$\lim_{x \to a} f(x) \overset{\text{limitado}}{\,} \underset{\underset{0}{\downarrow}}{g(x)} = 0$ } $f(x) \leq h(x) \leq g(x)$

$f(x) \xrightarrow{x \to a} L$, $g(x) \xrightarrow{x \to a} L$

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{x}$$

então

$h(x) \xrightarrow[x \to a]{} L$

$\forall N > 0 \quad \exists \delta > 0$

N

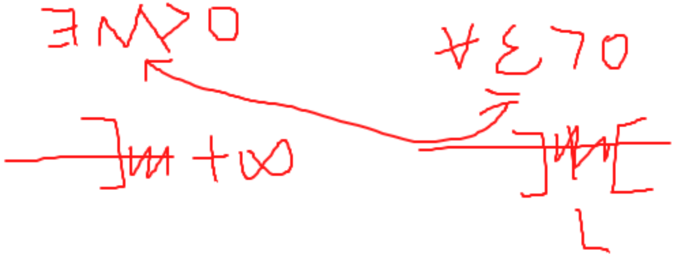
$]\,a\,[$

$]\, +\infty$

$+\infty$ não é um ponto

$+\infty - (+\infty)$, $\dfrac{+\infty}{+\infty}$

$$\lim_{x\to a} f(x) = +\infty \iff \forall N>0\ \exists \delta>0 \quad 0<|x-a|<\delta \implies f(x)>N$$



$$\lim_{x\to +\infty} f(x) = L \iff \forall \varepsilon>0\ \exists N>0 \quad x>N \implies |f(x)-L|<\varepsilon$$

$$\lim_{x\to +\infty} f(x) = +\infty \iff \forall N>0\ \exists M>0 \quad x>M \implies f(x)>N$$

$$\lim_{x\to+\infty} x = +\infty \qquad \lim_{x\to+\infty} \frac{x^2+1}{2x^2} = \lim_{x\to+\infty} \frac{\cancel{x^2}\left(1+\frac{1}{x^2}\right)}{\cancel{x^2}\cdot 2}$$

$$= \frac{1}{2}$$

$$\lim_{x\to+\infty} \frac{1}{x^2} = 0$$

Fixa $\varepsilon > 0$

$$\left|\frac{1}{x^2} - 0\right| = \frac{1}{x^2} < \varepsilon \iff x^2 > \frac{1}{\varepsilon} \iff |x| > \frac{1}{\sqrt{\varepsilon}}$$

tome $M>0$ tal que $M > \frac{1}{\sqrt{\varepsilon}}$

$$x > M \Rightarrow x > \frac{1}{\sqrt{\varepsilon}} \Rightarrow \left|\frac{1}{x^2} - 0\right| < \varepsilon$$

$$\lim_{x\to+\infty} \frac{x^3+3x+8}{x^2-5} = \lim_{x\to+\infty} \frac{\cancel{x^2}\cdot \overset{\nearrow +\infty}{x}\overbrace{\left(1+\frac{3}{x^2}+\frac{8}{x^3}\right)}^{\nearrow 1}}{\cancel{x^2}\underbrace{\left(1-\frac{5}{x^2}\right)}_{\searrow 1}} = +\infty$$

$$\lim_{x\to+\infty}\left(\sqrt{x^2+2x}-x\right) = \lim_{x\to+\infty} \sqrt{x^2\left(1+\frac{2}{x}\right)}-x$$

$|x>0 \quad |x|=x$

$$= \lim_{x\to+\infty} \overbrace{\sqrt{x^2}}^{=|x|}\sqrt{1+\frac{2}{x}} - x = \lim_{x\to+\infty} x\sqrt{1+\frac{2}{x}} - x$$

$$= \lim_{x\to+\infty} \overset{\nearrow +\infty}{x}\overbrace{\left(\sqrt{1+\underset{\searrow 0}{\frac{2}{x}}}-1\right)}^{\nearrow 0}$$

$+\infty \cdot 0$

indeterminado

$$\lim_{x\to+\infty}\left(\sqrt{\underbrace{x^2+2x}_{a}}-\underbrace{x}_{b}\right)\frac{\left(\sqrt{x^2+2x}+x\right)}{\left(\sqrt{x^2+2x}+x\right)}$$

$$=\lim_{x\to+\infty}\frac{\overbrace{(\cancel{x^2}+2x)}^{a^2}-\overbrace{\cancel{x^2}}^{b^2}}{\sqrt{x^2+2x}+x}=\lim_{x\to+\infty}\frac{\cancel{x}\cdot 2}{\cancel{x}\left(\underbrace{\sqrt{1+\frac{2}{x}}}_{\to 1}+1\right)}=\frac{2}{2}=1$$

$$x^2-x$$