3.6 Ex 2

$|f(x)-3| \leq 2|x-1|$. Calcule

$\lim_{x\to 1} f(x)$

$0 \leq |f(x)-3| \leq 2|x-1|$

$\downarrow_{x\to 1}$ $\quad$ $\downarrow_{x\to 1}$

0 $\quad$ 0

$\therefore$ $\swarrow$ pelo confronto

0

$\therefore \lim_{x\to 1} |f(x)-3| = 0$

$\Rightarrow \lim_{x\to 1} f(x)-3 = 0$

$\Rightarrow \lim_{x\to 1} f(x) = 3$

$\therefore \boxed{\lim_{x\to 1} f(x) = 3}$

$\lim_{x\to a} |g(x)| = 0$

$\Rightarrow \lim_{x\to a} g(x) = 0$

3.5 1. c)

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$h(x) = \left(\sqrt[3]{x+7}\right)^2 + \left(\sqrt[3]{x+7}\right) \cdot 2 + 4$$

$x \to 1$

$$2^2 + 2^2 + 2^2 = 12$$

$$\lim_{x \to 1} \frac{\sqrt[3]{x+7} - 2}{x-1} \cdot \frac{h(x)}{h(x)}$$

$$= \lim_{x \to 1} \frac{x+7-8}{x-1} \cdot \frac{1}{h(x)}$$

$$= \lim_{x \to 1} \frac{\cancel{x-1}}{\cancel{x-1}} \cdot \frac{1}{h(x)} = \frac{1}{12}$$

12

7.4 5)

$$g(x) = \log_a x \quad , a > 0 , a \neq 1$$

$$\boxed{g'(x) = \frac{1}{x \ln a}}$$

$$\ln x = \log_e x$$

$$\log_a x = \log_e a \cdot \log_e x$$
$$= \ln a \cdot \ln x$$

$$g(x) = \log_e x \cdot \log_a e$$
$$= \ln x \cdot \log_a e$$

$$g'(x) = (\log_a e \ln x)'$$
$$= \log_a e \cdot (\ln x)'$$
$$= \frac{\log_a e}{x} = \frac{1}{x \ln a}$$

3.5 2 c) $\lim_{x \to 0} \frac{f(x)}{x} = 1$

$\lim_{x \to 1} \frac{f(x^2-1)}{x-1}$

$= \lim_{x \to 1} \frac{f(x^2-1)}{x-1} \cdot \frac{x+1}{x+1}$

$= \lim_{x \to 1} \frac{f(x^2-1)}{x^2-1} \cdot (x+1) = 2$

$\to 1 \qquad \to 2$

$u = x^2-1$

$x \to 1 \Rightarrow u \to 0$

$u \to 0$

$\frac{f(u)}{u} \to 1$

$g(x) = \begin{cases} \frac{f(x)}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

g e' continua in 0

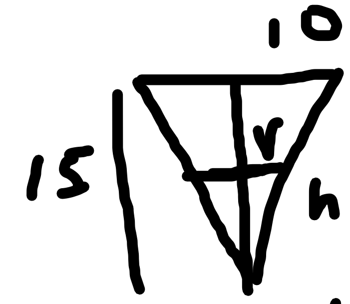
$g(x^2-1) \to g(1^2-1) = g(0)$

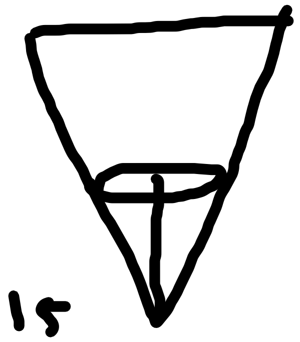
$= \frac{f(x^2-1)}{x^2-1}$

$$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$$

Velocidade

$$\frac{dV}{dt}$$



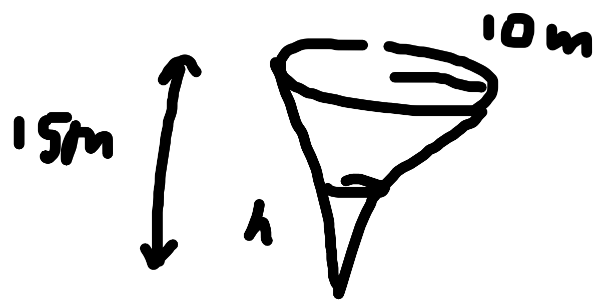


r(h)

$$\frac{10}{r} = \frac{15}{h} \qquad r = \frac{10}{15}h = \frac{2}{3}h$$

$$V(h) = \frac{\pi r^2 \cdot h}{3} = \pi \frac{4}{9} \cdot \frac{h^3}{3} = \frac{4\pi}{27}h^3$$

