assintota $+\infty$ $(-\infty)$

$$a = \lim_{\substack{x \to +\infty \\ -\infty}} \frac{f(x)}{x}$$

$$b = \lim_{\substack{x \to +\infty \\ -\infty}} f(x) - ax$$

Se a e b existem e são reais

$y = ax + b$ é a assintota p $+\infty$ $(-\infty)$

$$f^{(n+1)}(x) = \left(f^{(n)}\right)'(x).$$

$$f \rightsquigarrow f' \rightsquigarrow f'' \ldots$$

dom f' não precisa ser igual a dom f.

Def: Dizemos que f é de classe C^n se $f^{(n)}$ é contínua

Funções contínuas são de classe C^0.

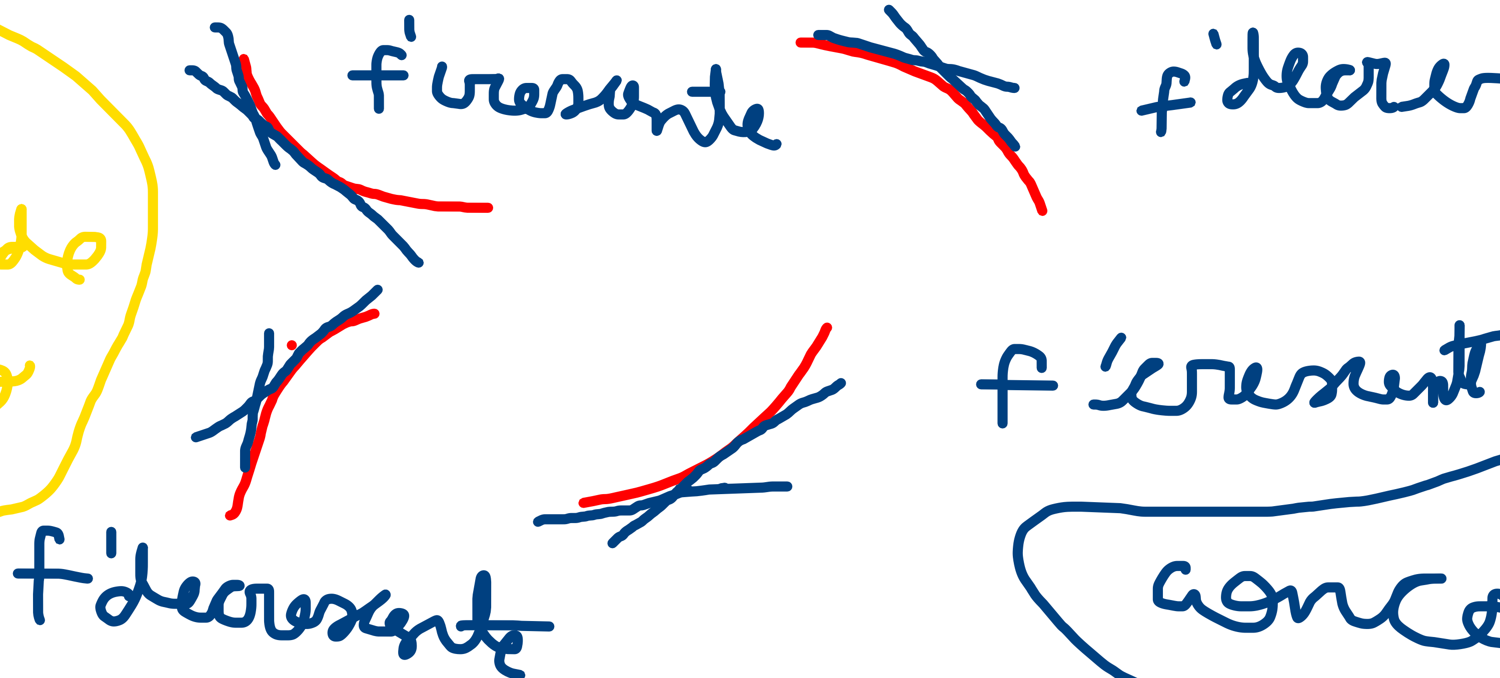
Def: Se f é de classe C^n $\forall n$ então f é de classe C^∞.

