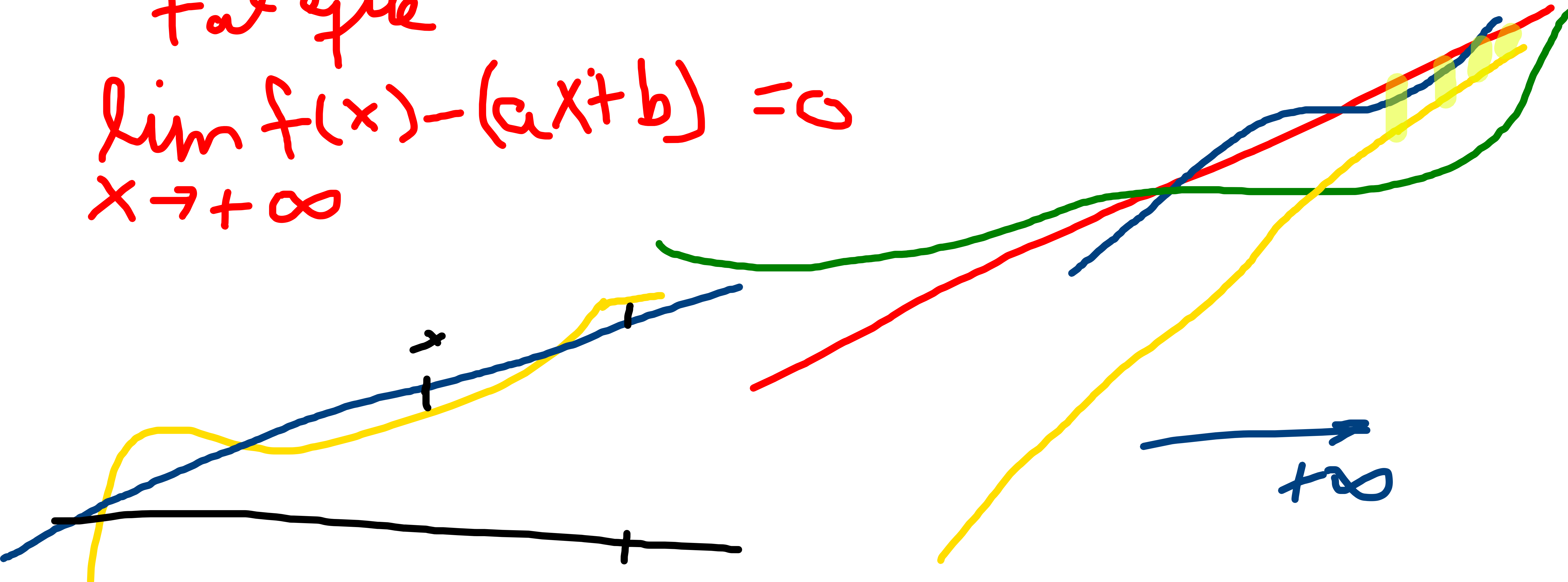
Assintota de f p/ $+\infty$

$r: \ y = ax + b$

tal que

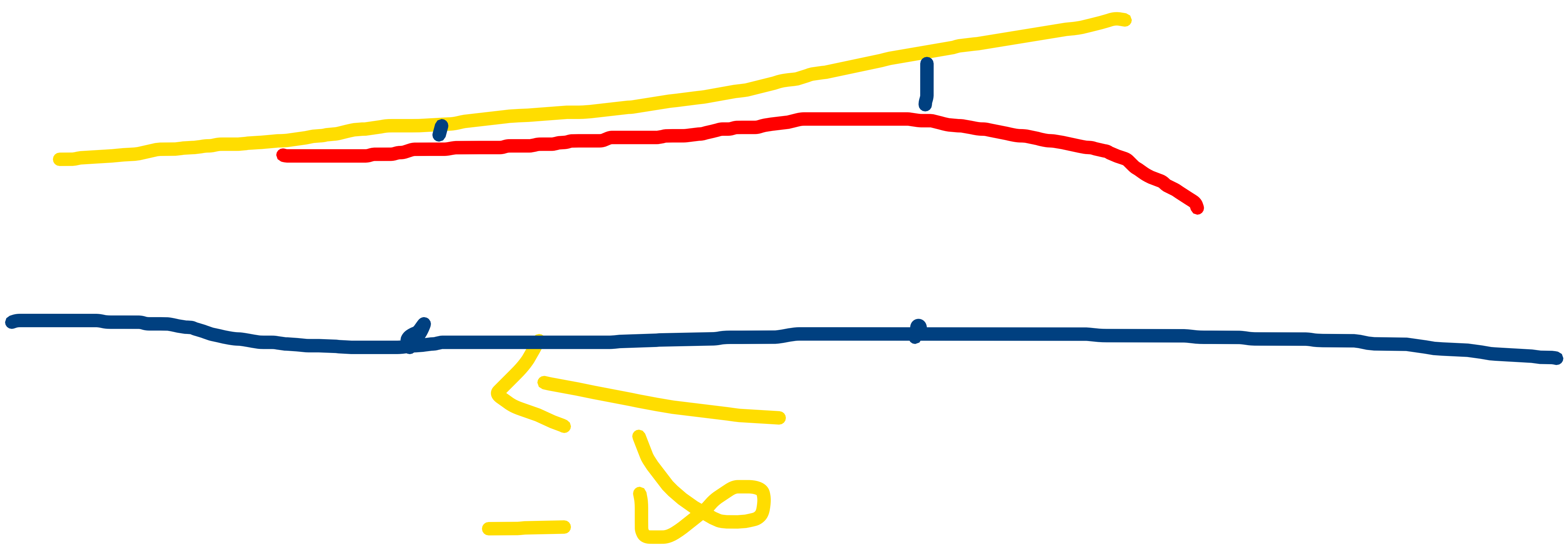
$$\lim_{x \to +\infty} f(x) - (ax+b) = 0$$



