PRO 5971

Statistical Process Monitoring
Attribute charts to monitor a process variance

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Outline

Preliminaries

Some attribute control charts to monitor a variance will be presented in next slides.

Most of them are extensions of those presented to monitor the process mean.

Some attribute control charts: np_{S^2}, G_{S^2}, $np_x^{\sigma^2}$, $np_{x(I)}^{\sigma^2}$, S_{tn}^2, $S_{tn(I)}^2$, $S_{tn(K)}^2$

The np_{s^2} control chart by Ho & Quinino (2013)

- The main idea: a direct extension of np_x chart to monitor the variability of the process but it reveals to have a poor performance
- Alternative Procedure:
  - Classify sequentially the items as approved or disapproved using a gauge until observing a approved items or b disapproved items
  - If a approved items are observed first, then the process is declared in-control
  - If b disapproved items are observed first, then the process is declared out-of-control

np_{s^2} chart proposed by Ho & Quinino (2013)

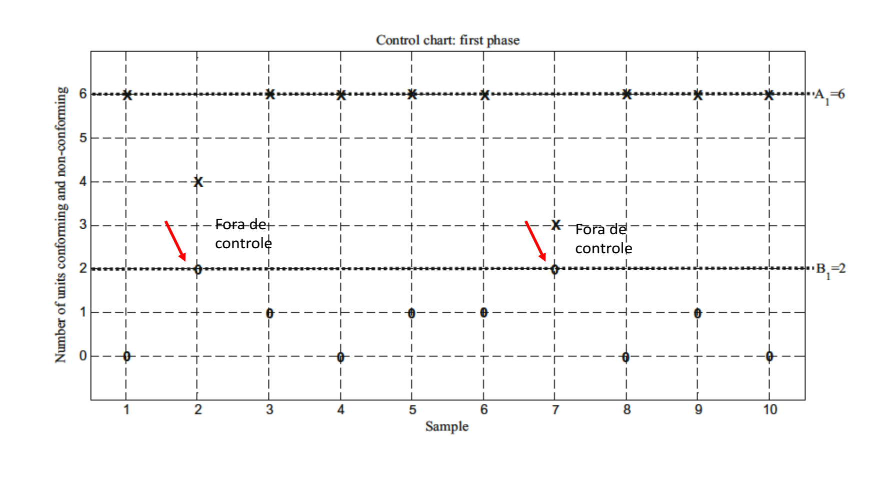

Figure 1: np_{s^2} chart- an example

np_{s^2} chart by Ho & Quinino (2013)

Let D, a binary variable with $D = 1$ if a approved is observed first and $D = 0$ if b disapproved is observed first;

Decision Rule: whenever $D = 0$, the process is judged as out-of-control;

Let p_0, the probability to be $\in$ [LWL, UWL] when the process is in-control

$$\alpha = P(D = 0) = 1 - \sum_{x=a}^{a+b-1} \binom{x-1}{a-1} p_0^a (1 - p_0)^{x-a}$$

And p_1, the probability to be $\in$ [LWL, UWL] when the process is out-of-control, β

$$\beta = P(D = 1) = \sum_{x=a}^{a+b-1} \binom{x-1}{a-1} p_1^a (1 - p_1)^{x-a}$$

np_{s^2} chart by Ho & Quinino (2013)

- Fixed a and b, it is not necessary inspect until $n = a + b - 1$ to take the decision
- The maximum inspected items is $(n = a + b - 1)$
- However, the number of inspection until to take a decision is a random variable
- Fixed α, some values of a, b, LWL/LDL and UWL/UDL may satisfy the restriction $ARL_0 = \tau$

Table 1: np_{s^2} chart - plans with $a+b-1 = 7$ and shift $\sigma_1 = 1.5 \times \sigma_0$

a	b	UDL	LDL	ARL_1
6	1	13.51	6.49	9.05
5	2	12.47	7.53	8.73
4	3	11.93	8.07	10.38
3	4	11.55	8.45	13.81
2	5	11.22	8.78	20.33
1	6	10.89	9.11	35.08

np_{s^2} chart by Ho & Quinino (2013)

