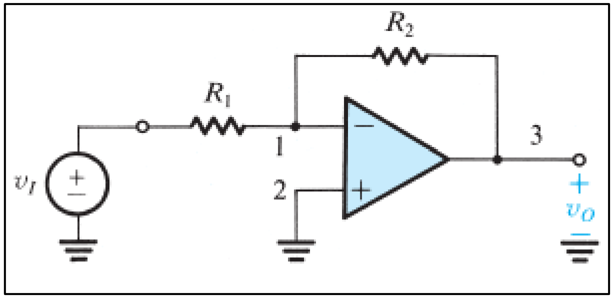
The Inverting Configuration with General Impedances

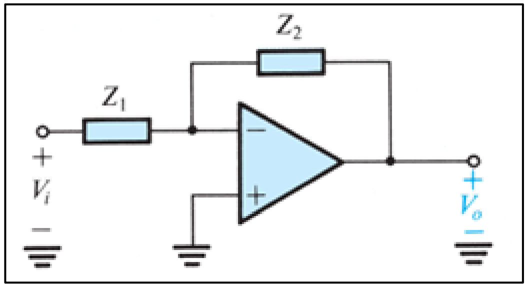
1 The op-amp circuit applications we have studied thus far utilized resistors in the op-amp feedback path and in connecting the signal source to the circuit (feed-in), that is, in the feed-in path. As a result, circuit operation has been (ideally) independent of frequency.



By allowing the use of capacitors together with resistors in the feedback and feed-in paths of op-amp circuits, a door is open to a very wide range of useful and exciting applications of the op amp.

The **op-amp-RC circuits** are considering two basic applications, namely, signal **integrators** and **differentiators**.

2 The inverting closed-loop configuration with impedances $Z_1(s)$ and $Z_2(s)$ replacing resistors R_1 and R_2, respectively is shown below.



$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

By replacing *s* by *jω* provides the transfer function, that is, the transmission magnitude and phase for a sinusoidal input signal of frequency ω.

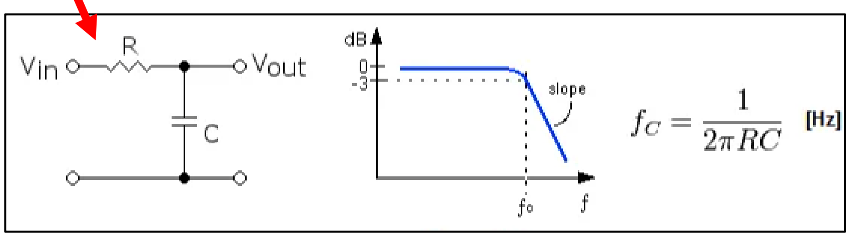
Note

A **single time constant network (STC)** is one that is composed of, or can be reduced to, one reactive component (capacitance or inductance) and one resistance.

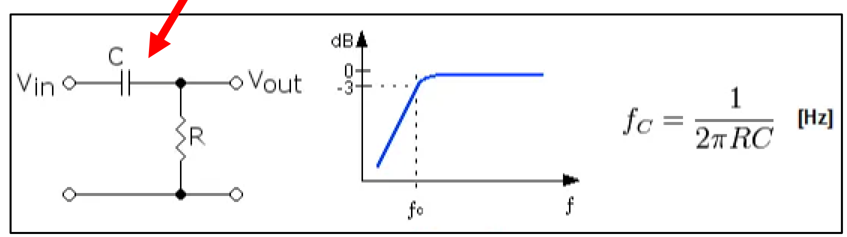
Table 1.2 Frequency Response of STC Networks

	Low-Pass (LP)	High-Pass (HP)
Transfer Function $T(s)$	$\frac{K}{1+(s/\omega_0)}$	$\frac{Ks}{s+\omega_0}$
Transfer Function (for physical frequencies) $T(j\omega)$	$\frac{K}{1+j(\omega/\omega_0)}$	$\frac{K}{1-j(\omega_0/\omega)}$
Magnitude Response $\lvert T(j\omega)\rvert$	$\frac{\lvert K\rvert}{\sqrt{1+(\omega/\omega_0)^2}}$	$\frac{\lvert K\rvert}{\sqrt{1+(\omega_0/\omega)^2}}$
Phase Response $\angle T(j\omega)$	$-\tan^{-1}(\omega/\omega_0)$	$\tan^{-1}(\omega_0/\omega)$
Transmission at $\omega=0$ (dc)	K	0
Transmission at $\omega=\infty$	0	K
3-dB Frequency	$\omega_0 = 1/\tau$; $\tau \equiv$ time constant $\tau = CR$ or L/R	

