PRO 5971

Statistical Process Monitoring: Attribute+Variable control charts

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Outline

Attribute+Variable control charts: Introduction

- The aim: Improve the performance of the attribute chart in monitoring of the process mean and/or variability
- Features:
  - The monitoring always starts using an attribute chart as it is cheaper, faster, easier, etc
  - If this chart does not signal, the process is declared in-control
  - Otherwise, depending on the proposal, the process may be declared out-of-control or measurements of quality characteristics are taken on the same inspected sample units or on another independent sample units and the decision on the process depends on the value of some quantitative statistic (like sample mean/variance of these measures) built with these measurements

Attribute+variable control charts: Introduction

CC by attributes

- Mean:
  - np
  - np_x
  - $\overline{X}_{rec}$, $\overline{X}_{att}$, $\overline{X_{tn}}$
- Variance: np_{S^2}; G_{S^2}; S^2_{tn}

CC by attribute+variable

- Mean:
  - np-$\overline{X}$
  - $np_x - \overline{X}$
  - ATTRIVAR1 and ATTRIVAR2
- Variance: MIXS^2, AVAI

PROCESS MEAN: np-Xbar (MIXED) chart proposed by Aslam et al. (2015)

- Attribute Chart:
  - The items are classified as non-conforming if they do not meet the specification limits: $Y = \#$ of non-conforming items in a sample of n
  - 2 sets of control limits: UCL_i and LCL_i, i=1, 2.
  - If $Y \in [LCL_2; UCL_2]$ - the process is in-control
  - If $Y \notin [LCL_1; UCL_1]$ - the process is out-of-control
  - Otherwise, use the VARIABLE chart
- Variable chart:
  - Measure all n items and obtain $\overline{X}$
  - If $\overline{X} \in [LCL_3; UCL_3]$ - the process is in control, otherwise out-of-control

np-Xbar chart proposed by Aslam et al. (2015)

Table 1. ARLs of the proposed control charts when $r_0 = 200$ and $p = 0.10$.

c	$n = 20$, $k_1 = 3.23$, $k_2 = 0.87$, $k_3 = 2.56$	$n = 30$, $k_1 = 3.49$, $k_2 = 0.04$, $k_3 = 2.88$	$n = 40$, $k_1 = 3.68$, $k_2 = 0.70$, $k_3 = 2.64$	$n = 50$, $k_1 = 3.50$, $k_2 = 0.29$, $k_3 = 2.81$
	ARL_1	ARL_1	ARL_1	ARL_1
0	200.00	200.00	200.00	200.00
0.1	87.11	70.65	61.72	51.60
0.2	31.94	21.15	15.53	11.53
0.3	12.64	7.58	5.09	3.76
0.4	5.74	3.39	2.31	1.80
0.5	3.06	1.91	1.43	1.22
0.6	1.93	1.34	1.12	1.04
0.7	1.42	1.11	1.03	1.00
0.8	1.183	1.02	1.00	1.00
0.9	1.074	1.00	1.00	1.00
1	1.02	1.00	1.00	1.00

Figure 1: Plans for n=20, 30 and 40, 50 with $ARL_0 = 200$ and USL=1.28

np-Xbar chart proposed by Aslam et al. (2015)

Table 7. Comparisons ARL_1 of the proposed control charts when $r_0 = 300$ and $p = 0.10$.

	$n = 40$			$n = 50$		
	Proposed control chart	Attribute chart $k = 3.19$	Variable chart $k = 2.93$	Proposed control chart	Attribute chart $k = 3.01$	Variable chart $k = 2.93$
c	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0	300.00	680.40	300.00	300.00	310.56	300.00
0.1	91.52	176.54	92.36	72.35	80.26	76.49
0.2	20.53	54.00	21.07	14.99	25.18	15.59
0.3	6.17	19.51	6.68	4.44	9.61	4.81
0.4	2.61	8.33	2.91	1.97	4.45	2.18
0.5	1.52	4.20	1.69	1.27	2.49	1.37
0.6	1.16	2.48	1.24	1.06	1.65	1.10
0.7	1.04	1.69	1.07	1.01	1.27	1.02
0.8	1.00	1.31	1.01	1.00	1.10	1.00
0.9	1.00	1.13	1.00	1.00	1.03	1.00
1	1.00	1.04	1.00	1.00	1.00	1.00