Polinomios, e^x, $\sin x$, $\cos x$ são de classe C^∞.

$f''(x) > 0 \quad \forall x \in]a,b[$

$\rightarrow f'(x)$ é crescente em $]a,b[$

$f''(x) < 0$ no int.
concavidade p/ baixo no int.



$f''(x) > 0$ no int.
concavidade p/ cima no int.

$$f(x) = \sqrt{x^2+1}$$

$$f'(x) = \frac{1}{2}\frac{1}{\sqrt{x^2+1}} \cdot 2x = \frac{x}{\sqrt{x^2+1}}$$

$=0 \Leftrightarrow x=0$

ass in totas $+\infty$

$$y = x$$

ass intota p/ $-\infty$

$$y = -x.$$

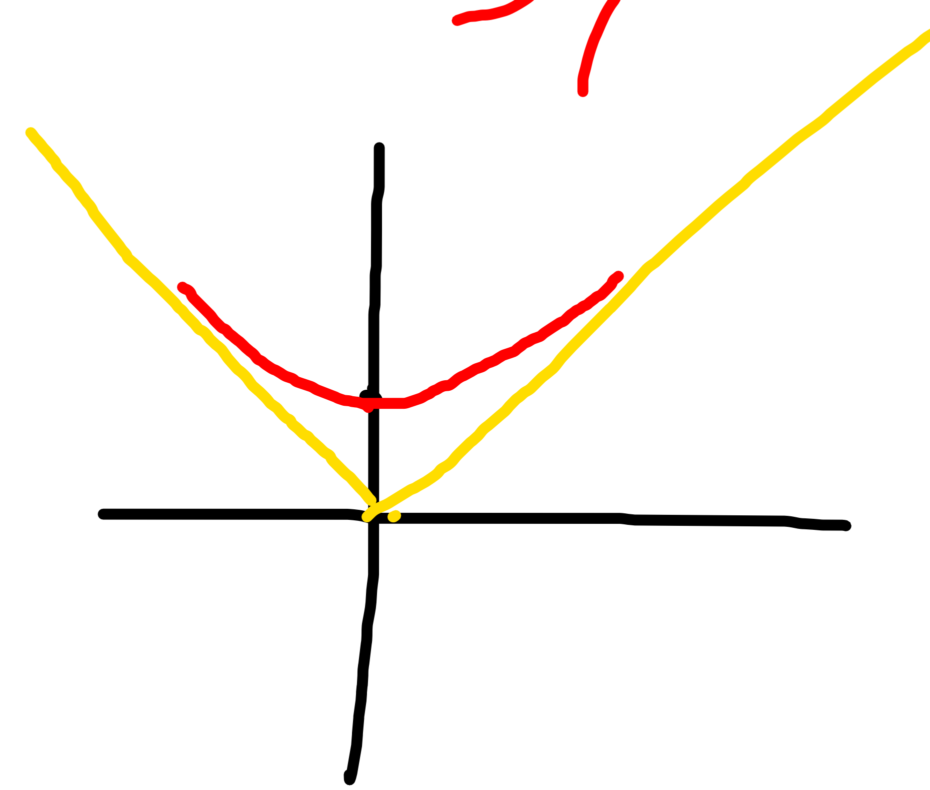
$$f''(x) = \frac{\sqrt{x^2+1} - x \cdot \frac{1}{2\sqrt{x^2+1}} \cdot 2x}{x^2+1} = \frac{\sqrt{x^2+1} - \frac{x^2}{\sqrt{x^2+1}}}{x^2+1} = \frac{\frac{x^2+1-x^2}{\sqrt{x^2+1}}}{x^2+1}$$

$\frac{a}{b}$ $\frac{c}{1}$

$$= \frac{1}{\sqrt{x^2+1}} \cdot \frac{1}{x^2+1} \quad \frac{a}{b} \cdot \frac{1}{c} > 0$$

