Finite Open Loop Gain
and
Closed-Loop Gain

Finite Open Loop Gain

Note

A **single time constant network (STC)** is one that is composed of, or can be reduced to, one reactive component (capacitance or inductance) and one resistance.

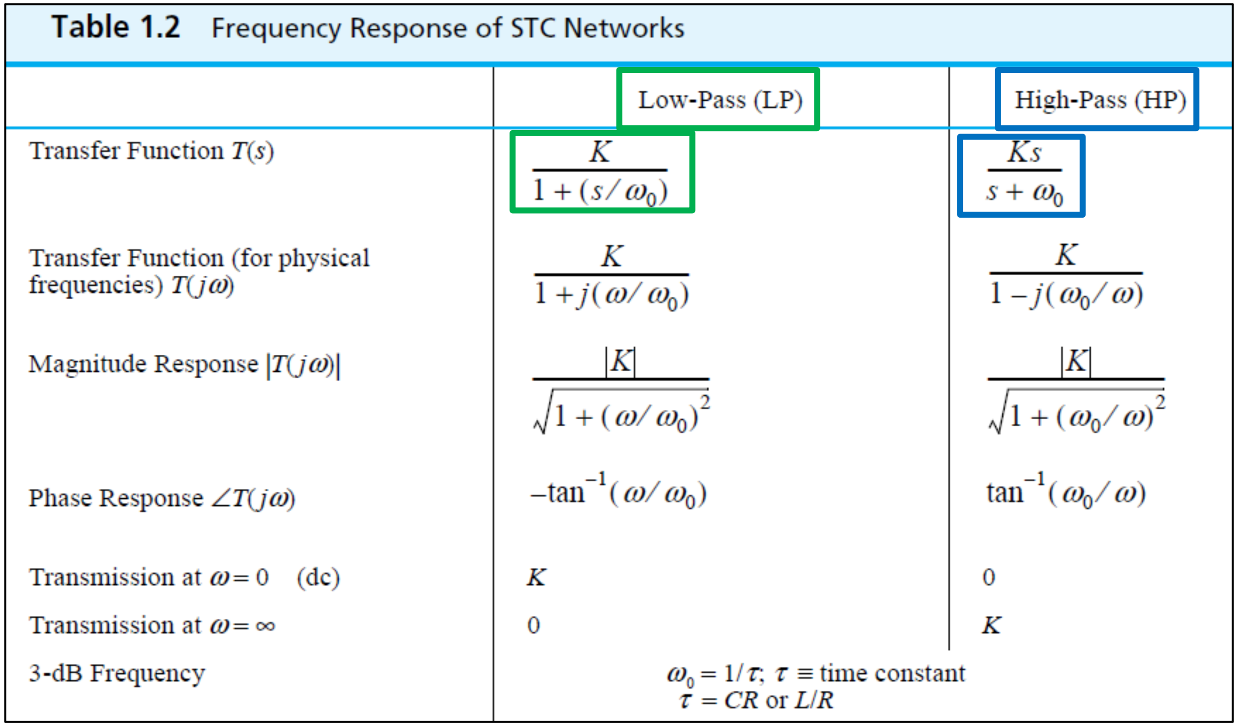
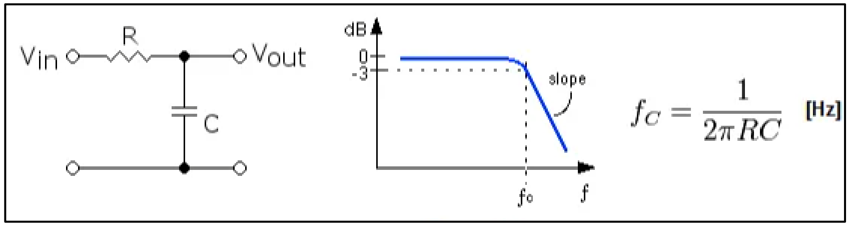
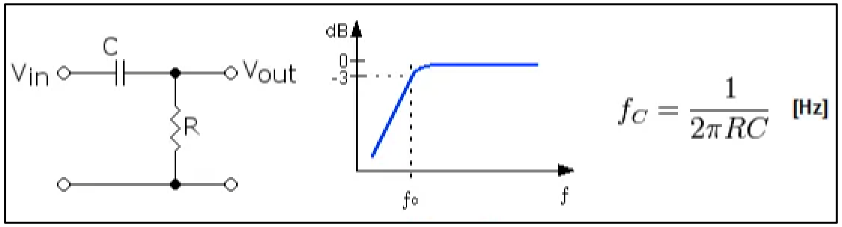


