VOICE LEADING IN SET-CLASS SPACE

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Introduction to Transformational Voice Leading

Figure 1 shows four pairs of chords and says something about the voice leading between Chord X and four versions of Chord Y. Chords X and Y_1 (Figure 1a) are identical—the pitches of the first chord are retained in the second. If we were to think of this progression transformationally, we might say that pitch transposition at T_0^p maps the first chord onto the second and thus maps each note in the first chord onto a corresponding destination in the second. These transformational mappings and the registral lines coincide, as each pitch in the first chord moves 0 semitones to its destination in the second.

In the second pair of chords (Figure 1b), one of the voices moves by semitone (G4–G♯4) while the others remain stationary. This voice leading would be generally understood as *smooth*. If we think of this progression transformationally, we would say that the relevant transformation is *fuzzy*-T_0^p, a transformation that deviates in some measurable way from a strict, crisp-T_0^p.[1] The asterisk in the example denotes the fuzziness of the T. The voice that does not participate in the actual transposition, the soprano

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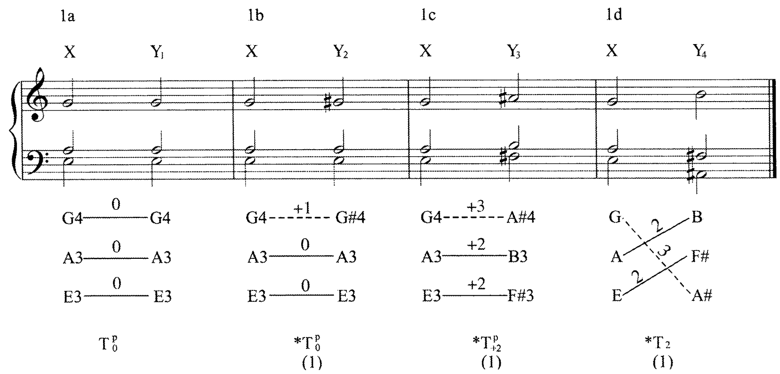


Figure 1. Transformational voice leading via transposition and fuzzy transposition in pitch and pitch-class space

in this case, deviates from the others by a semitone. That amount of deviation, defined as the *offset number*, is given in parentheses below the transposition number.[2] The voice leading thus involves *minimal offset*. As in Figure 1a, the transformational voice leading (i.e., the mappings induced by a transformation) and the registral lines coincide.

In Figure 1c, the second chord in the pair (Y_3) is transposed up two semitones from its position in the previous progression (Y_2). As a result, the voice leading from X to Y_3 is $*T^p_{+2}$. The offset number remains 1, because the soprano still deviates by only one semitone from the prevailing transformation. This voice leading is no longer smooth, because all of the voices are moving by two or three semitones, but it still involves minimal offset. The progression does not involve actual, crisp transposition, but it is *maximally uniform*; that is, it comes as close as possible to being an actual transposition, deviating by only one semitone of offset.[3] In this sense, the transformation in Figure 1b is also maximally uniform (as well as smooth). Once again, the transformational voices (mappings) and the registral lines coincide.

In Figure 1d, the two chords are no longer related by pitch transposition, but only by more abstract pitch-class transposition. $*T_2$ maps Chord X onto Chord Y_4, sending A onto B and E onto F♯, and also, with one pitch-class semitone of offset, sending G onto A♯ (motion by three semitones instead of the prevailing two). The transformational voice leading takes place in pitch-class space, mapping the pitch classes of X onto those of Y_4. These pitch-class mappings need not coincide with the registral lines, and this example involves voice crossing (i.e., the pitch-class voices cross in pitch space). The voice leading is still maximally uniform (approaches as nearly as possible an actual transposition), but the uniformity is felt in pitch class rather than in pitch space.

