Pg 243 (b)

11) f definida e derivável em $]-r, r[$, $r>0$. Suponha que

$f'(x) = x^2 + f^2(x)$ $\forall x \in]-r, r[$

$f(0) = 0$

a) Mostre que 0 é ponto de inflexão horizontal

b) Mostre que $f'(x) > 0$ $\forall x \neq 0$

c) Estude f com relação à concavidade.

d) Mostre que $f(x) > \frac{2}{3!} x^3$ para $0 < x < r$

e) Faça um esboço do gráfico de f.

a) $f'(0) = 0$ $\quad f'(x) \geq x^2 > 0$ $\forall x \neq 0$

$f''(x) = 2x + 2f(x)f'(x)$

$x < 0$

$f''(x) = \underbrace{2x}_{<0} + 2\underbrace{f(x)}_{<0}\underbrace{f'(x)}_{>0}$

< 0 $\quad < 0$

$f(x) < 0$

$x < 0$ pelo TVM

$f(x) < f(0)$

$\exists c$

$c \neq 0$

$0 < f'(c) = \frac{f(x) - f(0)}{x - 0}$

$= \frac{f(x)}{x}$

$\Rightarrow f(x) < 0$

$x > 0$ $\exists c$ entre 0 e x

$c > 0$

$0 < f'(c) = \frac{f(x) - f(0)}{x - 0} = \frac{f(x)}{x}$

$\Rightarrow f(x) > 0$

$x > 0$, $f''(x) = \underbrace{2x}_{>0} + 2\underbrace{f(x)}_{>0} \cdot \underbrace{f'(x)}_{>0}$

> 0

> 0

		0	
f'	+	0	+
f''	−	0	+

(a) P.I. (c)

$g(x) = \frac{2}{3!} x^3$ $\quad g'(x) = \frac{\cancel{3} \cdot 2}{\cancel{3} \cdot 2!} x^2 = x^2$

$f'(x) = x^2 + f^2(x) > x^2 = g'(x)$

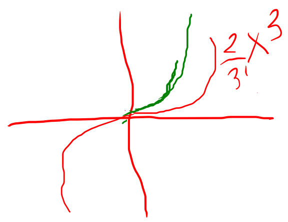
$\forall x > 0$

$f(0) = g(0)$ TVM

$f'(x) > g'(x) \Rightarrow$

$f(x) > g(x)$

$\forall x < r$



273. 19. Esboçar $y = \dfrac{x^2 - x + 1}{x^2}$ Dom $y = \mathbb{R} \setminus \{0\}$

$$\lim_{x \to -\infty} y(x) = \lim_{x \to -\infty} \frac{x^2 - x + 1}{x^2} = \lim_{x \to -\infty} \frac{\cancel{x^2}\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)}{\cancel{x^2}} = 1$$

$$\lim_{x \to +\infty} y(x) = \lim_{x \to +\infty} \frac{\cancel{x^2}\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)}{\cancel{x^2}} = 1$$

$$\lim_{x \to 0^-} y(x) = \lim_{x \to 0^-} \frac{x^2 - x + 1}{x^2} = +\infty$$

0^+

$$\lim_{x \to 0^+} y(x) = \lim_{x \to 0^+} \frac{x^2 - x + 1}{x^2} = +\infty$$

0^+

$$f(2) = \frac{3}{4} = \frac{27}{36}$$

$$f(3) = \frac{9 - 3 + 1}{9} = \frac{7}{9} = \frac{28}{36}$$

$$y' = \frac{(2x-1)x^2 - (x^2 - x + 1) \cdot 2x}{x^4}$$

$$= \frac{\cancel{2x^3} - x^2 - \cancel{2x^3} + 2x^2 - 2x}{x^4} = \frac{x^2 - 2x}{x^4} = \frac{x - 2}{x^3}$$

$x - 2 = 0$
$\Leftrightarrow x = 2$

$x^3 = 0$
$\Leftrightarrow x = 0$

$$y'' = \frac{1 \cdot x^3 - (x-2) \cdot 3x^2}{x^6} = \frac{x^3 - 3x^3 + 6x^2}{x^6} = \frac{-2x^3 + 6x^2}{x^6}$$

