$$\lim_{x\to 0} \frac{f(x)}{x} = 1$$

$$\lim_{x\to 1} \frac{f(x^2-1)}{x-1} \cdot \frac{(x+1)}{(x+1)}$$

$u \leftarrow$

$u = x-1$

$$\lim_{x\to 1} \underbrace{\frac{f(x^2-1)}{x^2-1}}_{\to 1} \; \underbrace{\frac{1}{x+1}}_{\to \frac{1}{2}}$$

$$\lim_{x\to 1} \frac{f(x^2-1)}{x^2-1} = 1$$

~~9~~.2.~~7~~

$$f(x) = \begin{cases} e^{\frac{-1}{x^2}} & \text{se } x \neq 0 \\ 0 & \text{se } x = 0 \end{cases}$$

$\boxed{3 > e > 2}$

a) calcule $f'(0)$

b) Determine f'

c) Esboce o gráfico esboçando os limites necessários

$e^u \quad u$

$\frac{u}{e^u} \to 0$

a) $$f'(0) = \lim_{h \to 0} \frac{f(h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{e^{-\frac{1}{h^2}} - 0}{h} = \lim_{h \to 0} \frac{e^{-\frac{1}{h^2}}}{h}$$

$$e^u = \sum_{n=0}^{+\infty} \frac{u^n}{n!}$$

$$e^u > \sum_{n=0}^{k} \frac{u^n}{n!}$$

$$\frac{1}{u} \to 0 \qquad \frac{u^2}{e^{u^2}} \to 0$$

$$\lim_{h \to 0^+} \frac{e^{-\frac{1}{h^2}}}{h} = \lim_{u \to +\infty} u e^{-u^2} = \lim_{u \to +\infty} \frac{u}{e^{u^2}} = 0$$

$\frac{1}{h} = u$

$$\lim_{h\to 0^-}\frac{e^{-\frac{1}{h^2}}}{h} = -\lim_{u\to 0^+}\frac{e^{-\frac{1}{u^2}}}{u}$$

$u=-h$

$$= -\underbrace{\lim_{u\to 0^+}\frac{e^{-\frac{1}{u^2}}}{u}} = -0 = 0$$

$$\therefore \boxed{\lim_{h\to 0}\frac{e^{-\frac{1}{h^2}}}{h} = 0}$$

$\therefore$ $f'(0)=0$

$$\left(-\frac{1}{x^2}\right)' = (-x^{-2})'$$

$$-1(-2)\cdot x^{-3}$$

b) $x\neq 0$ $\quad f'(x) = e^{-\frac{1}{x^2}}\cdot\left(-\frac{1}{x^2}\right)' = e^{-\frac{1}{x^2}}\cdot\frac{2}{x^3}$

		0	
$e^{-\frac{1}{x^2}}\cdot 2$	$+$		$+$
x^3	$-$	0	$+$
f'	$-$	0	$+$

$x^3=0$

$\Leftrightarrow x=0$

c) Esboce f

$$x \neq 0 \quad f''(x) = \frac{2\left[e^{-\frac{1}{x^2}}\left(\frac{2}{x^3}\right)\cdot x^3 - e^{-\frac{1}{x^2}}\, 3x^2\right]}{x^6}$$

$$= \frac{2e^{-\frac{1}{x^2}}}{x^6}\left[\underbrace{2 - 3x^2}\right] = 0 \iff 3x^2 = 2 \iff x = \pm\sqrt{\frac{2}{3}}$$

		$-\sqrt{\frac{2}{3}}$		0		$\sqrt{\frac{2}{3}}$	
x^3	$-$	$-$	$-$	0	$+$	$+$	$+$
f'	$-$	$-$	$-$	0	$+$	$+$	$+$
cres/dec	$\searrow$	$\searrow$	$\searrow$	$\rightarrow$	$\nearrow$	$\nearrow$	$\nearrow$
f''	$-$	0	$+$		$+$	0	$-$
	conc. baixo	PI			conc. cima	PI	

