$f(x) = \operatorname{sen} x$

Encontrar sen 1 com erro menor que 10^{-4}.

$f(x) = \operatorname{sen} x$ $\quad n \in \mathbb{N}$ $\quad f^{(4n)}(x) = \operatorname{sen} x$

$f'(x) = \cos x$ $\quad f^{(4n+1)}(x) = \cos x$

$f''(x) = -\operatorname{sen} x$ $\quad f^{(4n+2)}(x) = -\operatorname{sen} x$

$f'''(x) = -\cos x$ $\quad f^{(4n+3)}(x) = -\cos x$

$f^{(4)}(x) = \operatorname{sen} x$

$f^{(4n)}(0) = 0$

$f^{(4n+1)}(0) = 1$

$f^{(4n+2)}(0) = 0$

$f^{(4n+3)}(0) = -1$

$$\boxed{f(x) = \sum_{n=0}^{k} \frac{f^{(n)}(0)}{n!} x^n + \frac{f^{(k+1)}(\bar{x})}{(k+1)!} x^{k+1}}$$

$\exists \bar{x}$ entre 0 e x

k impar $\rightarrow$ $k = 2l+1$

$k = 2l+2$

$$\operatorname{sen} x = \left(\frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{(-1)^l x^{2l+1}}{(2l+1)!} \right) + \underline{\frac{f^{(2l+2)}(\bar{x})}{(2l+2)!} x^{2l+2}}$$

x entre 0 e 1

$$\operatorname{sen} 1 = \left(1 - \underbrace{\frac{1}{3!}}_{=k} + \frac{1}{5!} + \dots + \frac{(-1)^{\ell}}{(2\ell+1)!}\right) + \frac{f^{(2\ell+2)}(\bar{x})}{(2\ell+2)!}$$

550

0,001

Passa p/ outro lado e toma o módulo

$$\left|\operatorname{sen} 1 - \left(1 - \frac{1}{3!} + \dots + \frac{(-1)^{\ell}}{(2\ell+1)!}\right)\right| = \left|\frac{f^{(2\ell+2)}(\bar{x})}{(2\ell+2)!}\right| \leq \underbrace{\frac{1}{(2\ell+2)!}}_{} < 10^{-4}$$

$$\frac{1}{(2\ell+2)!} < 10^{-4} \iff 10^{4} < \underbrace{(2\ell+2)!}$$

$\ell = 3$ polinômio de Taylor de grau 7

$$(2 \cdot 3 + 2)! = 8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot \underbrace{4 \cdot 3 \cdot 2}$$

$\ell = 4$ Polinômio de Taylor de grau 9

$$(2 \cdot 4 + 2)! = 10! = \underline{10 \cdot 9} \cdot \underline{8} \cdot \underline{7} \cdot 6 \cdot 5 \cdot \underbrace{4 \cdot 3 \cdot 2} > 10^{6}$$

$$\therefore \boxed{10^{-6} < \frac{1}{10!}}$$

$0{,}841502$

$$\left| \operatorname{sen} 1 - \underbrace{\left(1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!}\right)}_{p} \right| < \cancel{10^{-5}} \; 10^{-6}$$

$0{,}841503 - 10^{-6}$

$< p - 10^{-5}$

$< \operatorname{sen} 1$

$< p + 10^{-5}$

$< 0{,}841507 + 10^{-6}$

$< 0{,}841508$

$\frac{1}{3!} = 0{,}166666\ldots$

$\frac{1}{5!} = 0{,}008333\ldots$

$\frac{1}{7!} = 0{,}000194\ldots$

$\frac{1}{9!} = 0{,}000002\ldots$

$1 \le 1 \le 1$

$-0{,}166667 \le -\frac{1}{3!} \le -0{,}166666$

$0{,}008333 \le \frac{1}{5!} \le 0{,}008334$

$-0{,}000195 \le -\frac{1}{7!} \le -0{,}000194$

$0{,}000002 \le \frac{1}{9!} \le 0{,}000003$

somando temos

$$0{,}841503 \le \underbrace{1 - \frac{1}{3!} + \frac{1}{5!} - \frac{1}{7!} + \frac{1}{9!}}_{p} \le 0{,}841507$$

