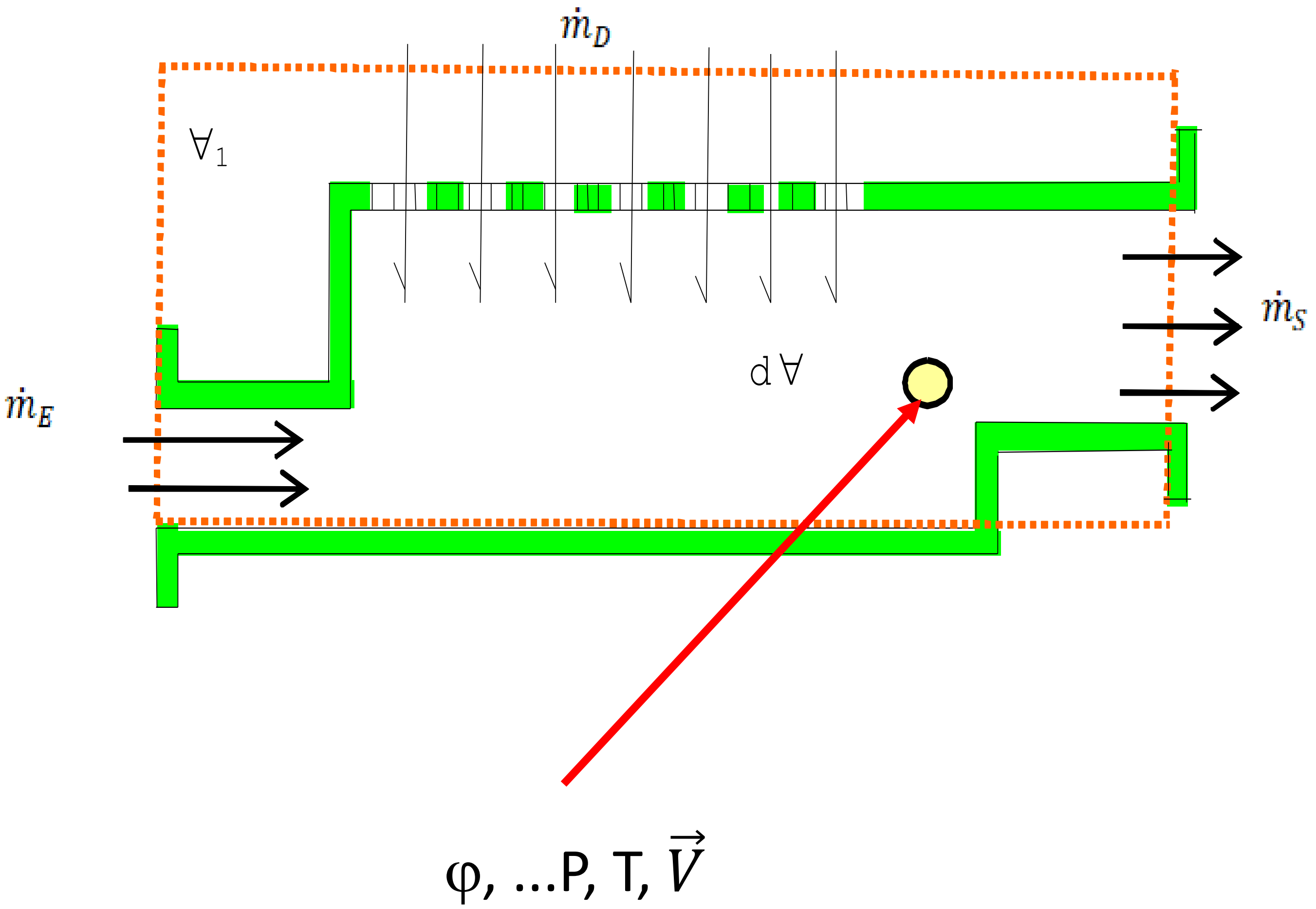
FENÔMENOS DE TRANSPORTE I
Balanço de massa diferencial

