Laurent Series: can represent any analytic function on the complex plane, and that includes W(z). They are given in generic form by:

$$f(z) = \underbrace{\sum_{n=0}^{\infty} a_n (z-z_0)^n}_{\text{Taylor}} + \underbrace{\sum_{n=1}^{\infty} \frac{b_n}{(z-z_0)^n}}_{\text{Principal Part}} = \sum_{n=-\infty}^{\infty} c_n (z-z_0)^n$$

for $R_1 < |z-z_0| < R_2$ and $c_n = \frac{1}{2\pi i} \oint_C \frac{f(z)\,dz}{(z-z_0)^{n+1}}$; $\forall n \in \mathbb{Z}$

In particular, for $n = 0, 1, 2 \ldots$; $(n \in \mathbb{N}) \Rightarrow c_n = \frac{f^{(n)}(z)}{n!}$

It's also plain to see that: $\begin{cases} \text{Positive Powers blow up at } z = \infty \\ \text{Negative Powers blow up as } |z-z_0| \to 0 \end{cases}$

On deriving a series representation of W(z), one must consider that there can be no singularity (blow-up) at $z = \infty$, since the flow must be uniform there. That means that the highest (positive) power such a series can have is just first order: $W(z) = U_\infty \Rightarrow F(z) = U_\infty z$

On the other hand, by placing the origin of the coordinates along with all singularities inside the body, one can represent W(z) by:

$$W(z) = A_0 + \frac{A_1}{z} + \cdots = \sum_{n=0}^{\infty} \frac{A_n}{z^n}$$

$$F(z) = A_0 z + A_1 \log(z) - \sum_{n=2}^{\infty} \frac{A_n}{(n-1)\, z^{(n-1)}} + \text{constant}$$

the presence of the term $A_1 \log(z)$ allows one to introduce circulation into the flow.

$\zeta = \xi + i\eta$

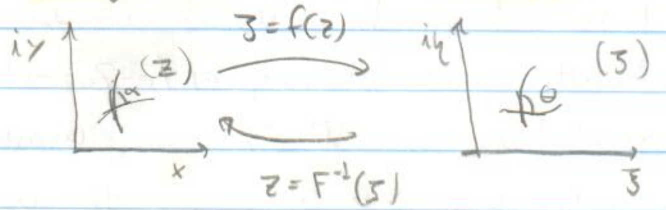
$$\frac{d\zeta}{dz} = \xi_{,x} + i\eta_{,x} = \eta_{,y} - i\xi_{,y}$$
$$= \xi_{,x} - i\xi_{,y} = \eta_{,y} + i\eta_{,x}$$

Mention the existence of the Blasius relations for forces and moments in terms of complex variables. Although they are beyond the scope of this introductory course.

Velocity Magnitude under conformal transformations

$$U^2 = |W(z)|^2 = |W(\zeta)|^2 \left|\frac{d\zeta}{dz}\right|^2$$

Meaning of the Derivatives of Analytic functions:

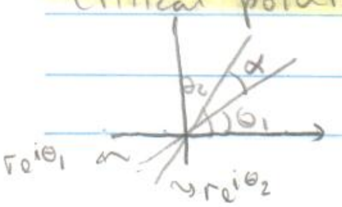


$$\delta\zeta = \frac{d\zeta}{dz}\,\delta z \Rightarrow \delta\zeta = \frac{df}{dz}\,\delta z$$

$$\begin{cases} |\delta\zeta| = \left|\frac{df}{dz}\right| \cdot |\delta z| \Rightarrow \text{isotropic stretching (local)} \\ \arg(\delta\zeta) = \arg\left(\frac{df}{dz}\right) + \arg(\delta z) \Rightarrow \text{isotropic rotation (local)} \end{cases}$$