7.15 18



V 0,1 m³/s

com que velocidade
a altura está subindo
quando $h = 5$

regra da cadeia

$$\frac{dV}{dt} = \frac{dV}{dh} \cdot \frac{dh}{dt}$$

$$\frac{dV}{dh} = \frac{4\pi h^2}{9}$$

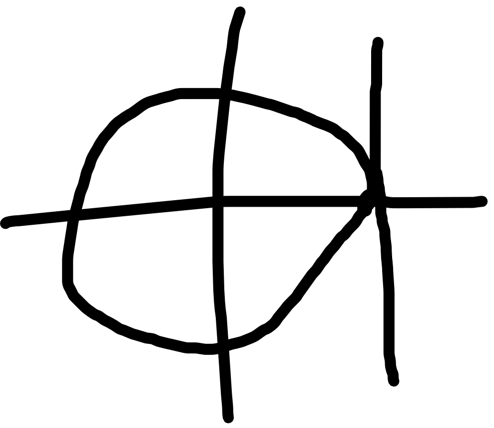
t_0: instante em que
$h = 5$.

$$\therefore \quad 0,1 = \frac{4\pi \cdot 25}{9} \cdot \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{0,9}{100\pi} = \frac{9}{10^3\pi}$$

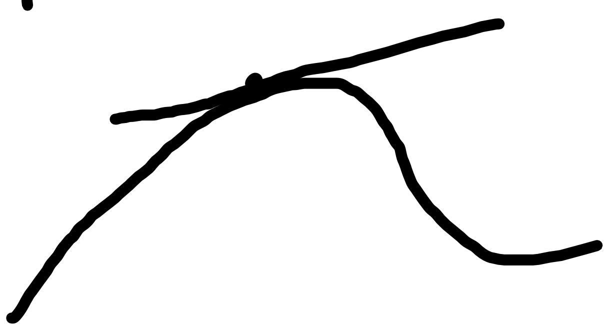
derivação implícita

$$\boxed{y = y(x)}$$



$$y = y(x)$$

$$f(x, y) = 0$$



$$y = \underline{y}(x)$$

$$(x^2 + y^2)' = (1)'$$

$$\boxed{2x + 2yy' = 0}$$

$$y' = -\frac{2x}{2y} = -\frac{x}{y}, \quad y \neq 0$$

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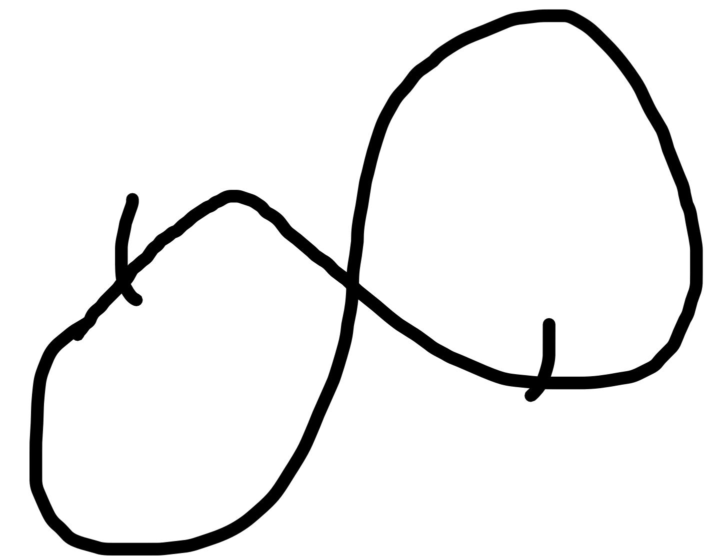
$y^3+y=x$ $\qquad y=y(x)$

