1) Resolver equações exponenciais

a) $\left(\frac{1}{5}\right)^x = 125 \Rightarrow (5^{-1})^x = 5^3 \Rightarrow 5^{-x} = 5^3$

$\Leftrightarrow$ se, e somente se,

$-x = 3$

$\boxed{x = -3}$

b) $125^x = 0{,}04 \Rightarrow (5^3)^x = \left(\frac{1}{25}\right) \Rightarrow 5^{3x} = \left(\frac{1}{5^2}\right) \Rightarrow 5^{3x} = 5^{-2}$

$\Leftrightarrow$

$3x = -2$

$\boxed{x = -2/3}$

c) $5^{3x-1} = \left(\frac{1}{25}\right)^{2x+3} \Rightarrow 5^{3x-1} = (5^{-2})^{2x+3} \Rightarrow 5^{3x-1} = 5^{-4x-6}$

$\Leftrightarrow$

$3x - 1 = -4x - 6$

$3x + 4x = 1 - 6$

$7x = -7$

$x = -7/7$

$\boxed{x = -1}$

d) $(2^x)^{x+4} = 32 \Rightarrow (2)^{x^2+4x} = 2^5$

$\Leftrightarrow$

$x^2 + 4x = 5$

$x^2 + 4x - 5 = 0$

$\begin{cases} \Delta = (4)^2 - 4(1)(-5) = 16 + 20 = 36 \\ x = \dfrac{-4 \pm \sqrt{36}}{2} = \begin{cases} x_1 = 1 \\ x_2 = -5 \end{cases} \end{cases}$

$\boxed{x = 1 \text{ ou } x = -5}$

e) $4^{x+1} - 9 \cdot 2^x + 2 = 0$

$(2^2)^{x+1} - 9 \cdot 2^x + 2 = 0$

$(2^{x+1})^2 - 9 \cdot 2^x + 2 = 0$

b+c

a

$(2^x \cdot 2^1)^2 - 9 \cdot 2^x + 2 = 0$

$a^b \cdot a^c$

$4 \cdot (2^x)^2 - 9 \cdot 2^x + 2 = 0$

Fazendo uma substituição de variáveis, consideramos

$z = 2^x$

$4z^2 - 9z + 2 = 0$

Resolvemos a quadrática

e depois descobrimos o valor de x, usando $z = 2^x$

$$4z^2 - 9z + 2 = 0$$

$$\Delta = (-9)^2 - 4(4)(2)$$
$$= 81 - 32 = 49$$

$$z = \frac{-(-9) \pm \sqrt{49}}{2(4)} = \begin{cases} z_1 = \frac{9+7}{8} = \frac{16}{8} = 2 \\ z_2 = \frac{9-7}{8} = \frac{2}{8} = \frac{1}{4} \end{cases}$$

Para $z_1 = 2 \Rightarrow z_1 = 2^x$

$2^1 = 2^x$

$\Leftrightarrow$

$\boxed{x = 1}$

Para $z_2 = \frac{1}{4} \Rightarrow z_2 = 2^x$

$\frac{1}{4} = 2^x$

$\frac{1}{2^2} = 2^x$

$2^{-2} = 2^x$

$\Leftrightarrow$

$\boxed{x = -2}$

ou seja,

$\boxed{x = 1 \text{ ou } x = -2}$

2) Sendo $\ln(a) = 2$; $\ln(b) = 5$ e $\ln\left(\frac{3}{5}\right) = -0{,}51$. Calcule

a) $\ln(a.b) = \ln(a) + \ln(b) = 2 + 5 = 7$

b) $\ln\sqrt{a.b} = \ln(a.b)^{1/2} = \frac{1}{2}\ln(a.b) = \frac{1}{2}\left[\ln(a) + \ln(b)\right] =$

$$= \frac{1}{2}[2+5] = \frac{7}{2} = 3{,}5$$

c) $\ln(a^2.b^3) = \ln(a^2) + \ln(b^3) = 2\ln(a) + 3\ln(b) =$

$$= 2(2) + 3(5) = 4 + 15 = 19$$

d) $\ln\left(\frac{3b^2}{5\sqrt{a^3}}\right) = \ln\left(\frac{3}{5} \cdot \frac{b^2}{\sqrt{a^3}}\right) = \ln\left(\frac{3}{5}\right) + \ln\left(\frac{b^2}{\sqrt{a^3}}\right) = \ln\left(\frac{3}{5}\right) + \ln(b^2) - \ln(\sqrt{a^3})$

$$= \ln\left(\frac{3}{5}\right) + 2\ln(b) - \ln(a)^{3/2} = \ln\left(\frac{3}{5}\right) + 2\ln(b) - \frac{3}{2}\ln(a) =$$

$$= -0{,}51 + 2(5) - \frac{3}{\cancel{2}}(\cancel{2}) = -0{,}51 + 10 - 3 = 6{,}49$$