- Objective Function:
- $(a^*, b^*, LDL^*, UDL^*) =$ $arg\ min[ARL_1(a; b; LDL; UDL; n; \delta]$ subject to

$$\begin{cases} a + b - 1 = n \\ ARL_0(a; b; LDL; UDL; n; \delta) = \tau \end{cases}$$

Table 2: Plans varying shift sizes δ; n denotes # average sample size; UDL=|LDL|

n	δ	a	b	UDL	ARL_1
6	1.1	5	2	2.47	114.3
	1.2	5	2	2.47	46.66
	1.5	5	2	2.47	8.72
	2	6	1	3.51	2.56
30	1.1	25	10	1.98	53.18
	1.2	25	9	1.98	13.93
	1.5	25	8	1.98	1.99
	2	26	7	2.14	1.05
50	1.1	41	10	1.8	35.84
	1.2	49	9	1.88	7.99
	1.5	43	8	1.97	1.33
	2	44	7	2.08	1

np_{s^2} chart proposed by Ho & Quinino (2013)

- Comparing the performance among np_{s^2}, S^2 and R:
  - If equal sample size is used in the 3 charts, np_{s^2} chart will not have good performance in terms of ARL
  - To make np_{s^2} chart competitive in terms of ARL, minimum sample sizes for S^2 and R charts (denoted as n^*) are searched such that the values ARL_1 (for a shift size) are lower than ARL_1 of np_{s^2} chart (for the same shift size)

Comparing

Table 3: Comparing np_{s^2}, R and S^2 charts - (n^* =ASS); $n = a + b - 1$

		$\delta = 1.1$			$\delta = 1.2$			$\delta = 1.5$			$\delta = 2$		
n	chart	np_{S^2}	R	S^2	np_{S^2}	R	S^2	np_{S^2}	R	S^2	np_{S^2}	R	S^2
6	n*	5.11	5	4	5.15	5	4	5.24	5	5	5.99	5	5
	ARL_1	114.3	113.89	117.83	46.66	47.32	49.85	8.72	8.95	8.03	2.56	2.74	2.52
9	n*	8.11	7	6	8.14	8	6	8.06	8	6	8.06	8	7
	ARL_1	100.56	102.57	98.34	37.53	36.83	37.08	6.2	6.16	6.61	1.87	1.91	1.85
12	n*	10.38	11	7	10.52	11	8	10.59	11	8	11.06	11	9
	ARL_1	88.9	88.88	91.27	30.8	30.82	29.53	4.79	4.9	4.86	1.55	1.57	1.52
15	n*	12.76	16	9	13.01	15	9	13.27	16	10	13.24	15	11
	ARL_1	80.22	79.31	80.14	26.21	26.71	26.76	3.86	3.8	3.84	1.35	1.35	1.33
20	n*	17.24	28	12	18.03	26	12	17.53	24	13	17.52	22	14
	ARL_1	68.76	68.61	68.03	20.57	20.25	20.77	2.91	2.92	2.92	1.18	1.17	1.18
30	n*	26.8	70	18	27.29	63	18	26.25	49	19	26.86	38	21
	ARL_1	53.16	53.14	52.36	13.93	13.92	14.04	1.98	1.96	2.02	1.05	1.05	1.04
40	n*	35.15	129	24	36.11	125	24	35.7	83	26	36.27	62	28
	ARL_1	42.98	42.96	42.41	10.28	10.27	10.39	1.56	1.56	1.55	1.01	1.01	1.01
50	n*	45.45	270	30	46.39	195	30	45.78	131	33	45.71	195	35
	ARL_1	35.84	35.84	35.46	7.99	8.32	8.13	1.33	1.33	1.32	1	1	1

np_{s^2} chart by Ho & Quinino (2013)

- Example- Monitoring the thickness variability of printed board:
  - Information of the process: $\mu = 0.06$ in and $\sigma = 0.004$ in;
  - The manager distrusts that the variability has increased to $\sigma_1 = 0.009$ in.
  - Designing np_{s^2} chart
    - $ARL_0 = 370$; $ARL_1 = 1.45$
    - Discriminating limits: UDL=0.0607; LDL=0.0513 (0.06 ± 0.0087)
    - Control limits: inspect sequentially until observing a=8 approved items or b=3 disapproved items

np_{s^2} chart by Ho & Quinino (2013)