Given the octave dislocations and the voice crossings in Figure 1d, is it still possible to understand the two chords as related by (fuzzy) transposition, to apprehend the mappings that result, and to retain a sense of them as voices, despite their lack of coincidence with the registral lines? The answer will depend on the musical context and the interests of the listener. Without too much difficulty, one might attend to the perfect fourth at the bottom of the first chord and listen for its projection into the second chord, with the lower note of the fourth moving up two semitones and the upper note of the fourth moving up an octave plus two semitones. It is also possible, although obviously more difficult, to imagine that the A♯ at the bottom of Chord Y_4 results from a transposition of G up two semitones to A, which is then moved down two octaves and minimally displaced onto A♯—the relevant ear training would involve hearing the progression of Figure 1d in relation to the progression of Figure 1c. We might say that the relatively straightforward progression of Figure 1c is elaborated by octave displacement in Figure 1d. Or, conversely, we might say that the more abstract pitch-class relations of Figure 1d are realized concretely, in pitch space, in Figure 1c. Either way, the music of Figure 1d permits us to retain a meaningful sense of transformational voice leading and transformational voices.

Figure 2 presents four more pairs of chords. The actual, registral voice leading of the first two chords (Figure 2a) is smooth, with A5 and E3

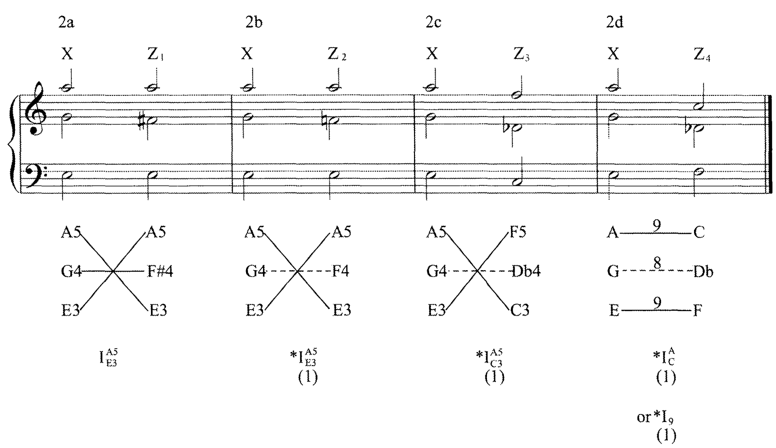


Figure 2. Transformational voice leading via inversion and fuzzy inversion in pitch and pitch-class space

retained as common tones while G4 moves a pitch semitone to F♯4. Thinking about the progression transformationally, we would note that the two chords are related by actual pitch inversion—the second is the inversion, in pitch space, of the first—and say that the inversion that exchanges A5 and E3 also sends G4 onto F♯4. This transformational voice leading is not smooth, because the mappings involve an exchange of tones, traversing large distances, rather than the simple retention of tones. The transformational voice leading calls our attention to an exchange of roles between A5 and E3: the pitch that was fourteen semitones away from the middle note in one chord is fifteen semitones away from it in the other, and vice versa. The registral lines move smoothly; the transformational voices capture the sense of inversional exchange between the chords. One can imagine the inner-voice G–F♯ as defining an axis of symmetry around which A5 and E3 balance. Alternatively, one can imagine A5 and E3 as defining a frame within which F♯4 balances G4. Within this frame, G4 creates a slight asymmetry that is rectified by the appearance of the balancing F♯4. If we assume an underlying desire for balance, we might say that the A5–E3 frame presses the G4 to move to F♯4. That sense of a motion toward inversional balance is captured by the proposed transformational voice leading.

Compared to this progression, the slightly altered progression in Figure 2b (the F♯4 in Chord Z1 becomes F4 in Chord Z2) involves fuzzy inversion, with the deviant voice offset by a semitone from the prevailing inversion. This voice leading thus involves *minimal offset*. It is *maximally balanced*; that is, it approaches as nearly as possible an actual inversion, deviating by only one semitone.[4] In this context, the F4 acts as a kind of "blue" F♯4, deviating by a semitone from the note that would create strict inversional symmetry.

In Figure 2c, Chord Z_3 is transposed down four semitones with respect to Chord Z_2. The inversion has changed accordingly, but the degree of offset is still 1. Now D♭4 substitutes for the inversionally desirable D4, a deviation of one semitone.

