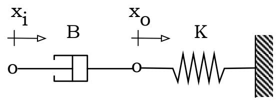
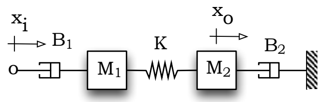
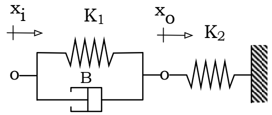
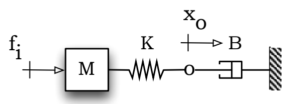
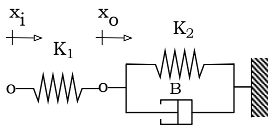
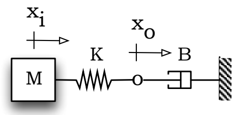
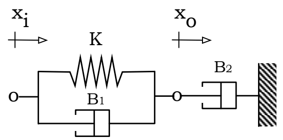
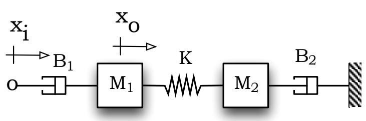
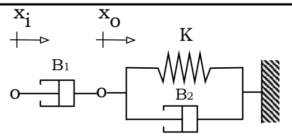
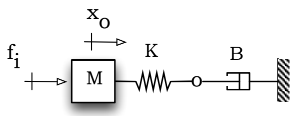
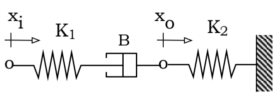
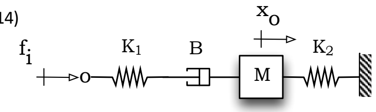
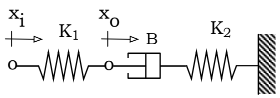
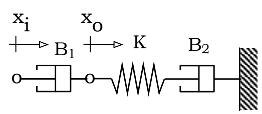
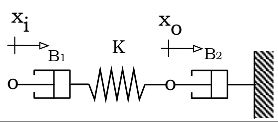
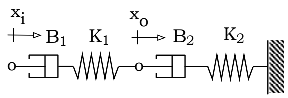
UNIVERSIDADE DE SÃO PAULO
ESCOLA DE ENGENHARIA DE SÃO CARLOS
DEPARTAMENTO DE ENGENHARIA MECÂNICA

SEM 0533 – MODELAGEM E SIMULAÇÃO DE SISTEMAS DINÂMICOS I
SEM 0232 – MODELOS DINÂMICOS

Respostas
Lista 1
Sistemas Mecânicos

1-) Obs: As F.T. não estão na forma padrão !

Modelo	F.T.	Modelo	F.T.
(1) 	$\frac{X_o}{X_i}(s) = \frac{Bs}{Bs+K}$	(9) 	$\frac{X_o}{X_i}(s) = \frac{B_1K}{M_1M_2s^3 + (M_1B_2 + M_2B_1)\,s^2 + (M_1K + M_2K + B_1B_2)\,s + (B_1+B_2)\,K}$
(2) 	$\frac{X_o}{X_i}(s) = \frac{Bs+K_1}{Bs+(K_1+K_2)}$	(10) 	$\frac{X_o}{F_i}(s) = \frac{K}{MBs^3 + MKs^2 + BKs}$
(3) 	$\frac{X_o}{X_i}(s) = \frac{K_1}{Bs+(K_1+K_2)}$	(11) 	$\frac{X_o}{X_i}(s) = \frac{K}{Bs+K}$
(4) 	$\frac{X_o}{X_i}(s) = \frac{B_1s+K}{(B_1+B_2)s+K}$	(12) 	$\frac{X_o}{X_i}(s) = \frac{B_1\left(M_2s^2 + B_2s + K\right)}{M_1M_2s^3 + (M_1B_2 + M_2B_1)\,s^2 + (M_1K + M_2K + B_1B_2)\,s + (B_1+B_2)\,K}$
(5) 	$\frac{X_o}{X_i}(s) = \frac{B_1s}{(B_1+B_2)s+K}$	(13) 	$\frac{X_o}{F_i}(s) = \frac{Bs+K}{MBs^3 + MKs^2 + BKs}$
(6) 	$\frac{X_o}{X_i}(s) = \frac{BK_1s}{B\,(K_1+K_2)\,s + K_1K_2}$	(14) 	$\frac{X_o}{F_i}(s) = \frac{1}{Ms^2+K_2}$
(7) 	$\frac{X_o}{X_i}(s) = \frac{K_1(Bs+K_2)}{B(K_1+K_2)s + K_1K_2}$	(15) 	$\frac{X_o}{X_i}(s) = \frac{B_1\,(B_2s+K)}{B_1B_2s + K\,(B_1+B_2)}$
(8) 	$\frac{X_o}{X_i}(s) = \frac{B_1K}{B_1B_2s + (B_1+B_2)K}$	(16) 	$\frac{X_o}{X_i}(s) = \frac{B_1K_1s + B_1K_1K_2}{B_1B_2(K_2+K_1)s + (K_1K_2)(B_1+B_2)}$