Mecânica dos Meios Contínuos – Hipótese do Contínuo

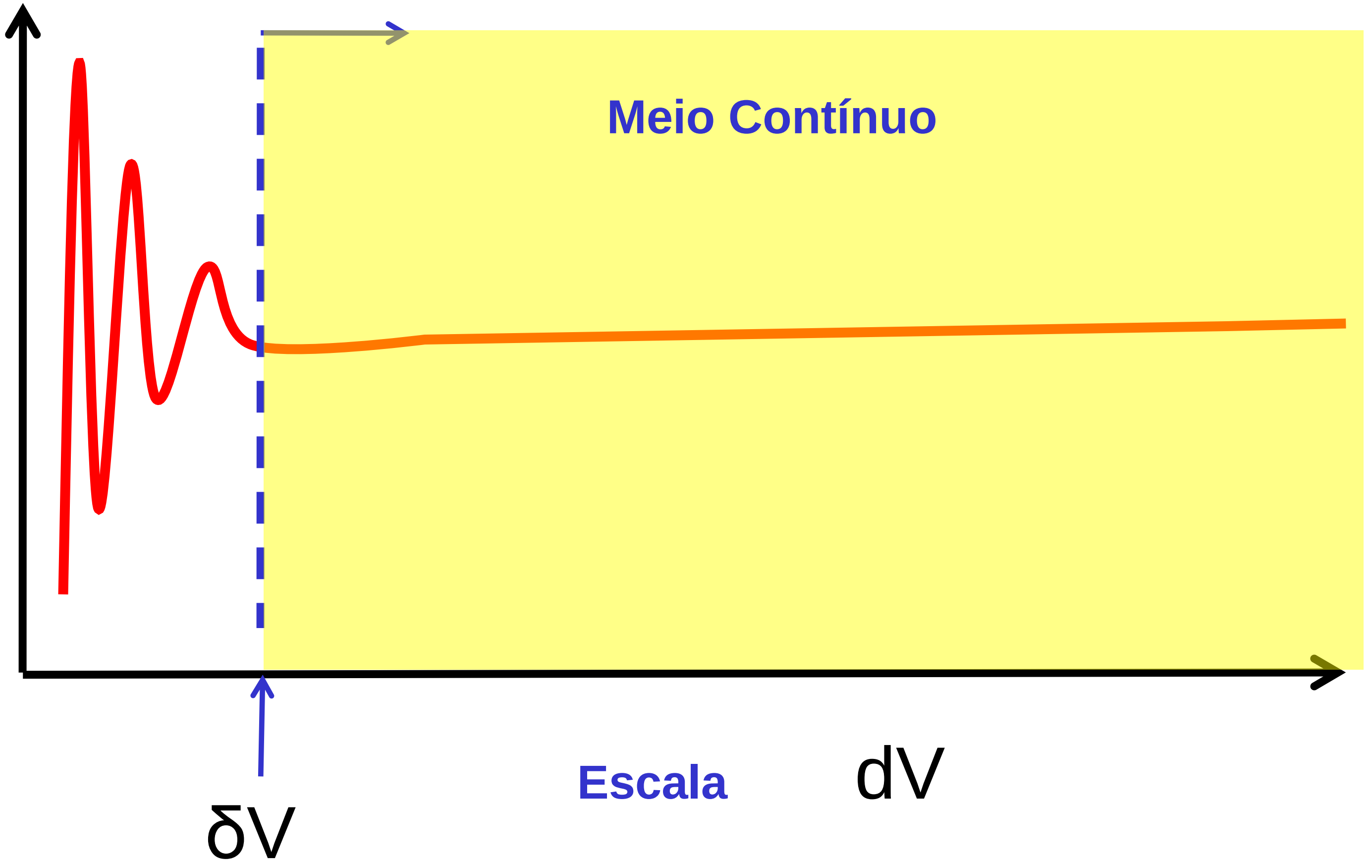


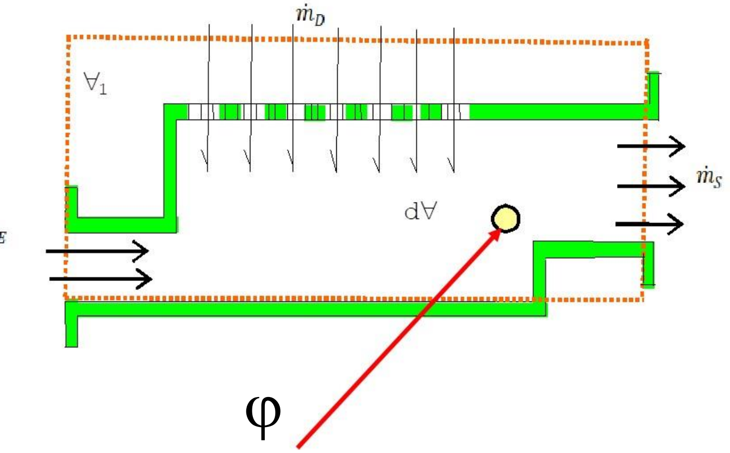
FENÔMENOS DE TRANSPORTE I

Mecânica dos Meios Contínuos – Hipótese do Contínuo

Propriedade

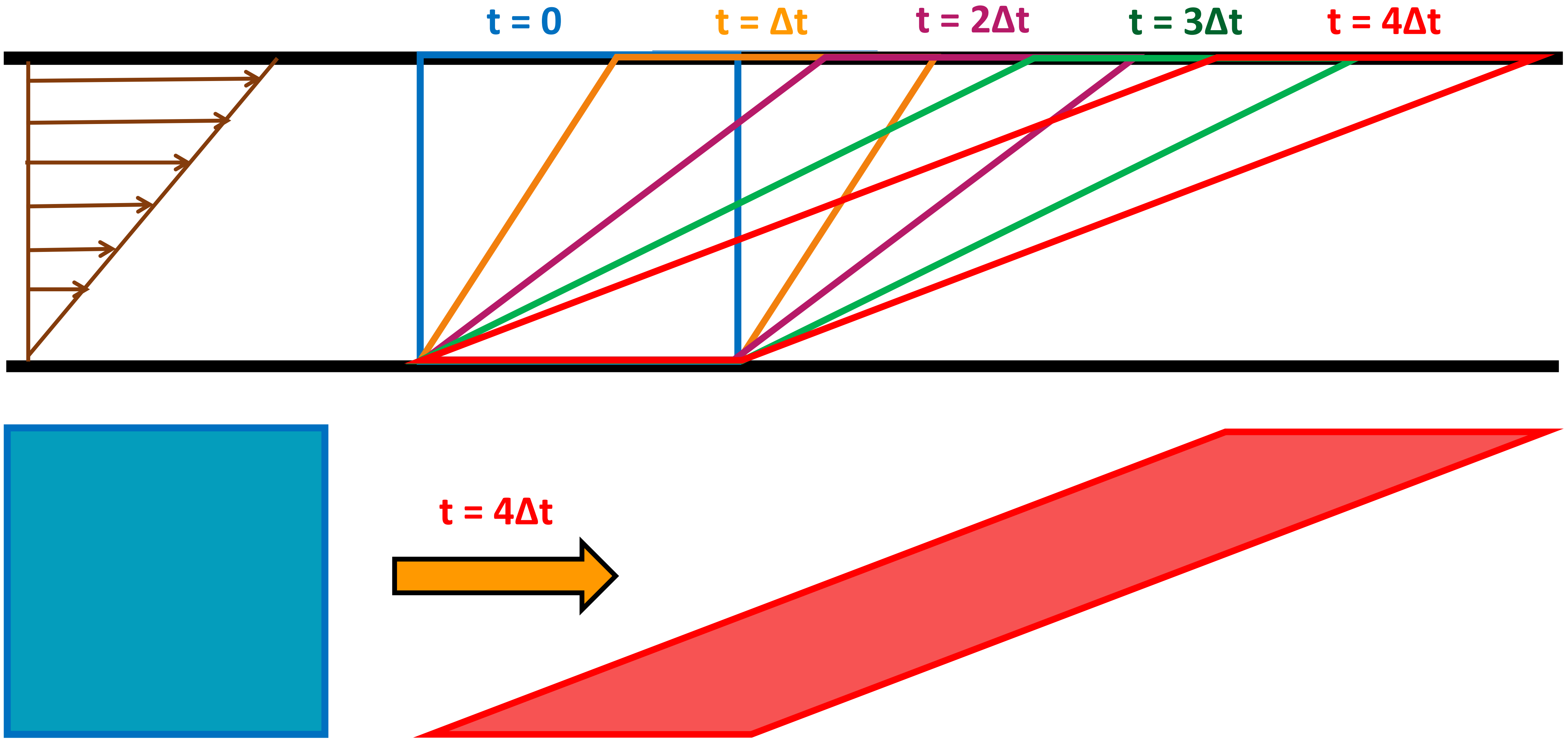
φ





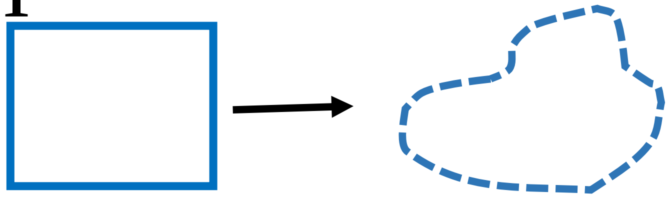