conformal mapping $\Rightarrow$ Preserves local angles

But it is non-conformal at points where $\boxed{f'(z) = 0}$
critical points

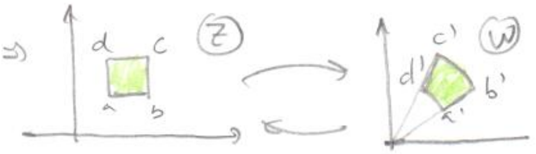


$\theta_2 - \theta_1 = \alpha$

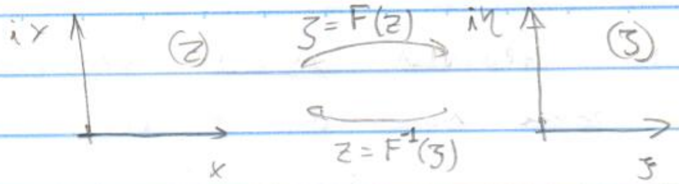
$\zeta = z^n$

$$\delta\zeta = n z^{(n-1)}\delta z$$
$$\left.\begin{array}{l} \zeta_1 = r^n e^{i(n\theta_1)} \\ \zeta_2 = r^n e^{i(n\theta_2)} \end{array}\right\} \underset{(z)}{\alpha} \xrightarrow{f(z)} \underset{(\zeta)}{n\alpha}$$

$W = \exp(z) \Rightarrow$



Mapping Flows:



$z = x + iy$ and $\zeta = \xi + i\eta$

$$\left.\begin{aligned}\xi &= \xi(x,y)\\ \eta &= \eta(x,y)\end{aligned}\right\} \Rightarrow \zeta = F(z) \rightleftharpoons z = F^{-1}(\zeta) \left\{\begin{aligned} x &= x(\xi,\eta)\\ y &= y(\xi,\eta)\end{aligned}\right.$$

$$F(z) = \Phi(x,y) + i\,\Psi(x,y) = \Phi_\zeta(\xi,\eta) + i\,\Psi_\zeta(\xi,\eta)$$ (Scalar)

$$\left\{\begin{aligned}\Phi_\zeta(\xi,\eta) &= \Phi[x(\xi,\eta), y(\xi,\eta)]\\ \Psi_\zeta(\xi,\eta) &= \Psi[x(\xi,\eta), y(\xi,\eta)]\end{aligned}\right\}\quad F(z) = \tilde{F}(\zeta) = \tilde{F}[\zeta(z)]$$ (F is a scalar)

$$J = \begin{vmatrix}\xi_{,x} & \xi_{,y}\\ \eta_{,x} & \eta_{,y}\end{vmatrix} \underset{\text{C.R.}}{=} (\xi_{,x})^2 + (\xi_{,y})^2 = (\eta_{,x})^2 + (\eta_{,y})^2$$

the transformation is analytic wherever $J \neq 0$

$J = \left|\frac{d\zeta}{dz}\right|^2$

$$\frac{dF}{dz} = \frac{d\tilde{F}}{d\zeta}\frac{d\zeta}{dz} \iff \frac{d\tilde{F}}{d\zeta} = \frac{dF}{dz}\frac{1}{(d\zeta/dz)}$$

Laplace operator: $\boxed{\nabla^2_{xy} = \left|\frac{d\zeta}{dz}\right|^2 \nabla^2_{\xi\eta}}$

$\tilde{F}(\zeta)$ exists at all points where $d\zeta/dz$ does not vanish

at the critical points of the transformation, $\tilde{F}(\zeta)$ ceases to be analytic.

$$\boxed{\frac{dz}{d\zeta} = \frac{1}{d\zeta/dz}}\ ;\quad \boxed{\tilde{W}(\zeta) = W[z(\zeta)]\frac{dz}{d\zeta}}$$

$\tilde{W}(\zeta) = \infty$ at the critical points, where $\frac{d\zeta}{dz} = 0$ and $\frac{dz}{d\zeta} = \infty$

therefore, we try to keep critical points out of the region of

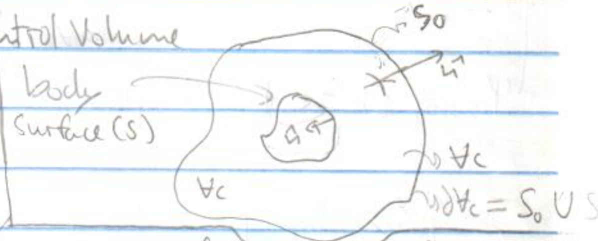
If Γ is preserved under conformal mappings, then the same holds for the lift

$$L = \rho U_\infty \Gamma \qquad \left\{ \Gamma + iQ = \oint_C W(z)\,dz = F(z_2) - F(z_1) \right.$$

F(z) is preserved

However, this does not hold for Moments.
Otherwise the zero moment about a cylinder with circulation would also apply to a Joukowski airfoil, which doesn't make any sense ...