Table 1.2 Frequency Response of STC Networks

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\dfrac{K}{1+(s/\omega_0)}$	$\dfrac{Ks}{s+\omega_0}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\dfrac{K}{1+j(\omega/\omega_0)}$	$\dfrac{K}{1-j(\omega_0/\omega)}$
Magnitude Response $\lvert T(j\omega)\rvert$	$\dfrac{\lvert K\rvert}{\sqrt{1+(\omega/\omega_0)^2}}$	$\dfrac{\lvert K\rvert}{\sqrt{1+(\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega = 0$ (dc)	K	0
Transmission at $\omega = \infty$	0	K
3-dB Frequency	$\omega_0 = 1/\tau$; $\tau \equiv$ time constant $\tau = CR$ or L/R	



$$f_C = \frac{1}{2\pi RC} \text{ [Hz]}$$

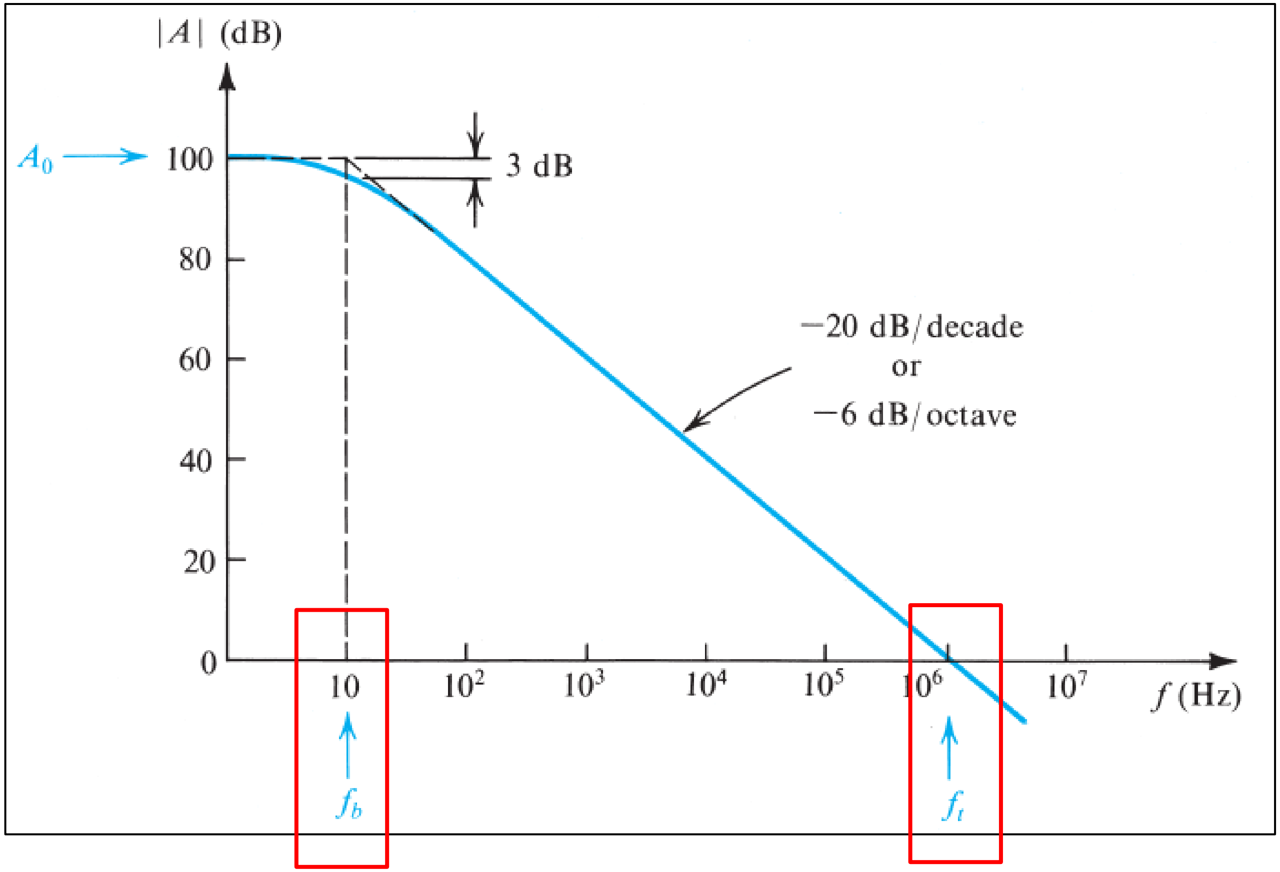


$$f_C = \frac{1}{2\pi RC} \text{ [Hz]}$$

low-pass network

high-pass network

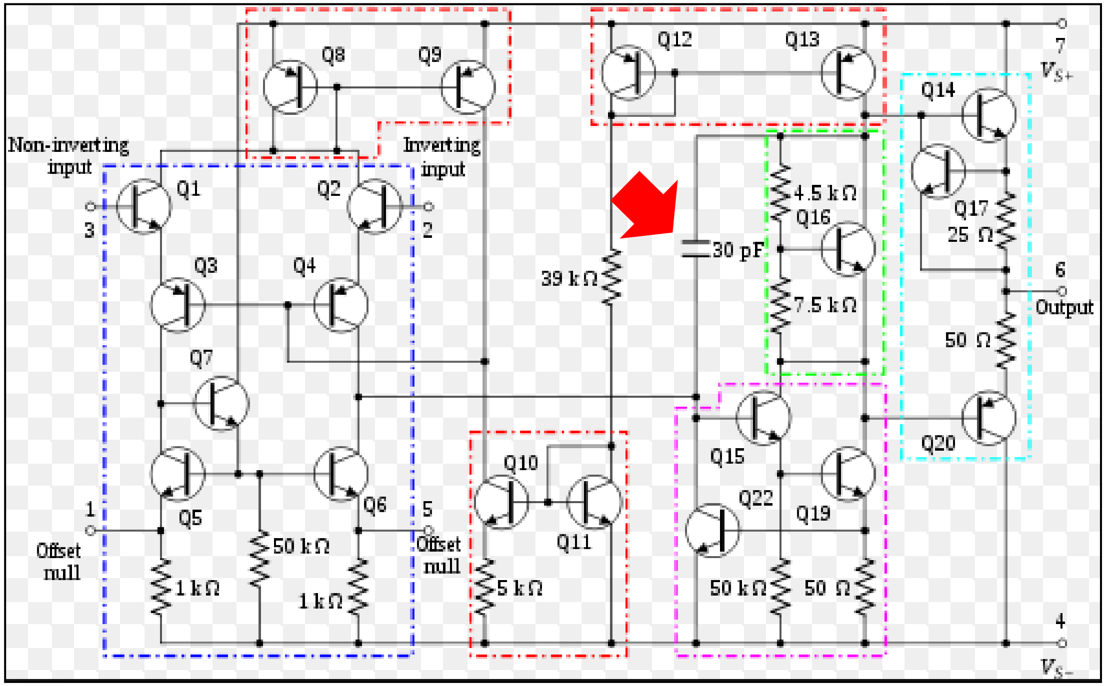
1 **The differential open-loop gain *A* of an op amp is not infinite;** rather, it is finite and decreases with frequency. The figure below shows a plot for |A|, with the numbers typical of some commercially available general-purpose op amps (such as the popular 741-type op amp, available from many semiconductor manufacturers).



Note that although the gain is quite high at dc and low frequencies, it starts to fall off at a rather low frequency (10 Hz in our example). The uniform –20-dB/decade gain rolloff shown is typical of **internally compensated** op amps. **These are units that have a network (usually a single capacitor)** included within the same IC chip whose function is to cause the op-amp gain to have the single-time-constant (STC) low-pass response shown.

This process of modifying the open-loop gain is termed **frequency compensation**, and its purpose is to ensure that op-amp circuits will be stable (as opposed to oscillatory).

The single capacitor included within the same IC chip whose function is to cause the op-amp gain to have the **single-time-constant (STC) low-pass response** shown.