FENÔMENOS DE TRANSPORTE I

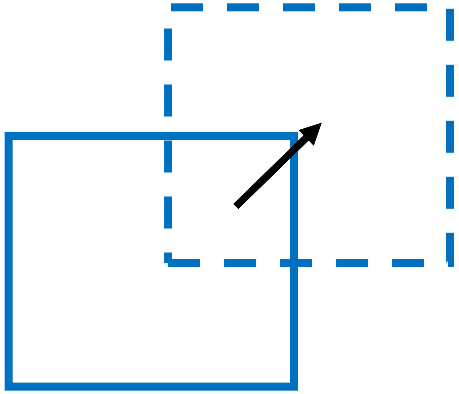
Campo de velocidades - Deformação



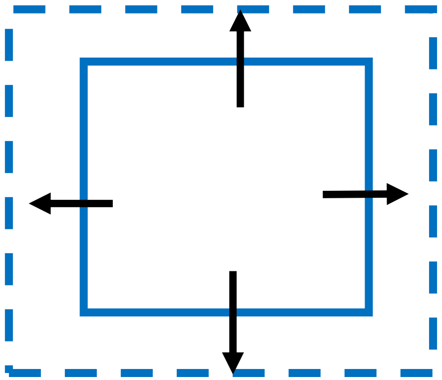
FENÔMENOS DE TRANSPORTE I

Campo de velocidades - Deformação

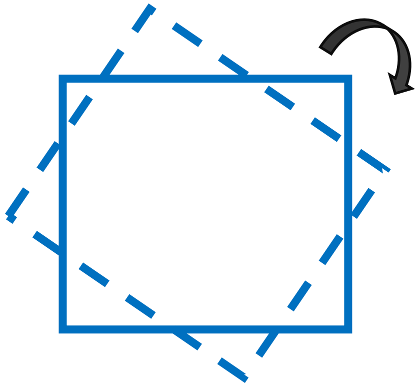




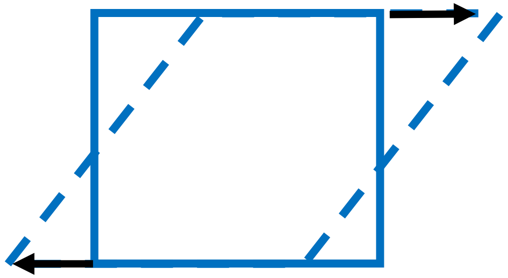
translação



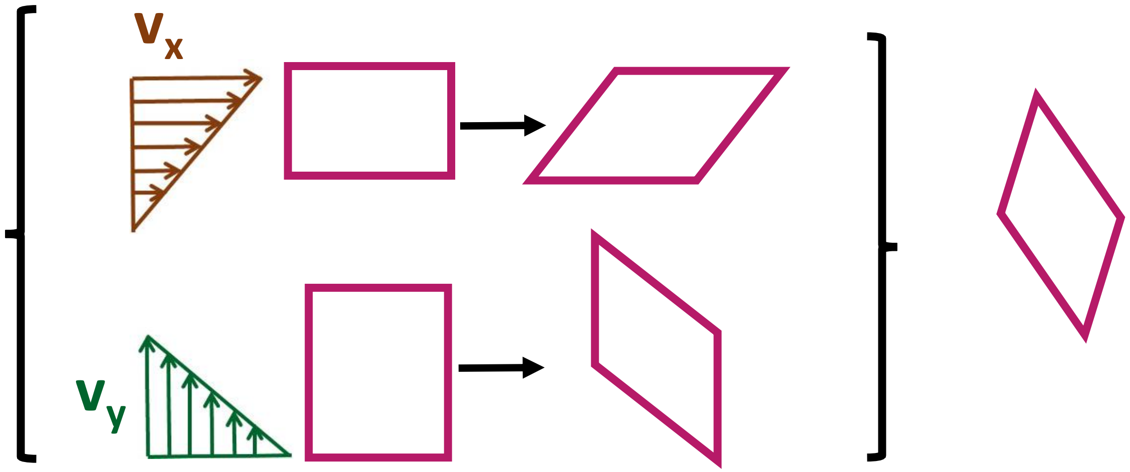