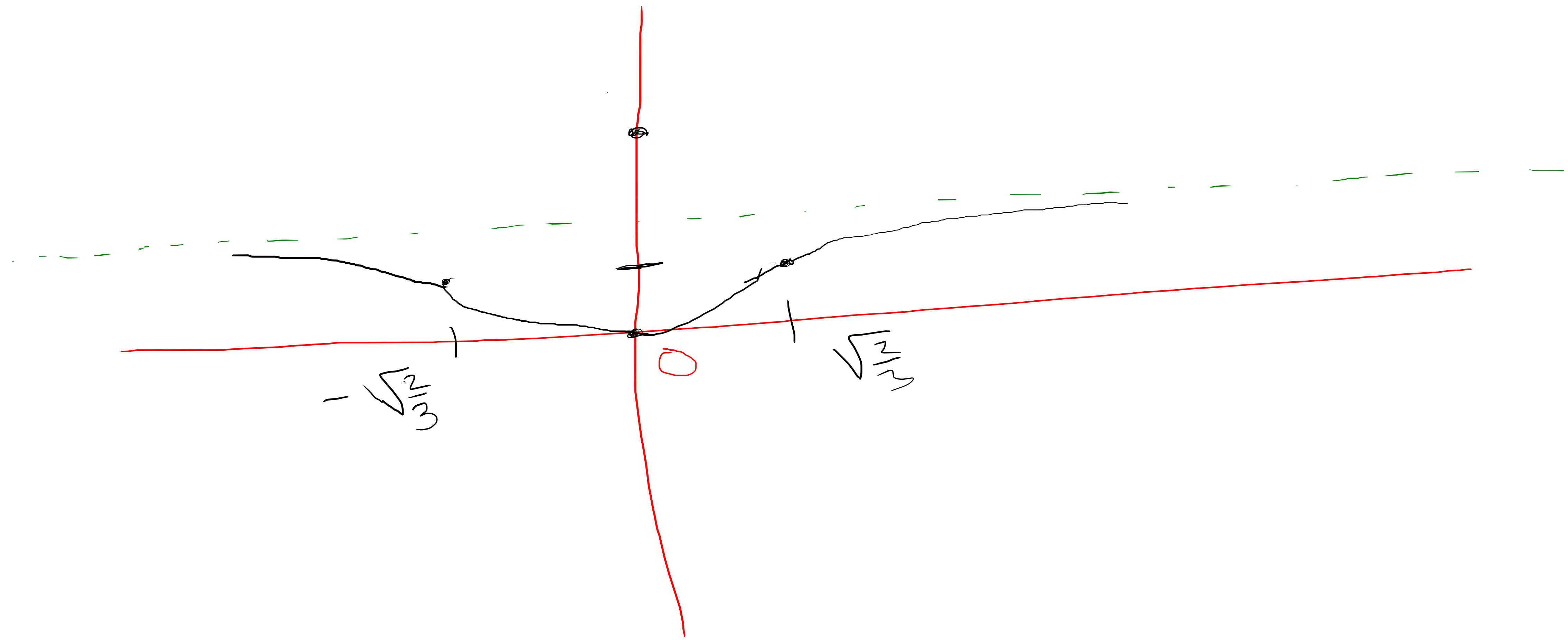
$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} e^{-\frac{1}{x^2}} = 1 \qquad -\frac{1}{x^2} \to 0^-$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} e^{-\frac{1}{x^2}} = 1 \qquad -\frac{1}{x^2} \to 0^-$$

$f(0) = 0$

$f\left(\sqrt{\tfrac{2}{3}}\right) = e^{-\frac{3}{2}}$

$f\left(-\sqrt{\tfrac{2}{3}}\right) = e^{-\frac{3}{2}}$



Mostre que $\underbrace{e^x}_{f(x)} > \underbrace{1 + x + \frac{x^2}{2} + \frac{x^3}{6}}_{g(x)}, \ \forall x > 0$

$\lim_{x \to +\infty} \frac{e^x}{x^2} = +\infty$

$\frac{e^x}{x^2} > \frac{x^3}{6} \cdot \frac{1}{x^2} = \frac{x}{6}$

$f'(x) = e^x$

$g'(x) = 1 + x + \frac{x^2}{2}$

$f''(x) = e^x$

$g''(x) = 1 + x$

$f'''(x) = e^x$

$g'''(x) = 1$

$\boxed{f'''(x) = e^x > 1 = g'''(x) \ \forall x > 0 \\ f''(0) = 1 = g''(0)}$ $\xRightarrow{TVM}$ $f''(x) > g''(x), \ \forall x > 0$

$\boxed{f''(x) > g''(x), \ \forall x > 0 \\ f'(0) = g'(0)}$ $\xRightarrow{TVM}$ $f'(x) > g'(x) \quad \forall x > 0$