Table 4: np_{s^2} chart- the last 17 inspection results

Sample	Classification of the thickness of the circuit board										a	b	Decision
1	GO	GO	GO	NG	GO	GO	GO	NG	NG		6	3	out-of-control
2	GO	GO	GO	GO	GO	GO	GO	GO			8	0	in-control
3	NG	GO	GO	GO	GO	GO	GO	GO	GO		8	1	in-control
4	GO	GO	NG	NG	GO	GO	GO	NG			5	3	out-of-control
5	GO	GO	GO	GO	GO	GO	GO	GO			8	0	in-control
6	GO	NG	GO	GO	GO	GO	GO	GO	NG	GO	8	2	in-control
7	GO	GO	GO	NG	NG	NG					3	3	out-of-control
8	NG	GO	GO	GO	GO	GO	GO	GO	GO		8	1	in-control
9	GO	GO	GO	GO	NG	NG	NG				4	3	out-of-control
10	GO	GO	GO	GO	GO	GO	GO	GO			8	0	in-control
11	NG	GO	NG	GO	GO	GO	GO	GO	GO	NG	7	3	out-of-control
12	GO	GO	NG	NG	NG						2	3	out-of-control
13	GO	NG	NG	GO	GO	GO	GO	GO	NG		6	3	out-of-control
14	GO	GO	GO	GO	NG	NG	NG				4	3	out-of-control
15	GO	NG	GO	GO	GO	GO	NG	GO	GO	GO	8	2	in-control
16	GO	GO	GO	NG	NG	NG					3	3	out-of-control
17	GO	GO	GO	GO	GO	GO	GO	GO			8	0	in-control

G_{S^2} control chart - by Bezerra et al. (2018)

- Motivation: inspired by the good performance of $\overline{X}_{rec}$ and $\overline{X}_{att}$ control charts
- Build a statistic like $\overline{X}_{att}$ based on the frequencies of the classifications and the warning limits
- Procedure: Classify the items into 3 classes using a device (like Figure2)
- L and U are the dimensions of the device

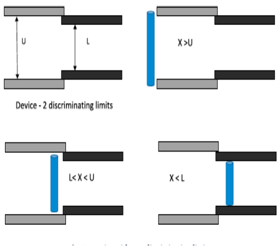

Figure 2: Device to classify

G_{S^2} control chart

- After the classification, we have n_1, # of items with $X < L$
- n_2, # of items in [L, U]
- n_3, # of items with $X > U$
- The statistic used to monitor is

$$G_{S^2} = (2-a)(z_L - t)^2\Phi(z_L)\, n_1 + t^2\left[\Phi(z_U) - \Phi(z_L)\right] n_2$$

$$+a(z_U - t)^2[1 - \Phi(z_U)]n_3$$

- Z_L and Z_U are standardized values of L and U; a, constant $\in\,]1;2[$ and t, also a constant $\in [0;1]$

G_{S^2} control chart - Density function

- Density Function: $U = 120$; $L = 110$, $a = 1.03$, $t = 0$; $\mu = 100$; In-control: $\sigma_0 = 10$; Out-of-control: $\sigma_1 = 20$

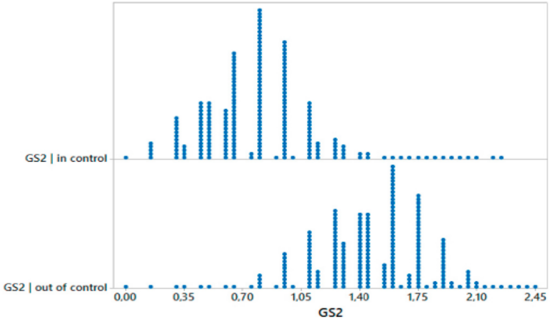

Figure 3: Density Function of G_{S^2}

G_{S^2} control chart

- Interest to detect increase in σ, thus if $G_{S^2} > CL_G$, the process is declared out-of-control
- The parameters of the control chart :a, t, z_U, z_L are optimized such that