Derivando imp. $\qquad f(x)$

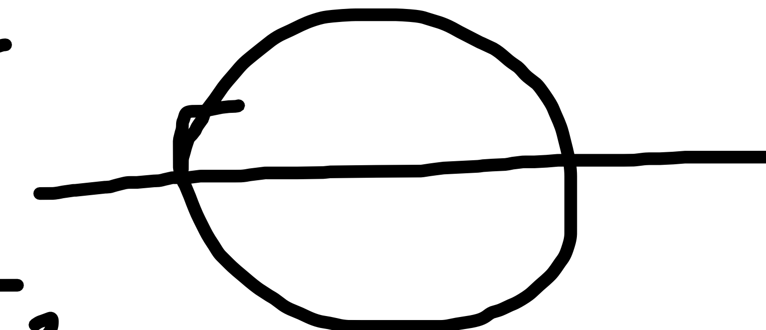


$3y^2y'+y'=1$

$y'(3y^2+1)=1$

$$\boxed{y'=\frac{1}{3y^2+1}}$$

$\sqrt{1-x^2}$



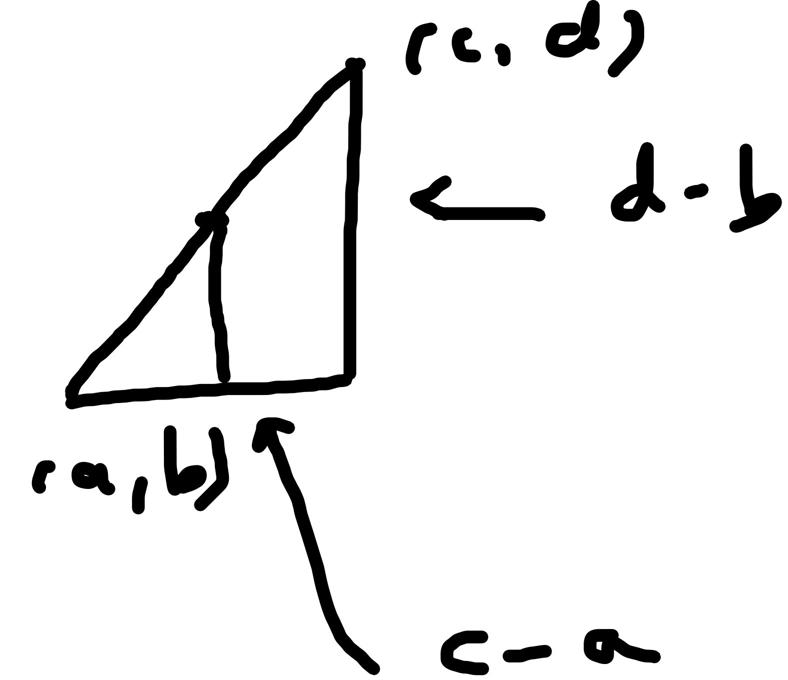
$-\sqrt{1-x^2}$

$\cdot\ (c, d)$

$\cdot\ (x, y)$

$\cdot\ (a, b)$

$$\boxed{\frac{d-b}{c-a} = \frac{y-b}{x-a}}$$



α tangente

$$\frac{y-b}{x-a} = \alpha$$

$(a, f(a))$

$f'(a)$

Encontre a reta tangente ao gráfico de $f(x)=x^3+2x-1$ no ponto $(1, \underline{f(x)})$.

$$f'(x) = 3x^2+2$$

$$f(1) = 1^3+2\cdot 1-1 = 2$$

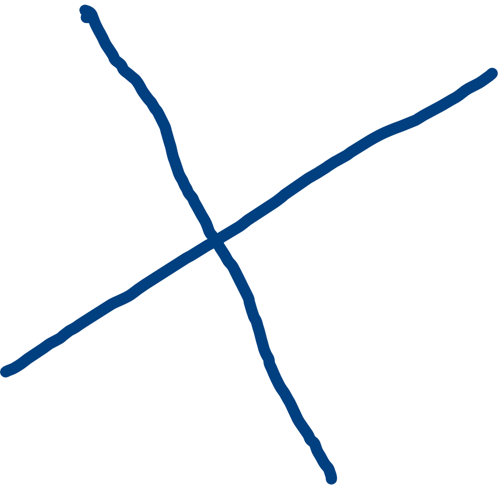
$$f'(1) = 3\cdot 1^2+2 = 5$$

$$\boxed{\frac{y-\overset{f(1)}{2}}{x-1} = \overset{f'(1)}{5}}$$

$$\boxed{y-2 = 5x-5}$$

7.16 reta tangente e a reta normal

$f(x) = x^2 - 3x$ no ponto de abscissa 0



$-\left(\frac{1}{-3}\right)$

$f'(x) = 2x - 3$

$f(0) = 0$, $f'(0) = -3$

$y - \underset{f(0)}{0} = -3(x - 0)$

$\boxed{y = -3x}$

inclinação da reta normal é

$-\left(\frac{1}{-3}\right) = \frac{1}{3}$

$y - 0 = \frac{1}{3}(x - 0)$

$y = \frac{1}{3}x$

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7. r passa pela origem tangente ao gráfico de $f(x) = x^3 + 2x^2 - 3x$.

Encontra r.

$f'(x) = 3x^2 + 4x - 3$

p/ t

r_t: $\boxed{(x-t)(3t^2 + 4t - 3) = y - t^3 - t^2 + 3t}$

$y - f(t)$

$(0,0) \in r_t$ então $-t(3t^2+4t-3) = -t^3 - t^2 + 3t$

$t = 0$ ou

$t \neq 0$

$-3t^2 - 4t + 3 = -t^2 - t + 3$

$\Leftrightarrow -3t^2 - 4t = -t^2 - t$

$\Leftrightarrow 2t^2 + 3t = 0$

$t \neq 0$

$\Leftrightarrow 2t + 3 = 0$

$\Leftrightarrow t = -\frac{3}{2}$

$\frac{27}{8} - \frac{9}{4} - \frac{9}{2}$

$= \frac{27 - 18 - 36}{8}$

$\frac{27}{4} - 6 - 3 =$

$\frac{27 - 36}{4} = -\frac{9}{4}$

$\boxed{t = 0}$

$-3x = y$

no ponto $(0,0)$

$t = -\frac{3}{2}$

$(x + \frac{3}{2})(\frac{-9}{4}) = y - \frac{27}{8}$

no ponto $(-\frac{3}{2}, f(-\frac{3}{2}))$