STC



STC



low-pass network

high-pass network

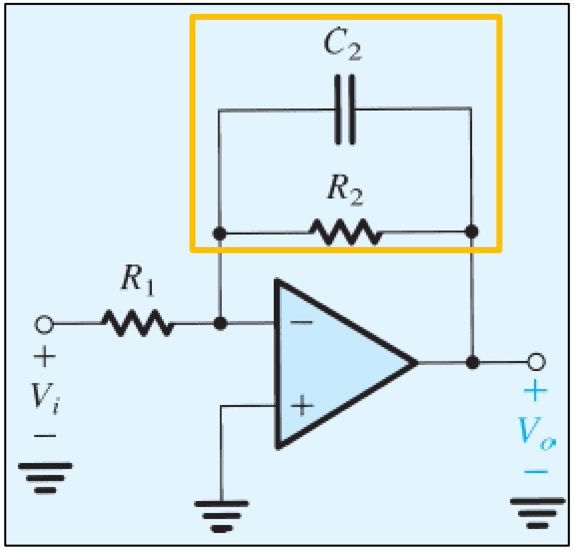
Exercise 1

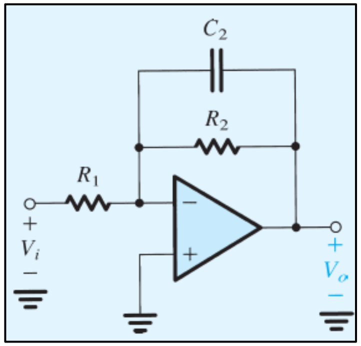
For the circuit below:

1) Derive an expression for the transfer function. Show that the **transfer function** is that of a low-pass STC (single time constant) circuit.

2) By expressing the transfer function in the standard form shown find the **dc gain** and the 3-dB frequency.

3) Design the circuit to obtain a **dc gain of 40 dB**, a **3-dB frequency of 1 kHz**, and an **input resistance of 1 kΩ**.





1 To obtain the transfer function of the circuit we substitute in the transfer function $Z_1 = R_1$ and since Z_2 is the parallel connection of two components, it is more convenient to work in terms of Y_2, that is, we use the following alternative form of the transfer function:

$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)} \rightarrow \frac{V_o(s)}{V_i(s)} = -\frac{1}{Z_1(s)Y_2(s)}$$

$$\begin{cases} Z_1 = R_1 \\ Y_2(s) = (1/R_2) + sC_2 \end{cases}$$

$$\rightarrow \frac{V_o(s)}{V_i(s)} = -\frac{1}{\frac{R_1}{R_2} + sC_2R_1} \rightarrow \frac{V_o(s)}{V_i(s)} = \frac{-R_2/R_1}{1 + sC_2R_2} \rightarrow \omega_0 = \frac{1}{C_2R_2} \rightarrow f_o = \frac{1}{2\pi R_2 C_2}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{-R_2/R_1}{1 + sC_2R_2}$$

$$\omega_0 = \frac{1}{C_2R_2}$$

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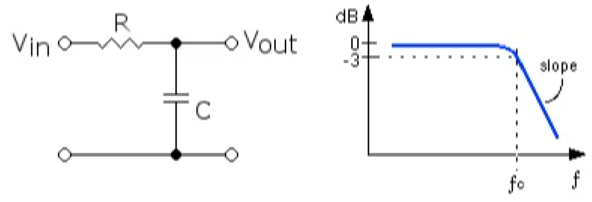
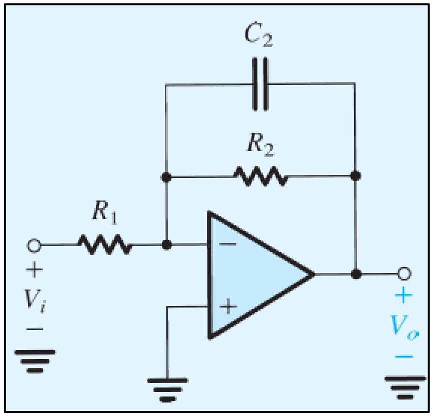