With Figure 2d, we shift from pitch space to pitch-class space and from pitch inversion to pitch-class inversion. Chord Z_4 has the same pitch classes as Chord Z_3 but is represented in two cases by octave-related pitches. The pitch-class mappings induced by the inversion (described either as I_C^A or I_9) are identified by the index number (sum) created by the mapped pitch class. The sum that connects G to D♭ deviates by a semitone from the sum that connects A and E to C and F. To put it in a slightly different way, the same inversion that balances A against C also balances E against F, but G and D♭ deviate by a single semitone from also balancing on that axis.

As a result, the voice leading in Figure 2d also involves the minimal offset of one semitone from a straight inversion at I_9. The pitch-class mappings in this case are identical with the registral lines, although this need not be the case. As with the transformational voice leading induced by transposition, inversional voice leading in pitch-class space can also lead to crossed voices in pitch space. The ear training required to hear the transformational voices operating in Figure 2d involves hearing it in relation to Examples 2a, 2b, and 2c. In those examples, the transformational voice leading did not coincide with the registral lines: the mappings involved traversing large distances and crossing in pitch space. In Figure 2d, the inversional near-symmetry exists only in pitch-class space, but the transformational voices now coincide with the registral lines.

In Figures 1 and 2, the chords labeled X all contain pitch classes [E, G, A], a member of set-class 3–7 [025]. The chords labeled Y_2, Y_3, Y_4, Z_2, Z_3, and Z_4—[E, G♯, A], [F♯, A♯, B], [E, F, A], and [C, D♭, F]—are all members of set-class 3–4 [015]. We might generalize that any member of set-class [025] has the potential to move to any member of set-class [015] via *T or *I with the minimal offset of one semitone. The progression from a member of set-class [025] to a member of set-class [015] can thus always be interpreted as either maximally uniform or maximally balanced (deviating from strict T or I by one semitone). More simply and directly, we might say that set-class [025] and set-class [015] are related by *minimal offset voice leading*.

When we say that [015] and [025] are connected by minimal offset voice leading, we mean that for any pitch-class set in [015], there will be a pitch-class set in [025] to which it can be connected by smooth voice leading, with one pitch class moving by semitone while the others are retained as common tones (as in Figure 1b). Furthermore, we mean that every pitch-class set in [015] can be connected to every pitch-class set in [025] by *T or *I with an offset of one semitone (as in Examples 1c, 1d, 2b, 2c, and 2d). Still more generally, we mean that whatever pitch class functions as the "1-element" or occupies the "1-position" within set-class [015] will be understood to move onto whatever pitch class functions as the "2-element" or occupies the "2-position" within set-class [025].[5] In this sense, the voice leading between set-classes involves the mapping of corresponding functions or elements, just as the voice leading between pitch-class sets involves the mapping of pitch classes and the voice leading between pitch sets involves the mapping of pitches.

At the level of the set-class, voice-leading uniformity and balance converge with voice-leading smoothness. If two pitch-class sets are related by *T or *I with minimum offset, then the set-class to which the sets belong will be related smoothly. As with [025] and [015], one element moves by the minimal distance while the others remain unchanged.

The minimal offset voice leading between [015] and [025], or between any two set-classes, may be realized with varying degrees of explicitness in the relationships among pitch-class sets belonging to those set-classes, and the relationships among the pitch-class sets may themselves be realized with varying degrees of explicitness in the relationships among the pitch sets that represent them. A smooth connection between set-classes entails a maximally uniform or maximally balanced voice leading among the pitch-class sets belonging to the set-classes. Conversely, a maximally smooth or balanced voice leading between pitch-class sets entails a smooth, parsimonious connection between the set-classes to which they belong. These relationships among set-classes and pitch-class sets constrain any possible pitch realization. Specifically, they provide a limit to the degree of smoothness obtainable in pitch space. For example, if two set-classes are related by voice-leading offset of 2 (and thus any sets within the set-classes are related by *T or *I to within an offset of 2), there can be no pitch realization that requires less than two pitch semitones of displacement from strict pitch transposition or pitch inversion.

