Teor: (Regra da cadeia)

$(f\circ g)'(x) = f'(g(x)).g'(x)$

Dem: $a(t) = \begin{cases} \dfrac{f(g(x)+t)-f(g(x))}{t}, & t\neq 0 \\ f'(g(x)), & t=0 \end{cases}$

$a(t)$ é contínua p/ $t=0$.

$$\frac{(f\circ g)(x+h)-(f\circ g)(x)}{h} = \begin{cases} \dfrac{f(g(x+h))-f(g(x))}{g(x+h)-g(x)}\cdot\dfrac{g(x+h)-g(x)}{h}, & g(x+h)\neq g(x) \\ 0, & g(x+h)=g(x) \end{cases}$$

$$= \begin{cases} a(g(x+h)-g(x))\cdot\dfrac{g(x+h)-g(x)}{h}, & g(x+h)\neq g(x) \\ 0 = a(g(x+h)-g(x))\cdot\dfrac{g(x+h)-g(x)}{h}, & g(x+h)=g(x) \end{cases}$$

$\therefore$

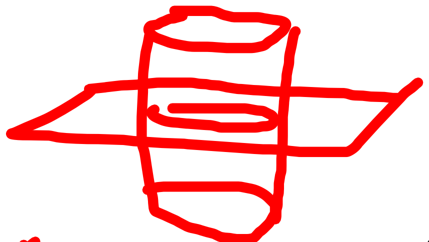
$$\lim_{h\to 0}\frac{(f\circ g)(x+h)-(f\circ g)(x)}{h} = \lim_{h\to 0} \underbrace{a(\underbrace{g(x+h)-g(x)}_{\to 0})}_{\to f'(g(x))}\cdot\underbrace{\frac{g(x+h)-g(x)}{h}}_{\to g'(x)}$$

$$= f'(g(x)).g'(x)$$

$$\frac{f(p+t)-f(p)}{t} \xrightarrow{t\to 0} f'(p)$$

$p = g(x)$

$$\frac{f(g(x+h))-f(g(x))}{\underbrace{g(x+h)-g(x)}_{t}}\cdot\frac{g(x+h)-g(x)}{h}$$



$V = \int A(x)\,dx$

$y(x)$ $\quad$ $z(y)$ $\quad$ $z(x)$ $\quad$ $\underline{V}(x)$ $\quad$ $x(t)$ $\quad$ $\underline{V}(t)$

$$\int f(x)\,dx$$

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Regola da catena

$$\frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$$

$$\frac{dV}{dt}(t_0)$$

$$\frac{dV(x_0)}{dx} \cdot \frac{dx(t_0)}{dt}$$

$$x_0 = x(t_0)$$

Σ

$V(x) = x^3 \qquad x > 0$

$x(t) = 2t^2 + 1 \qquad \frac{dx}{dt} = 4t$

$$\frac{dV}{dt}(1) = \left(\frac{dV}{dx} \cdot \frac{dx}{dt}\right)(1)$$

$\frac{dx}{dV} \quad \frac{dt}{dV}$

$\frac{dV}{dx} = 3x^2 \qquad x(1) = 3$

$$= \frac{dV}{dx}(3) \cdot \frac{dx}{dt}(1) = 27 \cdot 4$$

$$= 108$$

$$\lim_{x \to p} g(x) = L \Leftrightarrow \lim_{h \to 0} g(p+h) = L$$

$$\underline{g(x)} = \frac{f(x) - f(p)}{x - p} \xrightarrow[x \to p]{} f'(p)$$

$$g(p+h) = \frac{f(p+h) - f(p)}{h} \xrightarrow[h \to 0]{} f'(p)$$

$$V(x) = x^3 \qquad V'(x) = 3x^2$$

$$x(t) = 2t^2 + 1 \qquad x'(t) = 4t$$

$$V(x(t)) = (V \circ x)(t)$$

$$(V \circ x)'(t) =$$

$$V'(x(t)) . x'(t)$$

$$= 3x^2(t) . 4t$$

$$V'(x(1)) = 3 . 9 . 4$$

$$(2t^2 + 1)' = (2t^2)' + \underbrace{1'}_{=0}$$

$$= 2(t^2)' + 0$$

$$= 2 . 2t = 4t$$

$$\left(\operatorname{sen}(\underbrace{x^2+\cos x}_{u})\right)'$$

$$= \frac{\cos(x^2+\cos x)\left[2x-\operatorname{sen} x\right]}{\left[\ln(\underbrace{\cos(x^3)}_{u})\right]'}$$