1-) Obs: As F.T. não estão na forma padrão !

(a) $$k_{eq} = \frac{k_2k_3k_4k_5 + 2k_1k_3k_4k_5 + k_1k_2k_4K_5 + 2k_1k_2k_3k_5}{k_2k_3k_4 + k_2k_3k_5 + 2k_1k_3k_4 + 2k_1k_3k_5 + k_1k_2k_4 + k_1k_2k_5 + 2k_1k_2k_3}$$

(b) $$k_{eq} = k_{t1} + k_{t2} + k_1 l_1^2 + k_2 l_1^2 + k_3 l_2^2$$

(c) $$k_{eq} = \frac{3}{4}k$$

(d) $$k_{eq} = \sum_{i=1}^{4} \left(k_{ei}\cos^2\theta_i\right) \quad \text{onde:} \quad k_{e_1} = \left(\frac{k_5k_6k_7}{k_5k_6 + k_5k_7 + k_6k_7}\right) \; k_{e2} = \left(\frac{k_8k_9}{k_8 + k_9}\right) \; k_{e3} = \left(\frac{k_1k_2}{k_1 + k_2}\right) \; k_{e_4} = \left(\frac{k_3k_4}{k_3 + k_4}\right)$$

(e) $$k_{eq} = \frac{3E_1I_1}{l_1^3} + \frac{48E_2I_2}{l_2^3}$$

(f) $k_{eq} = 2\gamma A$ onde γ é o peso específico do líquido

3-) Obs: As F.T. não estão na forma padrão !

(a) Na forma matricial

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} K & -K \\ -K & K \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} B_1 \dot{x}_i \\ f_i \end{Bmatrix}$$

(b) $$\frac{X_1(s)}{X_i(s)} = \frac{B_1 M_2 s^2 + B_1 B_2 s + B_1 K}{M_1 M_2 s^3 + (M_1 B_2 + M_2 B_1)\, s^2 + (B_1 B_2 + M_1 K + M_2 K)\, s + B_1 K + B_2 K}$$

$$\frac{X_2(s)}{X_i(s)} = \frac{B_1 K}{M_1 M_2 s^3 + (M_1 B_2 + M_2 B_1)\, s^2 + (B_1 B_2 + M_1 K + M_2 K)\, s + B_1 K + B_2 K}$$

(c) $$\frac{F_0(s)}{F_i(s)} = \frac{M_1 K s}{M_1 M_2 s^3 + M_1 B_2 s^2 + (M_1 K + M_2 K)\, s + B_2 K}$$

4-) (a) Na forma matricial

$$\left[\begin{array}{cc} M_1 & 0 \\ 0 & M_2 \end{array}\right]\left\{\begin{array}{c} \ddot{u}_1 \\ \ddot{u}_2 \end{array}\right\}+\left[\begin{array}{cc} B_1+B_2 & -B_2 \\ -B_2 & B_2 \end{array}\right]\left\{\begin{array}{c} \dot{u}_1 \\ \dot{u}_2 \end{array}\right\}+\left[\begin{array}{cc} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{array}\right]\left\{\begin{array}{c} u_1 \\ u_2 \end{array}\right\}=\left\{\begin{array}{c} f_1(t) \\ f_2(t) \end{array}\right\}$$