$$f_C = \frac{1}{2\pi RC} \quad \text{[Hz]}$$

low-pass network



2 The capacitor behaves as an open circuit at dc. Thus at dc the gain is simply $(-R_2/R_1)$.

3 Now to obtain a dc gain of 40 dB, that is, 100 V/V, we select $R_2/R_1 = 100$.

For an input resistance of 1 kΩ, we select:

$R_1 = 1\ \text{k}\Omega$ → $R_2 = 100\ \text{k}\Omega$

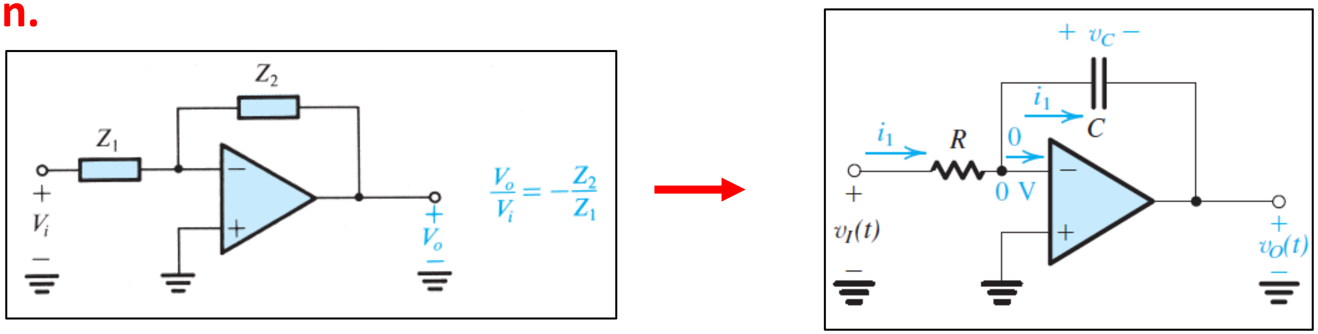
Finally, for a 3dB frequency at $f_0 = 1$ kHz, we calculate C_2:

$$\omega_0 = \frac{1}{C_2 R_2} \rightarrow 2\pi \times 1 \times 10^3 = \frac{1}{C_2 \times 100 \times 10^3} \rightarrow C_2 = 1.59\ \text{nF}$$

The Integrator

The Inverting Integrator

1 **By placing a capacitor in the feedback path and a resistor at the input, we obtain the circuit below. We shall now show that this circuit realizes the mathematical operation of integration.**



2 Let the input be a time-varying function $v_I(t)$. The virtual ground at the inverting op-amp input causes $v_I(t)$ to appear in effect across R, and thus the current $i_1(t)$ will be $v_I(t)/R$

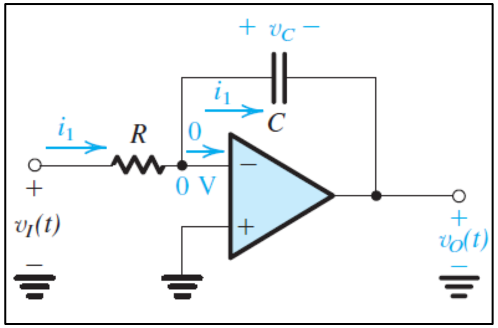
This current flows through the capacitor C, causing charge to accumulate on C. If we assume that the circuit begins operation at time $t = 0$, then at an arbitrary time t the current $i_1(t)$ will have deposited on C the following charge Q:

$$Q = \int_0^t i_I(t)dt$$

3 The capacitor voltage $v_c(t)$ will change by:

$$i_I(t) = C\frac{dV_C(t)}{dt} \rightarrow v_c(t) = \frac{1}{C}\int_0^t i_I(t)dt$$