Very brief note on the so-called Blasius' relations:
(Normal vectors point outward from control Volume
T. Reynolds: linear and angular momentum theorems.



$$\sum \vec{F} = \iiint_{\forall_c} \frac{\partial}{\partial t}(\rho \vec{U})\,d\forall + \oint_{S_0} \rho \vec{U}(\vec{U}\cdot\hat{n})\,ds + \oint_{S} \rho \vec{U}\underbrace{(\vec{U}\cdot\hat{n})}_{=0 \text{ on } S}\,ds + \oint_{S_0} p\hat{n}\,ds + \oint_{S} p\hat{n}\,ds$$

Bernoulli, H = constant: $P = H - \frac{\rho U^2}{2} - \rho\frac{\partial \Phi}{\partial t}$ — Steady flow at $S_0 \Rightarrow \left.\frac{\partial \Phi}{\partial t}\right|_{S_0} = 0$ — force $\vec{F}$ the fluid exerts upon the body

$$\oint_S p\hat{n}\,ds = \vec{F} = -\iiint_{\forall_c} \frac{\rho\,\partial \vec{U}}{\partial t}\,d\forall - \oint_{S_0} \rho\vec{U}(\vec{U}\cdot\hat{n})\,ds + \oint_{S_0} \frac{\rho U^2}{2}\hat{n}\,ds$$

$\vec{M}$ this is the moment the fluid exerts upon the body

$$\sum \vec{M} = \iiint_{\forall_c} \frac{\partial}{\partial t}(\rho \vec{r}\times\vec{U})\,d\forall + \oint_{S_0} \rho\,\vec{r}\times\vec{U}\,\vec{U}\cdot\hat{n}\,ds + \oint_{S} \rho\,\vec{r}\times\vec{U}\underbrace{\vec{U}\cdot\hat{n}}_{=0 \text{ on } S}\,ds + \oint_{S_0} \vec{r}\times\hat{n}P\,ds + \oint_{S} \vec{r}\times\hat{n}P\,ds$$

Similarly, by using Bernoulli's eq., we get:

$$\oint_S \vec{r}\times\hat{n}P\,ds = \vec{M} = -\iiint_{\forall_c} \frac{\rho\,\partial(\vec{r}\times\vec{U})}{\partial t}\,d\forall - \oint_{S_0} \rho\,\vec{r}\times\vec{U}(\vec{U}\cdot\hat{n})\,ds + \oint_{S_0} \vec{r}\times\hat{n}\left(\frac{\rho U^2}{2}\right)ds$$

see next back page → →

interest. Unless $W(z)=0$ at those points.

$$\Gamma_\zeta + i Q_\zeta = \oint_{C_\zeta} \tilde{W}(\zeta)\, d\zeta = \oint_{C_\zeta} W(z) \frac{1}{d\zeta/dz} \frac{d\zeta}{dz}\, dz = \int_{C_z} W(z)\, dz = \Gamma + iQ$$

Which means these quantities are preserved under the transformation, unless if there are sources or vortices that are situated at critical points of the transformation.

The requirement of an uniform flow at infinity ($z=\infty$) in a given direction α, i.e. $U_\infty e^{-i\alpha}$, further imposes conditions on the transformation:

$$\text{at } \zeta = \infty \Rightarrow \left\{ z = \zeta = \infty \quad \text{and} \quad \left.\frac{dz}{d\zeta}\right|_{\zeta=\infty} = 1 \right.$$

As a result of these conditions, the generic form of the transformation is given by:

$$z(\zeta) = \zeta + \sum_{n=1}^{\infty} \frac{c_n}{\zeta^n} \Rightarrow \frac{dz}{d\zeta} = 1 - \sum_{n=1}^{\infty} \frac{n\, c_n}{\zeta^{(n+1)}}$$

Which is, in a way, a "truncated" Laurent series.