Proximate, minimal-offset voice leading between set-classes represents a potential that pitch-class sets and pitch sets may realize to a greater or lesser extent; the relations among set-classes act as a deep, abstract constraint on the relations among the pitch-class sets and pitch sets that they comprise. Because of their power as deep-level potentials or constraints on transformational voice leading among pitch-class sets, the voice-leading relationships among set-classes merit further exploration.

In recent years, a number of theorists have begun to imagine and describe particular kinds of voice-leading spaces for set-classes.[6] Within these spaces, set-classes are related *parsimoniously*—those that are related smoothly, with relatively little semitonal *offset*, are located in close proximity, while those that are related by relatively high levels of voice-leading exertion are more widely separated.

Building on earlier work, in this article I construct a parsimonious voice-leading space for set-classes. Within this space, each set-class occupies a determinate location (normally defined by its prime form) and lies a fixed distance from the others (with distance measured by the amount of semitonal adjustment required to transform one into the other). Within this space, it becomes possible to interpret atonal harmonic progressions, including progressions among sets belonging to different set-classes. The result is an analytically practical approach to atonal harmony and voice leading.

A Parsimonious Voice-Leading Space for Set-Classes

A voice-leading space that contains all of the set-classes must necessarily be multidimensional. Specifically, the dyads can be modeled in one dimension, the trichords in two, the tetrachords in three, the pentachords in four, and the hexachords in five dimensions. The larger set-classes may be understood to occupy the same positions within the space as their

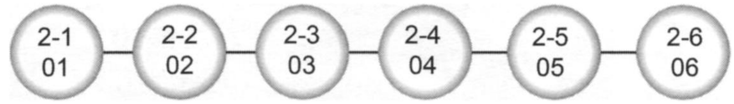


Figure 3. A parsimonious voice-leading space for dyad classes

smaller complements. Multidimensionality creates serious problems for conceptualization, visualization, and representation.

The approach taken here attempts to solve those problems by displaying the set-classes in *tiers*, with voice-leading connections both within a tier and between tiers. The tiers consist of *rows* and *columns* and are themselves gathered into *stacks*, which combine to form *complexes* of stacks. In this way, a five-dimensional space that contains set-classes of cardinality 1 through 6 can be grasped with relative ease. Essentially, I will be offering two- and three-dimensional slices and chunks of the underlying, multidimensional master space. This simplified rendering will make it possible to infer significant harmonic and voice-leading relationships from the position of the set-classes within the space and to explore those relationships analytically in musical compositions.

Within the integrated, five-dimensional space described here, all set-classes are given determinate locations fixed by a six-place *location vector*, $\langle a, b, c, d, e, f \rangle$, where a = universe (always equal to 0), b = complex, c = stack, d = tier, e = column, and f = row. A traditional prime form is interpreted here as a location vector, and every set-class is located in the position defined by its prime form. For example, set-class 6–20 [014589] may be found in the ninth row and eighth column of the fifth tier within the fourth stack located in the first complex, while set-class 4–22 [0247] may be found in the seventh row and fourth column of the second tier of the 0-stack, within the 0-complex (any variable not specified is assumed to be 0). In addition, set-classes may have other locations in the space.[7]

This approach will enable us to know the location(s) for every set-class within a single multidimensional space that contains them all. We will also know the shortest route between any two set-classes of any cardinality, both the length of the route and the set-classes it passes through. Finally, for any set-class, we will be able to visualize the sector it inhabits. Indeed, our approach involves visualizing and describing chunks, slices, and sectors of the master space, which is difficult to grasp in its entirety.

