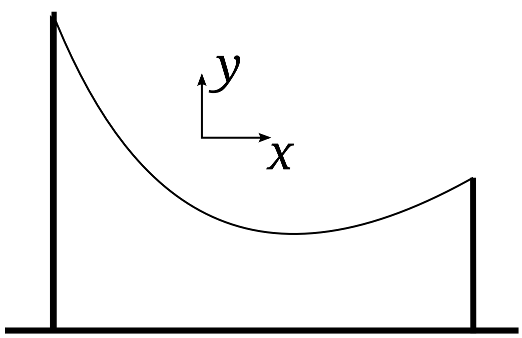
# PMR-5215 – Otimização Aplicada ao Projeto de Sistemas Mecânicos

## 2nd Assignment
## (groups of at most 2 students)

submit date: 2023-04-10

Catenary.

A cable with length *l* is suspended between two points:




Let *y(x)* be the cable height as a function of the horizontal coordinate *x*.
In this case, the total gravitational potential energy is given by:

$$E = \int \gamma\, y(x) \frac{ds}{dx}\, dx$$

where $\gamma$ is the cable *specific weight* by unity of length and *ds*/*dx* is the variation of the total length s and the horizontal displacement *x*.

Notice that $ds^2 = dy^2 + dx^2$, such that $\frac{ds}{dx} = \sqrt{1 + \frac{dy}{dx}}$ , that is, $E = \int \gamma\, y(x) \sqrt{1 + y'(x)^2}\, dx$

On the other hand, the total length *l* is given by:

$$l = \int \frac{ds}{dx} dx = \int \sqrt{1 + y'(x)^2}\, dx$$

a) Write the Lagrangean of the problem of finding the cable shape *y(x)* that minimizes the total gravitational potential energy subject to the constraint that its total length must be *l*.

b) Find the Lagrangean variation and the differential equation in *y(x)* that makes the Lagrangean stationary. Attempt to simplify at the maximum your final equation.

c) You should have found a $2^{nd}$ order differential equation in item b). Now find from the stationarity condition of the Lagrangean a *constant* value along the cable as a function solely of *y(x)* e *y'(x)* (that is, you must now find a first order diferential equation for *y(x)*).
Suggestion: Although the derivatives of the equation from item b) are in relation to the variable *x*, the

equation can be integrated in $y$ observing that $y''(x)=\frac{d}{dx}y'=\frac{dy}{dx}\frac{dy'}{dy}=y'\frac{dy'}{dy}$ and $\frac{d}{dy}\frac{y}{\sqrt{u}}=\frac{1}{\sqrt{u}}-\frac{yu'}{2u^{3/2}}$ . Alternatively, you may apply *Beltrami's* Identity.

d) Show that the catenary curve *y(x)* = *a* cosh((*x*-*b*)/*a*), where *a* e *b* are constants, is a solution of the differential equation from item c). $\cosh(x)=\frac{e^x+e^{-x}}{2}$ is the hyperbolic cosinus. Notice that $\frac{d}{dx}\cosh(x)=\sinh(x)=\frac{e^x-e^{-x}}{2}$ and $1+\sinh(x)^2=\cosh(x)^2$ .