$\boxed{f'(x) > g'(x) \ \forall x > 0 \\ f(0) = g(0)}$ $\xRightarrow{TVM}$ $f(x) > g(x) \ \forall x > 0$

12.6

$$\underbrace{\left|\sin x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!}\right)\right|}_{} < \boxed{\frac{|x|^7}{7!}}$$

$$\boxed{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} < \sin x \quad \forall x > 0}$$

TVM

q 3. (b)

estude a concavidade e ponto de inflexão de

$$f(x) = 2x^3 - x^2 - 4x + 1$$

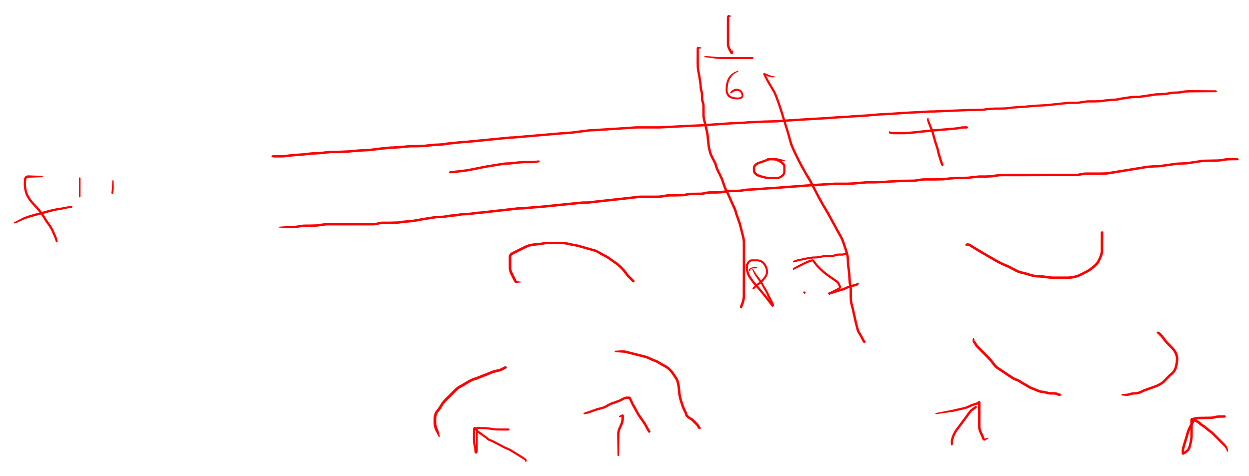
$$f'(x) = 6x^2 - 2x - 4$$

$$f''(x) = 12x - 2$$

$$f''(x) = 0 \iff \boxed{x = \frac{1}{6}}$$

$f''(x) < 0$
$\Rightarrow$ conc f
p/ baixo

$f''(x) > 0$
$\Rightarrow$ conc f
p/ cima



$x(3x+2)$

$g(x) = \sqrt[3]{x^3+x^2}$

$x \neq 0$ $\quad g'(x) = \left[(x^3+x^2)^{\frac{1}{3}}\right]' = \frac{1}{3}(x^3+x^2)^{-\frac{2}{3}} \cdot \underline{(3x^2+2x)}$

$x \neq -1$

$g'(0) = \lim_{h \to 0} \frac{\sqrt[3]{h^3+h^2} - 0}{h} = \lim_{h \to 0} \frac{\cancel{h}\sqrt[3]{1+\frac{1}{h}}}{\cancel{h}}$

$\therefore \nexists g'(0)$

$\lim_{h \to 0^-} \sqrt[3]{1+\frac{1}{h}} = -\infty$

$\lim_{h \to 0^+} \sqrt[3]{1+\frac{1}{h}} = +\infty$

$x \neq$

$x \neq 0$
$x \neq -1$

$$g''(x) = \frac{1}{3}\left[(x^3+x^2)^{-2/3} \cdot (3x^2+2x)\right]'$$

$$= \frac{1}{3}\left[-\frac{2}{3}(x^3+x^2)^{-5/3} \cdot (3x^2+2x)^2 + (x^3+x^2)^{-2/3} \cdot (6x+2)\right]$$

$$= \frac{1}{3}(x^3+x^2)^{-5/3}\left[-\frac{2}{3}(3x^2+2x)^2 + (x^3+x^2)(6x+2)\right]$$