diltação/contração



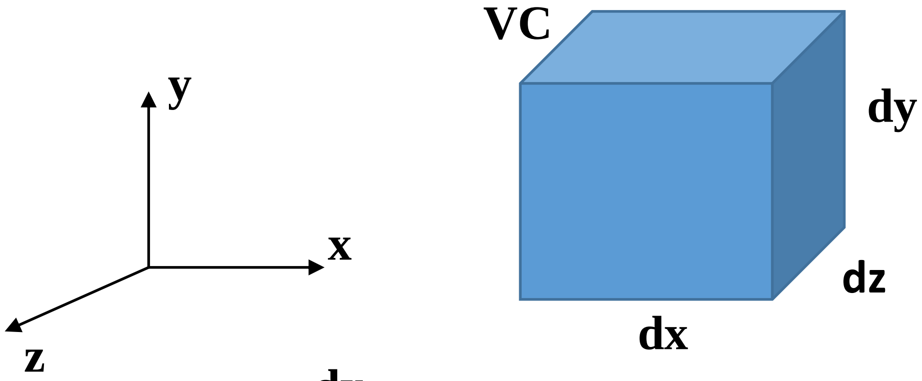
rotação



Cisalhamento (simples)

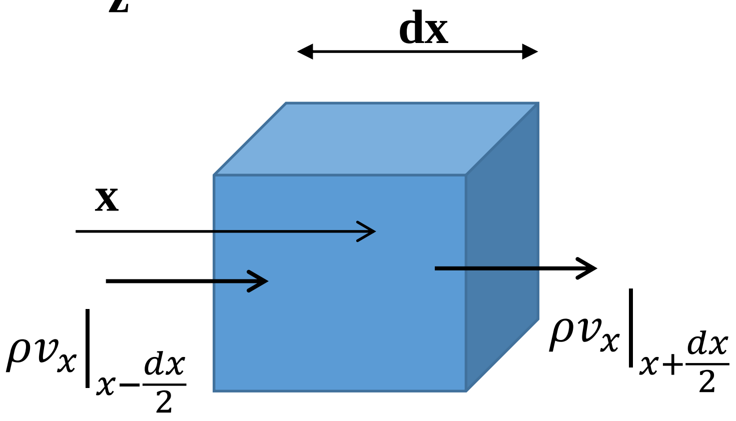


FENÔMENOS DE TRANSPORTE I
Balanço de massa microscópico



$$\left(\frac{\partial m}{\partial t}\right)_{VC} = \frac{\partial}{\partial t}\int_{VC} \rho dV = \left(\frac{\partial \rho}{\partial t}\right) dxdydz$$

$$\left(\frac{\partial m}{\partial t}\right)_{VC} = d\dot{m}_e \text{ - } d\dot{m}_s$$



$$f(x) \cong f(x_0) + \left(\frac{\partial f}{\partial x}\right)(x - x_0)$$

$$f\left(x + \frac{dx}{2}\right) \cong f(x) + \left(\frac{\partial f}{\partial x}\right)\left(x + \frac{dx}{2} - x\right)$$