		0	
f'	$-$	0	$+$
cres decr f	↘	→	↗
f''	$+$	$+$	$+$

$$f(0)=\sqrt{0^2+1}=1$$



$$f(x)=\frac{1-x}{(x-3)(x+2)} \qquad \mathrm{Dom}\, f=\mathbb{R}\setminus\{-2,3\}.$$

$$f'(x)=\frac{-1\,(x-3)(x+2)-(1-x)\,[\overbrace{x+2+x-3}^{2x-1}]}{[(x-3)(x+2)]^2}$$

$$x^2-2x+7=0$$

$$\Delta=4-28<0$$

$$=\frac{-(x^2-3x+2x-6)-(2x-1-2x^2+x)}{[(x-3)(x+2)]^2}$$

$$=\frac{-x^2+x+6+2x^2-3x+1}{[(x-3)(x+2)]^2}=\frac{x^2-2x+7}{[(x-3)(x+2)]^2}$$

$$>0$$

$$\begin{array}{r} x^2 - x - 6 \\ 2x - 2 \\ \hline 2x^3 - 2x^2 - 12x \\ -2x^2 + 2x + 12 \\ \hline \end{array}$$

$$f'(x) = \frac{x^2 - 2x + 7}{[(x-3)(x+2)]^2}$$

$$\begin{array}{r} x^2 - 2x + 7 \\ 4x - 2 \\ \hline 4x^3 - 8x^2 + 28x \\ -2x^2 + 4x - 14 \end{array}$$

$x - 3 + x + 2$

$$f''(x) = \frac{(2x-2)[(x-3)(x+2)]^2 - (x^2-2x+7)2(x-3)(x+2)[2x-1]}{[(x-3)(x+2)]^4}$$

$$= \frac{\cancel{(x-3)(x+2)}\left[(2x-2)[(x-3)(x+2)] - (x^2-2x+7)\cdot 2(2x-1)\right]}{\cancel{(x-3)(x+2)}[(x-3)(x+2)]^3}$$

$$= \frac{2x^3 - 4x^2 + 10x + 12 - 4x^3 + 10x^2 - 32x + 14}{[(x-3)(x+2)]^3} = \frac{-2x^3 + 6x^2 - 22x + 26}{[(x-3)(x+2)]^3}$$

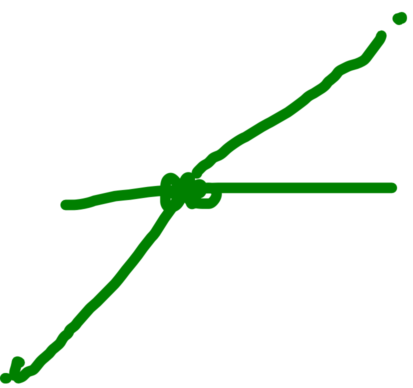
$$f''(x) = \frac{-2x^3+6x^2-22x+26}{[(x-3)(x+2)]^3} = \frac{-2}{[(x-3)(x+2)]^3}\cdot\underbrace{(x^3-3x^2+11x-13)}$$

