Monitoria 07/06

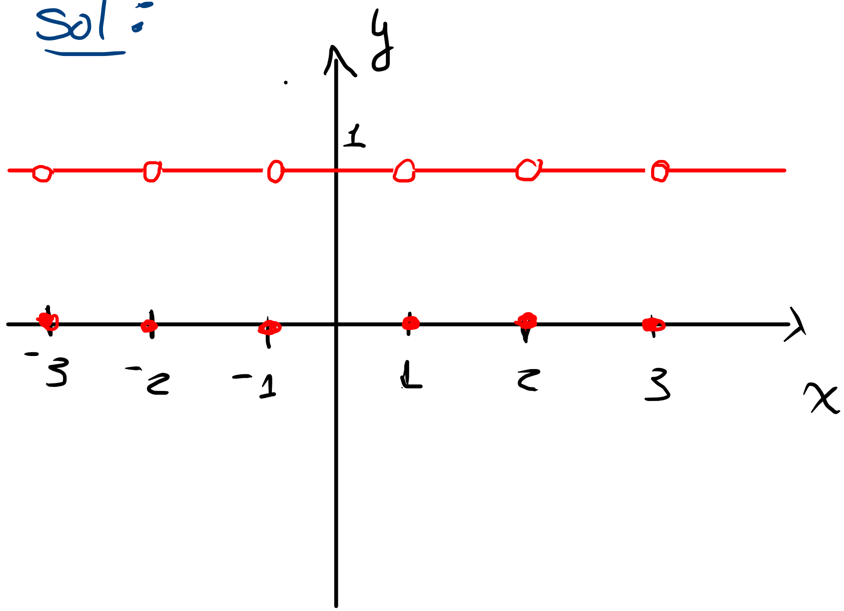
Listas para entrega: 4 listas

Média Simples das 3 maiores notas.

3.2

7- Exemplo de fç definida em $\mathbb{R}$, contínua em $\mathbb{R} \setminus \mathbb{Z}$ e descontínua em $\mathbb{Z}$.

sol:



$$f(x) = \begin{cases} 1, & \text{se } x \in \mathbb{R} \setminus \mathbb{Z} \\ 0, & \text{se } x \in \mathbb{Z} \end{cases}$$

3.3

2. Determine L para que f seja contínua no ponto dado.

Sol:

c) $f(x) = \begin{cases} \dfrac{\sqrt{x}-\sqrt{5}}{\sqrt{x+5}-\sqrt{10}}, & \text{se } x \neq 5 \\ L, & \text{se } x = 5 \end{cases}$ em $p=5$.

- $\lim\limits_{x \to 5} f(x) = f(5) = L$

$$\lim_{x\to 5} f(x) = \lim_{x\to 5}\left(\frac{\sqrt{x}-\sqrt{5}}{\sqrt{x+5}-\sqrt{10}}\right)$$

$\forall\, x \neq 5:$

$$\frac{\sqrt{x}-\sqrt{5}}{\sqrt{x+5}-\sqrt{10}} = \frac{(\sqrt{x}-\sqrt{5})}{(\sqrt{x+5}-\sqrt{10})}\cdot\frac{(\sqrt{x+5}+\sqrt{10})}{(\sqrt{x+5}+\sqrt{10})} =$$

$$= \frac{(\cancel{\sqrt{x}-\sqrt{5}})\cdot(\sqrt{x+5}+\sqrt{10})}{(x-5)} = \frac{\sqrt{x+5}+\sqrt{10}}{\sqrt{x}+\sqrt{5}}$$

$$\overbrace{(\cancel{\sqrt{x}-\sqrt{5}})\cdot(\sqrt{x}+\sqrt{5})}$$

$2\sqrt{10}$

$$\lim_{x \to 5} f(x) = \lim_{x \to 5} \left(\frac{\sqrt{x+5} + \sqrt{10}}{\sqrt{x} + \sqrt{5}} \right) = \frac{\sqrt{10}}{\sqrt{5}} = \boxed{\sqrt{2} = L}$$

$2\sqrt{5}$

7.11

8. $g: \mathbb{R} \to \mathbb{R}$ diferenciável tal que $g(1) = 2$ e $g'(1) = 3$. Calcule $f'(0)$, sendo f dada por $f(x) = e^x \cdot g(3x+1)$

Sol:

$$f'(x) = (e^x)' \cdot g(3x+1) + e^x \cdot (g(3x+1))' =$$

$$e^x \cdot g(3x+1) + e^x (g'(3x+1) \cdot 3) =$$

$$e^x (g(3x+1) + 3g'(3x+1)), \quad \forall x \in \mathbb{R}.$$

$$f'(0) = g(1) + 3 \cdot g'(1) = 11$$

25. Seja $y = \dfrac{1}{x^2+1}$, $x = x(t)$ derivável definida em $\mathbb{R}$. Verifique que, $\forall\, t \in \mathbb{R}$,

$$\frac{dy}{dt} = -2xy^2 \frac{dx}{dt}.$$

Sol: $y = y(x) \rightsquigarrow \dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt}$ (Regra da cadeia)

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}.$$

$$y = \frac{1}{x^2+1} \Rightarrow y^2 = \frac{1}{(x^2+1)^2} \quad (*)$$

$$\frac{dy}{dx} = \frac{-2x(1)}{(x^2+1)^2}$$

$$\frac{dy}{dt} = \frac{-2x}{(x^2+1)^2} \cdot \frac{dx}{dt} \overset{(*)}{=} -2x\,y^2 \cdot \frac{dx}{dt}$$

$$y = \frac{1}{x(t)^2+1}$$

$$\frac{dy}{dt} = -\frac{(x(t)^2+1)'}{(x(t)^2+1)^2} = -\frac{2x(t)\cdot x'(t)}{(x(t)^2+1)^2}$$

7.9

3. Seja $y = t^2 x$, $x = x(t)$ derivável. Calcule $\left.\frac{dy}{dt}\right|_{t=1}$, supondo $\left.\frac{dx}{dt}\right|_{t=1} = 2$

e $x(1) = 3$.

Sol: $y = t^2 \cdot x(t)$

$$\frac{dy}{dt}(t) = (2t) \cdot x(t) + t^2 \cdot \frac{dx}{dt}(t) \quad \forall t \in \mathbb{R}$$

$$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^{x} = e$$

$$\lim_{h \to 0} (1+h)^{1/h} = e$$

$$\left.\frac{dy}{dt}\right|_{t=1} = 2x(1) + 1 \cdot \left.\frac{dx}{dt}\right|_{t=1} = 6 + 2 = 8$$

6.3

d) $\lim_{x \to +\infty} \left(1+\frac{2}{x}\right)^{x+1}$

Sol:

$$\left(1+\frac{2}{x}\right)^{x+1} = \left(1+\frac{2}{x}\right)^{x} \cdot \left(1+\frac{2}{x}\right)$$

$$\frac{2}{x} = \frac{1}{u} \Rightarrow \boxed{u = \frac{x}{2}} \qquad x \to +\infty$$

$$u \to +\infty$$

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x} = \lim_{u\to+\infty}\left(1+\frac{1}{u}\right)^{2u} = e^{2}$$

$$\lim_{x\to+\infty}\left(1+\frac{2}{x}\right)^{x+1} = e^{2}\cdot\overbrace{\lim_{x\to+\infty}\left(1+\frac{2}{x}\right)}^{1} = e^{2}$$

$$\|$$

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}\cdot\lim_{x\to+\infty}\left(1+\overset{0}{\frac{2}{x}}\right)$$