Assintoto per $-\infty$

$r: y = ax + b$

$$\lim_{x \to -\infty} f(x) - (\underline{a}x + \underline{b}) = 0$$



Teor: f possui assíntota p/ $+\infty$ se e somente se

$\exists a \in \mathbb{R}$ $\quad a = \lim\limits_{x \to +\infty} \dfrac{f(x)}{x}$

e $\quad \exists b \in \mathbb{R}$ $\quad b = \lim\limits_{x \to +\infty} f(x) - ax$

$\tilde{a}x + \tilde{b} = ax + b$

Nesse caso, se existir, $y = ax + b$ é a assíntota.

calcule as assíntotas de

$$f(x) = \sqrt{x^2+1}$$

assíntota p/ $+\infty$

$$a = \lim_{x \to +\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x \to +\infty} \frac{|x|\sqrt{1+\frac{1}{x^2}}}{x}$$

$$= \lim_{x \to +\infty} \frac{\cancel{x}}{\cancel{x}} \sqrt{1+\underbrace{\frac{1}{x^2}}_{0}} = 1$$

$$b = \lim_{x \to +\infty} \left(\sqrt{x^2+1} - 1 \cdot x \right) \frac{\left(\sqrt{x^2+1} + x\right)}{\left(\sqrt{x^2+1} + x\right)}$$

$$b = \lim_{x \to +\infty} \frac{\cancel{x^2} + 1 - \cancel{x^2}}{|x|\sqrt{1+\frac{1}{x^2}} + x}$$

$x > 0$

$|x| = x$

$$= \lim_{x \to +\infty} \frac{1}{\underbrace{x}_{\to +\infty} \underbrace{\left(\sqrt{1+\frac{1}{x^2}} + 1\right)}_{\to 2}} = 0$$

assintota p/ $+\infty$

$$\boxed{r: \ y = x}$$

ASSINTOTA p/ $-\infty$

$$a = \lim_{x \to -\infty} \frac{\sqrt{x^2+1}}{x} = \lim_{x \to -\infty} \frac{|x|\sqrt{1+\frac{1}{x^2}}}{x}$$

$x<0$

$|x| = -x$

$$= \lim_{x \to -\infty} \frac{-1\cancel{x}}{\cancel{x}} \underbrace{\sqrt{1+\underbrace{\frac{1}{x^2}}_{0}}}_{1} = -1$$

$$b = \lim_{x \to -\infty} \sqrt{x^2+1} - (-x)$$

$$= \lim_{x \to -\infty} \left(\sqrt{x^2+1} + x\right) \frac{\left(\sqrt{x^2+1} - x\right)}{\left(\sqrt{x^2+1} - x\right)}$$

$$\sqrt{x^2+1} = \sqrt{x^2\left(1+\frac{1}{x^2}\right)} = \sqrt{x^2} \cdot \sqrt{1+\frac{1}{x^2}} = |x| \sqrt{1+\frac{1}{x^2}}$$

$$= \lim_{x \to -\infty} \frac{\cancel{x^2}+1-\cancel{x^2}}{|x|\sqrt{1+\frac{1}{x^2}} - x} = \lim_{x \to -\infty} \frac{1}{-x\sqrt{1+\frac{1}{x^2}} - x}$$