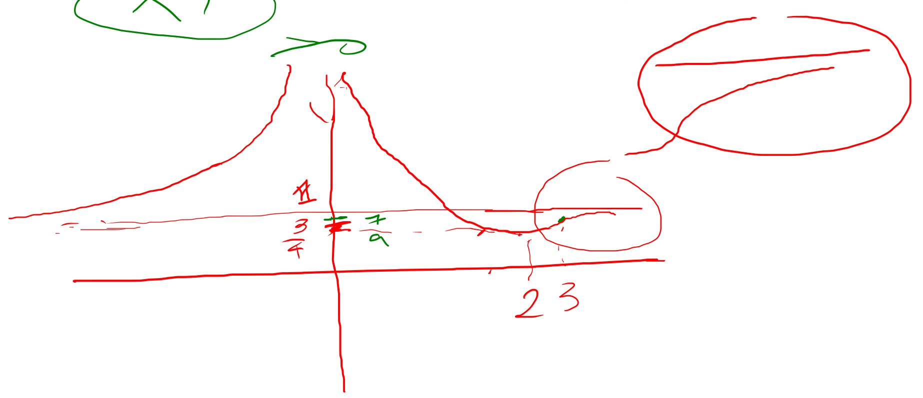
$$= \frac{2x^2(-x+3)}{x^6} = \frac{2(-x+3)}{x^4}$$

$-x + 3 = 0$
$\Leftrightarrow x = 3$

$x^4 > 0$

		0		2		3	
$x-2$	-	-	-	0	+	+	+
x^3	-	0	+	+	+	+	+
f'	+	∄	-	0	+	+	+
f''	+	∄	+	+	+	0	-
	∪		∪		∪		∩

P.I.



3. $y = \sqrt{x^2-4}$

$x \in \mathrm{Dom}\,y \iff x^2-4 \ge 0$

$\iff (x-2)(x+2) \ge 0$

$\iff x \ge 2$ ou $x \le -2$

$y(-2)=0$

$y(2)=0$

$y' = \frac{1}{2} \cdot \frac{1}{\sqrt{x^2-4}} \cdot 2x = \frac{x}{\sqrt{x^2-4}} > 0$

$x>2$
$x<-2$

$-2 \qquad 2$

$y'' = \frac{1 \cdot \sqrt{x^2-4} - x \, \frac{1}{2} \cdot \frac{1}{\sqrt{x^2-4}} \cdot 2x}{x^2-4}$

$\lim_{x \to +\infty} \sqrt{x^2-4} = +\infty$

$\lim_{x \to -\infty} \sqrt{x^2-4} = +\infty$

$= \frac{\sqrt{x^2-4} - \frac{x^2}{\sqrt{x^2-4}}}{x^2-4} = \frac{\cancel{x^2}-4-\cancel{x^2}}{(x^2-4)(\sqrt{x^2-4})} = \frac{-4}{(x^2-4)(\sqrt{x^2-4})} < 0$

$\forall x \in \mathrm{Dom}\,y$

x		-2		0		2	
x	$-$		$-$	0	$+$	$+$	$+$
y'	$-$	$\times$				$\times$	$+$
-4	$-$		$-$	$-$	$-$		$-$
y''	$-$	$\times$				$\times$	$-$
y	$\searrow$						$\nearrow$

$- \; - \; - \; - \; - \; -$

$+ \; + \qquad\qquad +$

$- \; - \qquad\qquad -$

assíntota p/ $+\infty$

$$a = \lim_{x \to +\infty} \frac{y(x)}{x} = \lim_{x \to +\infty} \frac{\sqrt{x^2-4}}{x} = \lim_{x \to +\infty} \frac{|x|\sqrt{1-\frac{4}{x^2}}}{x}$$

$x>0$

$$= \lim_{x \to +\infty} \frac{\cancel{x}}{\cancel{x}} \cdot \underbrace{\sqrt{1-\frac{4}{x^2}}}_{\to 1} = 1$$

$$b = \lim_{x \to +\infty} y(x) - ax = \lim_{x \to +\infty} \frac{\left(\sqrt{x^2-4} - x\right)\left(\sqrt{x^2-4}+x\right)}{\left(\sqrt{x^2-4}+x\right)}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^2}-4-\cancel{x^2}}{|x|\sqrt{1-\frac{4}{x^2}}+x} \overset{x>0}{=} \lim_{x \to +\infty} \frac{-4}{x\sqrt{1-\frac{4}{x^2}}+x} =$$

$$\lim_{x \to +\infty} \frac{\overbrace{-4}^{<0}}{\underbrace{x}_{\to +\infty}\underbrace{\left(\sqrt{1-\frac{4}{x^2}}+1\right)}_{\to 2>0}} = 0^-$$

assíntota p/ $-\infty$

$$a = \lim_{x \to -\infty} \frac{\sqrt{x^2-4}}{x} = \lim_{x \to -\infty} \frac{|x|\sqrt{1-\frac{4}{x^2}}}{x} \to 1$$