(b) F.T.:

$$\frac{U_1}{F_1}(s)=\frac{M_2 s^2+B_2 s+K_2}{M_1 M_2 s^4+(M_1 B_2+M_2 B_1+M_2 B_2) s^3+(M_1 K_2+B_1 B_2+M_2 K_1+M_2 K_2) s^2+(B_1 K_2+K_1 B_2) s+K_1 K_2}$$

$$\frac{U_2}{F_1}(s)=\frac{B_2 s+K_2}{M_1 M_2 s^4+(M_1 B_2+M_2 B_1+M_2 B_2) s^3+(M_1 K_2+B_1 B_2+M_2 K_1+M_2 K_2) s^2+(B_1 K_2+K_1 B_2) s+K_1 K_2}$$

$$\frac{U_1}{F_2}(s)=\frac{B_2 s+K_2}{M_1 M_2 s^4+(M_1 B_2+M_2 B_1+M_2 B_2) s^3+(M_1 K_2+B_1 B_2+M_2 K_1+M_2 K_2) s^2+(B_1 K_2+K_1 B_2) s+K_1 K_2}$$

$$\frac{U_2}{F_2}(s)=\frac{M_1 s^2+(B_1+B_2) s+(K_1+K_2)}{M_1 M_2 s^4+(M_1 B_2+M_2 B_1+M_2 B_2) s^3+(M_1 K_2+B_1 B_2+M_2 K_1+M_2 K_2) s^2+(B_1 K_2+K_1 B_2) s+K_1 K_2}$$

(c) $\left\{\begin{array}{c} U_1(s) \\ U_2(s) \end{array}\right\}=\left[\begin{array}{cc} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{array}\right]\left\{\begin{array}{c} F_1(s) \\ F_2(s) \end{array}\right\}$ $\quad\left[\begin{array}{cc} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{array}\right]=\left[\begin{array}{cc} \frac{N_1(s)}{D(s)} & \frac{N_2(s)}{D(s)} \\ \frac{N_2(s)}{D(s)} & \frac{N_3(s)}{D(s)} \end{array}\right]$

$N_1(s)=M_2 s^2+B_2 s+K_2$ $\quad N_3(s)=M_1 s^2+(B_1+B_2)\, s+(K_1+K_2)$

$N_2(s)=B_2 s+K_2$ $\quad D(s)=M_1 M_2 s^4+(M_1 B_2+M_2 B_1+M_2 B_2)\, s^3+(M_1 K_2+B_1 B_2+M_2 K_1+M_2 K_2)\, s^2+$
$(B_1 K_2+K_1 B_2)\, s+K_1 K_2$

4-) Continuação (d)

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \left\{ \begin{array}{c} \ddot{u}_1 \\ \ddot{u}_2 \end{array} \right\} + \begin{bmatrix} B_1 + B_2 & -B_2 \\ -B_2 & B_2 \end{bmatrix} \left\{ \begin{array}{c} \dot{u}_1 \\ \dot{u}_2 \end{array} \right\} + \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \left\{ \begin{array}{c} u_1 \\ u_2 \end{array} \right\} = \left\{ \begin{array}{l} K_1 x(t) + B_1 \dot{x}(t) \\ 0 \end{array} \right\}$$

