IOC5815 - Dinâmica do Fluido Geofísico I

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1 Homework 3

1. Derive the following equations in spherical coordinates: $\boxed{2}$
   (a) The conservation of mass equation can be derived from a material derivative point of view (Lagrangian) or as a Eulerian derivation based on a fixed point. Starting from these two equations, write the conservation of mass in spherical coordinates using the definitions on Fig. 2.1 (V). You should show the derivation of the divergence operator on a vector $\boldsymbol{F}$ ($\nabla \cdot \boldsymbol{F}$) in spherical coordinates also. See the link in the site for reference (it is not just a copy, try to really understand it!).
   (b) Momentum equation. Follow the derivation of Vallis from 2.43 up to 2.47, explaining all the steps. Include some sketches to illustrate the system of vectors, when necessary.
2. Consider an inviscid ocean subject to small perturbations, then the pressure can be expressed as a basic state (on hydrostatic balance) plus a perturbation: $p(x, y, z, t) = p_0(z) + p'(x, y, z, t)$. Assume f-plane with traditional approximation for the momentum equations. $\boxed{2}$
   (a) Case 1: Homogeneous ocean. Write down the vertical component of the Navier-Stokes equation using the pressure expression above and explain how the hydrostatic balance would affect the movement. Is the perturbation pressure in hydrostatic balance?
   (b) Case 2: Stratified ocean. Write down the vertical component of the momentum equation under the Boussinesq approximation and determine what is the necessary condition for the hydrostatic balance to be valid in this case.
3. Consider a homogeneous flow in geostrophic and hydrostatic balance on f-plane. Show that in this case, the geostrofic velocities are non-divergent and that the flow can only be two-dimensional, that is, the Taylor-Proudman theorem applies. $\boxed{2}$
4. (a) Make the Boussinesq approximation to the density equation for an incompressible fluid where the density field is expressed as $\rho(x, y, z, t) = \tilde{\rho}(z) + \rho'(x, y, z, t)$. Provide a physical interpretation to the resulting expression. $\boxed{2}$
   (b) Combine the above derived equation to the linearized momentum (non-hydrostatic) and continuity equation and derive a wave equation for the vertical velocity component of the fluid. Assume a oscillating solution for w and determine the dispersion relation.
5. Determine the dispersion relation for a continuosly stratified Boussinesq fluid but when the hydrostatic approximation is valid. Explain what is the difference between this and the previous case in terms of wave frequency. What are the implications of making a hydrostatic approximation in a Boussinesq fluid? $\boxed{2}$