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Then, on writing these expressions for Steady. 2-D flows, and by casting them in terms of complex variables, one gets:

$$X - iY = i\frac{\rho}{2}\oint_{C_0}[W(z)]^2\,dz$$ forces

$$M = -\frac{\rho}{2}\,\mathrm{Re}\left\{\oint_{C_0}[W(z)]^2 z\,dz\right\}$$ Moments

where C_0 is any arbitrary circuit that contains the body in its interior.

A few important notes on circulation and Lift, just before we delve into the matter of lift, proper.

From Karamcheti's book: "Principles of ideal fluid Aerodynamics"

1) hystorical Perspective = { D'Alembert's (1717 – 1783) Paradox
Kutta (1902) and Joukowski (1906)

2) the potential velocity is finite at any given contour, as long as there are no slope discontinuities in that contour. Whenever discontinuities appear, velocity blows up. (P.389)

unless :

3) However, there is a particular value of Γ for which the flow leaves the discontinuity smoothly. Because this particular value of Γ cancels out the flow around the discontinuity. (P.392)

4) Experimental evidence suggests that only certain bodies, which have sharp (pointed) trailing-edges, are suitable as lifting bodies or wings. (P390). Because this feature makes for a stable pressure recovery, and low pressure region. Despite the fact that it involves flow under reverse pressure gradient.

5) From the standpoint of mass conservation (Laplace's equation) alone, the potential flow solution is <u>not</u> unique, until (Γ) the circulation is specified.

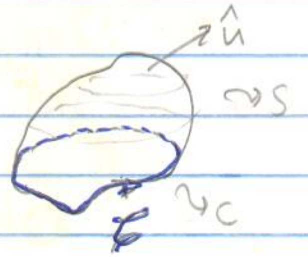
6) When the flow starts around an airfoil, it looks as if the Kutta condition doesn't apply. But, as soon as it has traveled about one chord length, the Kutta condition is established, along with the steady flow (P393)

$\vec{\omega} = 2\vec{\Omega}$

vorticity ↑ ↳ angular velocity

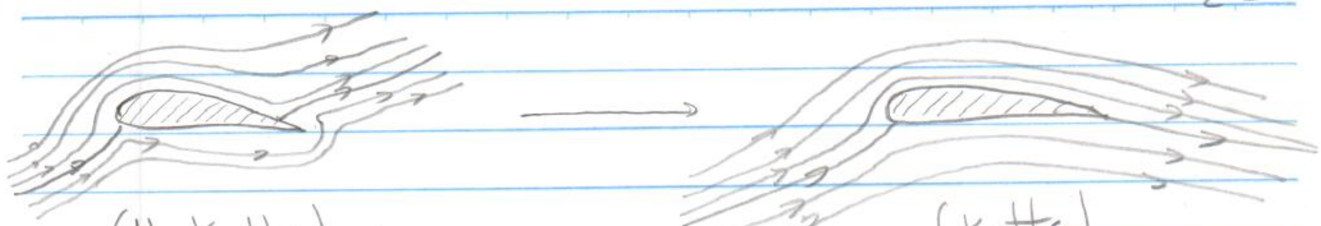
Stokes' Theorem

$$\Gamma \equiv \oint_{\mathcal{C}} \vec{q} \cdot \vec{dl} = \iint_{S} (\nabla \times \vec{q}) \cdot \hat{n} \, ds = \iint \vec{\omega} \cdot \hat{n} \, ds$$



In potential flows, we don't need to know the details of the vorticity field, which is incidentally generated by viscosity. On the contrary, all we actually need is a single, integral, measure o vorticity, namely, the circulation.

Now, it is worth adding that, in most cases we will not compute Γ by means of the above integral. Instead, we'll determine it from other, global, conditions, such as Kutta condition, itself.



