Atividade 2: SMA0300 Geometria Analítica

Exercício 1. Seja $E = (\vec{e_1}, \vec{e_2}, \vec{e_3})$ uma base orthonormal de V^3 e

$$\begin{aligned} \vec{f_1} &= \tfrac{\sqrt{2}}{2}\vec{e_1} - \tfrac{\sqrt{2}}{2}\vec{e_3} \\ \vec{f_2} &= \vec{e_2} \\ \vec{f_3} &= \tfrac{\sqrt{2}}{2}\vec{e_1} + \tfrac{\sqrt{2}}{2}\vec{e_3} \end{aligned}$$

(a) Mostre que $F = (\vec{f_1}, \vec{f_2}, \vec{f_3})$ é uma base de V^3.

(b) F é uma base ortonormal de V^3? Justifique sua resposta

(b) Encontre as coordenadas do vetor $\vec{v} = (1, 0, -1)_E$ na base F.

1- $E = (\vec{e}_1, \vec{e}_2, \vec{e}_3)$ base ortonormal de V^3.

$\vec{f}_1 = \frac{\sqrt{2}}{2}\vec{e}_1 - \frac{\sqrt{2}}{2}\vec{e}_3 = \left(\frac{\sqrt{2}}{2}; 0; -\frac{\sqrt{2}}{2}\right)_E$

$\vec{f}_2 = \vec{e}_2 = (0, 1, 0)_E$

$\vec{f}_3 = \frac{\sqrt{2}}{2}\vec{e}_1 + \frac{\sqrt{2}}{2}\vec{e}_3 = \left(\frac{\sqrt{2}}{2}; 0; \frac{\sqrt{2}}{2}\right)_E$

2,0

a) $F = (\vec{f}_1, \vec{f}_2, \vec{f}_3)$ é base V^3?

$\vec{f}_1, \vec{f}_2, \vec{f}_3$ é LI: $\begin{vmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{vmatrix} = \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1 \boxed{\neq 0}$

b) F será base ortonormal de V^3 se $\|\vec{f}_1\| = \|\vec{f}_2\| = \|\vec{f}_3\| = 1$ e $\vec{f}_1 \cdot \vec{f}_2 = \vec{f}_2 \cdot \vec{f}_3 = \vec{f}_1 \cdot \vec{f}_3 = 0$

$\begin{cases} \|\vec{f}_1\| = \sqrt{\frac{2}{4} + 0 + \frac{2}{4}} = \sqrt{1} = 1 \\ \|\vec{f}_2\| = \sqrt{0 + 1 + 0} = \sqrt{1} = 1 \\ \|\vec{f}_3\| = \sqrt{\frac{2}{4} + 0 + \frac{2}{4}} = \sqrt{1} = 1 \end{cases}$ e $\begin{cases} \vec{f}_1 \cdot \vec{f}_2 = \frac{\sqrt{2}}{2} \cdot 0 + 0 \cdot 1 + 0\left(-\frac{\sqrt{2}}{2}\right) = 0 + 0 + 0 = 0 \\ \vec{f}_2 \cdot \vec{f}_3 = 0 \cdot \frac{\sqrt{2}}{2} + 1 \cdot 0 + 0 \cdot \frac{\sqrt{2}}{2} = 0 + 0 + 0 = 0 \\ \vec{f}_1 \cdot \vec{f}_3 = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} + 0 \cdot 0 + \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{2}{4} + 0 - \frac{2}{4} = 0 \end{cases}$

$\boxed{\text{Então F é base ortonormal de } V^3.}$

4,0

c) $\vec{v} = (1, 0, -1)_E$ na base F

$(\vec{v})_F = M_{FE} \cdot (\vec{v})_E$ ou $(\vec{v})_E = M_{EF} \cdot (\vec{v})_F$, usarei essa.

$M_{EF} = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$ $(\vec{v})_E = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $(\vec{v})_F = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$

$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} \Rightarrow \begin{cases} \frac{\sqrt{2}}{2}\alpha + 0 \cdot \beta + \frac{\sqrt{2}}{2}\gamma = 1 & (I) \\ 0\alpha + 1\beta + 0\gamma = 0 & (II) \\ -\frac{\sqrt{2}}{2}\alpha + 0 \cdot \beta + \frac{\sqrt{2}}{2}\gamma = -1 & (III) \end{cases}$

4,0

(II): $\beta = 0$

(I)+(III): $2 \cdot \frac{\sqrt{2}}{2} \cdot \gamma = 0 \therefore \gamma = \frac{0}{\sqrt{2}} = 0$

(I): $\frac{\sqrt{2}}{2}\alpha + 0 + 0 = 1 \therefore \alpha = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\Rightarrow \boxed{\vec{v} = (\sqrt{2}, 0, 0)_F}$