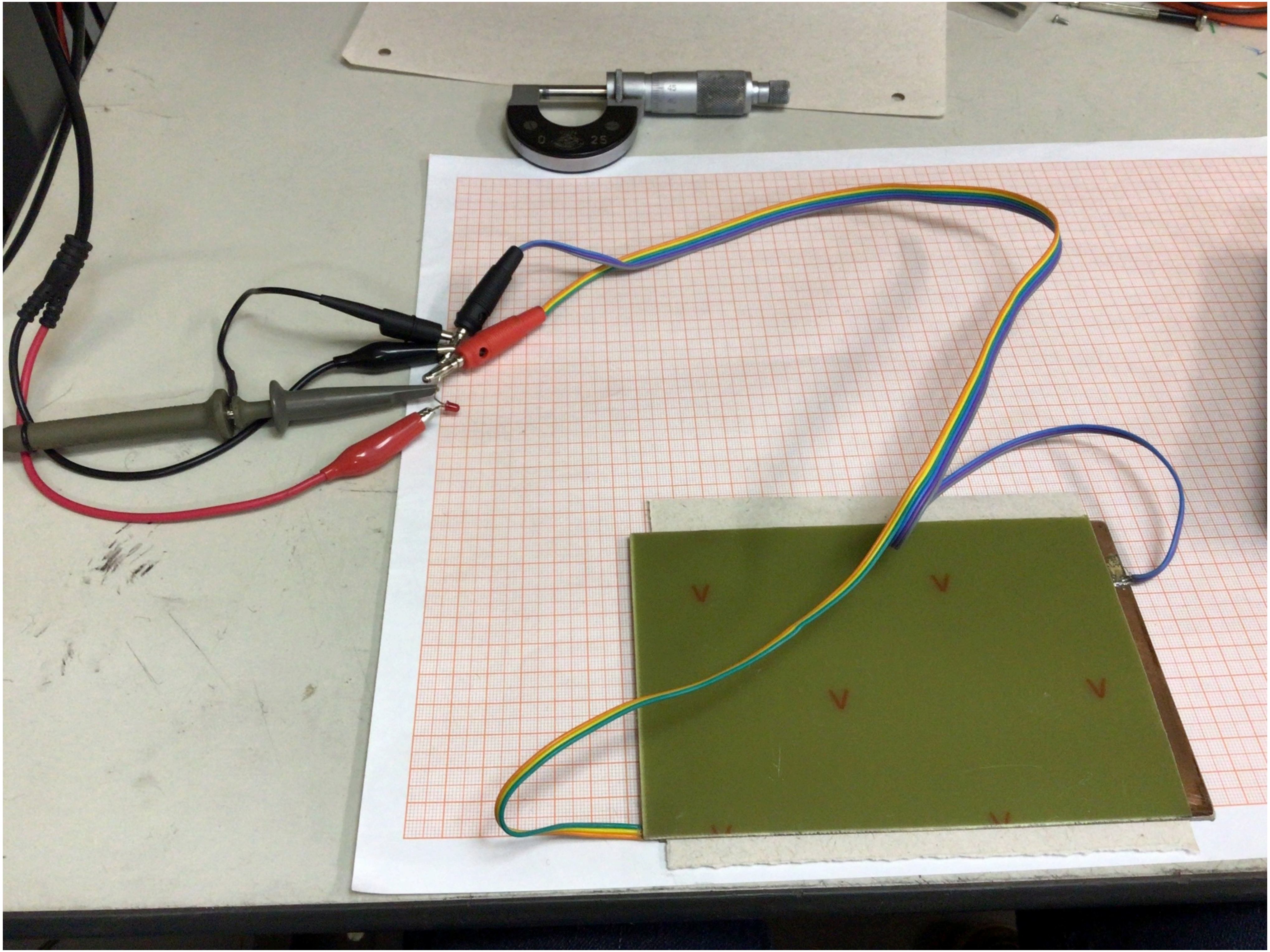


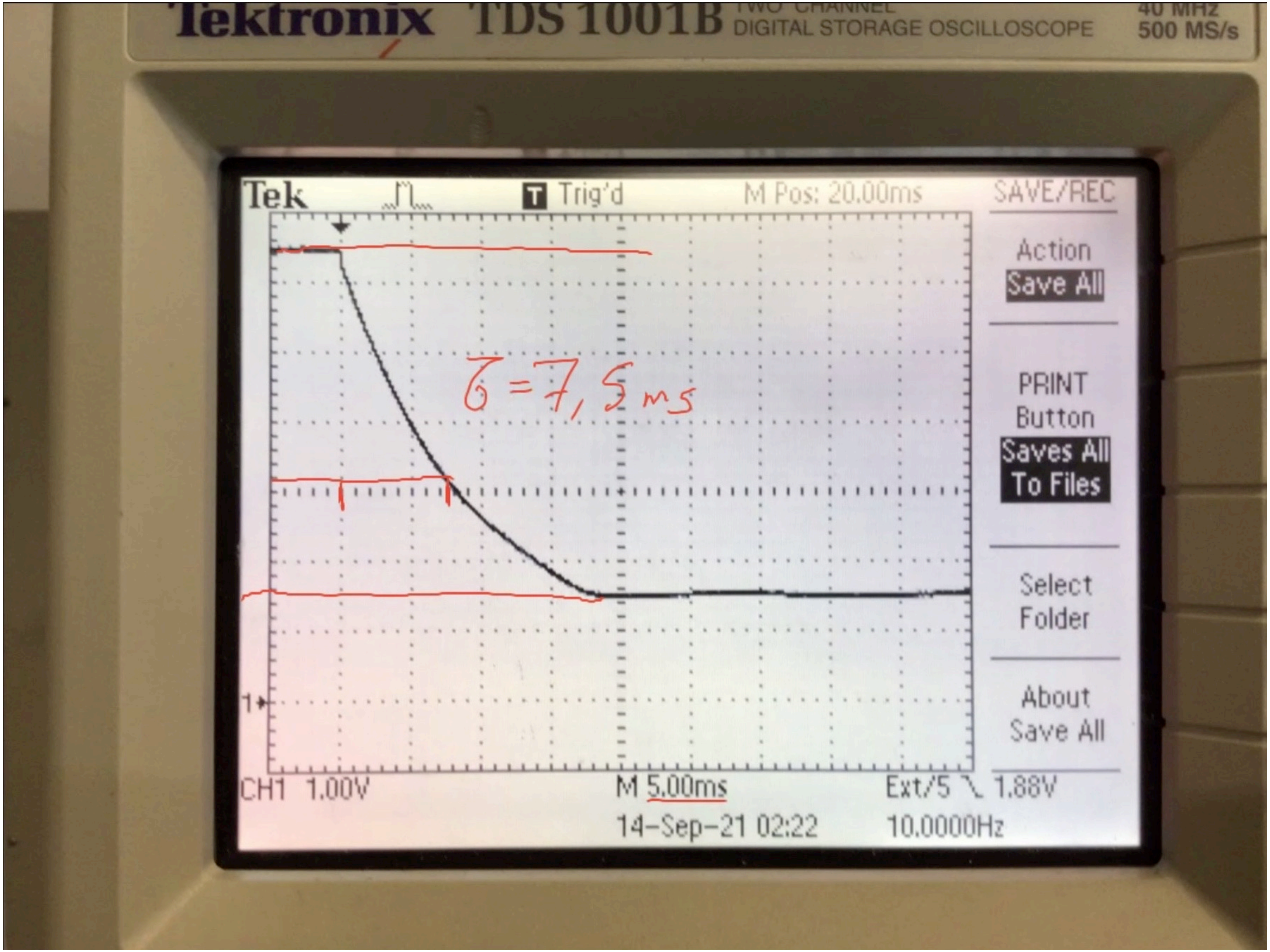