(No Kutta) (Kutta)

7) Circulation (Γ) owes it existence to the presence of viscosity — and of the non-slip condition, thereof. Those are real fluid properties that do not belong in the potential flow model. Instead, they are borne out of experiments — and, thus, they are brought in the model from outside. Remarkably, though, the right amount of circulation prescribed by the Kutta condition does <u>not</u> depend on the value of viscosity. That's why the potential model, alone, can handle the problem. (hypothesis of thin wakes – zero measure – negligible details)

8) Kutta Condition and Trailing edges (P. 399)

There are basically two types of T.E. the finite angle T.E. and the cusped T.E.

It can be shown that the discontinuities in potential flow (steady ones) must meet the conditions: $p_+ = p_-$ and $\vec{V}^+\cdot\hat{n} = \vec{V}^-\cdot\hat{n}$

Hence the discontinuities can only afect the tangential component of the flow velocity. Furthermore, Bernoulli eq. implies that

$$p^+ + \frac{\rho (u^+)^2}{2} = p_0 = p^- + \frac{\rho (u^-)^2}{2} \Rightarrow \|\vec{v}^-\| = \|\vec{v}^+\|.$$

Now, this <u>does not</u> rule out tangential discontinuities. For instance, the wake of a 3-D wing causes a discontinuity in the spanwise component of the tangent velocity, while the flow field meets all the above conditions. But when it comes to 2-D flows, the above conditions simply mean that there can be no velocity discontinuity whatsoever...

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Then what does this mean for either type of T.E?
Well, it's quite simple:

finite angle } ⇒ stagnation point

Cusped T.E. } ⇒ tangential velocity on Both sides.

——x——x——x ——x ——x ——x ——x

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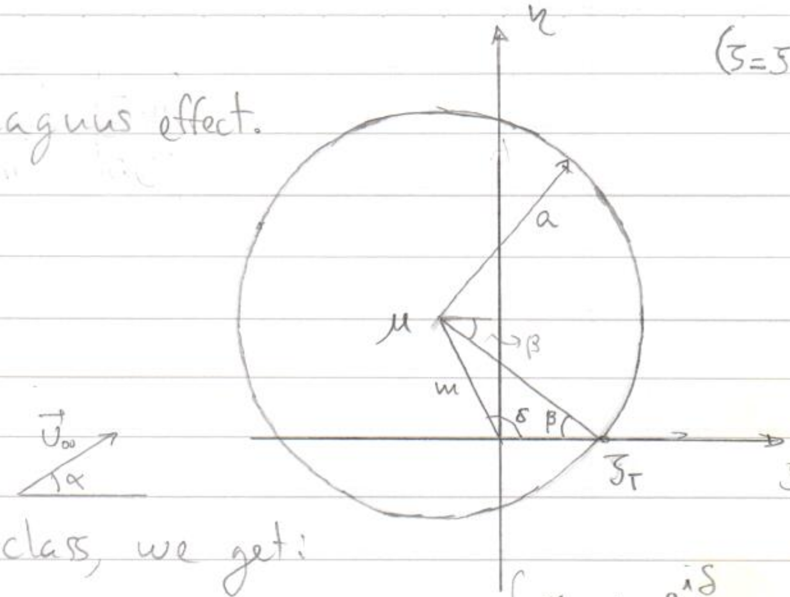
Joukowski Airfoil

$(\zeta = \xi + i\eta)$

Cylinder with Magnus effect.

$$\zeta = \mu + a e^{i\theta}$$

$$(0 \leq \theta \leq 2\pi)$$



$$\begin{cases} \mu = m e^{i\delta} \\ \zeta_T = \mu + a e^{-i\beta} \end{cases}$$