$x<0$

$$= \lim_{x \to -\infty} \frac{(-\cancel{x})\sqrt{1-\frac{4}{x^2}}}{\cancel{x}} = -1$$

$u^2 - v^2 = (u-v)(u+v)$

$x^2 - 4 - x^2$

$u \qquad v$

$$b = \lim_{x \to -\infty} y(x) - ax = \lim_{x \to -\infty} \frac{(\sqrt{x^2-4}+x)(\sqrt{x^2-4}-x)}{(\sqrt{x^2-4}-x)}$$

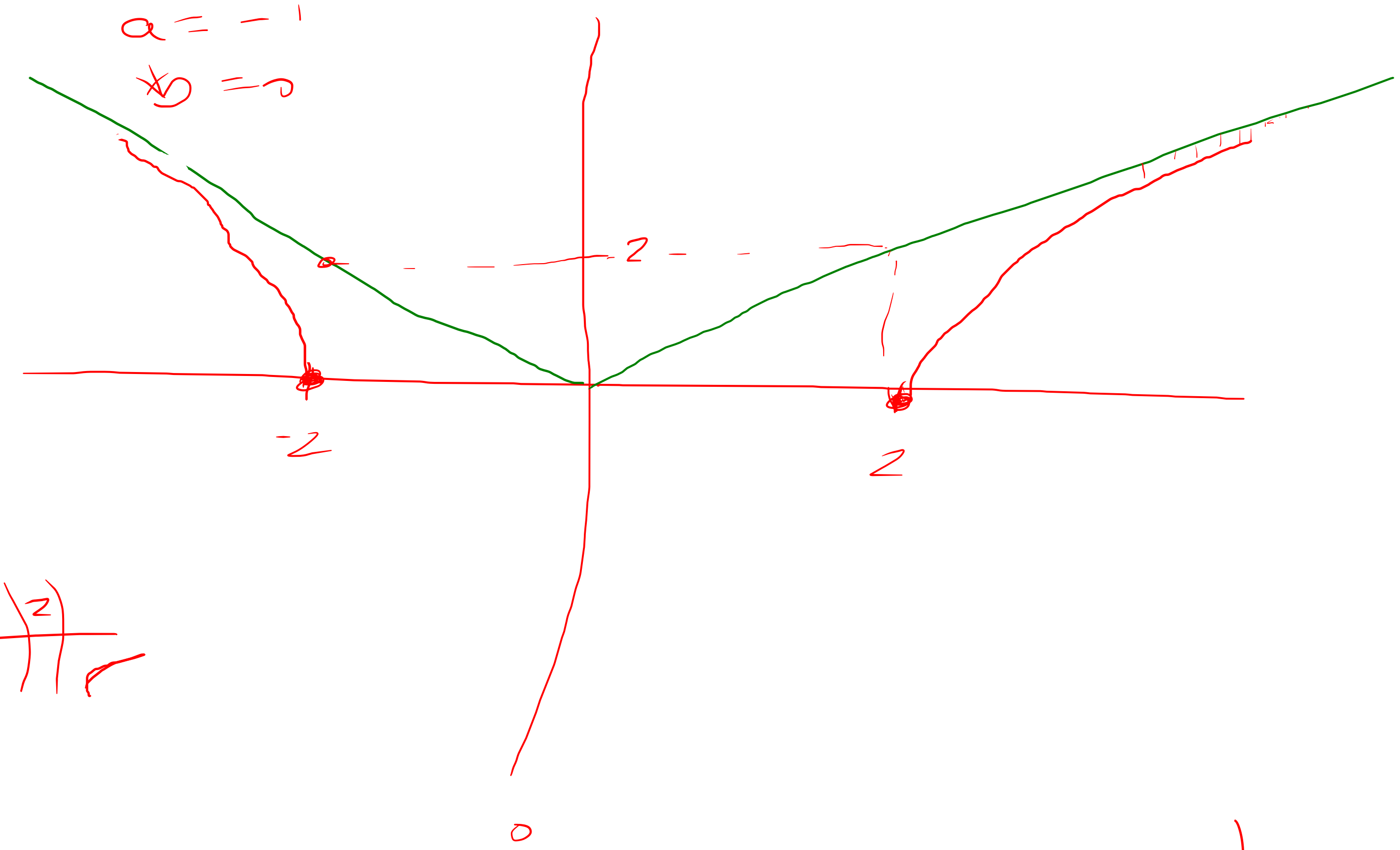
$x<0$

$$= \lim_{x \to -\infty} \frac{\cancel{x^2}-4-\cancel{x^2}}{|x|\sqrt{1-\frac{4}{x^2}}-x} = \lim_{x \to -\infty} \frac{-4}{-x\sqrt{1-\frac{4}{x^2}}-x} =$$

$$\lim_{x \to -\infty} \frac{\overset{<0}{-4}}{\underset{+\infty}{-x}\,\Big(\underset{2>0}{\sqrt{1-\frac{4}{x^2}}+1}\Big)} = 0^-$$