4 If the initial voltage (at t=0) on C is V_c , thus



$$v_c(t) = V_c + \frac{1}{C}\int_0^t i_I(t)dt$$

5 The output voltage $v_o(t)$ = - $v_c(t)$. Thus,

$$v_0(t) = -\frac{1}{RC}\int_0^t v_I(t)dt \; - V_c$$

Thus the circuit provides an output voltage that is proportional to the time integral of the input, with V_C being the initial condition of integration and **RC** is the **integrator time constant.**

6 Note that there is a negative sign attached to the output voltage, and thus this integrator circuit is said to be an **inverting integrator**. It is also known as a **Miller integrator.**

$$\frac{V_O(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

$$Z_1(s) = R$$

$$Z_2(s) = 1/sC$$

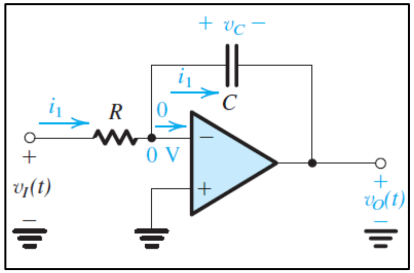
$$\rightarrow \frac{V_o(s)}{V_i(s)} = -\frac{1}{sCR} \rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = -\frac{1}{j\omega CR} \rightarrow$$

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{\omega CR}$$

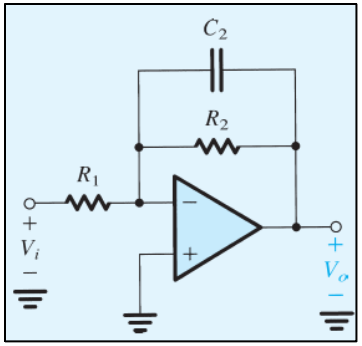
$$\phi = +90^\circ$$

7

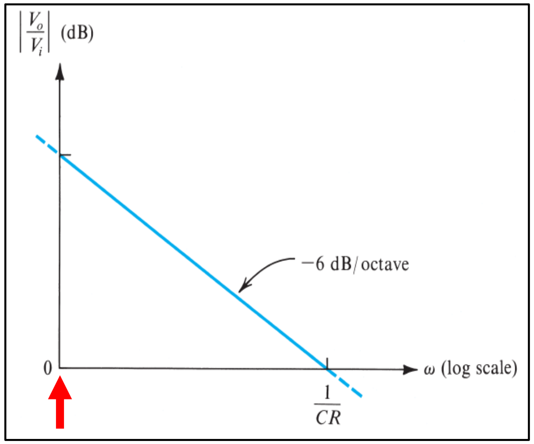
Comparison of the frequency response of the integrator to that of an STC low-pass network indicates that the integrator behaves as a low-pass filter.

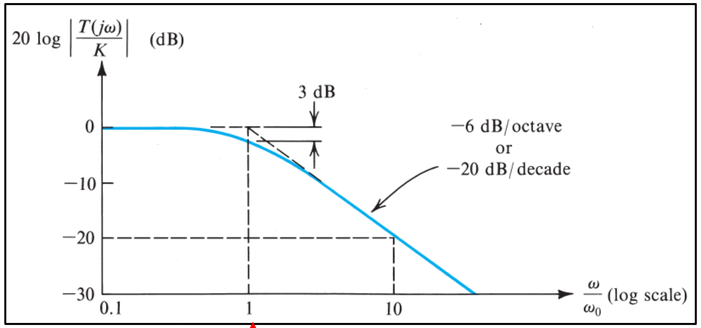


Integrator



Low-pass STC network ($k=R_2/R_1$)





corner frequency (f_c)