From the previous class, we get:

$$F(\tilde{\zeta}) = U_\infty \tilde{\zeta} + \frac{U_\infty a^2}{\tilde{\zeta}} + \frac{i\Gamma}{2\pi} \log\left(\frac{\tilde{\zeta}}{a}\right)$$

where $\zeta = \tilde{\zeta} e^{i\alpha} + \mu$, so we get: $\tilde{\zeta} = (\zeta - \mu) e^{-i\alpha}$

$$F(\zeta) = U_\infty (\zeta - \mu) e^{-i\alpha} + \frac{U_\infty a^2 e^{i\alpha}}{(\zeta - \mu)} + \frac{i\Gamma}{2\pi} \log\left(\frac{\zeta - \mu}{a e^{i\alpha}}\right) \quad (1)$$

Kutta condition:

$$W(z)\Big|_{z_T} = W(\zeta)\Big|_{\zeta_T} \frac{1}{\left(\frac{dz}{d\zeta}\right)_{\zeta_T}} \qquad \begin{cases} \left.\frac{dz}{d\zeta}\right|_{\zeta_T} = 0 \\ W(\zeta_T) = 0 \end{cases}$$

$$W(\zeta) = \frac{dF}{d\zeta} = U_\infty e^{-i\alpha} - \frac{U_\infty a^2 e^{i\alpha}}{(\zeta - \mu)^2} + \frac{i\Gamma}{2\pi} \frac{a e^{i\alpha}}{(\zeta - \mu)} \frac{1}{a e^{i\alpha}}$$

$$W(\zeta) = U_\infty e^{-i\alpha} + \frac{i\Gamma}{2\pi(\zeta - \mu)} - \frac{U_\infty a^2 e^{i\alpha}}{(\zeta - \mu)^2} \quad (2)$$

Then, for the Kutta condition to be met, we must have:

$$W(\zeta_T) = 0 \quad ; \quad (\zeta_T - \mu) = a e^{-i\beta}$$

$$W(\zeta_T) = U_\infty e^{-i\alpha} + \frac{i\Gamma e^{i\beta}}{2\pi a} - \frac{U_\infty \cancel{a^2} e^{i\alpha+2\beta}}{\cancel{a^2}} = 0 \quad (* e^{i\alpha})$$

$$U_\infty + \frac{i\Gamma}{2\pi a} e^{i(\beta+\alpha)} - U_\infty e^{i2(\alpha+\beta)} = 0$$

$$i\Gamma e^{i(\beta+\alpha)} = 2\pi a U_\infty \left(e^{i2(\alpha+\beta)} - 1\right)$$

$$-\Gamma = i2\pi a U_\infty \left(e^{i(\alpha+\beta)} - e^{i(\alpha+\beta)}\right) \frac{2i}{2i}$$

$$\boxed{\Gamma = 4\pi a U_\infty \sin(\alpha+\beta)}$$

This is the value of Γ that ensures the Kutta condition is met!

Notice we have not yet transformed the cylinder into an airfoil... but we've already had to tackle the all important Kutta condition. Then, as a result of the fact that the circulation, and hence, the lift are preserved through the transformation, we can compute it already:

$$L = \rho U_\infty \Gamma = 4\pi \rho U_\infty^2 a \sin(\alpha+\beta) \Rightarrow C_L = \frac{L}{\frac{1}{2}\rho U_\infty^2 c}$$

$$C_L = 8\pi \left(\frac{a}{c}\right) \sin(\alpha+\beta)$$

($c \Rightarrow$ chord)

Then, for small angles of attack, which is where the model works out, we can make

$$C_L \cong 8\pi\left(\frac{a}{c}\right)(\alpha+\beta)$$

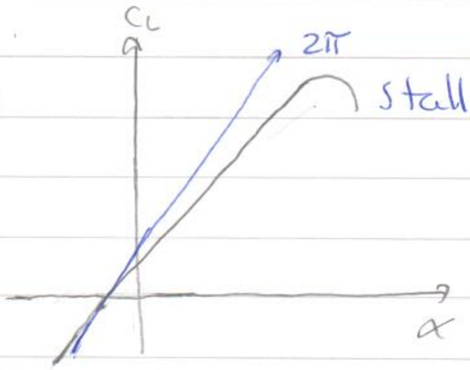
Moreover, for Joukowiski airfoils the chord is $c \cong 4a$, and we end up with:

$$\boxed{C_L \cong 2\pi(\alpha+\beta)}$$

This, in turn, implies that:

$$\alpha_0 = \alpha\Big|_{C_L=0} = -\beta \Rightarrow \boxed{\alpha_0 = -\beta}$$ $\beta \neq 0$ camber

which are exceedingly good approximations to actual experimental results.