Figure 2: Comparing with *np* and $\overline{X}$ charts

np-Xbar chart proposed by Aslam et al. (2015)

- For each sample n=30

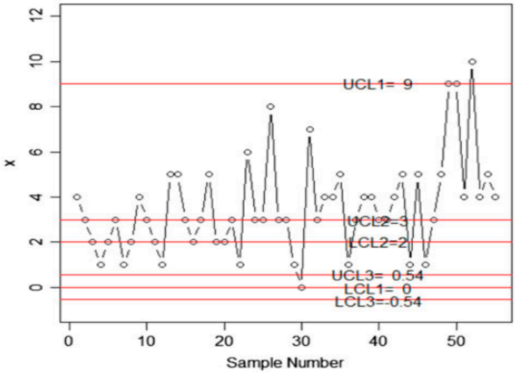


Figure 3: Example of chart proposed by Aslam et al. (2015)

Weakness of Aslam et al. (2015)'s proposal

Although the good performance of this proposal, one point is not clear: the measurements are made on same units or on new independent sample units?

To get the results shown in Aslam et al. (2015), the authors have necessarily assumed a new sample (thus the quality characteristic follows a normal distribution), otherwise the quality characteristic follows a truncated normal distribution if the measurement is made on the same units.

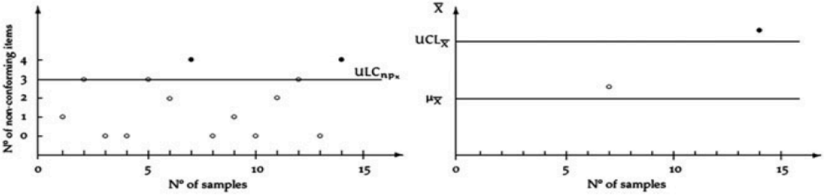
In terms of average sampling cost, Aslam et al. (2015)'s proposal certainly leads to an increase in the average sampling cost (this aspect is discussed in their paper).

A new paper considering measurement on the same units is the contribution of Leoni & Costa (2019)

PROCESS MEAN: The combined $np_x - \overline{X}$ chart by Sampaio et al. (2013)

- Use a two stages sampling scheme
- The sample of size n is split into two sub-samples: n_{np_x} e $n_{\overline{X}} = n - n_{np_x}$
- Phase 1: np_x chart is built plotting the results of the classification (using a gauge go-no go) of items of the first sub-sample
- Phase 2: If np_x chart signals, then values of the quality characteristic are taken from the units of second sub-sample and $\overline{X}$ is calculated
- If $\overline{X}$ also signals, then the process is declared out-of-control. An adjustment on the process is required, otherwise, the process goes on

The $np_x - \overline{X}$ chart by Sampaio et al. (2013)



The combined $np_x - \overline{X}$ control chart

Figure 4: Combined $np_x - \overline{X}$ chart

The $np_x - \overline{X}$ chart by Sampaio et al. (2013)

- Parameters of the combined $np_x - \overline{X}$ chart:
  - np_x chart: warning limits of the device (LWL;UWL), control limit (UCL_{np_x})
  - $\overline{X}$ chart: Control limits:$LCL_{\overline{X}}$ and $UCL_{\overline{X}}$
- Objective Function:

$$LWL^o; UWL^o; UCL^o_{np_x}; UCL^o_{\overline{X}} = argmin\left(ARL_1 = \frac{1}{1-\beta}\right)$$

$$\text{subject to } ARL_0 = \frac{1}{\alpha} = \tau$$

The $np_x - \overline{X}$ chart by Sampaio et al. (2013)

ATTENTION: Expression considering only increases in the process mean from μ_o to μ_1 as *np_x* has good performance for one-sided shift; LWL set at $-\infty$, UWL is searched. Fixed α

$$\alpha = P(Y > UCL_{np_x} | n_{np_x}, p_0 \cap \overline{X} > UCL_{\overline{X}} | \mu_0, n_{\overline{X}})$$

As these events are independent, then

$$\alpha = \alpha_{np_x} \times \alpha_{\overline{X}}$$

$p_0 = P(X > \mu_0 + k_w \sigma | \mu_0)$, the probability of an item be disapproved.