Figure 3 shows a parsimonious voice-leading space for the dyad classes, identified by their Forte names and their prime forms and arranged in a column (note that for reasons of space, the column is laid out horizontally in Figure 3). Every move up or down the column is a parsimonious move—one note of the dyad moves up or down by semitone. Notice that the set-classes are arranged between the extremes of *chromaticness* and *evenness*.[8]

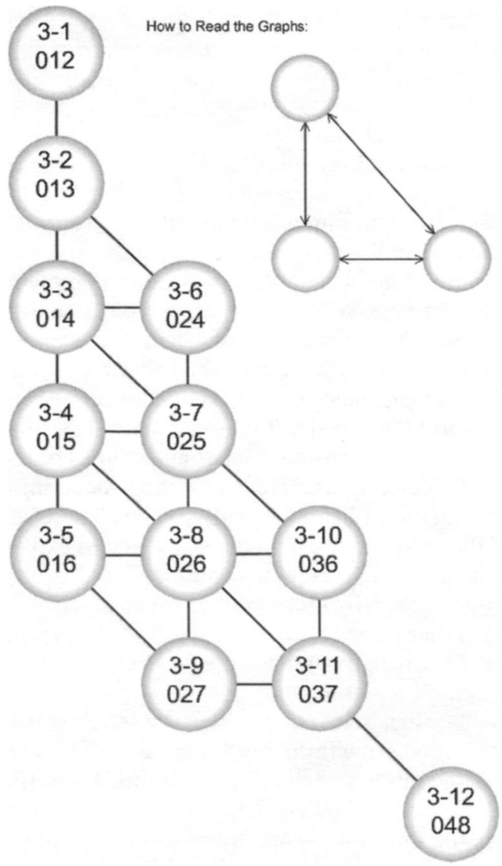


Figure 4. A parsimonious voice-leading space for trichord classes

At one end of the space, we find set-class 2–1 [01], the most chromatic of the dyads (the dyad whose notes are most tightly packed together), and at the other end, we find set-class 2–6 [06] (the dyad whose notes are most evenly dispersed through the space). The remaining set-classes can be characterized by their relative position with respect to these extremes. Set-class [02], for example, is only one degree of offset from the most chromatic dyad but four degrees of offset from the most even dyad.

Figure 4 shows a parsimonious voice-leading space for the familiar twelve T/I trichord classes.[9] Parsimonious moves occur in three ways, corresponding to the cardinality of the set-class: up or down within a column, right or left within a row, or along the diagonal (the possible moves are summarized at the top of Figure 4). With each move on the map, a single pitch class or, more accurately, a single functional member of the set-class moves up or down by semitone. In the diagonal move, one has to imagine the 0 moving down a semitone and the resulting trichord transposed up a semitone to its new position in the map (e.g., if the 0 in [014] moves down to 11, the resulting trichord (11, 1, 4) can be transposed up a semitone to the [025] position in the map). Within the larger space, all diagonal moves involve displacement of the 0.

As with the dyadic column in Figure 3, the trichordal tier in Figure 4 falls naturally into an arrangement that places the maximally chromatic and maximally even trichords at opposing extremes. Set-class 3–1 [012] and 3–12 [048] are separated by six semitones of offset, the largest possible distance in this tier. The other trichords are positioned between these extremes, and their degree of chromaticness and evenness is equivalent to their distance from the relevant extremes.

Also as with the dyadic space, the maximally chromatic and maximally even trichords are in direct, parsimonious connection with only one other set-class ([012] connects only to [013]; [048] connects only to [037]). Apart from these two, every trichord maintains a parsimonious connection with at least three other trichords of a different type. The most promiscuous is [026], which has parsimonious connections with six different set-classes. That is the theoretical maximum—a different set-class is produced by the motion of each of the three notes either up or down by semitone. Of course, every trichordal pitch-class set creates parsimonious connections with six other pitch-class sets, as each of the three constituent pitch classes moves up or down by semitone. In most cases, however, these moves produce either a dyad (as a pitch class moves onto a pitch class already represented in the set) or another trichord of the same type (as in the familiar parsimonious voice leading from a major to a minor triad). Among the trichords, only [026] can be led parsimoniously to six different set-classes. It thus occupies a unique position at the center of the trichordal voice-leading map. In their potential to connect parsimoniously with different set-classes, the remaining trichords range between the relatively impoverished [012] and [048] and the relatively extravagant [026].