Errors are related to thickness and to the angle of the trailing edge. In particular for potential flow solutions we adopt zero thickness trailing edges and, as for their angle, Joukowski airfoils will have cusped trailing edges.

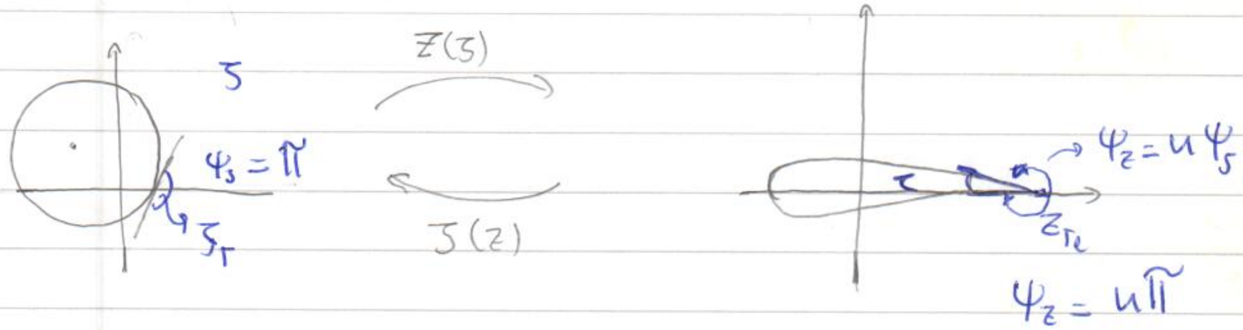
Joukowski transformation: $\boxed{Z = \zeta + \frac{C^2}{\zeta}}$

where the trailing edge $Z_t = \zeta_T = C \in \mathbb{R}$

$$\frac{dZ}{d\zeta} = 1 - \frac{C^2}{\zeta^2} \Rightarrow \left.\frac{dZ}{d\zeta}\right|_{\zeta_T} = 1 - \frac{\zeta_t^2}{\zeta_t^2} = 0$$

$$\frac{d^2 Z}{d\zeta^2} = +2\frac{C^2}{\zeta^3} \Rightarrow \left.\frac{d^2 Z}{d\zeta^2}\right|_{\zeta_T} = \frac{2}{\zeta_t} \neq 0 \Rightarrow n = 2$$

Trailing edge angle: $\tau = (2-n)\pi = 0$ <u>cuspid</u>



Pitching Moment on the airfoil:

Positive direction ⇒ nose-up (flip the sign of M_0 below)

From the Blasius' Relation for moments (P.454 Karamcheti)

$$M_0 = \mathrm{Re}\left[L\mu e^{-i\alpha} - i\,2\pi\rho U_\infty^2 C_1 e^{-2i\alpha}\right] \qquad (15.59)\ P.464$$

which gives us the moment with respect to the origin $Z=0$

In the most general case; $\mu = m e^{i\delta}$ and $C_1 \equiv \mathcal{C}^2 e^{i2\gamma}$

$$M_0 = -Lm\cos(\delta-\alpha) - 2\pi\rho U_\infty^2 C^2 \sin[2(\gamma-\alpha)]$$

$$= -Lm\cos(\delta-\alpha) + M_\mu$$

where $M_\mu = +2\pi\rho U_\infty^2 \mathcal{C}^2 \sin[2(\alpha-\gamma)]$ depends on the angle of attack, but it does not depend on the lift, nor does it depend on the circulation, for that matter. It is called Munk's moment.

