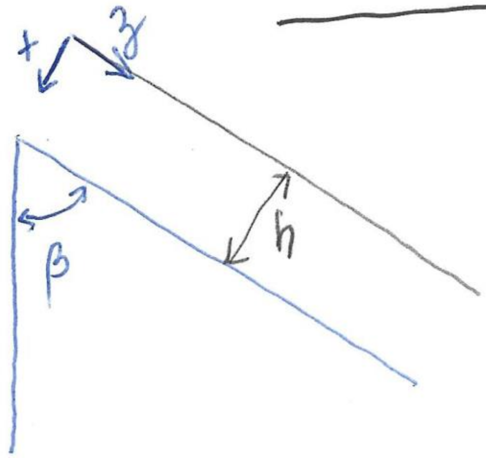


- laminar
- bidimensional
- desenvolvido
- incompressível
- newtoniano
- regime permanente

Continuidade: $\operatorname{div} \vec{V} = 0$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_z}{\partial z} = 0$$

desenvolvido: $\frac{\partial V_z}{\partial z} = 0$

$$\frac{\partial V_x}{\partial x} = 0 \rightarrow V_x = 0$$

N. Stokes: direção x

$$g_x = g \operatorname{sen}\beta \rightarrow 0 = -\frac{\partial p}{\partial x} + \rho g \operatorname{sen}\beta$$

$$P = \rho g \operatorname{sen}\beta\, x + f(z)$$

$$p/x = 0 \rightarrow P = P_{atm} \ (\forall z) \rightarrow f(z) = 0$$

$$\boxed{P = \rho g \operatorname{sen}\beta\, x}$$

direção z: $g_z = \rho g \cos\beta$

$$-\frac{\partial p}{\partial z} + \rho g \cos\beta + \mu \frac{\partial^2 V_z}{\partial x^2} = 0$$

$$\frac{d^2 V_z}{dx^2} = -\frac{\rho g \cos\beta}{\mu} \rightarrow V_z = -\frac{\rho g \cos\beta}{\mu} \frac{x^2}{2} + C_1 x + C_2$$

$$\text{C.C.} \begin{cases} x = h \rightarrow v_z = 0 \\ x = 0 \rightarrow \tau_{zx} = \tau_{xz} = 0 \end{cases}$$

atrito desprezível

$$\left\{ \underbrace{\mu_{ar}}_{\approx 0{,}02\,cp} \left.\frac{dv_z}{dx}\right|_{\substack{ar \\ x=0}} = \underbrace{\mu_L}_{1cp} \left.\frac{dv_z}{dx}\right|_{x=0} \right.$$

$$\frac{dv_z}{dx} \simeq 0$$

$$C_1 = 0$$

$$0 = -\frac{\rho g \cos\beta}{\mu}\frac{h^2}{2} + C_2$$

$$\hookrightarrow C_2 = \frac{\rho g \cos\beta}{2\mu} h^2$$

$$\boxed{v_z = \frac{\rho g \cos\beta\, h^2}{2\mu}\left(1 - \frac{x^2}{h^2}\right)}$$

$$\boxed{\langle v_z \rangle = \frac{1}{L.W}\int_0^W\!\!\int_0^L v_z\, dx\, dy = \frac{\rho g \cos\beta\, h^2}{3\mu}} \rightarrow \boxed{\dot{m} = \frac{\rho^2 g \cos\beta\, h^3 W}{3\mu}}$$

$$\boxed{\tau_{xz} = \tau_{zx} = \mu\frac{dv_z}{dx} = -\rho g \cos\beta\, x}$$

$$F_z\Big|_{x=h} = \int_0^L\!\!\int_0^W \tau_{zx}\Big|_{x=h} dy\, dz = -\rho g \cos\beta\, \underbrace{hLW}_{V}$$

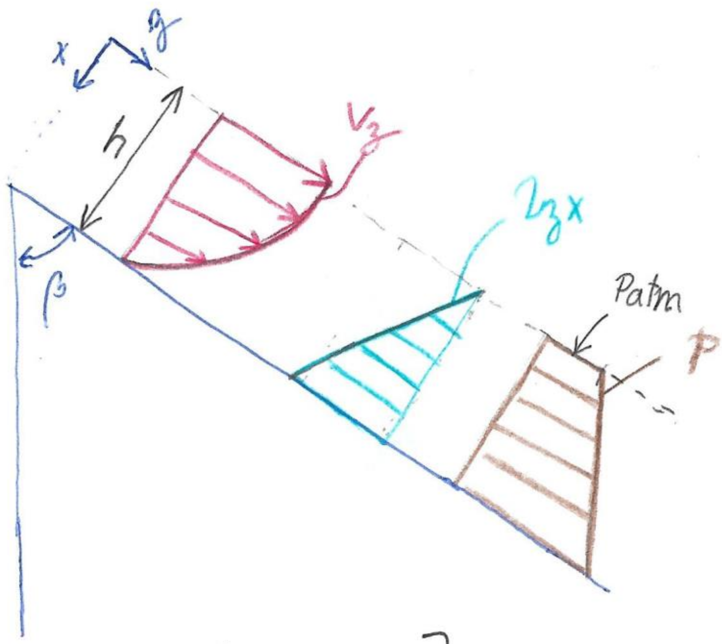
$$\boxed{F_z = -\rho g \cos\beta\, V = -mg\cos\beta}$$

$$F_x\Big|_{x=h} = \int_0^W\!\!\int_0^L p\, dz\, dy = LW(\rho g \,\text{sen}\beta\, h + P_{atm})$$

$$F_x = \rho g\,\text{sen}\beta\, V + P_{atm}.LZ \rightarrow \boxed{F_x = mg\,\text{sen}\beta + P_{atm}.A}$$

Se $\beta = 0 \rightarrow F_z = -mg$ e $F_x = P_{atm}.A$

Se $\beta = 90° \rightarrow F_z = 0$ e $F_x = \rho g V + P_{atm} A$



$$\rho \vec{g} = \begin{bmatrix} \rho g \operatorname{sen}\beta \\ 0 \\ \rho g \cos\beta \end{bmatrix}$$

$$\vec{\operatorname{grad}}\, p = \begin{bmatrix} \rho g \operatorname{sen}\beta \\ 0 \\ 0 \end{bmatrix}$$

$$\operatorname{div} \vec{\vec{\tau}} = \begin{bmatrix} 0 \\ 0 \\ -\rho g \cos\beta \end{bmatrix}$$

$$De = 4 r_H = \frac{4 . hW}{W} = 4h$$

$$Re = \frac{\rho \langle v \rangle De}{\mu} = \frac{\rho \langle v \rangle 4h}{\mu} = \frac{4 \dot{m}}{\mu W}$$

$$\dot{m} = \rho \langle v \rangle h W = \frac{\rho^2 g . \cos\beta \, h^3 W}{3\mu}$$

Laminar $\rightarrow$ $Re < 18$