$|\operatorname{sen} 1 - p| < 10^{-6}$

$p - 10^{-6} < \operatorname{sen} 1 < p + 10^{-6}$

$f(x) = \operatorname{arctg} x$

$f'(x) = \frac{1}{1+x^2}$

$f''(x) = -\frac{2x}{(1+x^2)^2}$

$f'''(x) = \frac{-2(1+x^2)^2 + (2x).2(1+x^2).2x}{(1+x^2)^4}$

$= -\frac{2(1+x^2) + 8x^2}{(1+x^2)^3} = \frac{-2+6x^2}{(1+x^2)^3}$

$\frac{1}{2n+1}$

achar série de Taylor

$\operatorname{arctg} x = \sum_{n=0}^{+\infty} a_n x^n$

$\frac{\pi}{4} = \operatorname{arctg} 1 = \sum_{n=0}^{+\infty} a_n$

$\pi = 4 \cdot \sum_{n=0}^{+\infty} a_n$

$\frac{\operatorname{arctg}\frac{1}{2}}{\operatorname{arctg}\frac{1}{2}}$

$\frac{1}{(2n+1)2^n}$

$\frac{1}{(2n+1)3^n}$

$$\operatorname{arctg} x = \sum_{n=1}^{+\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad x \in]-1, 1]$$

$$\boxed{\operatorname{arctg} 1 = \sum_{n=1}^{+\infty} \frac{(-1)^n}{2n+1}}$$

série alternée

$$\pi = 4 \sum_{n=1}^{+\infty} \frac{(-1)^n}{2n+1}$$

$$10^{-3} > \frac{1}{2k+3}$$

$$\iff 2k+3 > 10^3$$

$$\iff \frac{10^3 - 3}{2}$$

$$\pi = 4 \sum_{n=1}^{k} \frac{(-1)^n}{2n+1} + \sum_{n=k+1}^{+\infty} \frac{(-1)^n}{2n+1}$$

$$\left| \pi - 4 \sum_{n=1}^{k} \frac{(-1)}{2n+1} \right| \leq \left| \sum_{n=k+1}^{+\infty} \frac{(-1)^n}{2n+1} \right| \leq \left| \frac{(-1)^{k+1}}{2k+3} \right| = \frac{1}{2k+3}$$

$$\operatorname{tg}(x+y)=\frac{\sin(x+y)}{\cos(x+y)}$$

$$=\frac{\sin x\cos y+\sin y\cos x}{\cos x\cos y-\sin x\sin y}$$

$$=\frac{\cos x\,(\operatorname{tg}x\cos y+\sin y)}{\cos y\,(\cos x-\sin x\operatorname{tg}y)}$$

$$=\frac{\cancel{\cos x}\,\cancel{\cos y}\,(\operatorname{tg}x+\operatorname{tg}y)}{\cancel{\cos y}\,\cancel{\cos x}\,(1-\operatorname{tg}x\operatorname{tg}y)}$$

$$\operatorname{tg}(x+y)=\frac{\operatorname{tg}x+\operatorname{tg}y}{1-\operatorname{tg}x\operatorname{tg}y}$$

$$x=\operatorname{arctg}\frac{1}{2}$$

$$y=\operatorname{arctg}\frac{1}{3}$$

$$\operatorname{tg}\left(\operatorname{arctg}\frac{1}{2}+\operatorname{arctg}\frac{1}{3}\right)=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}=\frac{\frac{5}{6}}{\frac{5}{6}}=1$$

$$\Rightarrow \boxed{\operatorname{arctg} 1 = \underline{\operatorname{arctg}\frac{1}{2}} + \underline{\operatorname{arctg}\frac{1}{3}}}$$