The power can be obtained as

$$\beta = 1 - P(Y > UCL_{np_x} | n_{np_x}, p_1) \times P(\overline{X} > UCL_{\overline{X}} | \mu_1 = \mu_0 + \delta\sigma, n_{\overline{X}})$$

where $p_1 = P(X > \mu_0 + k_w \sigma | \mu_1 = \mu_0 + \delta\sigma)$,

The $np_x - \overline{X}$ chart by Sampaio et al. (2013)

- Some plans - with ARL_0=370 and shift δ=0.25

Table 1: Some plans of the combined $np_x - \overline{X}$ chart : ARL_0=370 and $\delta = 0.25$

$\overline{X}$		np_x		Combined np_x-$\overline{X}$			
n	ATS	n	ATS	n_1	n_2	ASS	ATS
3	106	5	93.95	2	2	2.12	102
				3	2	3.06	93.4
4	88.4	6	84.86	2	3	2.32	87.5
				4	2	4.05	83.9
				3	3	3.27	80.8
				4	3	4.16	73.9
5	75.8	8	70.27	4	3	4.16	73.9
				3	4	3.54	70.9
				5	3	5.1	68.7
				4	4	4.3	65.7
				5	4	5.21	61.7
6	66.1	9	64.91	4	4	4.3	65.7
				3	5	3.9	62.9
				6	3	6.09	63.8
				5	4	5.21	61.7
				4	5	4.54	58.9
				6	4	6.19	57.5
				5	5	5.45	55.7
				6	5	6.33	52.2

The $np_x - \overline{X}$ chart by Sampaio et al. (2013)

- Some plans - comparing AIC, unit cost equal 5.00 for attribute and variable inspection

Table 2: Some plans of the combined $np_x - \overline{X}$ chart

$\overline{X}$		np_x		Combined np_x-$\overline{X}$			
n	AIC	n	AIC	n_1	n_2	ASS	AIC
3	15	5	25	2	2	2.12	10.60
				3	2	3.06	18.40
4	20	6	30	2	3	2.32	11.62
				4	2	4.05	20.27
				3	3	3.27	16.35
				4	3	4.16	20.81
5	25	8	40	4	3	4.16	20.81
				3	4	3.54	17.70
				5	3	5.10	25.54
				4	4	4.30	21.54
				5	4	5.21	26.08
6	30	9	45	4	4	4.30	21.54
				3	5	3.90	16.92
				6	3	6.09	30.45
				5	4	5.21	26.08
				4	5	4.54	27.70
				6	4	6.19	30.98
				5	5	5.45	27.25
				6	5	6.33	31.70

PROCESS MEAN: ATTRIVAR charts by Ho & Aparisi (2016)

- Features of ATTRIVAR chart:
  - Development of a chart with a similar performance of $\overline{X}$ chart
  - To have a good performance for bilateral shift of mean
  - Low operational cost
  - 2 stages of inspection: by attributes and variables
  - The most of times the decision is taken considering only the results of the attribute inspection
  - A restriction for the proportion of times of variable inspection is included

ATTRIVAR charts by Ho & Aparisi (2016) - 2 possibilities

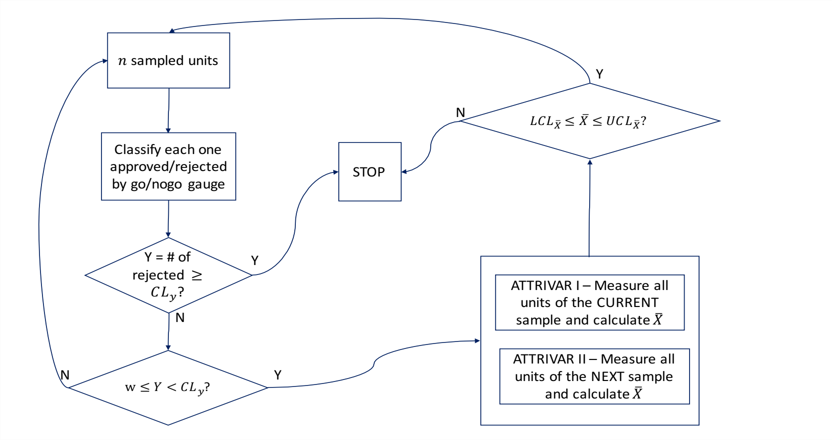


Figure 5: Inspection Procedure

ATTRIVAR charts proposed by Ho & Aparisi (2016)

