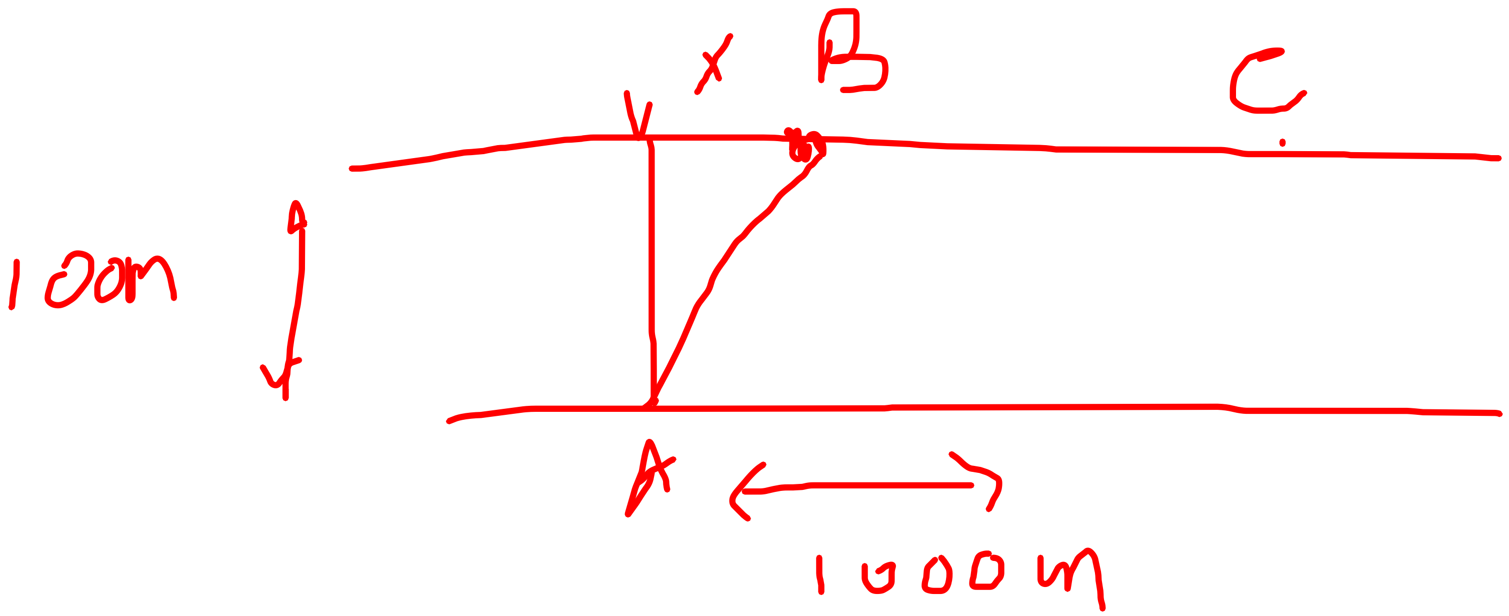


$$0 \le x \le 1000$$

$$f(x) = \frac{\sqrt{100^2 + x^2}}{10} + \frac{1000 - x}{15}$$

$$f'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{100^2 + x^2}} \cdot 2x \cdot \frac{1}{10} - \frac{1}{15} = 0$$

$$\Leftrightarrow \frac{x}{10} = \frac{1}{15} \cdot \sqrt{100^2 + x^2}$$

$x > 0$

$\iff \frac{3}{2}x = \sqrt{100^2 + x^2} \iff \frac{9}{4}x^2 = 100^2 + x^2$

$\iff \frac{5}{4}x^2 = 100^2 \iff x = \frac{2}{\sqrt{5}}100 = \boxed{\frac{200}{\sqrt{5}}}$

$\frac{\sqrt{2}\cdot 100}{15} < 10$

$\iff \sqrt{2}\cdot 10 < 15$

$\iff 2\cdot 10^2 < 15^2$

$\iff 200 < 225$ ✓

$f'(x) = \dfrac{\frac{1}{10}x - \frac{\sqrt{100^2+x^2}}{15}}{\sqrt{100^2+x^2}} > 0$

$100 \qquad \frac{200}{\sqrt{5}}$

x	0		$\frac{200}{\sqrt{5}}$		100
$f'(x)$		$-$	0	$+$	$+$
		↘		↗	

$f'(10) < 0$

$f'(100) = \dfrac{10 - \frac{\sqrt{2}\cdot 100}{15}}{\sqrt{100^2+100^2}} > 0$

Pg 277 17

semi esfera de raio r

$S(r,h) = 2\pi r^2 + \pi r^2 + 2\pi r h = 5\pi$

área da semi-esfera

$h > 0 \quad 5 - 3r^2 > 0$

$r^2 < \frac{5}{3}$

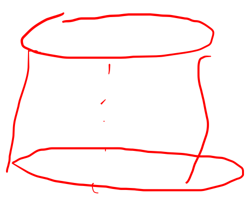
cilindro circular reto

área da sup do sólido é 5π.