$$a^o, t^o, z_L^o, z_u^o = argmin[ARL_1(a, t, z_l, z_U)]$$

- Restrict to:

$$a \in]1, 2[; t \in [0, 1], z_L \in [-w, 0]; z_U \in [0, w]$$

- Subject to:

$$ARL_0 = \tau$$

G_{S^2} control chart - Case 1 - with $t = 0$ and $|z_L| = z_U$

Table 5: Parameters of G_{S^2} - Case 1 - $t = 0$ and $|z_L| = z_U$

n	δ	z_L	a	CL	n	δ	z_L	a	CL
5	1.1	-1.8302	1.0205	0.2351	8	1.1	-1.7228	1.0790	0.4113
5	1.2	-1.8306	1.0062	0.2306	8	1.2	-1.7245	1.0796	0.4138
5	1.5	-1.8307	1.0337	0.2368	8	1.5	-1.7238	1.0470	0.4036
5	2	-1.8307	1.0063	0.2304	8	2	-1.7227	1.0072	0.3918
6	1.1	-1.9289	1.0334	0.2083	9	1.1	-1.7890	1.0647	0.3799
6	1.2	-1.9293	1.1621	0.2328	9	1.2	-1.7888	1.0176	0.3689
6	1.5	-1.9290	1.0083	0.2036	9	1.5	-1.7886	1.0183	0.3685
6	2	-1.9306	1.1133	0.2251	9	2	-1.7884	1.0430	0.3775
7	1.1	-1.9889	1.0107	0.2743	10	1.1	-1.8456	1.0206	0.3474
7	1.2	-1.9878	1.0079	0.2750	10	1.2	-1.8448	1.1044	0.3702
7	1.5	-1.9882	1.1040	0.2493	10	1.5	-1.8444	1.0398	0.3524
7	2	-1.9891	1.1206	0.2458	10	2	-1.8450	1.0049	0.3414

Table 6: Parameters of G_{S^2} - Case 2 - no restriction

n	δ	L	U	a	t	CL	n	δ	L	U	a	t	CL
5	1.1	-1.8365	1.8264	1.2743	0.1205	0.2930	8	1.1	-1.7287	1.7206	1.0326	0.0509	0.4054
5	1.2	-1.8213	1.8428	1.1376	0.0747	0.2502	8	1.2	-1.7297	1.7205	1.0401	0.1193	0.4785
5	1.5	-1.8102	1.8532	1.1934	0.0192	0.2591	8	1.5	-1.7212	1.7262	1.1023	0.0561	0.4103
5	2	-1.8099	1.8528	1.0928	0.0824	0.2519	8	2	-1.7121	1.7378	1.0373	0.0335	0.3994
6	1.1	-1.9097	1.9523	1.1277	0.2055	0.3788	9	1.1	-1.7919	1.7889	1.0425	0.1154	0.4579
6	1.2	-1.9481	1.9129	1.2531	0.2290	0.3970	9	1.2	-1.7734	1.8074	1.1832	0.0498	0.4039
6	1.5	-1.9528	1.9093	1.0581	0.1429	0.2899	9	1.5	-1.7800	1.7996	1.0543	0.0168	0.3696
6	2	-1.9242	1.9375	1.0167	0.1212	0.2783	9	2	-1.7937	1.7859	1.1028	0.0693	0.3942
7	1.1	-1.9871	1.9904	1.1536	0.1489	0.3550	10	1.1	-1.8754	1.8191	1.0031	0.0560	0.3644
7	1.2	-1.9875	1.9908	1.3598	0.0887	0.2672	10	1.2	-1.8343	1.8580	1.0160	0.0872	0.4171
7	1.5	-1.9890	1.9899	1.2616	0.1515	0.3232	10	1.5	-1.8426	1.8508	1.1827	0.1362	0.4612
7	2	-1.9891	1.9898	1.2758	0.1959	0.3899	10	2	-1.8416	1.8495	1.1173	0.0309	0.3659