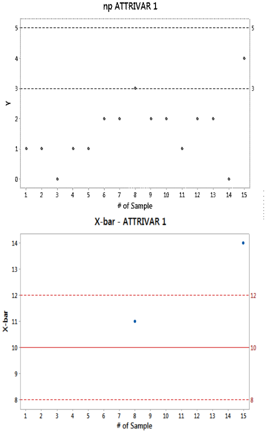


Figure 6: Example - ATTRIVAR1

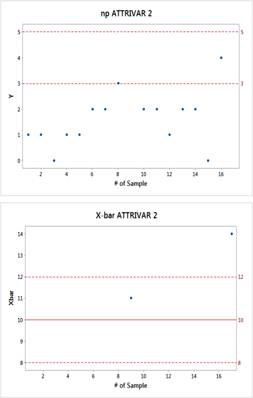


Figure 7: Example - ATTRIVAR2

ATTRIVAR charts by Ho & Aparisi (2016)

- Parameters of the chart:
  - Device: Discriminant limits LDL, UDL
  - Attribute chart: Control limit CL_y; Warning limit w
  - Variable chart: Control limits $UCL_{\overline{X}}$ and $LCL_{\overline{X}}$
- Restrictions:
  - $ARL_0 = \tau$
  - % of time that measurements of X is taken when the process is in-control $\%\overline{X}_{Max} \leq \pi$
- Objective Function: $(LDL^o, UDL^o, CL_y^o, w^o, UCL_{\overline{X}}^o, LCL_{\overline{X}}^o) = arg\ min\ (\ ARL_1)$
- Parameters searched by the genetic algorithm due to the complexity and high dimension

Chart A optimized for $\delta* = 0.5$, and Chart B for $\delta* = 2$, the in-control ARL = 370, n = 5, and $\bar{X}_{max} = 15\%$.

Table 3: Some plans - ATTRIVAR2

Shift ($\delta*$)	0	0.2	0.5	0.7	1	1.5	2	2.5	3
Chart A, $\delta* = 0.5$	370	206.7	43.5	17.6	6.4	2.7	2.1	1.9	1.6
Chart B, $\delta* = 2$	370	278.3	84.7	32.9	9.5	2.6	1.5	1.1	1

ATTRIVAR chart optimized for a small shift, its performance for a large shift is not good compared to the chart optimized for a large shift.

The same behavior occurs in the opposite case.

Therefore, the final user should decide which size shift is more critical for the process, for example, taking into account the process capability.

ATTRIVAR charts by Ho & Aparisi (2016)

Table 4: ATTRIVAR versus $\overline{X}$, n=3, 10 and $\%\overline{X}_{Max} = 25; 15; 5$

