$$2x^4 + 3x^2 + 2x - 5 \quad \lfloor\, x + 3$$

$$2x^3 - 6x^2 + 21x - 61$$

$$-2x^4 - 6x^3$$

$$-6x^3 + 3x^2$$

$$6x^3 + 18x^2$$

$$21x^2 + 2x$$

$$-21x^2 - 63x$$

$$-61x - 5$$

$$61x + 183$$

$$178 \quad \text{resto}$$

	x^4	x^3	x^2	x	1
-3	2	0	3	2	-5
		$2(-3)+0$	$(-6)(-3)+3$	$21(-3)+2$	$61)(-3)-5$
	2	-6	21	-61	178

$$2x^3 - 6x^2 + 21x - 61$$

$\{178\}$ resto

$$\lim_{x\to 1^+} \frac{x^2-3}{x^2+2x-3}$$

($x^2-3 \to -2$, $x^2+2x-3 \to 0$)

$$= \lim_{x\to 1^+} \frac{x^2-3}{(x-1)(x+3)}$$

($x^2-3 \to -2$, $x-1 \to 0^+$, $x+3 \to 4$)

$x > 1$

$x - 1 > 0$

$$= -\infty$$

$$\lim_{x\to+\infty} \frac{x^{\frac{3}{5}} + x^{\frac{4}{7}}}{2\sqrt{x}+2} = \lim_{x\to+\infty} \frac{x^{3/5}\left(1 + x^{\frac{4}{7}-\frac{3}{5}}\right)}{x^{1/2}\, 2\sqrt{1+\frac{2}{x}}}$$

($\frac{4}{7}-\frac{3}{5} < 0$)

$$= \lim_{x\to+\infty} \frac{x^{3/5-1/2}\left(1+x^{35}\right)}{\sqrt{1+\frac{2}{x}}}$$

($x^{3/5-1/2} \to +\infty$, $1+x^{35} \to 1$, $\sqrt{1+\frac{2}{x}} \to 1$)

$\frac{3}{5} > \frac{4}{7} \iff 21 > 20$

$$= +\infty$$

$$\lim_{x \to +\infty} \frac{\operatorname{sen} x}{x} = \lim_{x \to +\infty} \operatorname{sen} x \cdot \frac{1}{x} = 0$$

limitado

$\frac{1}{x} \to 0$

$\lim_{x \to +\infty} \operatorname{sen} x$ não existe

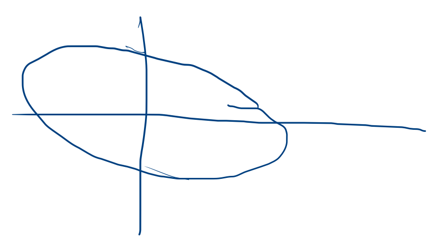
$\operatorname{sen}(2k\pi) = 0, \quad k \in \mathbb{Z}$

$\operatorname{sen} 0 = 0$

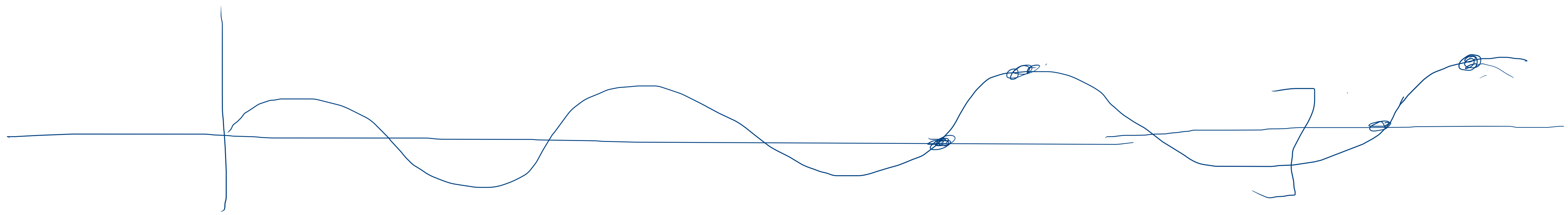
$\operatorname{sen}(2\pi) = 0$

$\operatorname{sen}(4\pi) = 0$

$\operatorname{sen}(-2\pi) = 0$



$$\operatorname{sen}\left(\frac{\pi}{2} + 2k\pi\right) = \operatorname{sen}\left(\frac{\pi}{2}\right) = 1$$