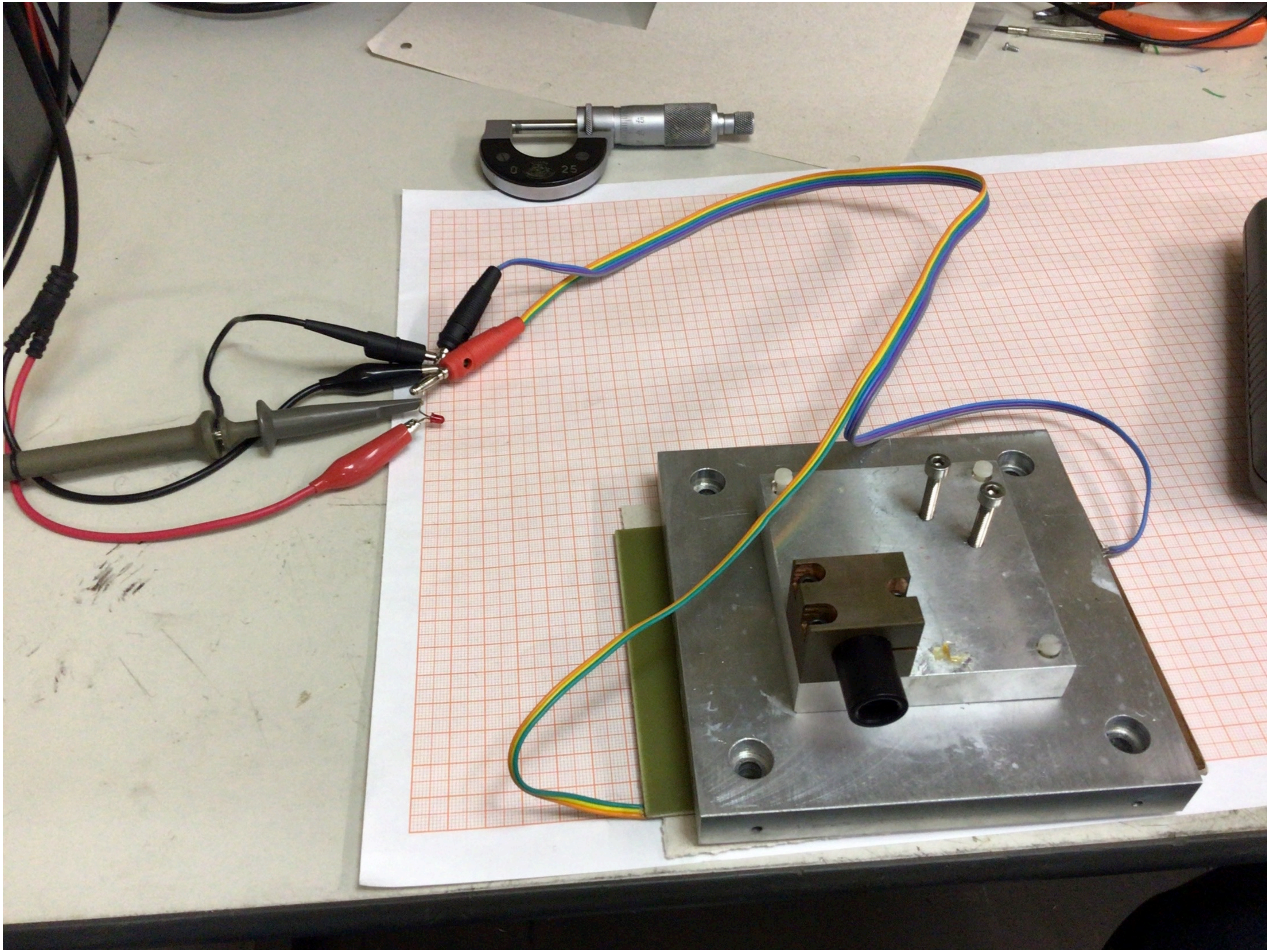
ETALON
0
25
15
10
5

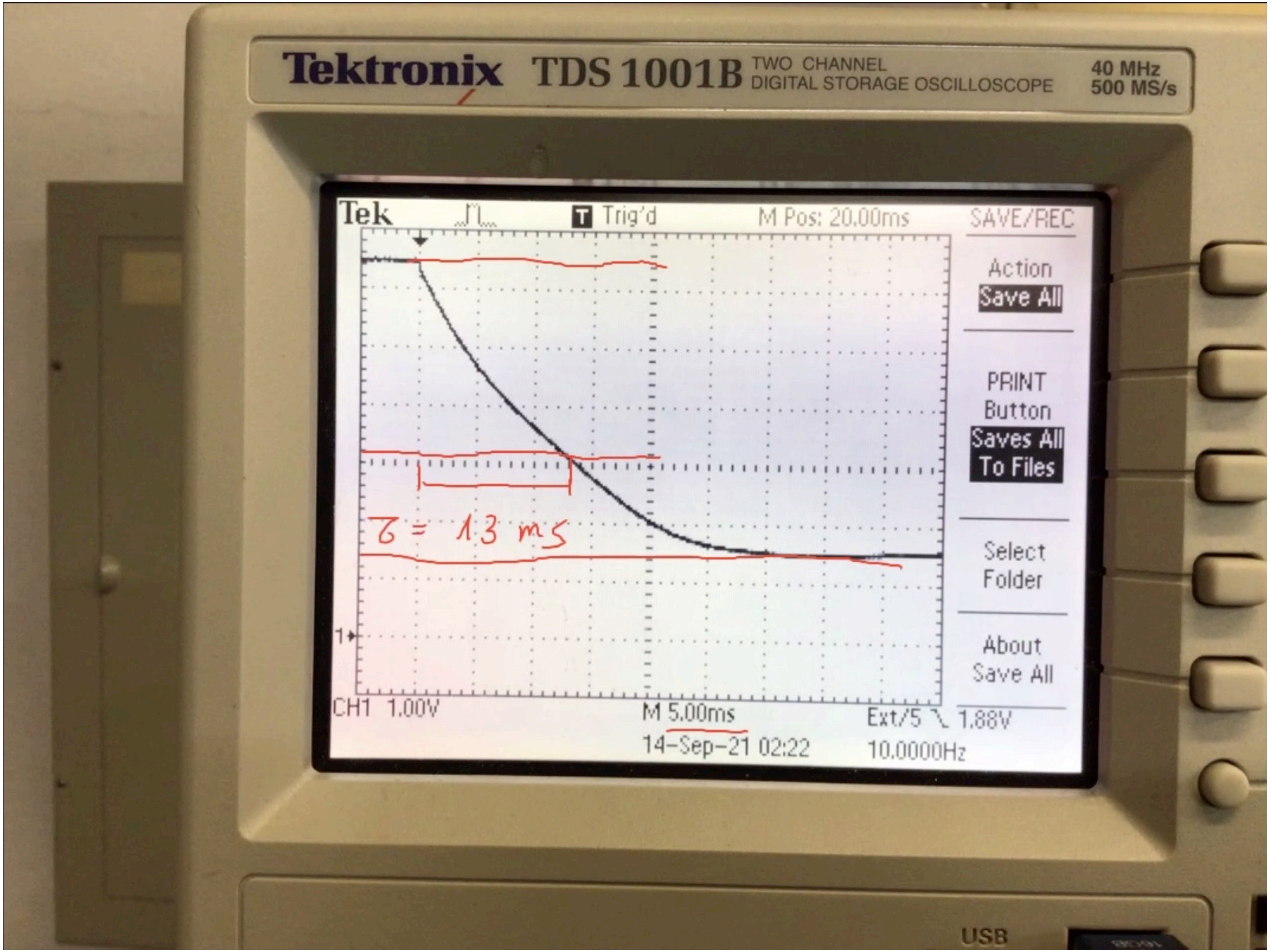



Tek
Trig'd
M Pos: 10.10ms
SAVE/REC
Action
Save All
PRINT
Button
Saves All
To Files
Select
Folder
About
Save All
CH1 1.00V
M 2.50ms
Ext/5 1.88V
14-Sep-21 02:16
10.0000Hz

