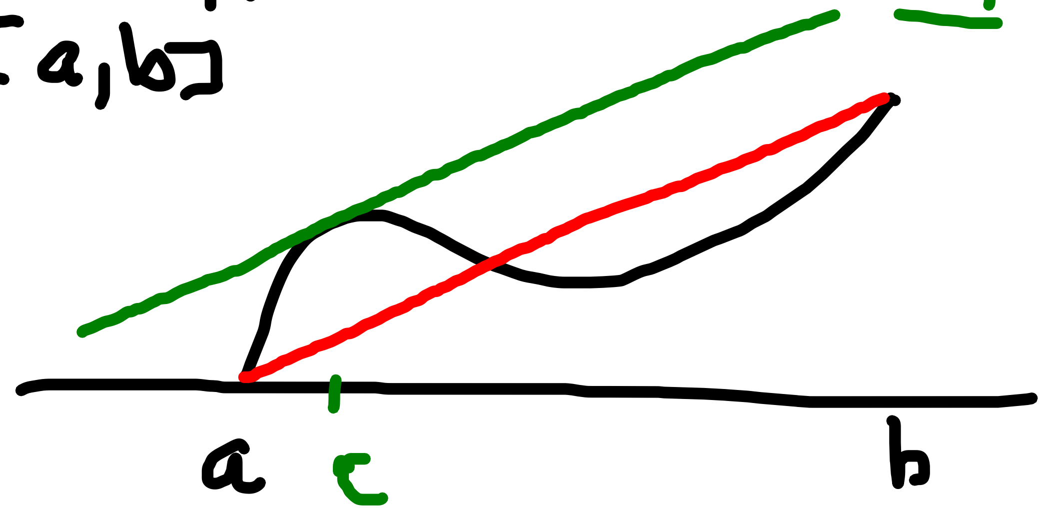
TVM Teo valor médio

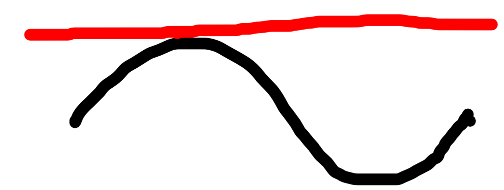
f derivável em $]a,b[$

f contínua em $[a,b]$

$$\exists\, c \in \,]a,b[\quad \frac{f(b)-f(a)}{b-a} = f'(c).$$



1º) $f(a)=f(b).$



$\exists\, c \in\,]a,b[$ tq c é ponto de máximo ou mínimo.

. .

Se $\{a, b\}$ são pontos de máximo e mínimo

então $f(x) = f(a) = f(b)\ \forall x \in\,]a,b[$

f é constante $\therefore$

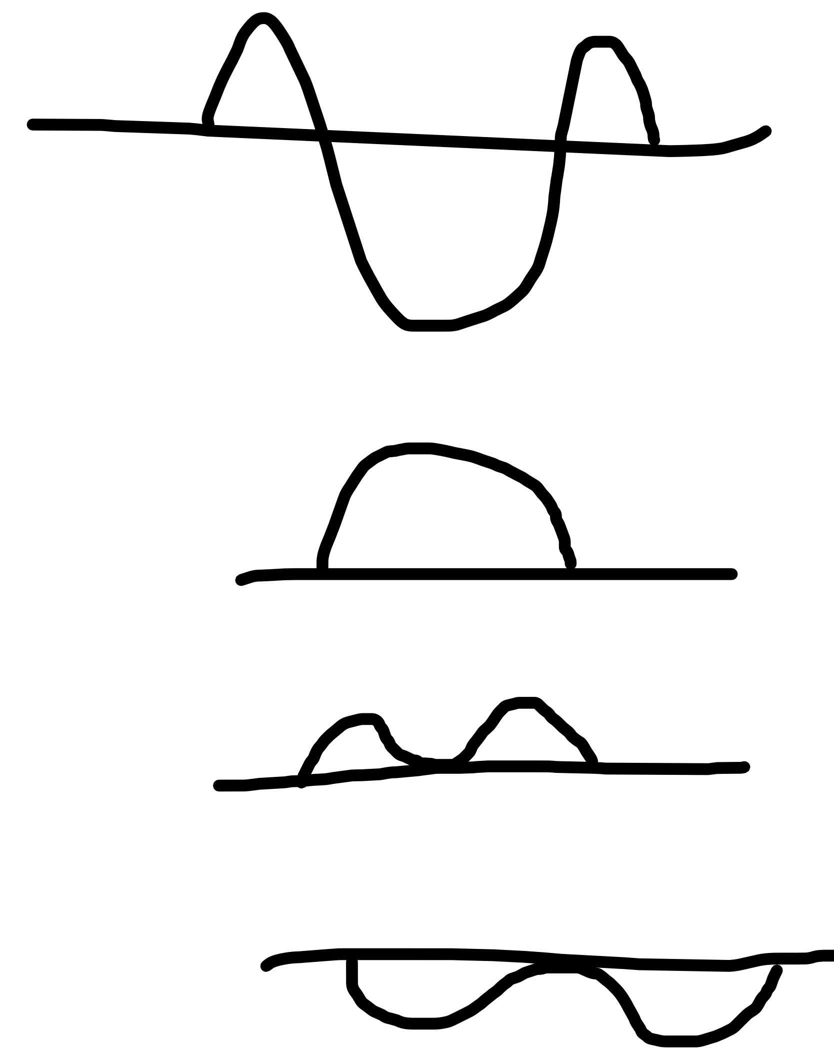
$$f'(x) = 0 = \frac{f(b) - f(a)}{b - a}$$