2 By analogy to the response of low-pass STC, the gain $A(s)$ of an internally compensated op amp may be expressed as:

$$A(s) = \frac{A_0}{1 + s/\omega_b} \longrightarrow \boxed{A(j\omega) = \frac{A_0}{1 + j\omega/\omega_b}}$$

where ***A_0*** **denotes the dc gain and *ω_b* is the 3-dB frequency (corner frequency or "break" frequency).**

For frequencies $\omega \gg \omega_b$ (about 10 times and higher) the equation can be approximated by:

$$A(j\omega) \simeq \frac{A_0\omega_b}{j\omega} \longrightarrow \boxed{|A(j\omega)| = \frac{A_0\omega_b}{\omega}}$$

from which it can be seen that the **gain |*A*| reaches unity (0 dB) at a frequency denoted by *ω_t*** and given by:

$$\longrightarrow \omega_t = A_0\omega_b$$

The frequency $f_t = w_t / 2\pi$ is usually specified on the data sheets of commercially available op amps and is known as the **unity-gain bandwidth.**

$$|A(j\omega)| = \frac{A_0\omega_b}{\omega} \longrightarrow \boxed{A(j\omega) \simeq \frac{\omega_t}{j\omega}}$$

$$\boxed{A(j\omega) \simeq \frac{\omega_t}{j\omega}}$$

3 *If f >> fb:*

- Doubling *f* (**an octave increase**) results in **halving the gain** (**a 6-dB reduction**)
- Increasing *f* by a factor of 10 (**a decade increase**) results in **reducing |*A*| by a factor of 10** (**a 20 dB reduction**).

An **op amp having this uniform –6-dB/octave (or equivalently –20-dB/decade) gain rolloff is said to have a single-pole model**.

Also, since this single pole *dominates* the amplifier frequency response, it is called **a dominant pole.**

Closed-Loop Gain

1

We next consider the effect of limited op-amp gain and bandwidth on the closed-loop transfer functions of the two basic configurations: the **inverting circuit** and the **noninverting circuit**. The closed-loop gain of the inverting amplifier, assuming a finite op-amp open-loop gain *A*, was derived before:

$$\boxed{\frac{V_o}{V_i} = \frac{-R_2/R_1}{1+(1+R_2/R_1)/A}}$$

Inverting Amplificer

$$\left.\begin{aligned} A(j\omega) &= \frac{A_0}{1+j\omega/\omega_b} \\ \omega_t &= A_0\omega_b \end{aligned}\right\} \longrightarrow \frac{V_o(s)}{V_i(s)} = \frac{-R_2/R_1}{1+\frac{1}{A_0}\left(1+\frac{R_2}{R_1}\right)+\frac{s}{\omega_t/(1+R_2/R_1)}}$$

For $A_o >> 1+R_2/R_1$, which is usualy the case: $\longrightarrow$

$$\boxed{\frac{V_o(s)}{V_i(s)} \simeq \frac{-R_2/R_1}{1+\frac{s}{\omega_t/(1+R_2/R_1)}}}$$

which is of the **same form as that for a low-pass STC network**. Thus, the inverting amplifier has an STC low-pass response with a dc gain of magnitude equal to R_2/R_1.

The closed-loop gain rolls off at a uniform –20dB/decade slope with a corner frequency (3-dB frequency) given by:

$$\boxed{w_b = w_{3dB} = \frac{w_t}{1+\frac{R_2}{R_1}}}$$

2

Similarly, analysis of the noninverting amplifier assuming a finite open-loop gain *A*, yields the closed-loop transfer function:

$$\boxed{\frac{V_o}{V_i} = \frac{1 + R_2/R_1}{1 + (1 + R_2/R_1)/A}}$$

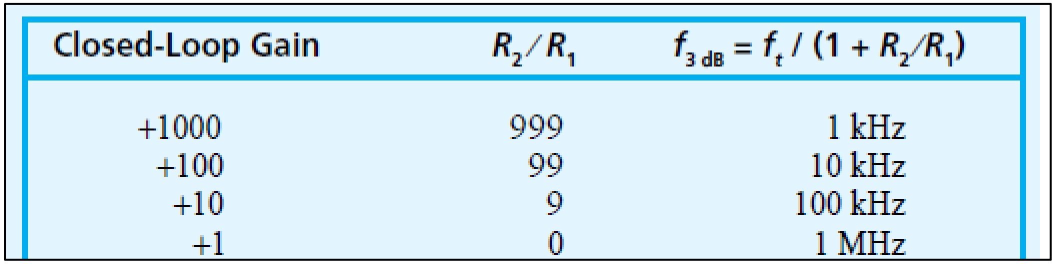
Non-Inverting Amplificer

Substituting for *A* and making the approximation $A_o >> 1 + R_2/R_1$ results in:

$$\frac{V_o(s)}{V_i(s)} \simeq \frac{1 + R_2/R_1}{1 + \dfrac{s}{\omega_t/(1 + R_2/R_1)}}$$

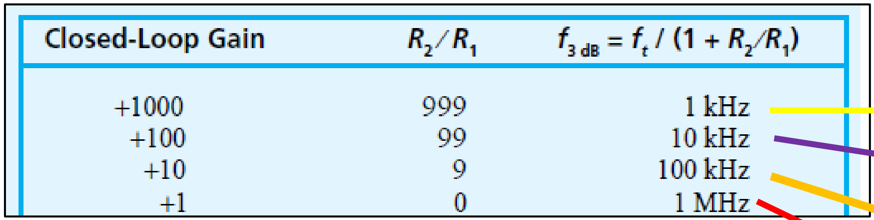
Thus **the noninverting amplifier has an STC low-pass response** with a dc gain of $(1 + R_2/R_1)$ and a 3dB frequency also given by:

$$\boxed{w_b = w_{3dB} = \frac{w_t}{1 + \dfrac{R_2}{R_1}}}$$

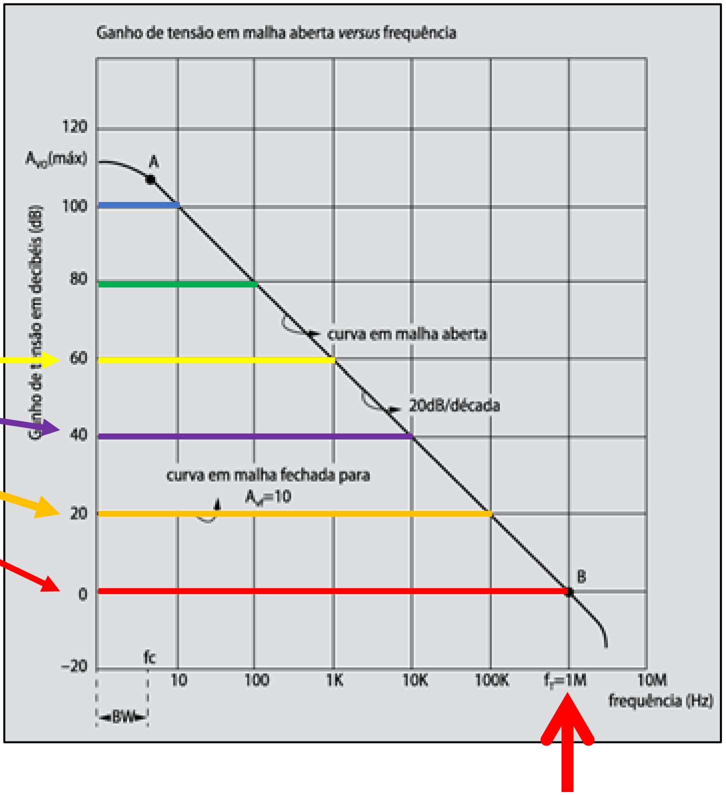


Closed-Loop Gain	R_2/R_1	$f_{3\,dB} = f_t/(1 + R_2/R_1)$
+1000	999	1 kHz
+100	99	10 kHz
+10	9	100 kHz
+1	0	1 MHz

Frequency Response (Inverting and Noninverting Amplifier (op amp 741)