$x < 0$

$|x| = -x$

$x<0 \Rightarrow |x| = -x$

$$= \lim_{x \to -\infty} \frac{1}{\underbrace{-x}_{\to +\infty}\underbrace{\left(\sqrt{1+\frac{1}{x^2}}+1\right)}_{\to 2}} = 0$$

assintota p/ $-\infty$

$r: y = -x$

Dem: $\Rightarrow$

$$\lim_{x\to-\infty} f(x)-(ax+b)=0$$

$$\lim_{x\to-\infty}\frac{ax+b}{x}=\lim_{x\to-\infty}\frac{a\cancel{x}}{\cancel{x}}+\frac{b}{x}=a$$

$$\lim_{x\to-\infty}\frac{f(x)}{x}=\lim_{x\to-\infty}\frac{f(x)-(ax+b)+(ax+b)}{x}$$

$$=\lim_{x\to-\infty}\underbrace{\frac{f(x)-(ax+b)}{x}}_{0}+\underbrace{\frac{ax+b}{x}}_{a}=a$$

$$\lim_{x\to-\infty} f(x)-ax = \lim_{x\to-\infty} f(x)-ax-b+b$$

$$= \lim_{x\to-\infty} \underbrace{f(x)-(ax+b)}_{0} + b = b$$

$\Leftarrow$

$a, b \in \mathbb{R}$ tais que

$$a = \lim_{x \to -\infty} \frac{f(x)}{x}$$

$$b = \lim_{x \to -\infty} f(x) - ax$$

$$\lim_{x \to -\infty} f(x) - (ax+b) = \lim_{x \to -\infty} \underbrace{f(x) - ax}_{\downarrow \, b} - b = 0$$

Ex.

$f(x) = \ln x$

assintota p/ $+\infty$

$$a = \lim_{x \to +\infty} \frac{\ln x}{x} \overset{\infty/\infty}{\underset{L'H}{=}} \lim_{x \to +\infty} \frac{\frac{1}{x}}{1} = \lim_{x \to +\infty} \frac{1}{x} = 0$$

$$b = \lim_{x \to +\infty} \ln x - 0x = \lim_{x \to +\infty} \ln x = +\infty$$

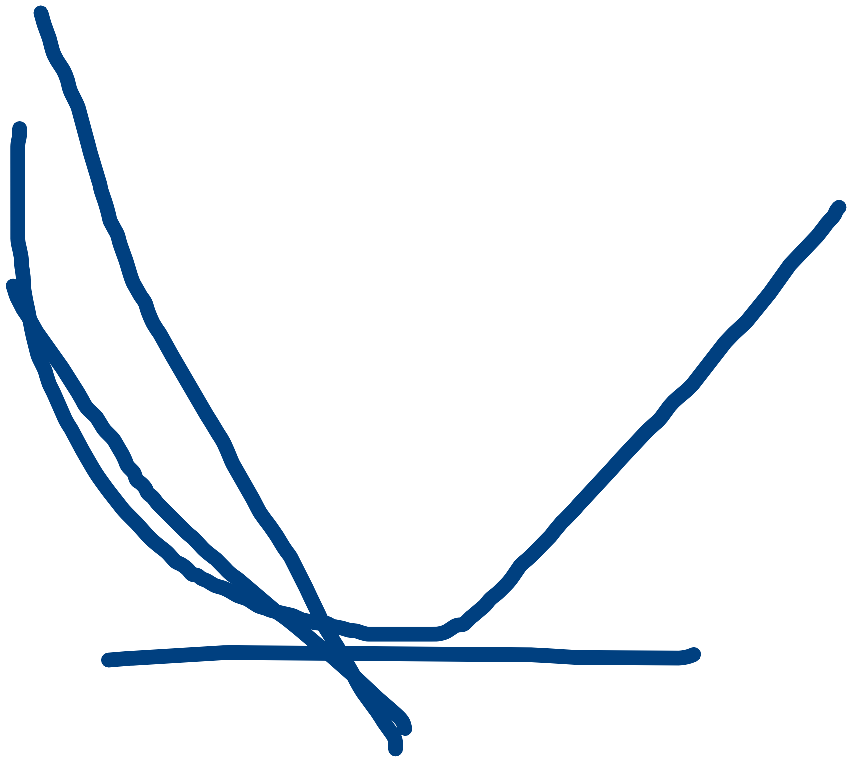
$\therefore$ assintota p/ $+\infty$ n existe

EX :

$$f(x) = x^2$$

assintota p/ $-\infty$

$$a = \lim_{x \to -\infty} \frac{x^2}{x} = \lim_{x \to -\infty} x = -\infty$$



Portanto x^2 não tem assintota p/ $-\infty$.