$c \in\,]a,b[$ qualquer

a) $\exists x \in]a,b[$

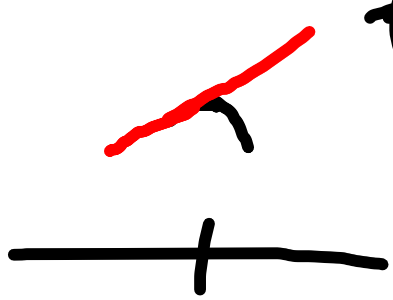
$f(x) > f(a)$

b) $\exists x \in]a,b[\quad f(x) < f(a)$



a) c ponto de máximo

$f(c) = f(x) > f(a)$



$$\lim_{h \to 0^-} \frac{\overbrace{f(c+h)}^{\leq f(c)} - f(c)}{\underbrace{h}_{<0}} \overset{\leq 0}{} \geq 0$$

$$\lim_{h \to 0^+} \frac{\overbrace{f(c+h)}^{\leq f(c)} - f(c)}{\underbrace{h}_{>0}} \overset{\leq 0}{} \leq 0$$

$\therefore f'(c) \leq 0$

e $f'(c) \geq 0 \Rightarrow f'(c) = 0$

b)

$$\lim_{h \to 0^-} \frac{f(c+h) - f(c)}{h} \leq 0 \Rightarrow f'(c) \leq 0$$

≥ 0

< 0

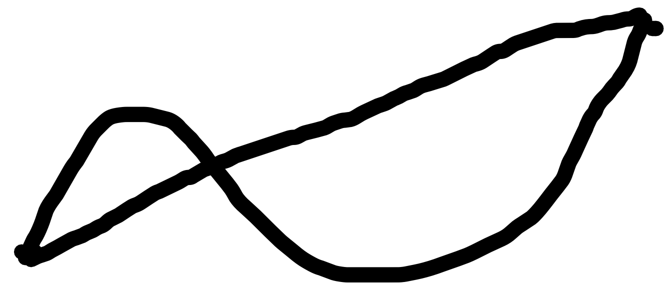
$$\lim_{h \to 0^+} \frac{f(c+h) - f(c)}{h} \geq 0 \Rightarrow f'(c) \geq 0$$

≥ 0

> 0

$\therefore f'(c) = 0.$

2)



$$g(x) = f(x) - \left[f(a) + \frac{f(b)-f(a)}{b-a}(x-a) \right]$$

$$g(a) = f(a) - [f(a) + 0] = 0$$

$$g(b) = f(b) - [\cancel{f(a)} + f(b) - \cancel{f(a)}]$$

$$= f(b) - f(b) = 0$$

Rolle caso 1: $\exists\, c \in\,]a, b[$

$$g'(c) = 0 = \frac{g(b) - g(a)}{b - a}$$

$$g(x) = f(x) - \left[f(a) + \frac{f(b) - f(a)}{b-a} \cdot (x - a) \right]$$

$$g'(x) = f'(x) - \left[0 + \frac{f(b) - f(a)}{b-a} \cdot 1 \right]$$

$$= f'(x) - \frac{f(b) - f(a)}{b-a}$$

$\therefore$

$$0 = g'(c)$$

$$= f'(c) - \frac{f(b) - f(a)}{b-a}$$

$$\therefore \boxed{f'(c) = \frac{f(b) - f(a)}{b-a}}$$

consequência do TVM

$f'(x)<0$ $\forall x>a$
então
f é estritamente
decrescente.

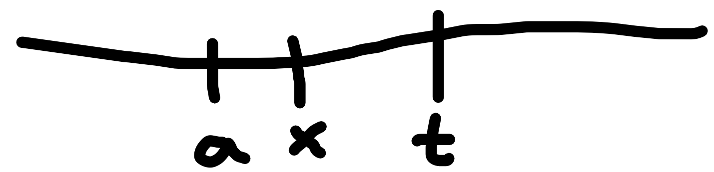
$f'(x)>0$ $\forall x>a$

então f é estritamente
crescente.

$\exists c \in \,]x,t[$:

$$\frac{f(t)-f(x)}{t-x} = f'(c) > 0$$

$t-x > 0$



$$\therefore \quad f(t)-f(x) > 0$$

$$f(t) > f(x)$$

TVM de Cauchy

Usa p/ provar

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

L'Hospital

1) $\displaystyle\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ se $\displaystyle\lim_{x \to a} f(x) = 0$

$\displaystyle\lim_{x \to a} g(x) = 0$

e $\displaystyle\lim_{x \to a} \frac{f'(x)}{g'(x)}$ existe

2) $$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to a} f(x) = \infty, \quad \lim_{x \to a} g(x) = \infty$$

$$\lim_{x \to a} \frac{f'(x)}{g'(x)} \text{ existe.}$$