Table 7: Comparing G_{S^2}: Case 1, 2 and S^2 charts

n	δ	Case 1	Case 2	S^2	n	δ	Case 1	Case 2	S^2
5	1.1	130.90	130.32	106.41	8	1.1	112.87	111.75	84.93
5	1.2	59.62	59.11	42.35	8	1.2	46.19	45.94	29.42
5	1.5	12.99	12.88	8.01	8	1.5	8.68	8.68	4.85
5	2	3.99	3.97	2.51	8	2	2.62	2.62	1.66
6	1.1	119.47	119.12	98.03	9	1.1	105.87	105.43	79.84
6	1.2	51.54	51.06	36.93	9	1.2	40.99	40.63	26.66
6	1.5	10.35	10.35	6.60	9	1.5	7.42	7.41	4.28
6	2	3.08	3.05	2.11	9	2	2.26	2.24	1.52
7	1.1	113.90	113.33	90.87	10	1.1	99.43	98.88	75.25
7	1.2	47.23	46.88	32.80	10	1.2	36.88	36.77	24.36
7	1.5	9.07	9.01	5.60	10	1.5	6.41	6.39	3.83
7	2	2.71	2.70	1.84	10	2	1.98	1.97	1.41

G_{S^2} control chart - Sample size to match with S^2 chart

Table 8: Minimum sample size for G_{S^2} chart to get a similar ARL_1 in S^2 control chart

n_{S^2}	$\delta = 1.10$	$\delta = 1.20$	$\delta = 1.50$	$\delta = 2.0$
5	9	9	9	9
6	11	11	10	10
7	12	12	12	11
8	15	14	13	12
9	16	16	15	15
10	18	17	17	16
11	19	19	18	17
12	21	20	19	19
13	23	22	21	20
14	24	24	23	21
15	26	25	24	23

G_{S^2} control chart -Numerical Example

- Monitor the variability of the printed circuit board thickness
- $\sigma_0 = 0.004$ in; $\sigma_1 = 0.008$ in; the sample size n=15 items
- Case 1: $a = 1.0317; z_U = 1.8144$ leading to $L = 0.05274$ and $U = 0.06726$in; and $CL_G = 0.4866$
- Case 2: $a = 1.1535; z_L = |1.8167|; z_U = 1.8121$ leading to $L = 0.05273$ in and $U = 0.06725$ in; and $CL_G = 0.8629$
- In both cases, $ARL_1 = 1.449$

G_{S^2} control chart -Numerical Example

Table 9: Results/decision of 15 samples G_{S^2} control chart

#Sample	n_1	n_2	n_3	G_{S^2}	Decision ($CL = 0.4866$)
1	0	14	1	0.1182224	In-control
2	1	14	0	0.1109574	In-control
3	3	8	4	0.8057615	Out-of-control
4	1	14	0	0.1109574	In-control
5	2	13	0	0.2219147	In-control
6	0	15	0	0.0000000	In-control
7	2	11	2	0.4583594	In-control
8	1	14	0	0.1109574	In-control
9	0	14	1	0.1182224	In-control
10	11	0	4	0.5838468	Out-of-control
11	1	13	1	0.2291797	In-control
12	1	12	2	0.3474021	In-control
13	4	8	3	0.7984965	Out-of-control
14	2	10	3	0.5765818	Out-of-control
15	0	14	1	0.1182224	In-control

Weakness of np_{s^2} and G_{S^2} charts

A large number of parameters to be searched to get a target value of ARL_0

High computational time spent to find the parameters mainly for the G_{S^2} chart

The statistic G_{S^2} is a discrete variable

Gauges used to classify the items have to be properly ordered; not found in the market

S^2_{tn} control chart - Preliminaries

Motivation: by the good performance of $\overline{X}_{tn}$, the idea is a direct extension to monitor the process variance

The gauges used to classify are found in the marked, so they do not have to be ordered.