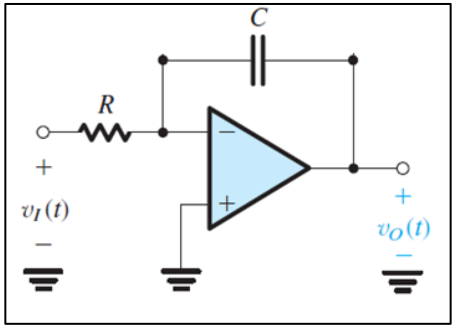
$$f_c = \frac{1}{2\pi R_2 C_2}$$

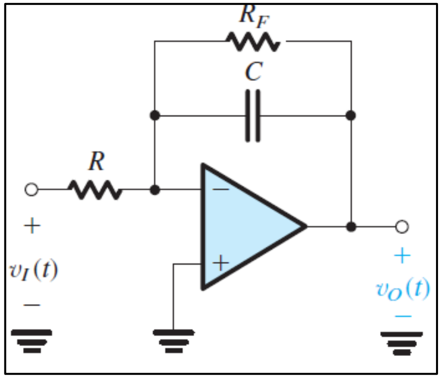
Observe also that at ω = 0, the magnitude of the integrator transfer function is infinite. This indicates that at dc the op amp is operating with an open loop.

$$\left|\frac{V_o}{V_i}\right| = \frac{1}{\omega CR}$$

***R_2* causes the frequency of the integrator pole to move from its ideal location at ω = 0 to one determined by the corner frequency of the STC network (*R_F*,*C*).**

8 The **dc problem of the integrator circuit can be alleviated by connecting a resistor R_F** across the integrator capacitor C, as shown below, and thus the **gain at dc will be $-R_F/R$** rather than infinite. Such a resistor provides a dc feedback path. Specifically, the integrator transfer function becomes:





$$\frac{V_o(s)}{V_i(s)} = -\frac{1}{sCR}$$

$$\frac{V_o(s)}{V_i(s)} = -\frac{R_F/R}{1 + sCR_F}$$

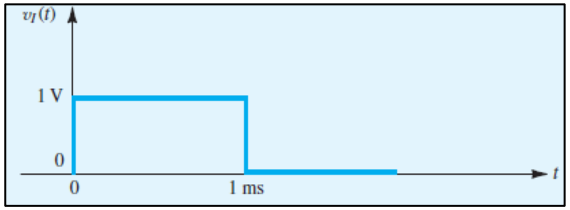
9 **Unfortunately, however, the integration is no longer ideal, and the lower the value of R_F, the less ideal the integrator circuit becomes**. This is because R_F causes the frequency of the integrator pole to move from its ideal location at ω = 0 to one determined by the corner frequency of the STC network (R_F,C).

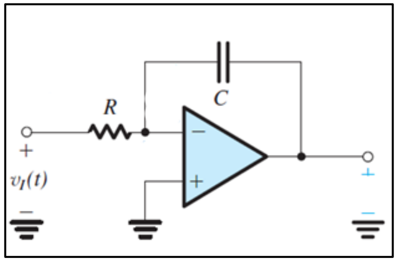
The lower the value we select for R_F , the higher the corner frequency will be and the more nonideal the integrator becomes. Thus selecting a value for R_F presents the designer with a trade-off between dc performance and signal performance.

$$f_o = \frac{1}{2\pi C R_F}$$

Exercise 2

Find the output produced by a Miller integrator in response to an input pulse of 1V height and 1ms width as shown below. Let *R* = 10 kΩ and *C* = 10 nF.
If the integrator capacitor is shunted by a 1-MΩ resistor, how will the response be modified?
The op amp is specified to saturate at ± 13V.

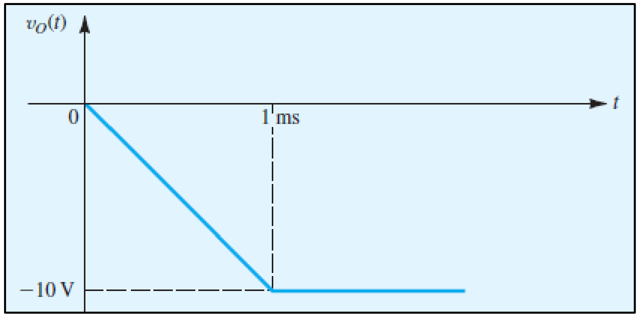




1 In response to a 1V, 1ms input pulse, the integrator output, if V_C = 0, will be:

$$v_o(t) = -\frac{1}{RC}\int_0^t v_I(t)dt - V_c \rightarrow \quad v_o(t) = -\frac{1}{RC}\int_0^t 1dt \quad 0 \le t \le 1\text{ms}$$