Figure 5 expands the trichordal map to reveal trichords that are capable of parsimonious self-mapping. In Figure 5 and subsequent figures, self-mapping is denoted with an asterisk (*), and location vectors other than the prime form are denoted with an equal sign (=). Set-class 3–11 [037], for example, maps onto two other forms of itself. On the map, [037] is connected with [=047] and [=038]—these are the familiar P and L transformations of neo-Riemannian theory. In contrast, 3–2 [013], 3–5 [016], and 3–7 [025] are capable of only one parsimonious self-mapping. In each case, the parsimonious voice leading connects inversionally related forms of a set-class.

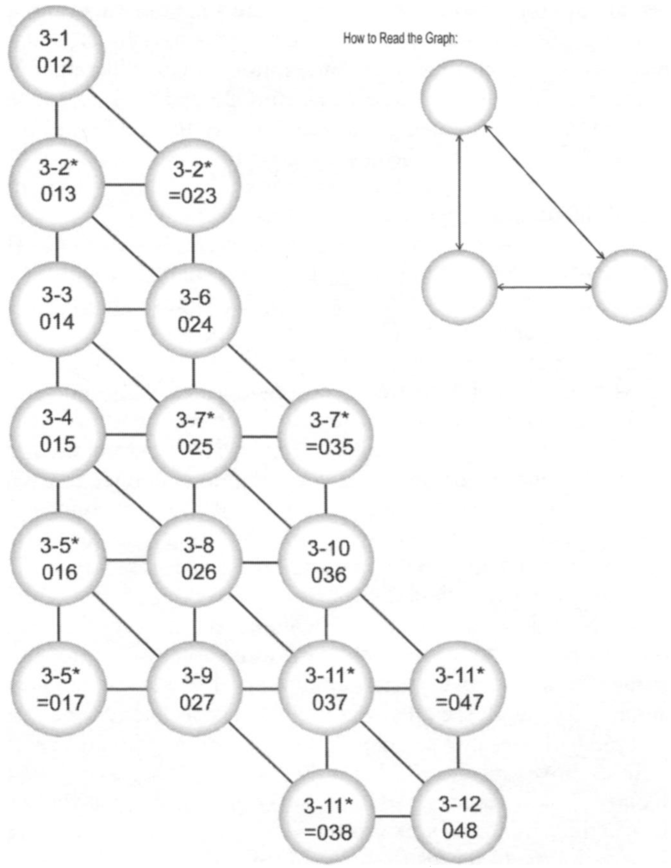


Figure 5. A parsimonious voice-leading space for trichord classes, expanded to reveal self-mapping (self-mapping denoted with *; location vector other than prime form denoted with =)

Figures 3, 4, and 5 contain set-classes of a single cardinality. However, if we permit pitch classes to split (one pitch class diverges onto two) or fuse (two pitch classes converge onto one), it becomes possible for a dyad to move (parsimoniously) to a trichord and vice versa and, by extension, to connect any two set-classes that differ in size.[10] Accordingly, Figure 6 brings the dyads and trichords into a single shared space (and includes the singleton, as well).

As with the trichordal maps in Figures 4 and 5, there are still three kinds of parsimonious connections: up or down within a column, right or left within a row, or along the diagonal. Diagonal moves from dyads to trichords involve the 0 splitting onto 0 and 11, with the result then transposed up a semitone into its position in the map (e.g., 02 → E02 → 013).