S^2_{tn} control chart by Quinino et al. (2020)

- Similarly, the statistic S^2_{tn} is calculated using the n simulated truncated values as:

$$S^2_{tn} = \frac{\sum_{r=1}^{N_A} x^2_{Ar} + \sum_{j=1}^{N_B} x^2_{Bj} + \sum_{k=1}^{N_C} x^2_{Ck} + \sum_{z=1}^{N_D} x^2_{Dz} + \sum_{s=1}^{N_E} x^2_{Es}}{n-1}$$

  where $x^2_{\nu\rho} = (x_{\nu\rho} - \bar{X}_{tn})^2$, $\nu = A, B, C, D, E$ and $\rho = r, s, k, z, s$.
- The statistic $\chi^2_{tn} = \frac{(n-1)S^2_{tn}}{\sigma^2_0}$ is used to draw the control chart and the control limit is $UCL_{S^2_{tn}} = \chi^2_{1-\alpha;n-1}$.
- The control limit of S^2_{tn} are equal to the control limit of traditional S^2 chart.
- However, the approximation to some basic distributions for S^2_{tn} in case of out-of-control process is unknown. Thus, the Monte Carlo simulation was made in this study.

Comparing S^2 and S^2_{tn}

Table 10: S^2 versus S^2_{tn}

n	δ	s^2	S^2_{tn}	n	δ	s^2	S^2_{tn}	n	δ	s^2	S^2_{tn}	n	δ	s^2	S^2_{tn}
5	1	370.4	372.58	7	1	370.4	385.51	9	1	370.4	369.82	11	1	370.4	349.16
	2	11.48	29.9		2	8.11	20.99		2	6.21	15.65		2	5.01	12.4
	3	4.05	11.59		3	2.85	7.54		3	2.23	5.35		3	1.87	4.11
	4	2.52	7.08		4	1.85	4.49		4	1.52	3.2		4	1.33	2.5
	5	1.93	5.18		5	1.48	3.28		5	1.27	2.39		5	1.16	1.89
	7	1.48	3.56		7	1.21	2.32		7	1.1	1.75		7	1.05	1.45
	9	1.3	2.88		9	1.11	1.93		9	1.05	1.5		9	1.02	1.28
	12	1.17	2.39		12	1.06	1.64		12	1.02	1.32		12	1.01	1.17
	14	1.13	2.19		14	1.04	1.54		14	1.01	1.26		14	1	1.13
	16	1.1	2.06		16	1.03	1.47		16	1.01	1.32		16	1	1.1
6	1	370.4	365.5	8	1	370.4	371.47	10	1	370.4	371.2	12	1	370.4	361.27
	2	9.52	24.94		2	7.04	18.12		2	5.55	13.86		2	4.56	10.96
	3	3.34	9.24		3	2.5	6.3		3	2.03	4.66		3	1.74	3.64
	4	2.11	5.52		4	1.66	3.74		4	1.41	2.8		4	1.27	2.24
	5	1.66	4.06		5	1.36	2.77		5	1.21	2.11		5	1.12	1.73
	7	1.31	2.83		7	1.14	1.99		7	1.07	1.58		7	1.03	1.35
	9	1.18	2.31		9	1.07	1.67		9	1.03	1.37		9	1.01	1.21
	12	1.1	1.94		12	1.03	1.45		12	1.01	1.23		12	1	1.12
	14	1.07	1.8		14	1.02	1.38		14	1.01	1.18		14	1	1.09
	16	1.05	1.7		16	1.01	1.32		16	1	1.15		16	1	1.07

The improved control chart: $S^2_{tn(I)}$ and $S^2_{tn(K)}$

These improvements are proposed by Yamauchi et al. (2022).

The sampling scheme used for $S^2_{tn(I)}$ control chart is identical of the applied for $\overline{X}_{tn(I)}$:

1. Inspect a sample of size n_a and compute the statistic $S^{2_a}_{tn}$
2. If $\chi^2_a = \dfrac{(n_a - 1)S^{2_a}_{tn}}{\sigma^2_0} > \chi^2_{1-\alpha;n_a-1}$ then the process is declared out-of-control, adjusted and go to Step 1; otherwise go to Step 3.
3. Inspect a sample of size n_b; compute the statistic $S^{2_b}_{tn}$; if $\chi^2_b = \dfrac{(n_b - 1)S^{2_b}_{tn}}{\sigma^2_0} > \chi^2_{1-\alpha;n_b-1}$ then the process is declared out-of-control, adjusted and go to Step 1.