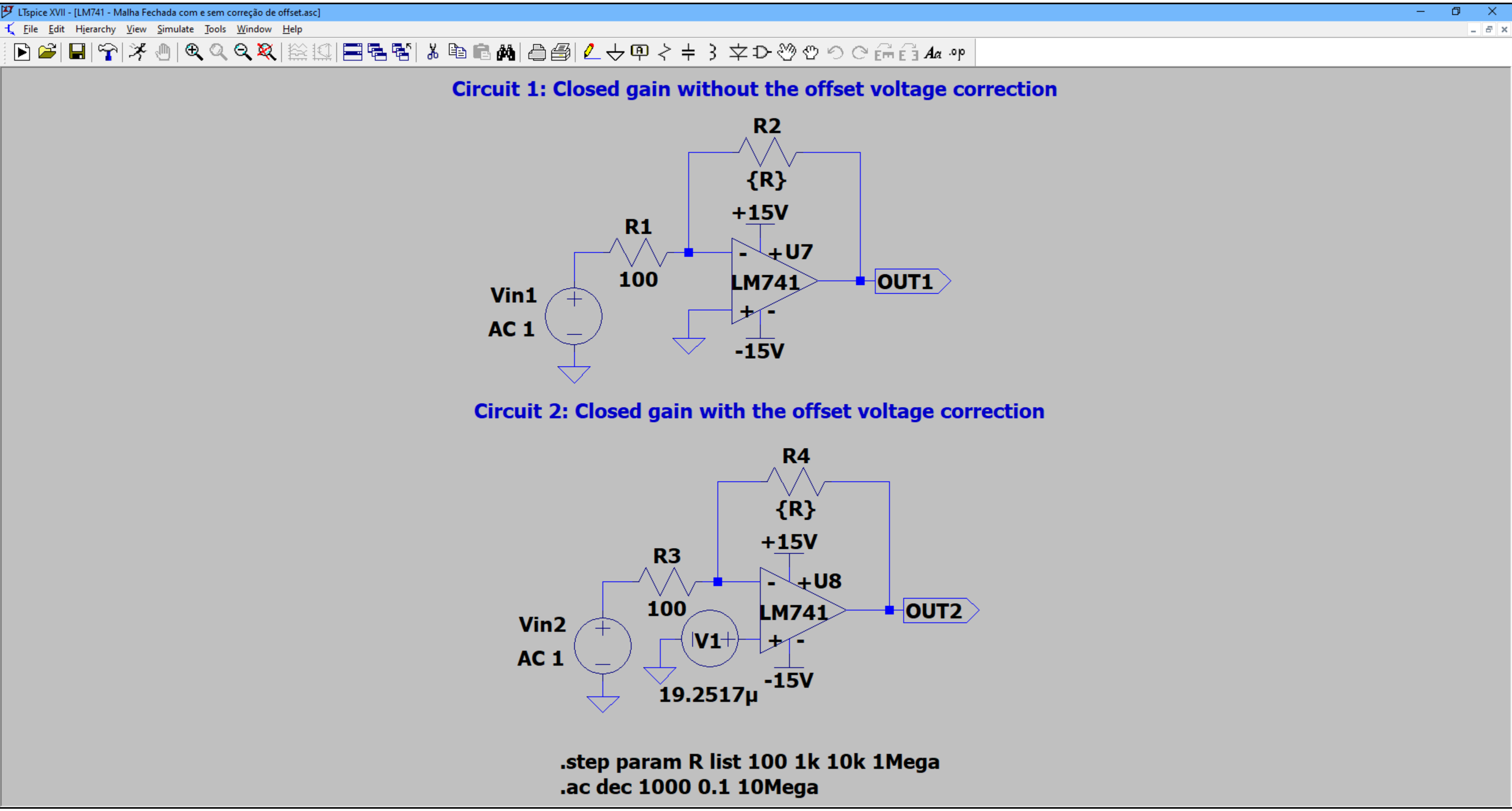


Closed-Loop Gain	R_2/R_1	$f_{3\,dB} = f_t / (1 + R_2/R_1)$
+1000	999	1 kHz
+100	99	10 kHz
+10	9	100 kHz
+1	0	1 MHz



Influence of the Offset Voltage in the Closed-Loop Gain

Frequency Response
Close-Loop Gain
(the op amp LM741 has a very LOW offset voltage)