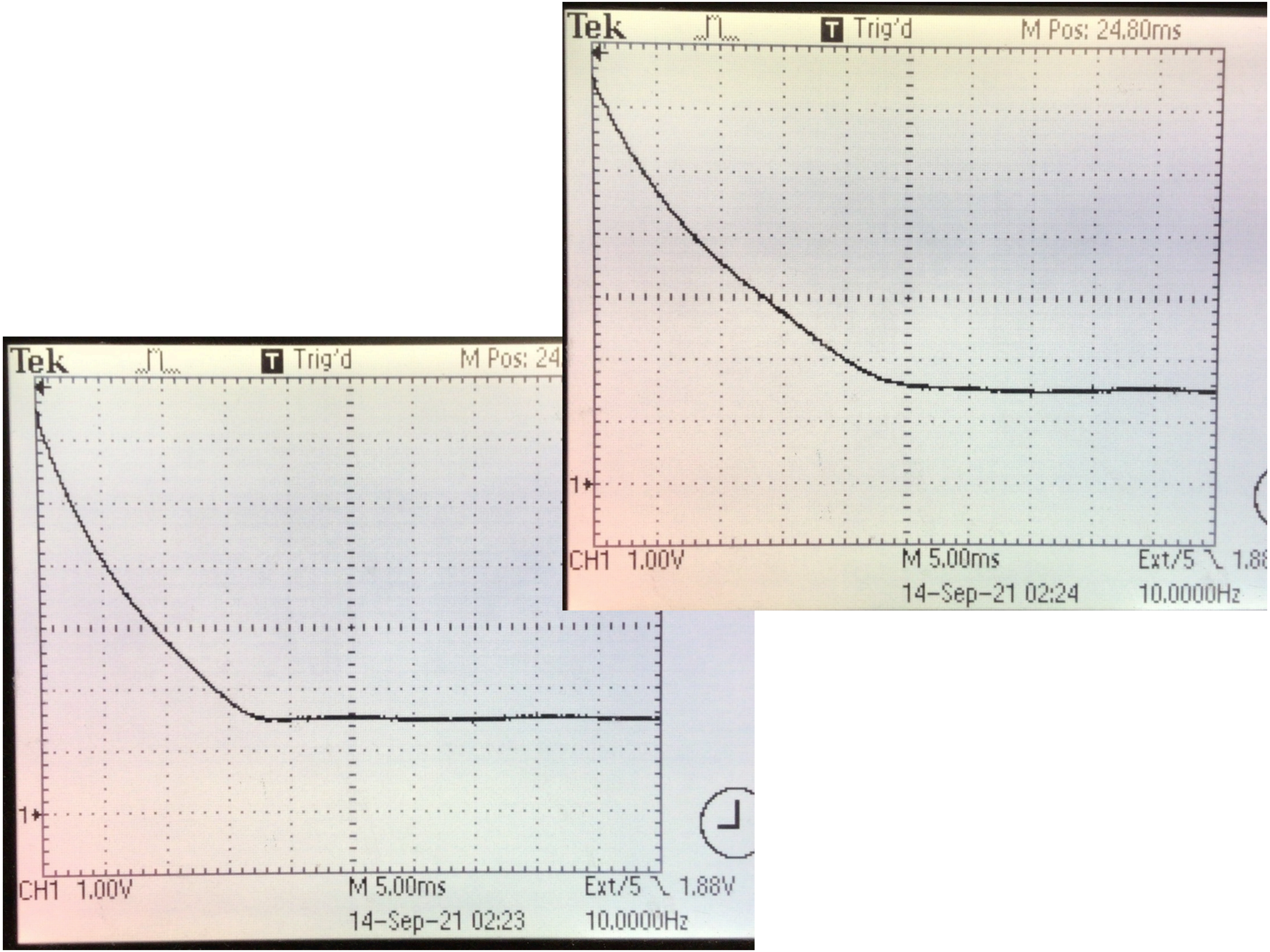



Tektronix TDS 1001B TWO CHANNEL DIGITAL STORAGE OSCILLOSCOPE
40 MHz 500 MS/s
Tek
Trig'd
M Pos: 20.00ms
SAVE/REC
Action
Save All
PRINT Button
Saves All To Files
Select Folder
About Save All
τ = 7,5 ms
CH1 1.00V
M 5.00ms
Ext/5 1.88V
14-Sep-21 02:22
10.0000Hz


0 25



Tektronix
TDS 1001B
TWO CHANNEL
DIGITAL STORAGE OSCILLOSCOPE
40 MHz
500 MS/s
Tek
Trig'd
M Pos: 20.00ms
SAVE/REC
Action
Save All
PRINT
Button
Saves All
To Files
Select
Folder
About
Save All
τ = 13 ms
CH1 1.00V
M 5.00ms
Ext/5 1.88V
14-Sep-21 02:22
10.0000Hz
USB



Tek
Trig'd
M Pos: 24.80ms
CH1 1.00V
M 5.00ms
Ext/5 1.88V
14-Sep-21 02:24
10.0000Hz
Tek
Trig'd
M Pos: 24
CH1 1.00V
M 5.00ms
Ext/5 1.88V
14-Sep-21 02:23
10.0000Hz