$V(r,h) = \frac{2}{3}\pi r^3 + \pi r^2 \cdot h$

Vol semi esfera

Vol cilindro



Determine r e h para que o volume seja máximo.

$0 < r < \sqrt{\frac{5}{3}}$

$h = \frac{5\pi - 3\pi r^2}{2\pi r} = \frac{5 - 3r^2}{2r}$

$V(r) = \frac{2}{3}\pi r^3 + \pi r^2 \cdot \left(\frac{5 - 3r^2}{2r}\right) = \frac{2}{3}\pi r^3 + \frac{\pi}{2}(5r - 3r^3)$

$$V(r) = \frac{2}{3}\pi r^3 + \frac{5}{2}\pi r - \frac{3}{2}\pi r^3 = \left(\frac{4-9}{6}\right)\pi r^3 + \frac{5}{2}\pi r$$

$$= -\frac{5}{6}\pi r^3 + \frac{5}{2}\pi r$$

$$V'(r) = -\frac{5}{2}\pi r^2 + \frac{5}{2}\pi = 0$$

$$\Leftrightarrow \frac{5}{2}\pi\left(-r^2+1\right) = 0 \Leftrightarrow r = \pm 1$$

	-1		0		1		$\frac{5}{3}$	
$-r^2+1$	$-$	0	$+$	$+$	0	$-$	$-$	$-$
V'				$+$	0	$-$		

$\nearrow$ $\searrow$

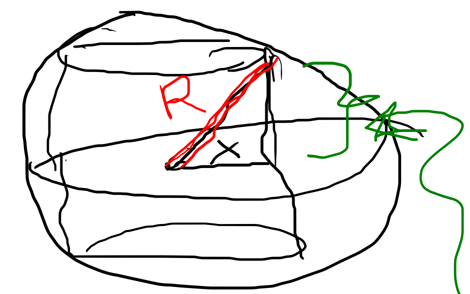
Ponto de máximo

Volume máximo

$r=1$

$h=1$ (usando a fórmula)

5. Determine a altura do cilindro circular reto de volume máximo inscrito na esfera de raio R dado



$0 < x < R$

$$\boxed{h(x) = 2\sqrt{R^2 - x^2}}$$

dobro de

$$V(x) = \pi x^2 \cdot 2\sqrt{R^2 - x^2}$$

$$V'(x) = 4\pi x\sqrt{R^2 - x^2} + \pi x^2 \cdot \cancel{2} \cdot \frac{1}{\cancel{2}} \frac{1}{\sqrt{R^2 - x^2}} \cdot -2x$$

$$= 4\pi x\sqrt{R^2 - x^2} - 2x^3\pi \frac{1}{\sqrt{R^2 - x^2}}$$

$$V'(x) = \frac{4\pi x(R^2 - x^2) - 2x^3\pi}{\sqrt{R^2 - x^2}} = \frac{2\pi\left[2xR^2 - 2x^3 - x^3\right]}{\sqrt{R^2 - x^2}} = \overbrace{\frac{2\pi}{\sqrt{R^2 - x^2}}}^{>0} \underbrace{\left[2xR^2 - 3x^3\right]}_{*}$$

$$2xR^2 - 3x^3 = 0 \iff x(2R^2 - 3x^2) = 0$$

$$\iff x = 0 \text{ ou } 3x^2 = 2R^2 \iff x = \pm\sqrt{\tfrac{2}{3}}R$$

		$-\sqrt{\frac{2}{3}}R$		0		$\sqrt{\frac{2}{3}}R$		R	
x	−	−	−	0	+	+	+	+	
$2R^2 - 3x^2$	−	0	+	+	+	0	−	−	−
$V'(x)$					+	0	−		

Ponto de máximo

$V\left(\sqrt{\frac{2}{3}}R\right)$ é o valor máximo $\Rightarrow$

$$h = 2\sqrt{R^2 - \tfrac{2}{3}R^2} = 2\sqrt{\tfrac{1}{3}R^2} = \frac{2}{\sqrt{3}}R$$

19. Preço unitário p — q quantidade demandada

$L(4) = 4 \cdot 4 - 3{,}5 \cdot 4$
$= 16 - 14 = 2$

$p = \sqrt{20-q}$ $\qquad 0 \leq q \leq 20$

$L(20) = 0 - 3{,}5 \cdot 20$
< 0

Para produzir e vender uma unidade gasta em torno R\$ 3,50

Qual a quantidade a ser produzida para que o lucro seja máximo

lucro
$0 \leq q \leq 20$

$L(q) = \sqrt{20-q} \cdot q - 3{,}5q$

$L'(q) = \frac{1}{2} \frac{1}{\sqrt{20-q}} \cdot (-1) \cdot q + \sqrt{20-q} \cdot 1 - 3{,}5$