Sample size $n = 3$					Sample size $n = 10$				
$\%\overline{X}_{Max}$	δ	$\overline{X}$	ATTRIVAR 1	ATTRIVAR 2	$\%\overline{X}_{Max}$	δ	$\overline{X}$	ATTRIVAR 1	ATTRIVAR 2
25	0.25	184.06	189.84 (3.1%)	209.53 (13.8%)	25	0.25	73.21	79.55 (8.7%)	92.39 (26.2%)
25	0.5	60.64	61.29 (1.1%)	72.39 (19.4%)	25	0.5	12.82	14.18 (10.6%)	18.11 (41.3%)
25	1	9.76	10.19 (4.4%)	11.49 (17.7%)	25	1	1.77	1.99 (12.4%)	3.04 (71.8%)
25	1.25	4.95	5.16 (4.2%)	6.06 (22.4%)	25	1.25	1.21	1.36 (12.4%)	1.99 (64.5%)
25	1.5	2.91	3 (3.1%)	3.78 (29.9%)	25	1.5	1.04	1.06 (1.9%)	1.63 (56.7%)
25	2	1.47	1.51 (2.7%)	1.98 (34.7%)	25	2	1	1.02 (2%)	1.09 (9%)
15	0.25	184.06	189.84 (3.1%)	209.53 (13.8%)	15	0.25	73.21	77.82 (6.3%)	99.76 (36.3%)
15	0.5	60.64	61.29 (1.1%)	72.39 (19.4%)	15	0.5	12.82	14.31 (11.6%)	20.28 (58.2%)
15	1	9.76	10.19 (4.4%)	11.49 (17.7%)	15	1	1.77	2.01 (13.6%)	3.11 (75.7%)
15	1.25	4.95	5.2 (5.1%)	6.06 (22.4%)	15	1.25	1.21	1.36 (12.4%)	2.07 (71.1%)
15	1.5	2.91	3.03 (4.1%)	3.78 (29.9%)	15	1.5	1.04	1.08 (3.8%)	1.63 (56.7%)
15	2	1.47	1.61 (9.5%)	1.98 (34.7%)	15	2	1	1 (0%)	1.09 (9%)
5	0.25	184.06	192.85 (4.8%)	219.49 (19.2%)	5	0.25	73.21	89.55 (22.3%)	119.62 (63.4%)
5	0.5	60.64	64.78 (6.8%)	78.1 (28.8%)	5	0.5	12.82	19.27 (50.3%)	24.77 (93.2%)
5	1	9.76	11.65 (19.4%)	12.38 (26.8%)	5	1	1.77	2.66 (50.3%)	3.65 (106.2%)
5	1.25	4.95	5.83 (17.8%)	6.49 (31.1%)	5	1.25	1.21	1.76 (45.5%)	2.36 (95%)
5	1.5	2.91	3.36 (15.5%)	3.87 (33%)	5	1.5	1.04	1.3 (25%)	1.66 (59.6%)
5	2	1.47	1.73 (17.7%)	2.07 (40.8%)	5	2	1	1.04 (4%)	1.16 (16%)

ATTRIVAR charts by Ho & Aparisi (2016)- comparing costs

Comparison: Inspection Cost for variables = $10\times$ cost for attributes

Table 5: Ratio of the average inspection costs - ATTRIVAR versus $np_x - \overline{X}$

	np_x-$\overline{X}$				ATRIVAR 1 -Ratio		ATTRIVAR 2 - Ratio	
Cases	n_1	n_2	ASS	$\%\overline{X}$	$\%\overline{X}_{max} = 5$	$\%\overline{X}_{max} = 15$	$\%\overline{X}_{max} = 5$	$\%\overline{X}_{max} = 15$
$\%\delta^* = 0.25$; n=3					$\%\overline{X} = 3.64$	$\%\overline{X} = 7.74$	$\%\overline{X} = 2.38$	$\%\overline{X} = 2.38$
	2	3	2.32	10.7	61.71	91	86.44	84.16
	3	2	3.06	0.3	123.43	182	172.88	168.33
$\%\delta^* = 1.0$; n=5					$\%\overline{X} = 3.93$	$\%\overline{X} = 7.18$	$\%\overline{X} = 4.47$	$\%\overline{X} = 13.98$
	4	5	4.54	10.8	60.34	82.03	62.48	223.52
	5	4	5.21	5.3	97.07	131.96	100.51	198.71

ATTRIVAR charts by Ho & Aparisi (2016) - Comparing costs

Comparison: Inspection Cost for variables $= 10\times$ cost for attributes and $n = 20$

% of times for variable inspection in Aslam et al. (2015)'s proposal is always higher than ATTRIVAR 1 and ATTRIVAR 2

Average cost of Aslam et al. (2015)'s proposal is always more expensive than ATTRIVAR

Table 6: Ratio of average inspection costs - ATTRIVAR versus Aslam et al. (2015)'s proposal