(e)

$$\bar{H}_{11}(s) = \frac{U_1}{X}(s) = \frac{M_2B_1s^3+(M_2K_1+B_1B_2)s^2+(K_1B_2+K_2B_1)s+K_1K_2}{M_1M_2s^4+(M_1B_2+M_2B_1+M_2B_2)s^3+(M_1K_2+B_1B_2+M_2K_1+M_2K_2)s^2+(B_1K_2+K_1B_2)s+K_1K_2}$$

$$\bar{H}_{21}(s) = \frac{U_2}{X}(s) = \frac{B_1B_2s^2+(B_1K_2+B_2K_1)s+K_1K_2}{M_1M_2s^4+(M_1B_2+M_2B_1+M_2B_2)s^3+(M_1K_2+B_1B_2+M_2K_1+M_2K_2)s^2+(B_1K_2+K_1B_2)s+K_1K_2}$$

$$\left\{ \begin{array}{c} U_1(s) \\ U_2(s) \end{array} \right\} = \begin{bmatrix} \bar{H}_{11}(s) & \bar{H}_{21}(s) \end{bmatrix} \left\{ \begin{array}{l} X(s) \\ 0 \end{array} \right\}$$

5-) (a)

$$M_1\ddot{x} + (B_1 + B_3)\,\dot{x} + K_1 x - B_2\dot{z} = 0$$
$$M_2\ddot{x} + K_2 x + M_2\ddot{z} + B_2\dot{z} + K_2 z = f(t)$$

(b)

$$M_1\ddot{x} + (B_1 + B_2 + B_3)\,\dot{x} + K_1 x - B_2\dot{y} = 0$$
$$-B_2\dot{x} + M_2\ddot{y} + B_2\dot{y} + K_2 y = f(t)$$

6-) (i)

$$M_1\ddot{x}_1 + K_1 x_1 - B\dot{x}_2 - (K_2 + K_3)\,x_2 = M_1 g + f_i(t)$$
$$M_2\ddot{x}_1 + M_2\ddot{x}_2 + B\dot{x}_2 + (K_2 + K_3)\,x_2 = M_2 g$$

(ii)

$$x_{1_0} = \frac{(M_1 + M_2)\,g}{K_1}$$
$$x_{2_0} = \frac{M_2 g}{K_2 + K_3}$$

8-) (a)

$$M_1\ddot{x}_1 + B_1\dot{x}_1 + K_1\,(x_1 - x_2) = f_i(t)$$
$$M_2\ddot{x}_2 + B_2\dot{x}_2 + K_2 x_2 = K_1\,(x_1 - x_2)$$

(b)

$$M_1\ddot{x}_1 + B_1\dot{x}_1 + K_1\,(x_1 - x_2) = f_i(t)$$
$$M_2\ddot{x}_2 + B_2\dot{x}_2 + K_2 x_2 + M_2 g = K_1\,(x_1 - x_2)$$

9-) para $x_3 = 0$

$$M_1\ddot{u}_1 + B_1\dot{u}_1 + (K_1 + K_2)\,u_1 - K_2 u_2 = f_i(t)$$
$$-K_2 x_1 + M_2\ddot{u}_2 + B_2\dot{u}_2 + K_2 u_2 = 0$$

10-) $J_1\dot{\omega}_1 + B_1\omega_1 + K_1\theta_1 - K_2\left(\theta_2 - \theta_1\right) = 0$

$J_2\dot{\omega}_2 + B_2\omega_2 + K_2\left(\theta_2 - \theta_1\right) - T_i(t) = 0$

F.T. : por conta do estudante !

11-) (a) $\left(I_m + I_p\right)\ddot{\theta} = T - RF$

$m_r\ddot{x} = F - c\dot{x} - kx$

(b) $I_{1e} = N^2 I_1 \quad T_{1e} = NT_1$ (propriedades equivalentes)

$$I_{1e}\ddot{\theta}_2 = T_{1e} + k_T\left(\theta_3 - \theta_2\right)$$
$$I_2\ddot{\theta}_3 = -k_T\left(\theta_3 - \theta_2\right) - c_T\dot{\theta}_3$$

12-) $J\ddot{\theta} + B_1\dot{\theta} + \left(K_1 + K_2R^2\right)\theta - K_2Rx = Rf_a(t)$

$-K_2R\theta + M\ddot{x} + B_2\dot{x} + \left(K_2 + K_3\right)x = 0$

F.T. : por conta do estudante !