$\varepsilon = \frac{1}{2}$

$\forall N > 0$

suponga que N e t.q $x > N \Rightarrow |\operatorname{sen} x - L| < \frac{1}{2}$

$k_0 \quad 2k_0\pi > N$

$\frac{\pi}{2} + 2k_1\pi > N$

$$1 = |\operatorname{sen}(2k_0\pi) - \operatorname{sen}(\tfrac{\pi}{2} + 2k_1\pi)|$$

$$\leq |\operatorname{sen}(2k_0\pi) - L| + |L - \operatorname{sen}(\tfrac{\pi}{2} + 2k_1\pi)|$$

$$< \frac{1}{2} + \frac{1}{2} = 1$$

$$\lim_{x \to 0^+} \operatorname{sen} \frac{1}{x} = L$$

$$x_k = \frac{1}{2k\pi}$$

$$y_k = \frac{1}{\frac{\pi}{2} + 2k\pi}$$

$$\varepsilon = \frac{1}{2}$$

Supõe que existe $\delta > 0$

Fixa k_0

$$0 < \frac{1}{2k_0\pi} < \delta \qquad \frac{1}{\delta} < 2k_0\pi$$

Fixa k_1

$$0 < \frac{1}{\frac{\pi}{2} + 2k_1\pi} < \delta \qquad \frac{1}{\delta} < \frac{\pi}{2} + 2k_1\pi$$

$$0 < |x - 0| < \delta \Rightarrow \left|\operatorname{sen}\frac{1}{x} - L\right| < \frac{1}{2}$$

$$1 = \left|\operatorname{sen}\frac{1}{x_{k_0}} - \operatorname{sen}\frac{1}{y_{k_1}}\right| \leq \left|\operatorname{sen}\frac{1}{x_{k_0}} - L\right| + \left|L - \operatorname{sen}\frac{1}{y_{k_1}}\right|$$

$$< \frac{1}{2} + \frac{1}{2} = 1$$

$$\lim_{x \to -\infty} \sqrt{x^2+2x} + \sqrt[3]{x^3-4x^2}$$

$$= \lim_{x \to -\infty} |x| \left(\sqrt{1+\frac{2}{x}} \right) + x \sqrt[3]{1-\frac{4}{x}}$$

$$\begin{cases} x<0 \\ |x|=-x \end{cases}$$

$$= \lim_{x \to -\infty} -x \sqrt{1+\frac{2}{x}} + x \sqrt[3]{1-\frac{4}{x}}$$

$$= \lim_{x \to -\infty} x \left(-\sqrt{1+\frac{2}{x}} + \sqrt[3]{1-\frac{4}{x}} \right)$$

$-\infty$

$\to 0$

$$\lim_{x\to-\infty} \sqrt{x^2+2x} + \sqrt[3]{x^3-4x^2}$$

$$= \lim_{x\to-\infty} \underbrace{\sqrt{x^2+2x}+x}_{f(x)} + \underbrace{(-x + \sqrt[3]{x^3-4x^2})}_{g(x)} = -\frac{7}{3}$$

$*\ f(x) \to -1$ $\qquad *\ g(x) \to -\frac{4}{3}$

$$*\ \lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{(\sqrt{x^2+2x}+x)(\sqrt{x^2+2x}-x)}{(\sqrt{x^2+2x}-x)}$$

$$= \lim_{x\to-\infty} \frac{x^2+2x-x^2}{\underbrace{|x|}_{-x}\sqrt{1+\frac{2}{x}}-x} = \lim_{x\to-\infty} \frac{2x}{-x\underbrace{(\sqrt{1+\frac{2}{x}}+1)}_{2}}$$

$$= -\frac{2}{2} = -1$$

$$* \lim_{x \to -\infty} g(x) = \lim_{x \to -\infty} -x + \sqrt[3]{x^3 - 4x^2}$$

$$= \lim_{x \to -\infty} \left(\sqrt[3]{x^3 - 4x^2} - x\right) \frac{h(x)}{h(x)}$$

$$h(x) = \left(\sqrt[3]{x^3 - 4x^2}\right)^2 + \left(\sqrt[3]{x^3 - 4x^2}\right) x + x^2$$

$$= \lim_{x \to -\infty} \frac{x^3 - 4x^2 - x^3}{h(x)}$$

$$= \lim_{x \to -\infty} \frac{x^2(-4)}{x^2\left(\left(\sqrt[3]{1 - \frac{4}{x}}\right)^2 + \sqrt[3]{1 - \frac{4}{x}} \cdot 1 + 1^2\right)} = -\frac{4}{3}$$

$$1 + 1 + 1 = 3$$

$$f(x) = \begin{cases} 2x+3, & x<2 \\ x^2+a, & x>2 \end{cases}$$

$x_0 = 2$

Encontre a tal que f é contínua em $\mathbb{R}$.

$$\lim_{x\to 2^+} f(x) = \lim_{x \to 2^+} x^2 + a = 2^2 + a = 4 + a$$

$$= f(2) = 2 \cdot 2 + 3 = 7 \quad \therefore \boxed{a = 3}$$

$$\lim_{x\to 2^-} f(x) = \lim_{x \to 2^-} 2x + 3 = 2 \cdot 2 + 3 = 7$$

$$f(x) = \begin{cases} 2x+3, & x \le 2 \\ x^2+3, & x>2 \end{cases}$$ é contínua em 2.