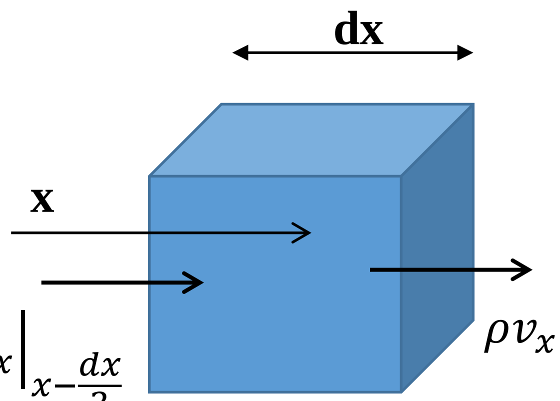
$$f\left(x + \frac{dx}{2}\right) \cong f(x) + \left(\frac{\partial f}{\partial x}\right)\left(\frac{dx}{2}\right)$$

$$\rho v_x\big|_{x-\frac{dx}{2}} = \rho v_x - \left(\frac{\partial \rho v_x}{\partial x}\right)\frac{dx}{2}$$

$$\rho v_x\big|_{x+\frac{dx}{2}} = \rho v_x + \left(\frac{\partial \rho v_x}{\partial x}\right)\frac{dx}{2}$$

FENÔMENOS DE TRANSPORTE I
Balanço de massa microscópico

$$\left(\frac{\partial m}{\partial t}\right)_{VC} = \frac{\partial}{\partial t}\int_{VC}\rho dV = \left(\frac{\partial \rho}{\partial t}\right)dxdydz \qquad \left(\frac{\partial m}{\partial t}\right)_{VC} = d\dot{m}_e - d\dot{m}_s = \left(\frac{\partial \rho}{\partial t}\right)dxdydz$$



$$\rho v_x\Big|_{x-\frac{dx}{2}} = \frac{d\dot{m}_{ex}}{dydz} \qquad \rho v_x\Big|_{x+\frac{dx}{2}} = \frac{d\dot{m}_{sx}}{dydz}$$

$$\frac{d\dot{m}_{ex} - d\dot{m}_{sx}}{dydz} = \rho v_x\Big|_{x-\frac{dx}{2}} - \rho v_x\Big|_{x+\frac{dx}{2}}$$

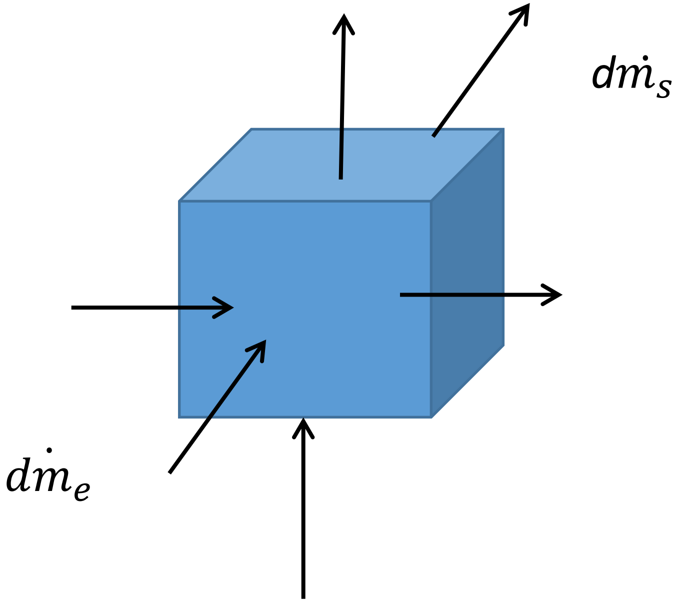
$$\frac{d\dot{m}_{ex} - d\dot{m}_{sx}}{dydz} = \rho v_x - \left(\frac{\partial \rho v_x}{\partial x}\right)\frac{dx}{2} - \rho v_x - \left(\frac{\partial \rho v_x}{\partial x}\right)\frac{dx}{2} = -\left(\frac{\partial \rho v_x}{\partial x}\right)dx$$