série alternée

$$= \sum_{n=1}^{+\infty} \frac{(-1)^n}{2n+1}\left(\frac{1}{2}\right)^{2n+1} + \sum_{n=1}^{+\infty} \frac{(-1)^n}{2n+1}\left(\frac{1}{3}\right)^{2n+1}$$

$$\pi = 4\sum_{n=1}^{+\infty} \frac{(-1)^n}{2n+1}\left(\frac{1}{2}\right)^{2n+1} + 4\sum_{n=1}^{+\infty} \frac{(-1)^n}{2n+1}\left(\frac{1}{3}\right)^{2n+1}$$

$$\left|\pi - \left(4\sum_{n=1}^{k_1} \frac{(-1)^n}{2n+1}\left(\frac{1}{2}\right)^{2n+1} + 4\sum_{n=1}^{k_2} \frac{(-1)^n}{2n+1}\left(\frac{1}{3}\right)^{2n+1}\right)\right|$$

$$\leq \left|4\sum_{n=k_1+1}^{+\infty} \frac{(-1)^n}{2n+1}\left(\frac{1}{2}\right)^{2n+1}\right| + \left|4\sum_{n=k_2+1}^{+\infty} \frac{(-1)^n}{2n+1}\left(\frac{1}{3}\right)^{2n+1}\right| <$$

$$\leq 4\cdot\frac{1}{2k_1+3}\cdot\frac{1}{(2)^{2k_1+3}} + 4\cdot\frac{1}{(2k_2+3)}\cdot\frac{1}{3^{2k_2+3}} < 10^{-3}$$

$$\pi = 4\sum_{n=1}^{+\infty} a_n + 4\sum_{n=1}^{+\infty} b_n = 4\sum_{n=1}^{k_1} a_n + 4\sum_{n=k_1+1}^{+\infty} a_n + 4\sum_{n=1}^{k_2} b_n + 4\sum_{n=k_2+1}^{+\infty} b_n$$

$$\pi - 4\sum_{n=1}^{k_1} a_n - 4\sum_{n=1}^{k_2} b_n = 4\sum_{n=k_1+1}^{+\infty} a_n + 4\sum_{n=k_2+1}^{+\infty} b_n$$

$$\left|\pi - \left(4\sum_{n=1}^{k_1} a_n + 4\sum_{n=1}^{k_2} b_n\right)\right| = \left|4\sum_{n=k_1+1}^{+\infty} a_n + 4\sum_{n=k_2+1}^{+\infty} b_n\right|$$

$$\leq \left|4\sum_{n=k_1+1}^{+\infty} a_n\right| + \left|4\sum_{n=k_2+1}^{+\infty} b_n\right|$$

$K_2 = 5$

$2K_2 + 3 = 13$

$4 \cdot \frac{1}{13} \cdot \frac{1}{3^{13}}$

8×4

3^9

3^6

9^3

$2^{10} > 10^3$

$\frac{13 \cdot 3^{13}}{4} > 3^{14} = 3^6 \cdot 3^6 \cdot 9$

$> 36 \cdot 10^4$

81×9

729

$\boxed{K_2 = 5 \quad \frac{4}{2K_2+3} \cdot \frac{1}{(3)^{2K_2+3}} < \frac{1}{36} 10^{-4}}$

$\frac{1}{36} 10^{-4} + \frac{1}{9} 10^{-4}$

$\frac{1+4}{36} \leq \frac{5}{36} \cdot 10^{-4} < 10^{-4}$

$2^4 = 16$

$2^8 \quad 16 \times 16 = 96 \quad 16 \quad 256$

$K_1 = 6$ $\quad 2K_1 + 3 = 15$

$4 \cdot \frac{1}{15} \cdot \frac{1}{3^{15}} < \frac{1}{9} \cdot 10^{-4}$

$\frac{15}{4} 3^{15} = \frac{15}{4} \cdot 2^{10} \cdot 2^5 > 3 \cdot 10^3 \cdot 30 \geq 9 \cdot 10^4$