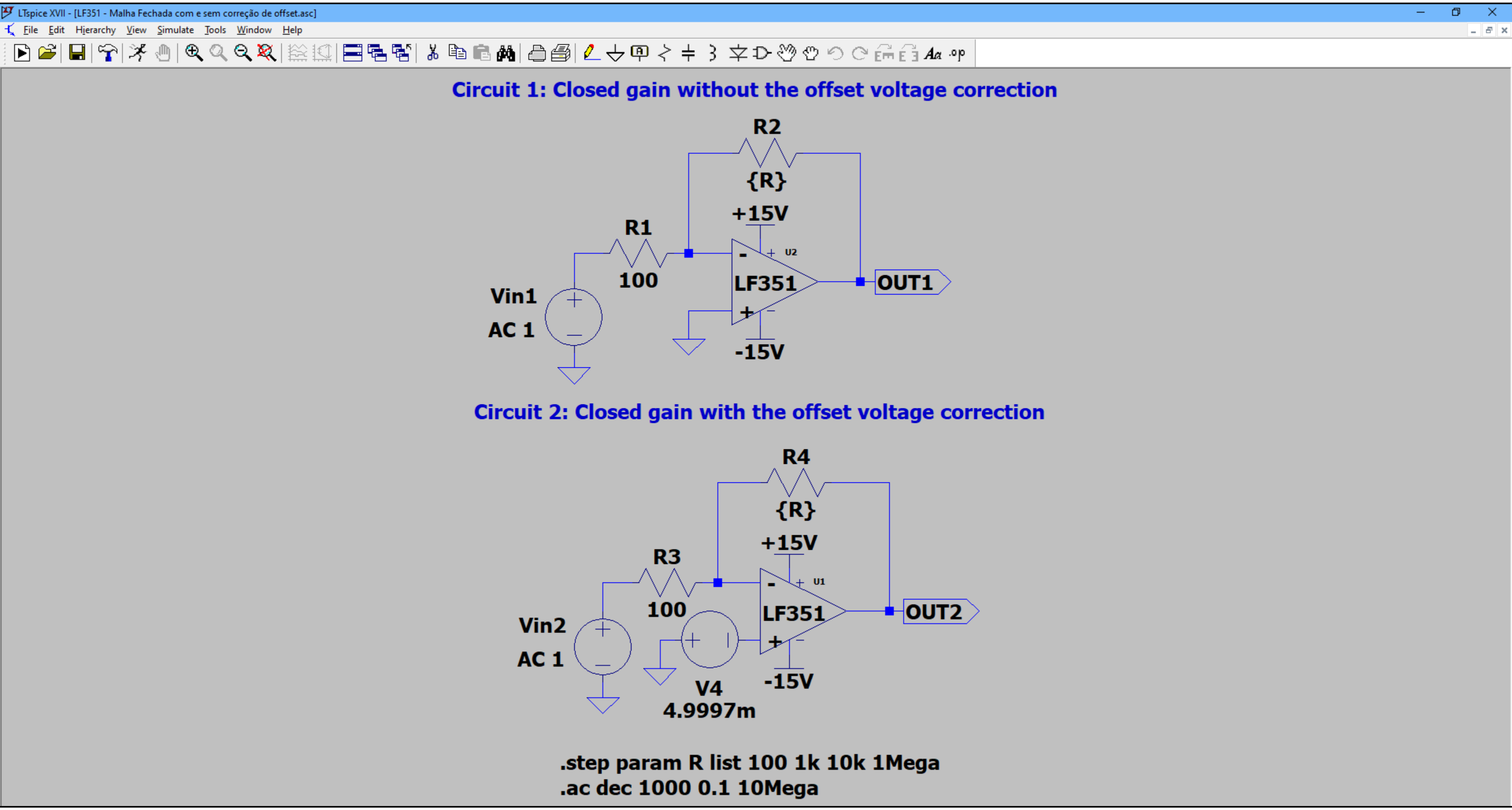


Circuit 1: Closed gain without the offset voltage correction

Circuit 2: Closed gain with the offset voltage correction

```
.step param R list 100 1k 10k 1Mega
.ac dec 1000 0.1 10Mega
```

Frequency Response
Close-Loop Gain
(the op amp LF351 has a HIGH offset voltage)



Circuit 1: Closed gain without the offset voltage correction

Circuit 2: Closed gain with the offset voltage correction

```
.step param R list 100 1k 10k 1Mega
.ac dec 1000 0.1 10Mega
```