Now let us compute the moment with respect to any point z on the plane of the airfoil, by making:

$$z = \mu + h e^{i\varphi} \qquad \text{(generic point)}$$

$$M_z = M_\mu + Lh\cos(\varphi-\alpha)$$

$$M_z = +2\pi\rho U_\infty^2 \mathcal{C}^2 \left\{ \sin[2(\alpha-\gamma)] + \frac{2ah}{\mathcal{C}^2}\sin(\alpha+\beta)\cos(\varphi-\alpha) \right\}$$

where we can also make: $2\sin(A)\cos(B) = \sin(A+B) + \sin(A-B)$

$$2\sin(\alpha+\beta)\cos(\varphi-\alpha) = \sin(\beta+\varphi) + \sin(2\alpha+\beta-\varphi)$$

$$M_z = 2\pi\rho U_\infty^2 \mathcal{C}^2 \left\{ \sin[2(\alpha-\gamma)] + \frac{ah}{\mathcal{C}^2}\left[\sin(\beta+\varphi) + \sin(2\alpha+\beta-\varphi)\right] \right\}$$

For M_z to become independent of the angle of attack α, we must have:

$$\left.\begin{cases} 2\alpha - 2\gamma = k_1\pi \\ 2\alpha + \beta - \varphi = k_2\pi \end{cases}\right\} \Rightarrow \varphi = \beta + 2\gamma + (k_1-k_2)\pi = \beta + 2\gamma + k\pi$$

where $k \in \mathbb{N}$

Hence, for $0 < \varphi < 2\pi$ we must have

$$\boxed{\varphi = \beta + 2\gamma + \pi}$$

which, in turn, leads to:

$$M_z = 2\pi \rho U_\infty^2 \ell^2 \left\{ \sin[2(\alpha-\gamma)] + \frac{ah}{\ell^2}\left[\sin(2\beta+2\gamma+\pi) + \sin(2\alpha-2\gamma-\pi)\right]\right\}$$

$$M_z = 2\pi \rho U_\infty^2 \ell^2 \left\{ \sin[2(\alpha-\gamma)] - \frac{ah}{\ell^2}\left[\sin(2\beta+2\gamma) + \sin(2\alpha-2\gamma)\right]\right\}$$

And for $h = \frac{\ell^2}{a}$ we get: $z = \mu + \frac{\ell^2}{a} e^{i(\beta+2\gamma+\pi)}$

$$M_z = 2\pi \rho U_\infty^2 \ell^2 \left\{ \cancel{\sin[2(\alpha-\gamma)]} - \sin[2(\beta+\gamma)] - \cancel{\sin[2(\alpha-\gamma)]}\right\}$$

$$M_z = -2\pi \rho U_\infty^2 \ell^2 \sin[2(\beta+\gamma)]$$

which is independent of α. Therefore, we get:

$$\begin{cases} z_{AC} \equiv \mu - \frac{\ell^2}{a} e^{i(\beta+2\gamma)} \\ M_{AC} = -2\pi \rho U_\infty^2 \ell^2 \sin[2(\beta+\gamma)] \end{cases}$$

In particular, for Joukowski Airfoils, we have

$\rightarrow \rightarrow$

Z

$$C_1 \equiv \ell^2 e^{i2\gamma} = C^2 = \zeta_T^2 \in \mathbb{R}$$

For Joukowski Airfoils, $\gamma = 0$, and the above formulals become simply:

$$\begin{cases} Z_{AC} \equiv \mu - \dfrac{\zeta_T^2}{a} e^{i\beta} \\ M_{AC} = -2\pi\rho U_\infty^2 \zeta_T^2 \sin(2\beta) \end{cases}$$

$$C_{mac} \equiv \frac{M_{AC}}{\frac{1}{2}\rho U_\infty^2 c^2} \Rightarrow C_{mac} = +4\pi \frac{\zeta_T^2}{c^2} \sin(2\alpha_0)$$

$c \Rightarrow$ airfoil chord $\neq C^2 = \zeta_T^2$

$\beta = -\alpha_0$; $\alpha_0 \leq 0 \therefore C_{mac} \Rightarrow$ nose-down

For thin airfoils Z_{AC} is very close to the so-called quarter-chord point, which sits on the chord line, at a $c/4$ distance from the leading edge.