			0.2	0.4	0.6	1.0
Aslam et al. (2015)		$\%\overline{X}$	71.44	71.44	71.44	71.44
		ARL_1	53.31	8.05	2.34	1.03
ATTRIVAR 1	$\%\overline{X}_{Max} = 25\%$	$\%\overline{X}$	10.57	19.43	10.44	11.08
		ARL_1	56.97	10.28	3.78	1.28
		Ratio	20.37	31.96	20.20	21.04
	$\%\overline{X}_{Max} = 15\%$	$\%\overline{X}$	10.57	10.61	10.66	10.78
		ARL_1	56.97	11.89	4.42	1.35
		Ratio	20.37	20.42	20.49	20.64
	$\%\overline{X}_{Max} = 5\%$	$\%\overline{X}$	0.85	1.58	3.34	3.29
		ARL_1	145.91	30.06	6.34	1.90
		Ratio	7.65	8.61	10.91	10.85
ATTRIVAR 2	$\%\overline{X}_{Max} = 25\%$	$\%\overline{X}$	19.43	19.43	19.43	24.43
		ARL_1	79.23	17.43	4.84	1.90
		Ratio	30.69	30.69	30.69	36.90
	$\%\overline{X}_{Max} = 15\%$	$\%\overline{X}$	10.66	10.65	10.66	12.04
		ARL_1	89.16	17.45	5.46	2.00
		Ratio	19.79	19.78	19.79	21.50
	$\%\overline{X}_{Max} = 5\%$	$\%\overline{X}$	3.67	3.66	4.28	3.61
		ARL_1	119.63	24.40	7.68	2.53
		Ratio	11.10	11.09	11.86	11.03

PROCESS VARIANCE: MIX S^2 chart: Introduction

- The aim: to develop a chart to monitor the variability with a better performance than np_{S^2} chart
- MIX S^2 chart: Inspection in two stages
- Stage 1: Attribute np_{S^2} chart is used
- Stage 2: If np_{S^2} chart signals, the variable S^2 chart is built
- Decision Criterion: If both charts signal, the process is said out-of-control

MIX S^2 chart by Ho & Quinino (2016)

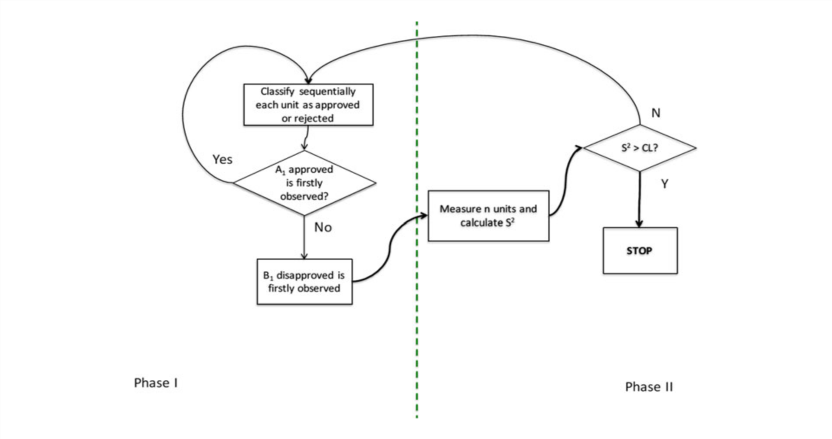


Figure 8: Inspection Procedure

MIX S^2 chart by Ho & Quinino (2016)

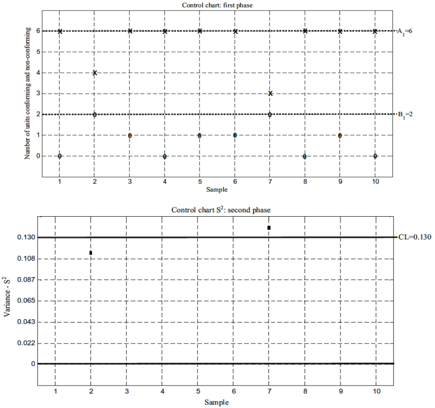


Figure 9: An example of MIX S^2 chart

MIX S^2 chart by Ho & Quinino (2016)

To declare out of control: b items firstly classified as disapproved and Sample Variance larger than control limit, fixed α

$$\alpha = P\left(\sum_{y=b}^{a+b-1} \binom{y-1}{b-1} p_0^b (1-p_0)^{y-b} \cap S^2 > CL \middle| \sigma = \sigma_0\right)$$

p_0, probability to be classified as disapproved according to the UWL and LWL.