$u = \cos \underbrace{x^3}_{v}$

$u' = -\operatorname{sen} v$

$= (-\operatorname{sen} x^3)v'$

$v = x^3 \quad v' = 3x^2$

$$\frac{1}{\cos x^3} \cdot (-\operatorname{sen} x^3) \cdot 3x^2$$

$$\left[\sec(\underbrace{x^2+1}_{u})\right]'$$

$u = x^2 + 1$

$u' = 2x$

$$(\sec u \cdot \operatorname{tg} u) \cdot u'$$

$$= \sec(x^2+1)\operatorname{tg}(x^2+1) \cdot 2x$$

$(e^x)' = e^x$

$w' = -\frac{1}{(x+1)^2} \cdot 1$

$$\Big[\ln\big(\underbrace{3^{(x^2+\operatorname{sen} x)}}_{v} + \overset{w}{\frac{1}{x+1}}\big)\Big]'$$

$u' = v' + w'$

$=$

$(\ln u)' =$

$$\frac{1}{u} \cdot u'$$

$$v = e^{\overbrace{\ln 3 \cdot [x^2 + \operatorname{sen} x]}^{a}}$$

$= \frac{1}{u} \cdot u' = \frac{1}{u} \cdot (v' + w')$

$= \frac{1}{u} (v \cdot a' + w')$

$v' = e^a \cdot a' = v \cdot a'$

$= \frac{1}{3^{x^2+1} + \frac{1}{x+1}} \cdot \left(3^{x^2+\operatorname{sen} x} \cdot \ln 3 \, (2x + \cos x) - \frac{1}{(x+1)^2} \right)$

$a' = \ln 3 (x^2 + \operatorname{sen} x)' = \ln 3 (2x + \cos x)$

$(e^x)' = e^x$

$\alpha \neq 0$

$(x^\alpha)' = (e^{\ln x \cdot \alpha})' = (f \circ g)'(x)$

$f(x) = e^x \quad f'(x) = e^x$

$g(x) = \ln x \cdot \alpha \quad g'(x) = \frac{1}{x} \cdot \alpha$

$= f'(g(x)) \cdot g'(x) = e^{\ln x \cdot \alpha} \cdot \frac{1}{x} \cdot \alpha = x^\alpha \cdot \frac{1}{x} \cdot \alpha$

$= \alpha x^\alpha \cdot x^{-1} = \alpha x^{\alpha - 1}$

$x = e^{\ln x}$

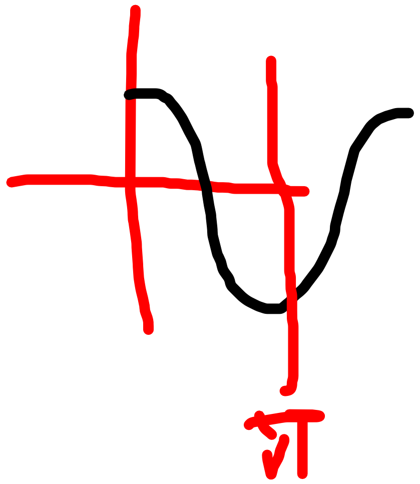
$x^\alpha = (e^{\ln x})^\alpha = e^{(\ln x \alpha)}$

$(a^x)' = (e^{\ln a \cdot x})' = (f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

$f(x) = e^x \quad f'(x) = e^x$

$g(x) = \ln a \cdot x \quad g'(x) = \ln a$

$= e^{\ln a \cdot x} \cdot \ln a = a^x \cdot \ln a$



$\cos x : [0, \pi] \to [-1, 1]$

$\arccos x : [-1, 1] \to [0, \pi]$

$f \circ f^{-1}(x) = x$

função inversa

$(f \circ f^{-1})'(x) = f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1 = x'$