$g(x) = x^3 - 3x^2 + 11x - 13$

$g'(x) = 3x^2 - 6x + 11 = 0$ $\quad$ $\Delta = 36 - 4\cdot3\cdot11 < 0$

$g'(x) > 0$

$g \nearrow$



$\therefore$ g tem uma única raiz

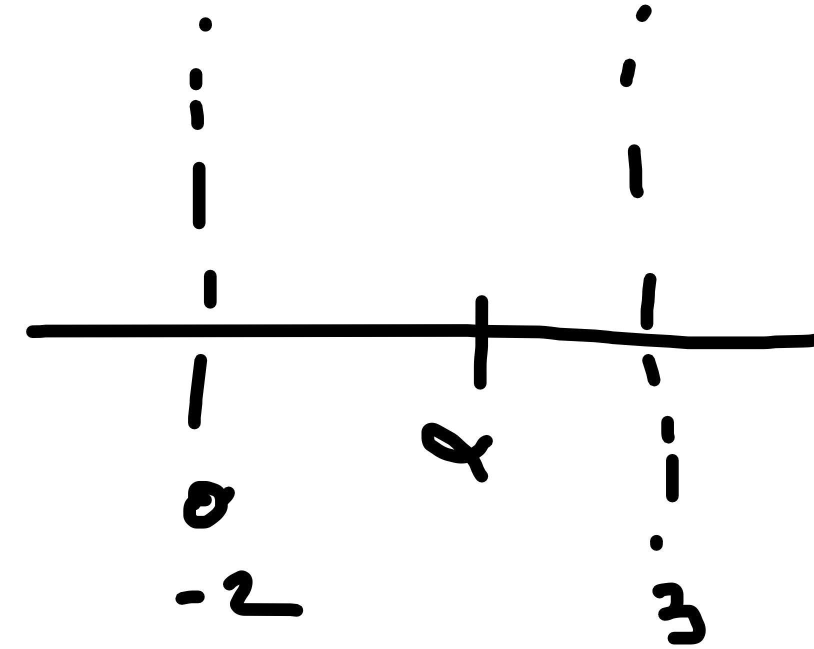
$1 \quad 2$

$g(1) = -4$ $\quad$ α raiz

$g(2) = 5$

$1 < \alpha < 2$ $\quad$ $g(\alpha) = 0$

	$-\infty$		-2		α		3		$+\infty$
$f'(x)$		$+$	$\times$	$+$	$+$	$+$	$\times$	$+$	
cres/decr		$\nearrow$	$\times$	$\nearrow$	$\nearrow$	$\nearrow$	$\times$	$\nearrow$	
$g(x)$		$-$	$-$	$-$	0	$+$	$+$	$+$	
$(x-3)(x+2)$		$+$	0	$-$	$-$	$-$	0	$+$	
-2		$-$	$-$	$-$	$-$	$-$	$-$	$-$	
f''		$+$	$\times$	$-$	0	$+$	$\times$	$-$	



$-\infty$ $\quad -2^-$ $\quad -2^+$ $\quad$ Ponto de inflexão $\quad 3^-$ $\quad 3^+$ $\quad +\infty$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\cancel{x}\left(\frac{1}{x}-1\right)}{\cancel{x}\cdot x\left(1-\frac{3}{x}\right)\left(1+\frac{2}{x}\right)} = 0^+$$

$\left(\frac{1}{x}-1\right) \to -1$

$x \to -\infty$, $\left(1-\frac{3}{x}\right) \to 1$, $\left(1+\frac{2}{x}\right) \to 1$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{\cancel{x}\left(\frac{1}{x}-1\right)}{\cancel{x}\, x\left(1-\frac{3}{x}\right)\left(1+\frac{2}{x}\right)} = 0^-$$

$\left(\frac{1}{x}-1\right) \to -1$

$x \to +\infty$, $\left(1-\frac{3}{x}\right) \to 1$, $\left(1+\frac{2}{x}\right) \to 1$

$$\lim_{x\to -2^-} \frac{\overbrace{(1-x)}^{\to 3}}{\underbrace{(x-3)}_{\to -5}\underbrace{(x+2)}_{\to 0^-}} = +\infty$$

$$\lim_{x\to -2^+} \frac{\overbrace{(1-x)}^{\to 3}}{\underbrace{(x-3)}_{\to -5}\underbrace{(x+2)}_{\to 0^+}} = -\infty$$

$x > -2$

$x+2 > 0$

$$\lim_{x\to 3^-} \frac{\overbrace{1-x}^{\to -2}}{\underbrace{(x-3)}_{\to 0^-}\underbrace{(x+2)}_{\to 5}} = +\infty$$

$$\lim_{x\to 3^+} \frac{\overbrace{1-x}^{\to -2}}{\underbrace{(x-3)}_{\to 0^+}\underbrace{(x+2)}_{\to 5}} = -\infty$$

- dom f
- f' analisa sinal f'

analisa/decre

f''

analisa concavidade

limites $\pm\infty$

Pontos 'encostados' no dom de f

procura assíntotas

Plota os pontos