$$\rightarrow \boxed{v_o(t) = -10t}$$

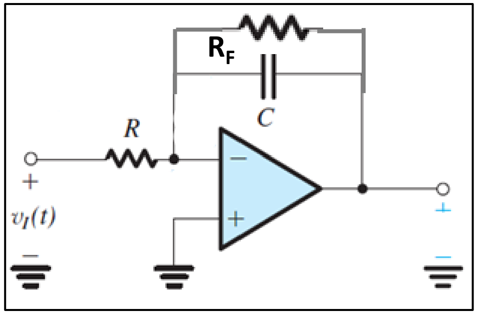


Charging a capacitor with a constant current produces a linear voltage across it !

2 The current in the resistor produces a constant current in the capacitor:

$I_R = I_C$ = 1V/10kΩ = 0,1mA

3 Next consider the situation with resistor connected R_F = 1MΩ across *C*.
As before, the 1V pulse will provide a constant current *I* = 0.1 mA. Now, however, this current is supplied to an STC network composed of R_F in parallel with *C*.



$$v_o(t) = v_{ofinal} - (v_{ofinal} - v_{oinitial})e^{-t/\tau}$$

$$v_{oinitial} = 0$$
$$v_{ofinal} = IR_f = 0.1x10^6 = 100$$
$\tau = CR_F = 10 \times 10^{-9} \times 10^6 = 10ms$

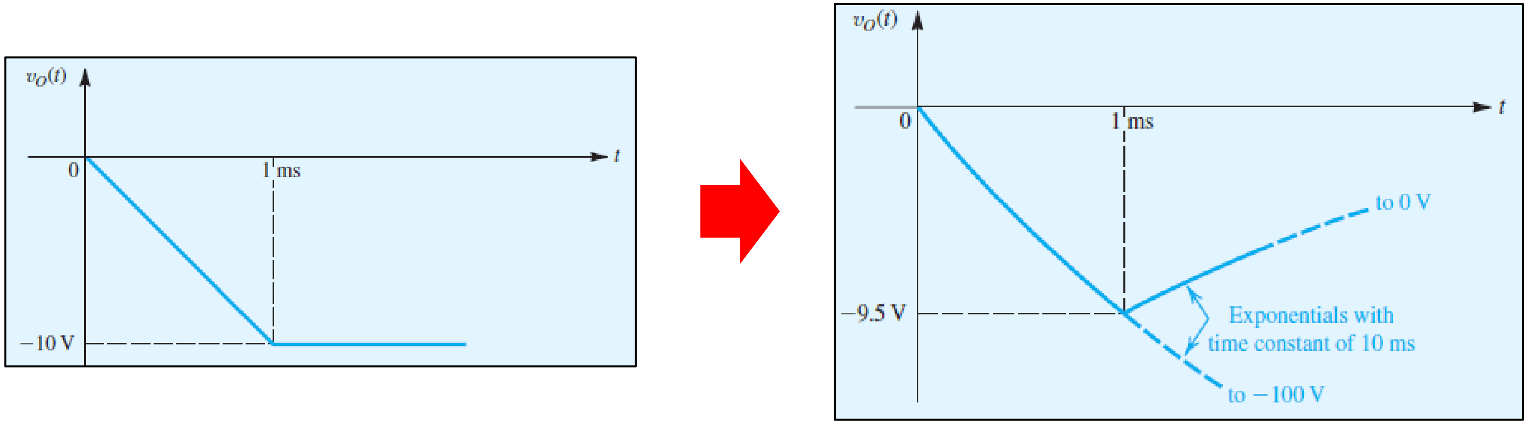
→ $$v_o(t) = -100(1 - e^{-t/0{,}01})$$

$0 \le t \le 1ms$

→ $$v_o(1ms) = -100\left(1 - e^{0{,}001/0{,}01}\right) = -9{,}5V$$

4 **The output waveform is shown below, from which we see that including R_F causes the ramp to be slightly rounded such that the output reaches only −9.5 V, 0.5 V short of the ideal value of −10 V**.

Furthermore, for t >1 ms, the capacitor discharges through R_F with the relatively long time-constant of 10 ms.