As these events are independent

$$\alpha = P\left(\sum_{y=b}^{a+b-1} \binom{y-1}{b-1} p_0^b (1-p_0)^{y-b}\right) P\left(S^2 > CL \middle| \sigma = \sigma_0\right) = \alpha_1 \alpha_2$$

The type II error β: np_{S^2} signals but S^2 chart not or np_{S^2} does not signal

$$\beta = P\left(\sum_{y=b}^{a+b-1} \binom{y-1}{b-1} p_1^b (1-p_1)^{y-b}\right) P\left(S^2 > CL \middle| \sigma = \sigma_1\right) +$$

$$+ P\left(\sum_{y=a}^{a+b-1} \binom{y-1}{a-1} p_1^{y-a} (1-p_1)^a\right)$$

MIX S^2 chart proposed by Ho & Quinino (2016)

- Parameters of the chart: # of approved items a; # of disapproved items b; Discriminating limits UDL and LDL
- Objective Function:

$$(a^o, b^o, UDL^o, LDL^o) = argminARL_1(a, b, UDL, LDL)$$

subject to:

$$ARL_0 = \frac{1}{\alpha} = 370$$

$$ARL_1^{MixS^2} < ARL_1^{S^2}$$

$$\alpha_2 = \frac{\alpha}{\alpha_1}$$

$$ASC^{MixS^2} < ASC^{S^2}$$

ASC=average sampling cost; α_1 and α_2, errors of type I of charts at stages 1 and 2; values of α, α_1 and α_2 in the interval [0; 1]

MIX S^2 chart proposed by Ho & Quinino (2016)

Table 7: Examples of plans of MIX S^2 chart

Plans	a	b	LDL	UDL	α_1	α_2	ARL_1	ANI_0	ANI_1	n	Cost
A	20	10	8.49	11.51	0.0028	0.9763	2.391	23,011	26.547	2	$29.12
B	14	4	8.00	12.00	0.0063	0.4266	6.217	14.654	16.229	4	$20.75
C	5	2	7.72	12.28	0.0072	0.3741	8.889	5.1126	5.5745	2	$ 6.17
D	6	2	7.48	12.52	0.0028	0.9708	7.773	6.0692	6.4574	2	$7.25
E	13	9	8.59	11.41	0.0300	0.9025	3.891	15.449	19.133	3	$21.70

n= sample size for S^2 chart;

shift $\delta = 1.5$;

attribute and variable inspection costs: $1.00 and $4.00

MIX S^2 chart proposed by Ho & Quinino (2016)

Table 8: Best plans of MIX S^2 chart

δ	a	b	LDC	UDC	α_1	α_2	LC	ARL_1	ANI_0	ANI_1	n	ARL_1 of S^2	Max ASC Mix S^2
1.1	6	3	7.93	12.07	0.00280	0.9810	0.0006	106.616	6.2388	6.3772	2	106.9340	$6.434
1.3	6	3	7.93	12.07	0.00280	0.9810	0.0006	20.293	6.2388	6.7128	2	21.0908	$7.014
1.5	6	2	7.48	12.52	0.00278	0.9708	0.0013	7.773	6.0692	6.4574	2	8.02752	$7.253
1.7	6	2	7.48	12.52	0.00278	0.9708	0.0013	4.108	7.9959	7.5864	2	4.36487	$8.091
2.0	7	1	6.46	13.54	0.00280	0.9651	0.0019	2.420	6.9972	6.4359	2	2.5153	$9.000

n=5 for S^2 chart

MIX S^2 chart proposed by Ho & Quinino (2016)

Table 9: Comparing the plans MIX S^2 chart with its main competitor - economical scenario:

c_a	c_m	a	b	MIX S^2	S^2
1	4	7	1	9	20
1	3.5	7	1	8.58	17.5
1	3	7	1	8.15	15
1	2.5	7	1	7.72	12.5
1	2	7	1	7.29	10
1	1.5	7	1	7	7.5
1	1.45	7	1	7	7.25
1	1.4	7	1	6.999	7
1	1.35	6	2	6.963	6.75
1	1.25	6	2	6.87	6.25

c_a=inspection cost by attribute; c_m=measurement cost

Alternating variable attribute inspections (AVAI) was proposed by Silva et al. (2022)

It is a mixed procedure which employs attribute and variable control charts:

- Step 1: Classify n_a items according to the warning limit z_a and obtain Y, (# of items which quality characteristic value is higher than z_a).
- Step 2: If $Y > UCL_{n_a}$ then the process is declared out-of-control, adjusted and to go step 1, otherwise to go to step 3;
- Step 3: Take a sample of size $n_b < n_a$ and calculate $\chi_b^2 = \frac{(n_b - 1)S_{n_b}^2}{\sigma_0^2}$, $S_{n_b}^2$, its sample variance. If $\chi_b^2 > UCL_{n_b}$, the process is declared out-of-control, adjusted and go to step 1;
- Step 4: Go to step 1.