$$= \frac{1}{3}(x^3+x^2)^{-5/3}\left[-\frac{2}{3}(9x^4+12x^3+4x^2) + (6x^4+2x^3+6x^3+2x^2)\right]$$

$$= \frac{1}{3}(x^3+x^2)^{-5/3}\left[\cancel{-6x^4} \cancel{-8x^3} - \frac{8}{3}x^2 \cancel{+6x^4} \cancel{+8x^3} + 2x^2\right]$$

$$= \frac{1}{3}(x^2)^{-5/3}(x+1)^{-5/3}\left(-\frac{2}{3}x^2\right)$$

		0		-1	
$x+1$	$-$	$-$	$-$	0	$+$
$-\frac{2}{3}x^2$	$-$	$-$	$-$	$-$	$-$
$f''(x)$	$+$	$\cancel{3}$	$+$	$\cancel{3}$	
	∪		∪	P.I	∩

		-1		$-\frac{2}{3}$		0	
den x^3+x^2	+	0	−	−	−	0	+
num $3x^2+2x$	+	+	+	0	−	0	+
f'	+		−	0	+		+
	↗		↘		↗		↗
$f''(x)$	+		+	+	+	−	−

pico

P.I

$g(-1)=0$

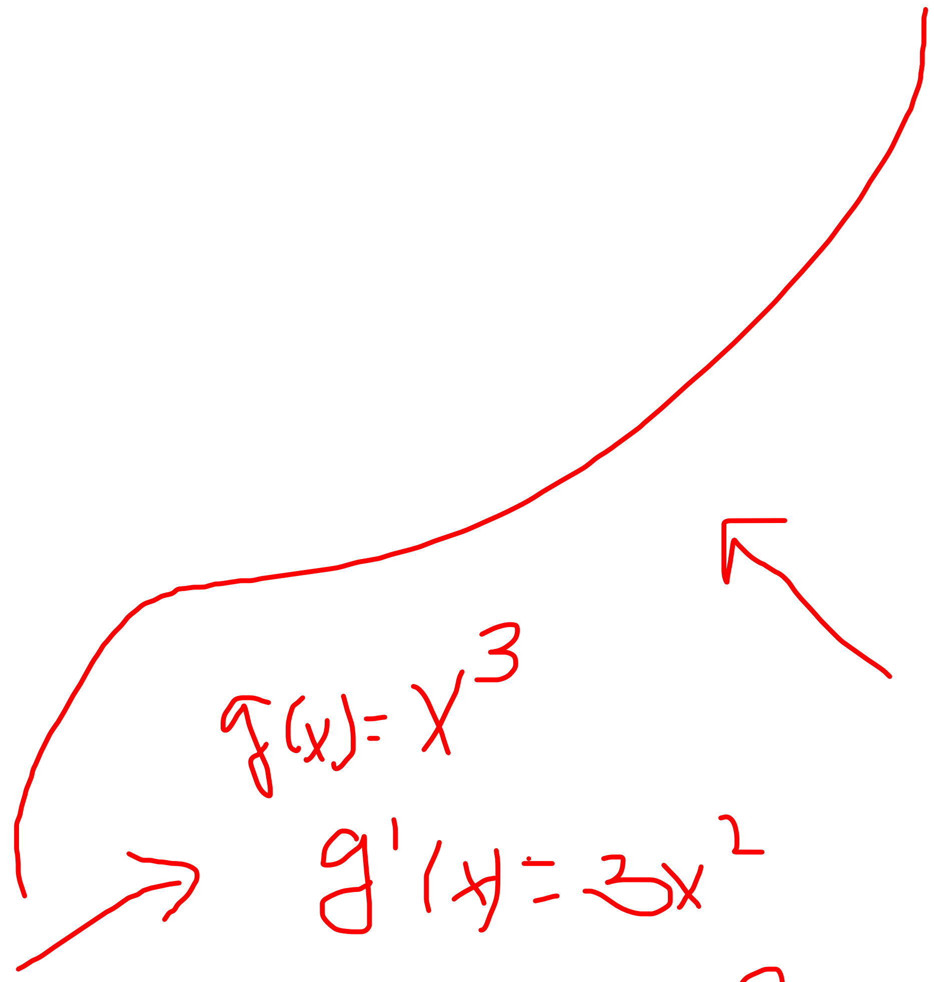
$g(0)=0$

$g\left(-\frac{2}{3}\right)=\sqrt[3]{-\frac{8}{27}+\frac{4}{9}}$

$=\frac{\sqrt[3]{4}}{3}$

$f'(x)=0$

Ponto critico

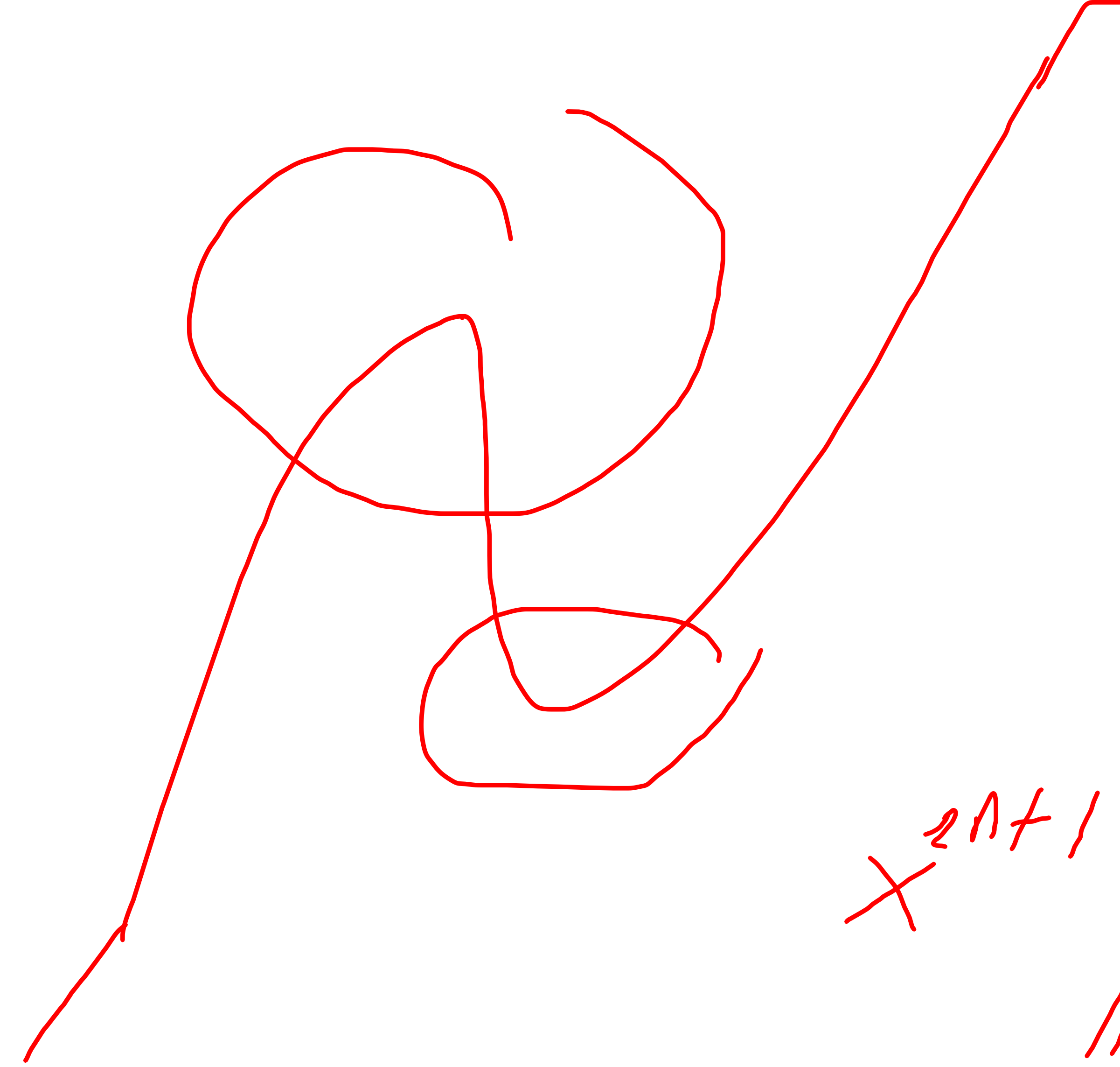


$g(x)=x^3$

$g'(x)=3x^2$

$h(x)=x^3+x$

$\rightarrow h'(x)=3x^2+1>0$



x^{2n+1}

$\mathbb{R}$