The improved control chart: $S^2_{tn(I)}$ and $S^2_{tn(K)}$

The control chart $S^2_{tn(K)}$ applies Klein's supplementary rule (Klein 2000):

- In equally spaced time intervals random samples of size n and obtained the statistic S^2_{tn}.
- Let $\chi^{2^{i-1}}_{tn}$ and $\chi^{2^{i}}_{tn}$ be two successive values of $\chi^2_{tn} = \frac{(n-1)S^2_{tn}}{\sigma^2_0}$.
- The process is declared out of control if $\chi^{2^{i-1}}_{tn} > UCL_{S^2_{tn}}$ and $\chi^{2^{i}}_{tn} > UCL_{S^2_{tn}}$.

Comparing

Table 11: ARL_1 values for the S^2, S^2_{tn}, $S^2_{tn(K)}$ and $S^2_{tn(I)}$

Shift δ_1	S^2	S^2_{tn}	$S^2_{tn(K)}$	$S^2_{tn(I)}$	S^2	S^2_{tn}	$S^2_{tn(K)}$	$S^2_{tn(I)}$
	Sample size n=7				Sample size n=8			
1	370	353	370	382	370	355	371	377
1.1	177	227	**204**	227	170	226	**197**	**223**
1.2	96	147	**121**	147	90	142	**113**	143
1.3	58	103	**79**	**100**	53	98	**72**	**91**
1.4	38	75	**56**	**74**	35	69	**50**	**66**
1.5	27	57	**41**	**54**	24	52	**36**	**48**
1.6	20	44	**31**	**41**	17	39	**28**	**36**
1.7	15	36	**25**	**33**	13	32	**22**	**28**
1.8	12	30	**20**	**26**	10	26	**18**	**22**
1.9	9.7	25	**17**	**21**	8.5	21	**15**	**18**
2	8.1	21	**15**	**18**	7	18	**13**	**15**
3	2.9	7.5	**6**	**6.1**	2.5	6.3	**5.2**	**5.1**
5	1.5	3.3	3.4	**2.7**	1.4	2.8	3	**2.4**
7	1.2	2.3	2.8	**2.1**	1.1	2	2.5	**1.8**
9	1.1	1.9	2.5	**1.8**	1.1	1.7	2.4	**1.6**

Sample sizes

Table 12: Minimum sample size for $S^2_{tn(I)}$ (with $n_a = 2n - 2$; $n_b = 2$) and $S^2_{tn(K)}$ control charts to have equal performance of the standard S^2 chart

shift	n = 2		n = 3		n = 4		n = 5		n = 6		n = 7		n = 8	
δ_2	$S^2_{tn(K)}$	$S^2_{tn(I)}$	$S^2_{tn(K)}$	$S^2_{tn(I)}$	$S^2_{tn(K)}$	$S^2_{tn(I)}$	$S^2_{tn(K)}$	$S^2_{tn(I)}$	$S^2_{tn(K)}$	$S^2_{tn(I)}$	$S^2_{tn(K)}$	$S^2_{tn(I)}$	$S^2_{tn(K)}$	$S^2_{tn(I)}$
1.1	4	6	6	9	7	11	9	14	10	18	11	19	13	>20
1.2	4	6	6	9	7	11	9	13	10	15	11	17	13	19
1.3	4	6	6	8	7	11	9	12	10	14	11	16	13	18
1.4	4	6	6	8	7	10	9	12	10	13	11	15	13	16
1.5	4	5	6	8	7	10	9	11	10	13	11	14	13	16
1.6	4	5	6	8	7	9	9	11	10	12	11	14	13	16
1.7	4	5	6	8	7	9	9	11	10	12	12	14	13	15
1.8	4	5	6	7	7	9	9	11	10	12	12	14	13	15
1.9	4	5	6	7	7	9	9	10	11	12	12	13	14	15
2.0	4	5	6	7	8	9	9	10	11	12	13	13	14	15

$\mathbf{np}_x$-type charts to monitor variance: $\mathbf{np}_X^{\sigma^2}$, $\mathbf{np}_{X(I)}^{\sigma^2}$

These approaches are proposed by Silva et al. (2022)

I - $\text{np}_X^{\sigma^2}$ control chart:

- Classify n items according to the warning limit w_u and get the statistic Y (the number of items which value of quality characteristic is larger than w_u)
- If $Y > UCL_{np_x}$ then the process is declared out-of-control.