$\therefore \boxed{(f^{-1})'(x) = \dfrac{1}{f'(f^{-1}(x))}}$

Ex: $e^{\ln x} = x$

$f(x) = e^x \quad f^{-1}(x) = \ln x$

$(\ln x)' = \dfrac{1}{e^{\ln x}} = \dfrac{1}{x}$

$\arcsin x : [-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$

$f(x) = \sin x \quad f^{-1}(x) = \arcsin x$

$f'(x) = \cos x$

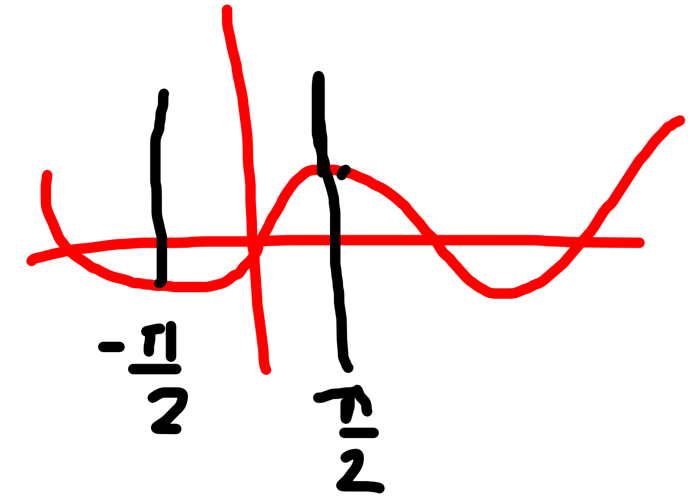
$\cos x \geq 0 \ \forall x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$(\arcsin x)' = \dfrac{1}{\cos(\arcsin x)} = \dfrac{1}{\sqrt{1 - \underbrace{\sin^2(\arcsin x)}_{x^2}}}$

$= \dfrac{1}{\sqrt{1 - x^2}}$

$(e^x)' = e^x$

$f'(f^{-1}(x)) = f'(\ln x) = e^{\ln x} = x$



$\sin x : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$

$\cos^2 x = 1 - \sin^2 x$

$|\cos x| = \sqrt{1 - \sin^2 x}$

$\parallel$

$\cos x$

$(\operatorname{tg} x)' = \underline{\sec^2 x} = \underline{1+\operatorname{tg}^2 x}$

$1+\operatorname{tg}^2 x = \frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\therefore \; 1+\operatorname{tg}^2 x = \underline{\sec^2 x}.$

$f(x) = \operatorname{tg} x$

$f^{-1}(x): \operatorname{arctg} x :]-\infty, +\infty[\to]-\frac{\pi}{2}, \frac{\pi}{2}[$

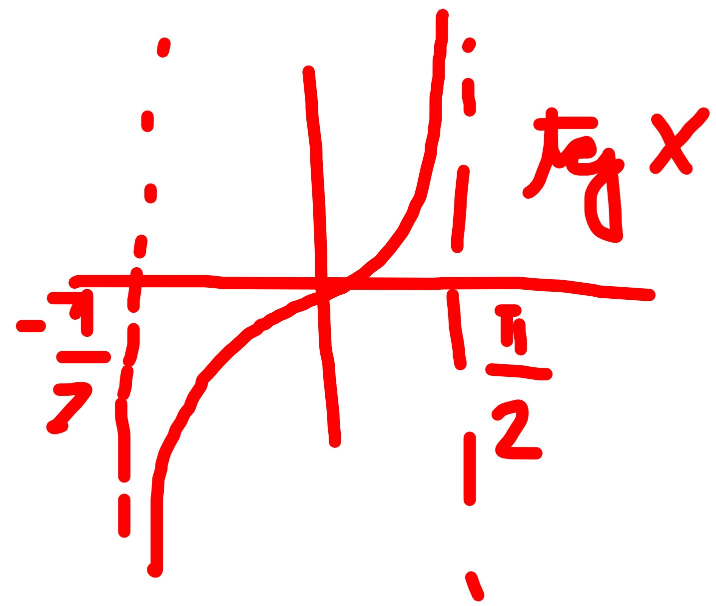
$(\operatorname{arctg} x)' = \frac{1}{\sec^2(\operatorname{arctg} x)} = \frac{1}{1+\operatorname{tg}^2(\operatorname{arctg} x)} = \underline{\frac{1}{1+x^2}}$

Ex: $\left(\sqrt[5]{x}\right)' = \frac{1}{5\left(\sqrt[5]{x}\right)^4} = \frac{1}{5x^{4/5}}$

$= \frac{1}{5} x^{-\frac{4}{5}}$

$f(x) = x^5$

$f'(x) = 5x^4$



$\boxed{+,\ -,\ \times,\ \div \quad \circ}$

$\left(\frac{x^{\alpha+1}}{\alpha+1}\right)' = x^\alpha$

$\frac{1}{1+x^2} = (\operatorname{arctg} x)'$

$$\left[(\underbrace{x^{100}+2x}_{u})^{280}\right]' = 280u^{279}\cdot u'$$

$$u^{280}$$

$$= 280(x^{100}+2x)^{279}\cdot(100x^{99}+2)$$

$$(x^x)' = (e^{\underbrace{\ln x\cdot x}_{u}})' = e^{u}\cdot u'$$

$$= x^x\cdot\left(\frac{1}{x}\cdot x+\ln x\cdot 1\right)$$

$$= x^x(1+\ln x)$$