$a = -1$

$b = 0$



$a = 1$

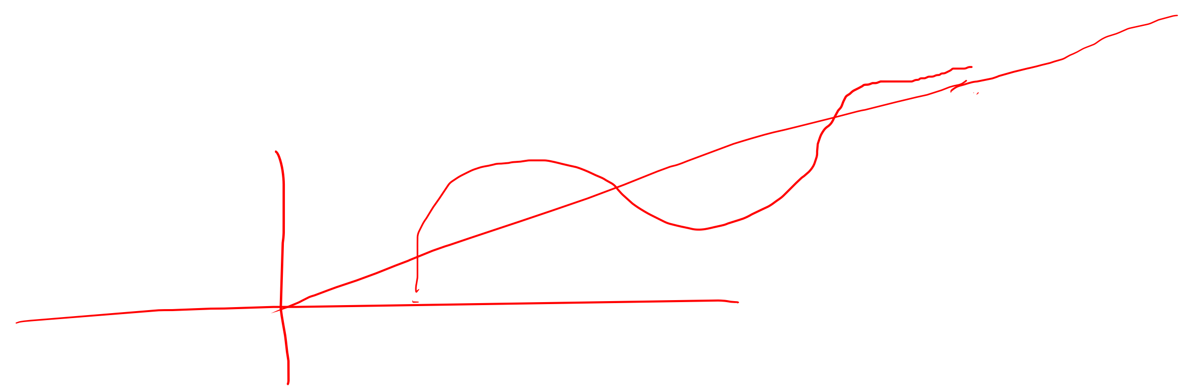
$b = 0$

$$\frac{x^2+\sin x}{x} = x + \frac{\sin x}{x}$$

$a = 1$

$b = 0$

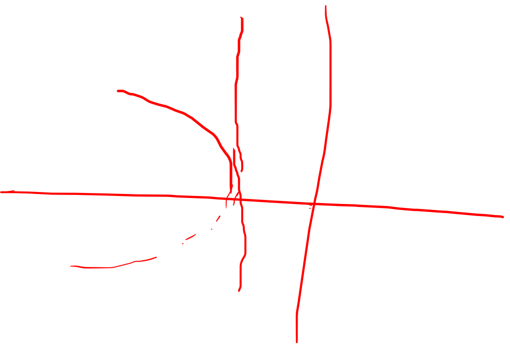
$y = x$



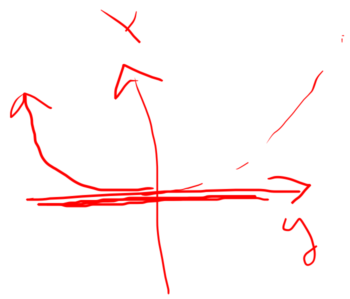
$$y = \sqrt{x^2 - 4} \qquad x \le -2$$

$$y^2 = x^2 - 4$$

$$y^2 + 4 = x^2$$



$$\boxed{x = -\sqrt{y^2 + 4}}$$ função inversa



$$\frac{dx}{dy} = -\frac{1}{2} \cdot \frac{1}{\sqrt{y^2+4}} \cdot 2y$$

$$= -\frac{y}{\sqrt{y^2+4}}$$

$$\frac{dx}{dy}(0) = 0$$

Pg 268

Exemplo 7

$$f(x) = \sqrt[3]{x^3 - x^2}$$

assíntota p/ $+\infty$

$$a = \lim_{x \to +\infty} \frac{f(x)}{x} = 1$$

$$b = \lim_{x \to +\infty} f(x) - 1 \cdot x = -\frac{1}{3}$$

$y = x - \frac{1}{3}$ assíntota p/ $+\infty$

Para $-\infty$ $y = x - \frac{1}{3}$ assíntota p/ $-\infty$

		0		$\frac{2}{3}$				1	
f'	+		−		+		+		+
f	↗		↘		↗		↗		↗
	+		+		+		+		−

		0		1	
f''	+		+		

$f(x) = \sqrt[3]{x^3 - x^2}$

$f(0) = 0$

$f\left(\frac{2}{3}\right) = \sqrt[3]{\frac{8}{27} - \frac{4}{9}} = \frac{\sqrt[3]{-4}}{3} = -\frac{\sqrt[3]{4}}{3}$

$f(1) = 0$