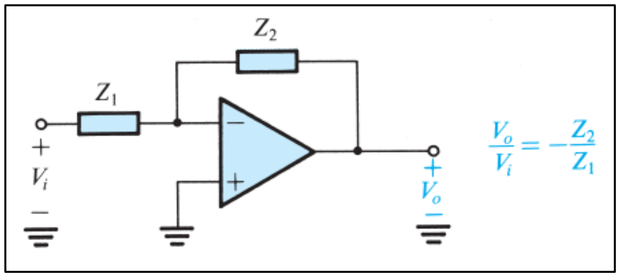
This example hints at an important application of integrators, namely, their use in providing triangular waveforms in response to square-wave inputs !

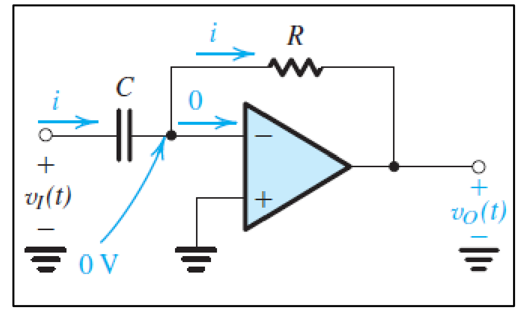
Integrators have many other applications, including their use in the design of active filters.

The Differentiator

The Op Amp Differentiator

1 **Interchanging the location of the capacitor and the resistor of the integrator circuit results in the circuit that performs the mathematical function of differentiation.**





2 To see how this comes about, let the input be the time-varying function, and note that the virtual ground at the inverting input terminal of the op amp causes to appear in effect across the capacitor *C*. Thus the current through *C* will be C(dv_I/dt), and this current flows through the feedback resistor *R* providing at the op-amp output the following voltage:

$$i = C\frac{d(V_C)}{dt}$$

$$V_o = -V_C$$

$$v_O(t) = -CR\frac{dv_I(t)}{dt}$$

3 The frequency-domain transfer function of the differentiator circuit can be found by substituting $Z_1(s)$=1/sC and $Z_2(s)$= R in the transfer function of an inverting configuration with general impedances:

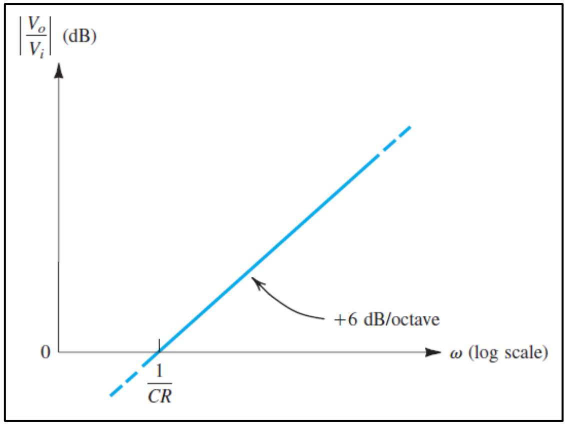
$$\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

$$Z_1(s) = R$$

$$Z_2(s) = 1/sC$$

$$\rightarrow \frac{V_o(s)}{V_i(s)} = -sCR \rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = -j\omega CR \rightarrow \begin{cases} \left|\frac{V_o}{V_i}\right| = \omega CR \\ \phi = -90^\circ \end{cases}$$

4 The Bode plot of the magnitude response can be found by noting that for an octave increase in ω, the magnitude doubles (increases by 6 dB). Thus the plot is simply a straight line of slope +6 dB/octave (+20 dB/decade) intersecting the 0 dB line where ***RC*** is the **differentiator time-constant**.



5 The differentiator circuit suffer from stability problems and **are generally avoided in practice**. This is due to the spike introduced at the output every time there is sharp change in $v_I(t)$. Such a change could be interference coupled electromagnetically (“picke up”) from adjacent signal sources.

When the circuit is used, it is usually necessary to connect a small-valued resistor in series with the capacitor. This modification, unfortunately, turns the circuit into a nonideal differentiator.

For this reasons and because they suffer from stabily problems, differentiator circuits are generally avoided in practice.