$\mathbf{np}_x$-type charts to monitor variance: $\mathbf{np}_X^{\sigma^2}$, $\mathbf{np}_{X(I)}^{\sigma^2}$

II- $\mathrm{np}_{X(I)}^{\sigma^2}$ control chart:

- 1 - Classify n_a items according to the warning limit z_a and get the statistic Y_a (the number of items which value of quality characteristic is larger than z_a)
- 2 - If $Y_a > UCL_{n_a}$ then the process is declared out-of-control, adjusted and go to step I. Otherwise go to step 3
- 3 - Classify $n_b < n_a$ items according to the warning limit z_b and get the statistic Y_b (the number of items which value of quality characteristic is larger than z_b). If $Y_b > UCL_{n_b}$ then the process is declared out-of-control, adjusted and go to step I.
- 4 - Go to step 1.

Table 13: Comparison of the performance of the control charts: S^2, $np_x^{\sigma^2}$ and $np_{x(I)}^{\sigma^2}$ control charts.

				ARL_1					
δ	S^2 chart	$np_x^{\sigma^2}$	$np_{x(I)}^{\sigma^2}$	S^2 chart	$np_x^{\sigma^2}$	$np_{x(I)}^{\sigma^2}$	S^2 chart	$np_x^{\sigma^2}$	$np_{x(I)}^{\sigma^2}$
1.00	370.398	370.364	370.362	370.398	370.341	370.364	370.398	370.303	370.395
1.10	98.340	96.239	95.919	91.272	88.886	90.602	85.294	83.167	85.351
1.20	37.080	35.132	35.316	32.889	30.799	32.077	29.528	27.605	29.161
1.30	17.830	16.359	16.716	15.406	13.889	14.827	13.530	12.136	13.226
1.40	10.194	9.102	9.484	8.690	7.595	8.311	7.555	6.554	7.357
1.50	6.610	5.781	6.145	5.608	4.792	5.366	4.863	4.123	4.751
1.60	4.706	4.057	4.397	3.995	3.367	3.850	3.474	2.907	3.429
1.70	3.595	3.072	3.393	3.065	2.566	2.991	2.681	2.233	2.690
1.80	2.898	2.467	2.775	2.487	2.081	2.471	2.192	1.830	2.249
1.90	2.435	2.073	2.374	2.107	1.770	2.139	1.874	1.575	1.973
2.00	2.114	1.805	2.103	1.846	1.561	1.918	1.656	1.407	1.793
Sample	$n_{S^2} = 6$	$n_{np_x} = 10$	$ASS = 7.515$	$n_{S^2} = 7$	$n_{np_x} = 12$	$ASS = 8.517$	$n_{S^2} = 8$	$n_{np_x} = 14$	$ASS = 9.52$
Size			$n_a = 14$ $n_b = 1$			$n_a = 16$ $n_b = 1$			$n_a = 18$ $n_b = 1$
UCL	$UCL_{S^2} = 3.641$	$UCL_{np_x} = 2$	$UCL_{n_a} = 3$	$UCL_{S^2} = 3.344$	$UCL_{np_x} = 2$	$UCL_{n_a} = 3$	$UCL_{S^2} = 3.121$	$UCL_{np_x} = 2$	$UCL_{n_a} = 3$
			$UCL_{n_b} = 0$			$UCL_{n_b} = 0$			$UCL_{n_b} = 0$
Discriminant	-	$w_u = 2.1734$	$z_a = 1.945$	-	$w_u = 2.2512$	$z_a = 2.01$	-	$w_u = 2.3144$	$z_a = 2.06$
limit			$z_b = 3.405$			$z_b = 3.36$			$z_b = 3.405$

Themes for the Seminars

Find other contributions (any kind) to improve the standard control charts

If you have to make a choice, which of the proposals you will choose? Justify

Is there room for other improvement? Do you have other idea?

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