Sem peso: $\tau_1 = \rightarrow \quad 5/_{2,7} = 1,85 \quad / \quad 5/_3 = 1,7$

$\sim 3 \times 2,5\,ms \quad 7,5\,ms$

$R = 10\,M\Omega \qquad C = \frac{7,5 \cdot 10^{-3}}{10^7}$

$= 7,5 \cdot 10^{-10} F = 750\,pF$

$A = 0,1\,m \times 0,14\,m$

$\varepsilon_0 = 8,8 \cdot 10^{-12} \frac{F}{m}$

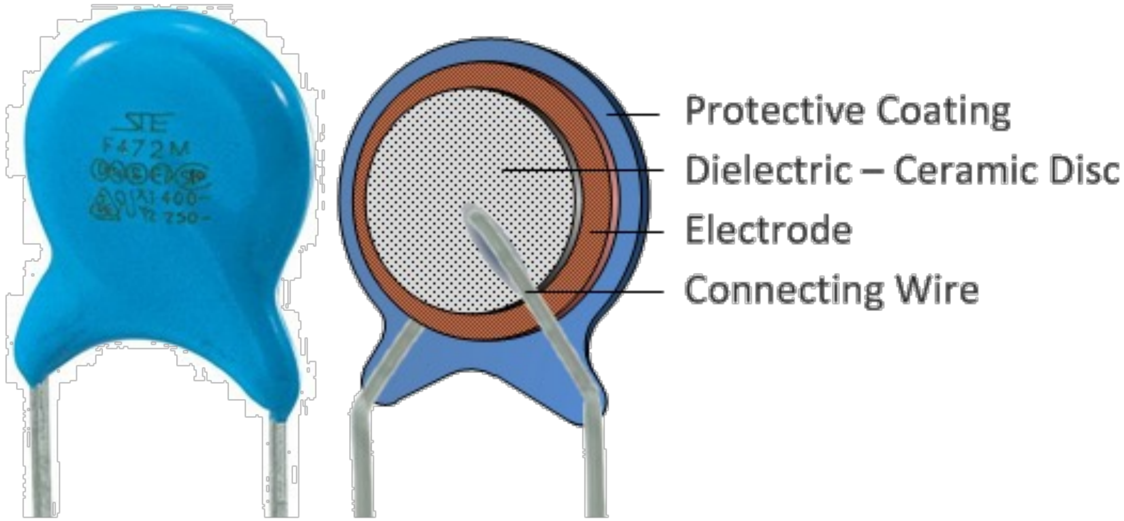
$C = 8,8 \cdot 10^{-12} \frac{F}{m} \cdot \frac{1,4 \cdot 10^{-2}\,m^2}{10^{-4}\,m}$