$x_0 < 2$

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} 2x+3 = 2x_0+3 = f(x_0)$$

↑ contínua

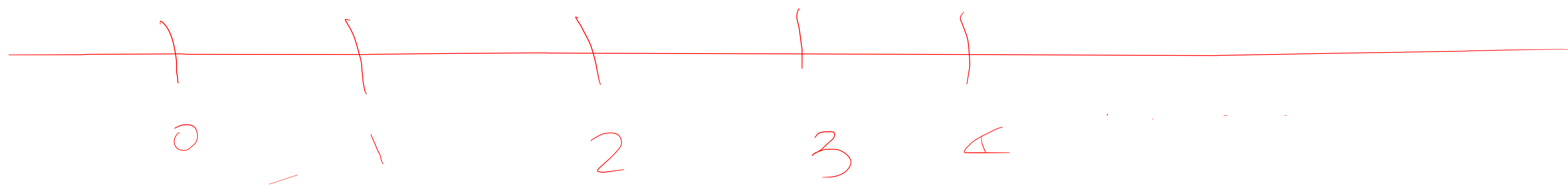
$x_0 > 2$

$$\lim_{x \to x_0} f(x) = \lim_{x \to x_0} x^2+3 = x_0^2+3 = f(x_0)$$

↑ contínua

$\therefore$ f é contínua em $\mathbb{R}$

Sequência



$$(x_n : n \in \mathbb{N})$$

$$f : \mathbb{N} \to \mathbb{R} \qquad f(n) = x_n$$

$$\lim_{n \to +\infty} x_n = L \iff \forall \varepsilon > 0 \ \exists n_0 \in \mathbb{N} \ \ n \geq n_0 \Rightarrow |x_n - L| < \varepsilon$$

$\sqrt{2}, \quad \sqrt{\sqrt{2}+2}, \quad \sqrt{\sqrt{\sqrt{2}+2}+2}, \ \ldots$

$x_1 = \sqrt{2}$

$x_2 = \sqrt{x_1 + 2} = \sqrt{\sqrt{2} + 2}$

$x_{n+1} = \sqrt{x_n + 2}$ $\qquad \lim_{n \to +\infty} x_n$

$x_n = \left(1 + \frac{1}{n}\right)^n$

$x_n \le a$

$\forall n \in \mathbb{N}$

è limitato

$$e = \lim_{n \to +\infty} x_n$$

$n < m$

$x_n < x_m$

strettamente crescente

$$e = \lim_{n \to +\infty} \left(\sum_{k=0}^{n} \frac{1}{k!} \right)$$

$$e = 1 + 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \dots\dots$$

$$e = \lim_{x \to 0} (1+x)^{1/x}$$

$$x_1 = \sqrt{2}$$

$$x_{n+1} = \sqrt{x_n + 2}$$

$$x_1 = \sqrt{2} \leq 2 \quad ✓$$

$$x_n \leq 2 \Rightarrow x_{n+1} \leq 2$$

$x_n \leq 2 \Rightarrow x_2 + 2 \leq 4$

$\Rightarrow x_{n+1} = \sqrt{x_2 + 2} \leq \sqrt{4} = 2$

$x_n \leq 2 \forall n$ $\Rightarrow x_{n+1} \leq 2$ ✓

$x_n < x_{n+1} = \sqrt{x_n + 2}$

$\Leftrightarrow x_n^2 < x_n + 2$

$\Leftrightarrow x_n^2 \leq 2x_n = x_n + x_n < x_n + 2$

$\therefore x_n < x_{n+1}$

$\lim_{n \to +\infty} x_n$ existe

$\lim_{n \to +\infty} x_n = L$

$\Downarrow$

$\lim_{n \to +\infty} x_{n+1} = L$

$\|$

$\lim_{n \to +\infty} \sqrt{x_n + 2} = \sqrt{\lim_{n \to +\infty} x_n + 2}$

$= \sqrt{L + 2}$

$L = \sqrt{L+2}$

$\Rightarrow L^2 = L + 2$

$\Leftrightarrow L^2 - L - 2 = 0$

$\Leftrightarrow L = \dfrac{1 \pm \sqrt{1+8}}{2} = 2$ ou -1

$\therefore$ $L = 2$