$$\left\{\begin{array}{l} d\dot{m}_{ex} - d\dot{m}_{sx} = \left(\frac{\partial \rho}{\partial t}\right)_x dxdydz = -\left(\frac{\partial \rho v_x}{\partial x}\right)dxdydz \\ d\dot{m}_{ey} - d\dot{m}_{sy} = \left(\frac{\partial \rho}{\partial t}\right)_y dxdydz = -\left(\frac{\partial \rho v_y}{\partial y}\right)dydxdz \\ d\dot{m}_{ez} - d\dot{m}_{sz} = \left(\frac{\partial \rho}{\partial t}\right)_z dxdydz = -\left(\frac{\partial \rho v_z}{\partial z}\right)dzdxdy \end{array}\right.$$

$$d\dot{m}_e - d\dot{m}_s = \left(\frac{\partial \rho}{\partial t}\right)dxdydz = -\left(\frac{\partial \rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y} + \frac{\partial \rho v_z}{\partial z}\right)dxdydz$$

FENÔMENOS DE TRANSPORTE I

Equação da continuidade



$$d\dot{m}_e - d\dot{m}_s = \left(\frac{\partial \rho}{\partial t}\right) dxdydz = -\left(\frac{\partial \rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y} + \frac{\partial \rho v_z}{\partial z}\right) dxdydz$$

$$\frac{\partial \rho}{\partial t} + \left(\frac{\partial \rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y} + \frac{\partial \rho v_z}{\partial z}\right) = 0$$

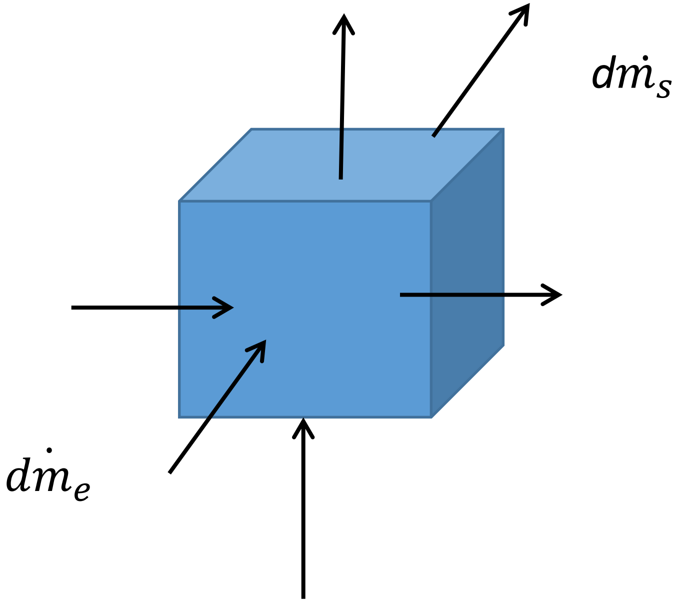
$$\frac{\partial \rho}{\partial t} + div(\rho \vec{v}) = 0$$

Equação da continuidade

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{v}) = 0$$

FENÔMENOS DE TRANSPORTE I

Equação da continuidade



$$\frac{\partial \rho}{\partial t} + \nabla.(\rho\vec{v})=0$$

$$\boxed{\frac{\partial \rho}{\partial t} + div(\rho\vec{v})=0}$$ **Euler**

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \vec{v}.\overrightarrow{grad}\rho$$ **Lagrange/Euler**

$$\frac{D\rho}{Dt} = -div(\rho\vec{v}) + \vec{v}.\overrightarrow{grad}\rho = -\rho div\vec{v} - \vec{v}.\overrightarrow{grad}\rho + \vec{v}.\overrightarrow{grad}\rho$$

$$\boxed{\frac{D\rho}{Dt} = -\rho\, div\vec{v}}$$ **Lagrange**

Escoamento incompressível: $\boxed{\frac{D\rho}{Dt} = 0 = \ div\,\vec{v}=0}$