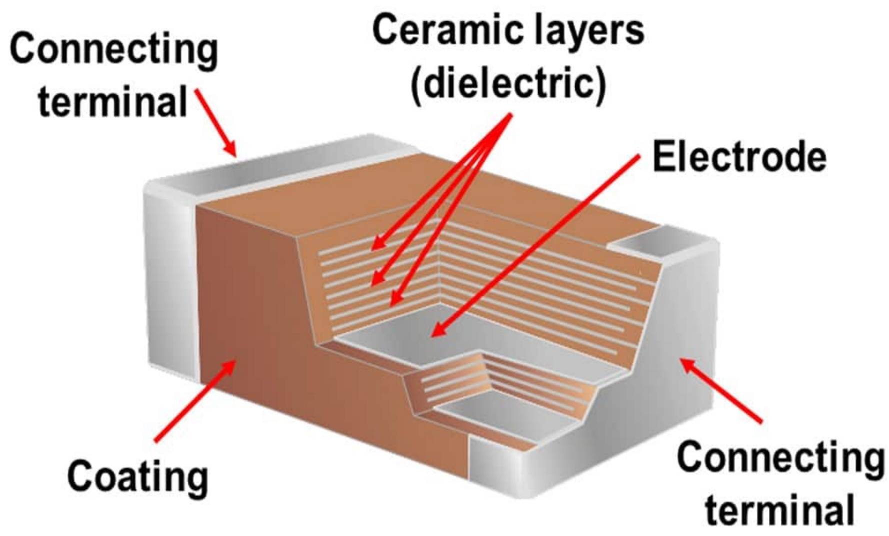
$d = 0,1 \cdot 10^{-3}\,m$

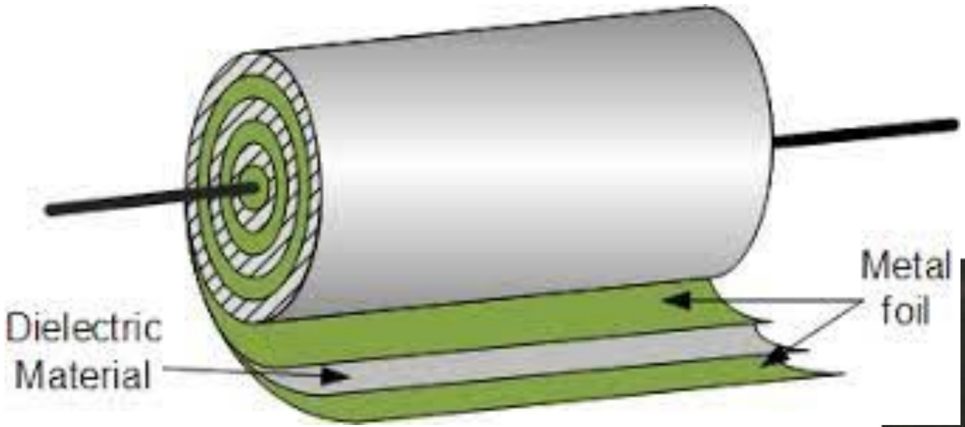
$= 12,32 \cdot 10^2\,pF = 1,2\,nF$

$\tau = 12\,ms$

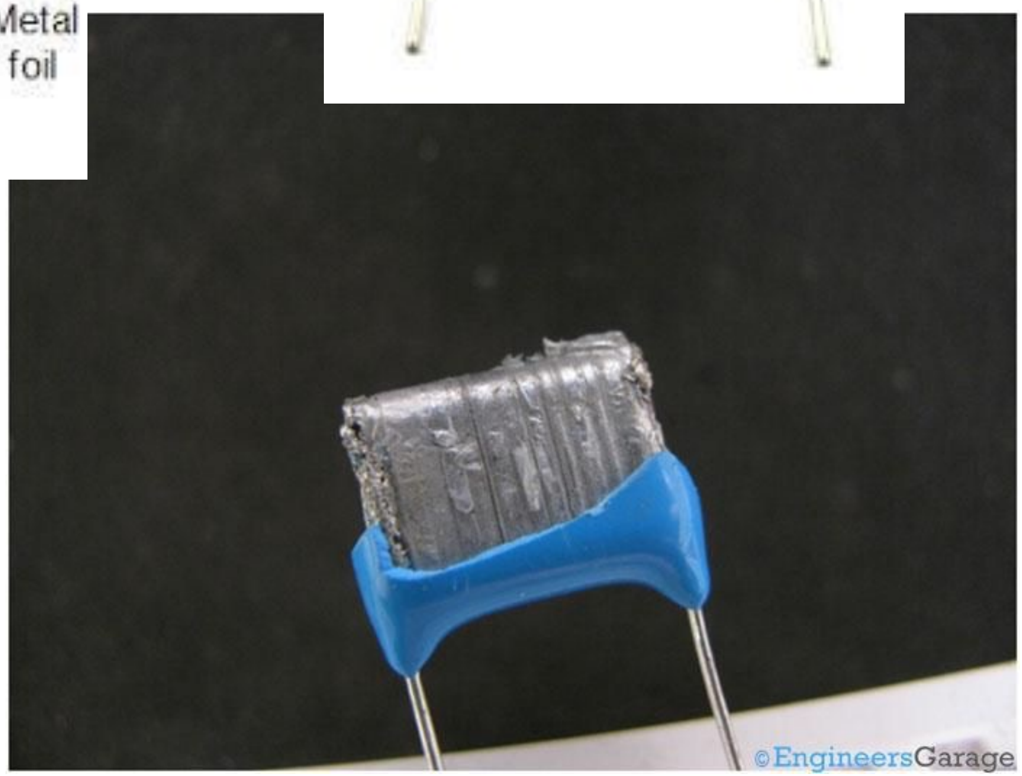
Tipos de capacitores:

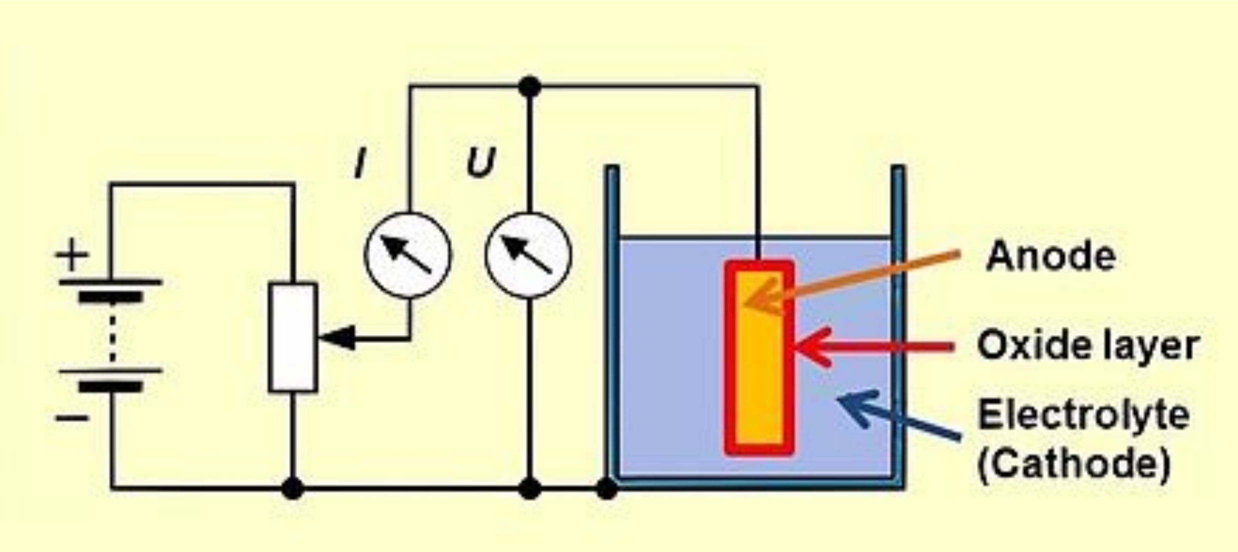


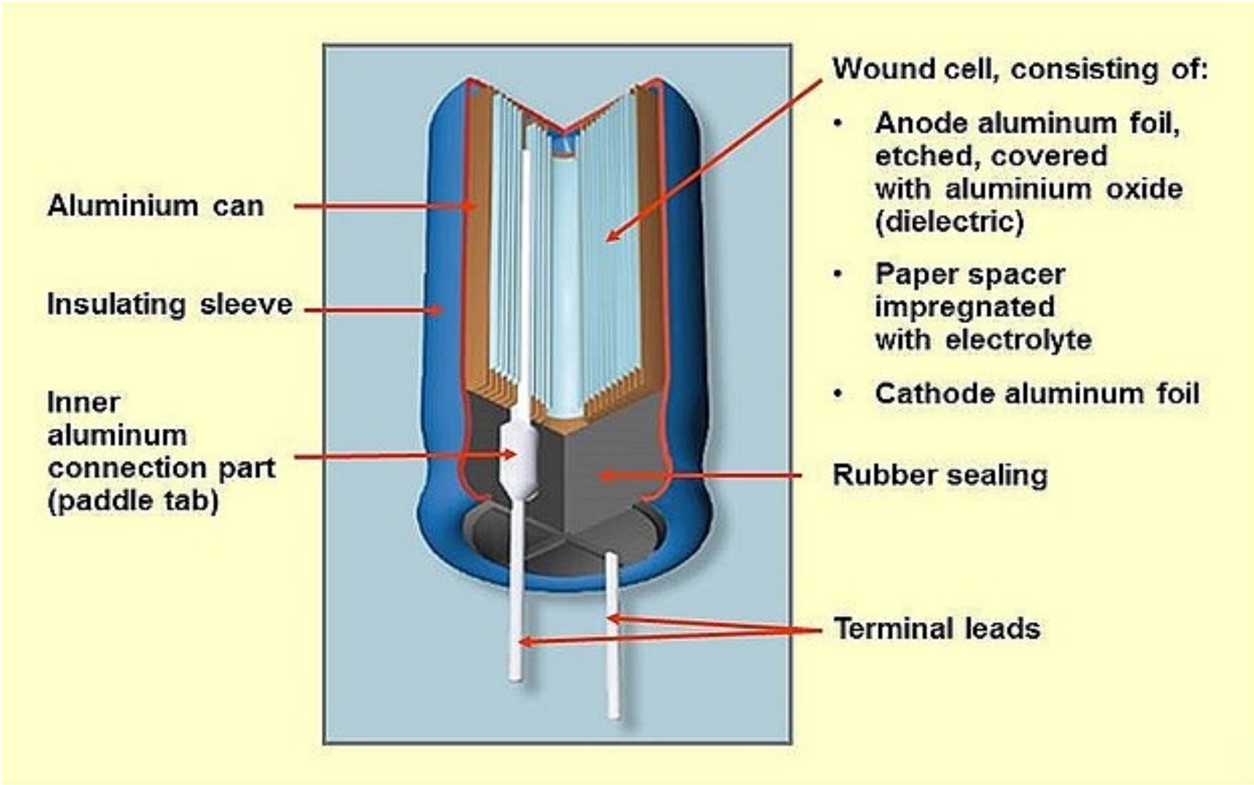



Metal
foil
Dielectric
Material


CL21 225J
250V DC


©EngineersGarage


I
U
+
−
Anode
Oxide layer
Electrolyte
(Cathode)


Wound cell, consisting of:
• Anode aluminum foil, etched, covered with aluminium oxide (dielectric)
• Paper spacer impregnated with electrolyte
• Cathode aluminum foil
Aluminium can
Insulating sleeve
Inner aluminum connection part (paddle tab)
Rubber sealing
Terminal leads

Aplicação: Esfera dielétrica em campo uniforme

Inicial

Aplicação: Esfera dielétrica em campo uniforme