$L'(q) = 0$

$\Leftrightarrow$

$$\frac{-q + 2(20-q) - 7\sqrt{20-q}}{2\sqrt{20-q}} = 0$$

$\Leftrightarrow -q + 40 - 2q - 7\sqrt{20-q} = 0$

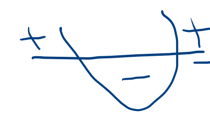
$\Leftrightarrow -3q + 40 = 7\sqrt{20-q}$

$\Leftrightarrow 9q^2 - 240q + 1600 = 49(20-q) = 980 - 49q$

$\Leftrightarrow 9q^2 - 191q + 620 = 0$

$$\begin{array}{r} 191 \\ \times 191 \\ \hline 191 \\ 191 \\ 19 \\ \hline 36481 \end{array} \qquad \begin{array}{r} 620 \\ \times 36 \\ \hline 22320 \end{array}$$

$$q = \frac{191 \pm \sqrt{36481 - 22320}}{18} = \frac{191 \pm \sqrt{14161}}{18}$$



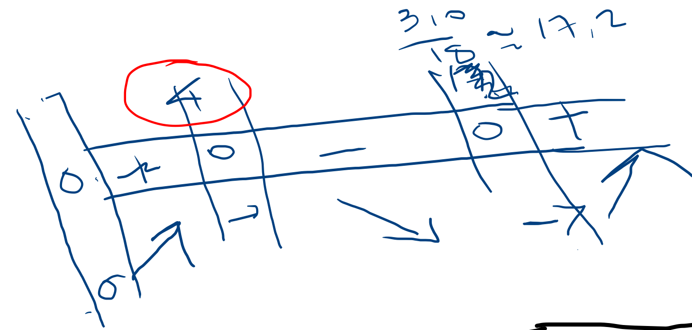
candidatos a máximo 4 e 20

$L(4) > L(20)$

$\frac{191 - 119}{18} = \frac{72}{18} = 4$

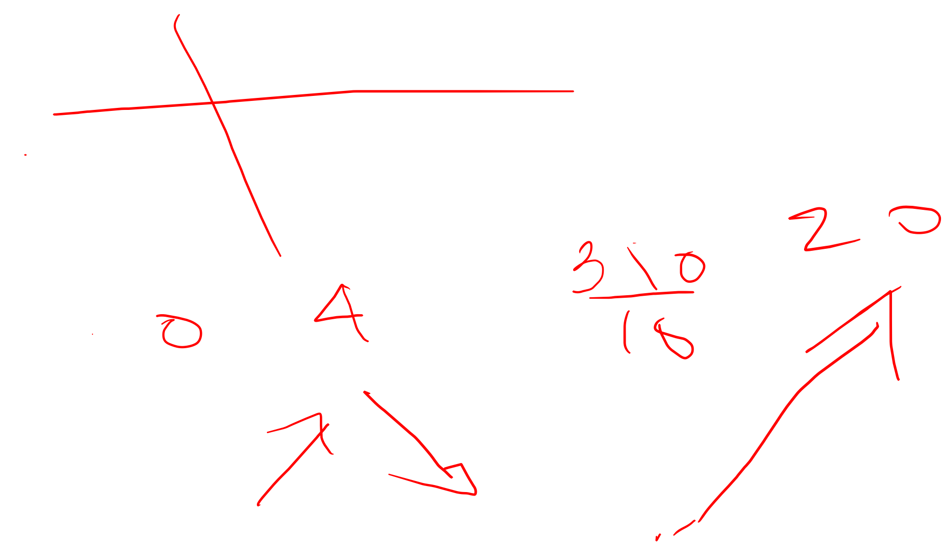
$\frac{191 + 119}{18} = \frac{310}{18} = 17{,}2$

$\frac{310}{18} \approx 17{,}2$



4 e 20

$f(4) > f(20)$

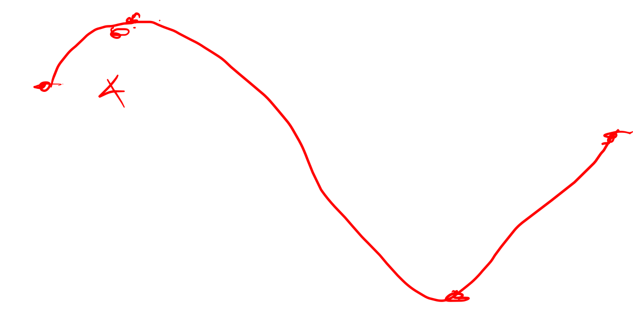


o lucro

é máximo

ótimo com $q = 4$.

$\boxed{\text{Polinômio (Taylor)}}$



Senos e cossenos

(Fourier)

arctg x