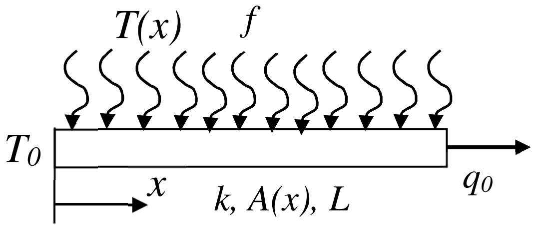
Variational Calculus

1) Consider the optimization problem below that aims to maximize the thermal conductivity of a bar subject to a distributed heat source, and boundary conditions shown in the figure.

$$\begin{aligned} &\underset{A(x),\,T(x)}{\text{Min}} \quad -\tfrac{1}{2}\int_0^L kA(x)\left(\frac{dT(x)}{dx}\right)^2 dx + \int_0^L fT dx + q_0 T(L) \\ &\text{tal que} \quad \int_0^L A dx = V_0 \quad \text{(volume)} \end{aligned}$$

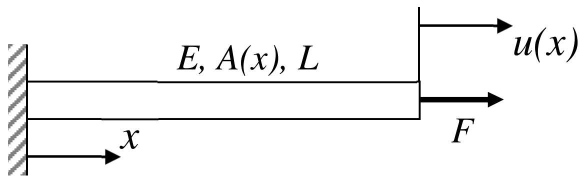


Derive the optimality equation for the optimization problem, as well as the thermal equilibrium equation for the bar. For that:

a) Write the Lagrangian function (L) of the problem;
b) Variate the Lagrangian function and impose the stationarity condition ($\delta L = 0$);
c) Isolate the terms referring to the variationals of the design variable functions (δA e δu) and determine the required equations

2) Consider the optimization problem below that aims to minimize the flexibility of a bar subjected to a force as shown in the figure.

$$\begin{aligned} &\underset{A(x),\,u(x)}{\text{Min}} \quad \tfrac{1}{2}\int_0^L EA(x)\left(\frac{du(x)}{dx}\right)^2 dx - Fu(L) \\ &\text{tal que} \quad \int_0^L A dx = V_0 \quad \text{(volume)} \end{aligned}$$



Derive the optimality equation from the optimization problem as well as the bar equilibrium equation. For that:

a) Write the Lagrangian function (L) of the problem;
b) Variate the Lagrangian function and impose the stationarity condition ($\delta L = 0$);
c) Isolate the terms referring to the variationals of the design variable functions (δA e δu)) and determine the required equations