AVAI - Comparing

Table 10: *ARL* values of control charts: S^2 versus $np_{x(I)}^{\sigma^2}$ versus $np_x - S^2$ mixed control chart - $n_{S^2} = 3, 4$.

	ARL_1					
δ	S^2	$np_{x(I)}^{\sigma^2}$	P1: $np_x - S^2$	S^2	$np_{x(I)}^{\sigma^2}$	P2: $np_x - S^2$
1.0	370.398	370.367	370.299	370.398	370.544	370.392
1.1	132.699	131.388	131.415	117.832	112.876	116.938
1.2	60.785	59.186	59.195	49.85	46.145	48.692
1.3	33.107	31.620	31.626	25.748	23.261	24.719
1.4	20.443	19.149	19.155	15.354	13.65	14.509
1.5	13.856	12.744	12.752	10.178	8.964	9.492
1.6	10.078	9.120	9.13	7.307	6.409	6.744
1.7	7.741	6.907	6.921	5.574	4.891	5.108
1.8	6.206	5.472	5.489	4.459	3.928	4.067
1.9	5.147	4.495	4.517	3.703	3.284	3.37
2.0	4.387	3.803	3.829	3.167	2.835	2.883
Sample Size	$n_{S^2} = 3$	$ASS = 3.002$ $n_a = 4$ $n_b = 2$	$ASS = 3.503$ $n_a = 5$ $n_b = 2$	$n_{S^2} = 4$	$ASS = 5.010$ $n_a = 9$ $n_b = 1$	$ASS = 5.004$ $n_a = 7$ $n_b = 3$
Estimated Inspection Cost	$c_{S^2} = 12$	$AIC = 3.00$	$AIC = 6.51$	$c_{S^2} = 16$	$AIC = 5.01$	$AIC = 9.51$
UCL	$UCL_{S^2} = 5.915$	$UCL_{n_a} = 0$ $UCL_{n_b} = 0$	$UCL_{n_a} = 0$ $UCL_{n_b} = 11.022$	$UCL_{S^2} = 4.719$	$UCL_{n_a} = 2$ $UCL_{n_b} = 0$	$UCL_{n_a} = 1$ $UCL_{n_b} = 6.729$
Discriminant limit	-	$z_a = 3.325$ $z_b = 3.31$	$z_a = 3.32$	-	$z_a = 2.04$ $z_b = 3.5$	$z_a = 2.445$

Themes for Seminars: Attribute CC to monitor mean and variance

- Some contributors of attribute CC to monitor process mean and variability
  - Steiner, S., Geyer, P., and Wesolowsky, G. (1994). Control charts based on grouped data.International Journal of Production Research, 32(1):75–91.
  - Steiner, S., Geyer, P., and Wesolowsky, G. (1996). Shewhart control charts to detect mean and standard deviation shifts based on grouped data. Quality and reliability engineering international, 12:345–353.
  - Stevens, W. L. (1948). Control by gauging. Journal of the Royal Statistical Society, 10(1):54–108.
  - Wu, Z. and Jiao, J. (2008). A control chart for monitoring process mean based on attribute inspection. International Journal of Production Research, 46:4331–4337.
  - Wu, Z., Khoo, M., Shu, L., and Jiang, W. (2009). An np control chart for monitoring the mean of a variable based on an attribute inspection. International Journal of Production Economics, 121:141–147
- List the common points, differences, weakness and strength of these papers

Themes for Seminars

- Find other contributions in the literature related to mix control charts to improve the standard control charts as $\overline{X}$ or S^2 or R
- How to plan attribute or mixed chart to have equal performance (in term of ARL, Sampling Cost) of a "pure" variable chart?

References

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Ho, L. L. & Quinino, R. C. (2016), 'Combining attribute and variable data to monitor process variability: MIX s^2 control chart', *Int J Adv Manuf Technol* **87**. DOI 10.1007/s00170-016-8702-5.

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Silva, L. A., Ho, L. L. & Quinino, R. C. (2022), A mixed control chart to monitor the process variance: A new case using alternating inspections, *in* 'LIV Simpósio Brasileiro de Pesquisa Operacional', SBPO.