The singleton occupies 0 dimensions; the dyads, one dimension (a single column); and the trichords, two dimensions (a tier, comprising columns and rows). The tetrachords occupy three dimensions: a stack, which comprises tiers, themselves comprising columns and rows (see Figure 7).[11] This map represents the set-classes with the minimum possible redundancy (only 4–19 occurs twice) and with each possible parsi-

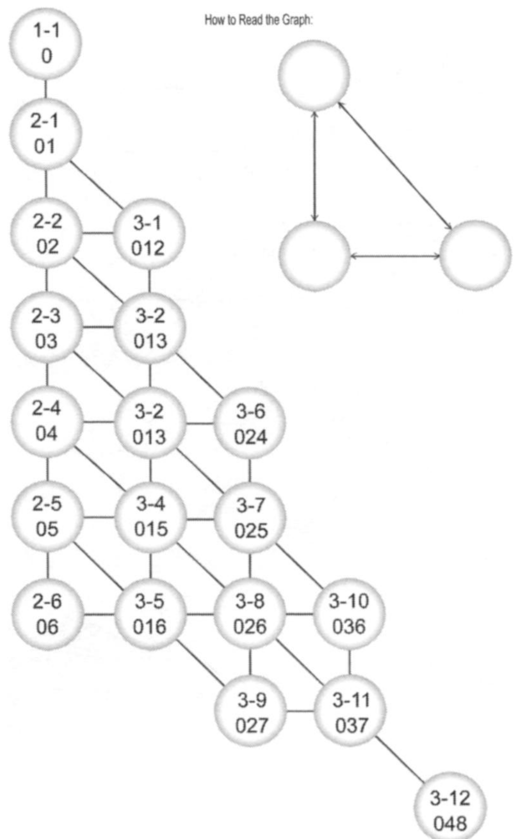


Figure 6. A parsimonious voice-leading space for dyad classes and trichord classes

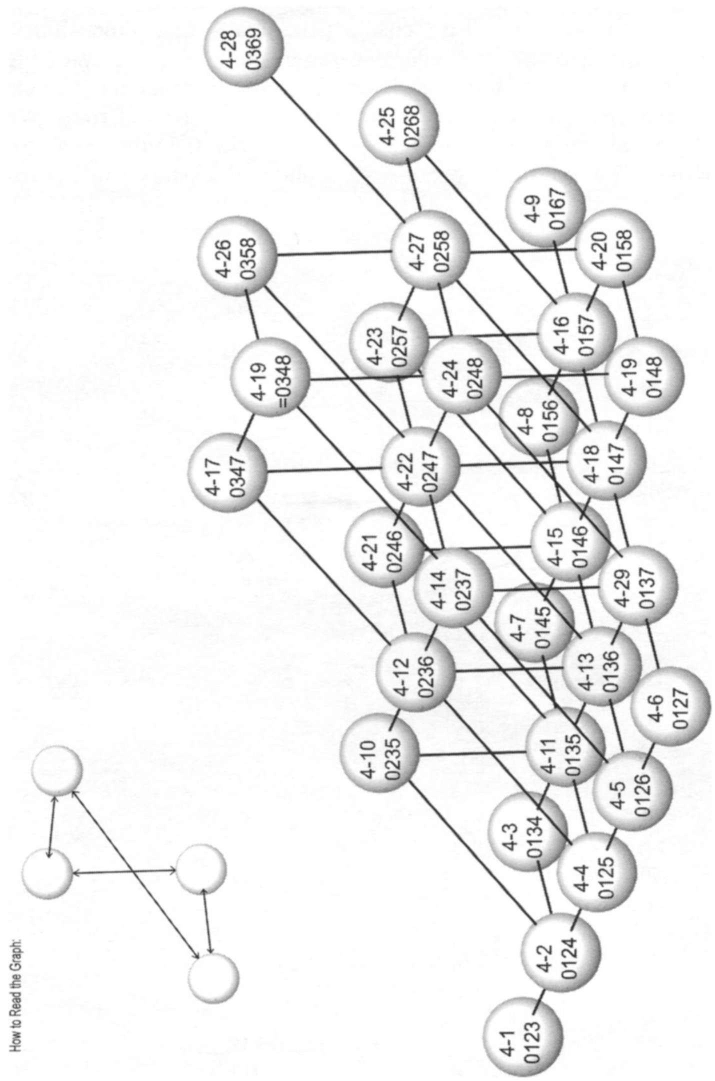


Figure 7. A parsimonious voice-leading space for tetrachord classes

monious connection between set-classes represented only once. The tetrachords are displayed in three tiers. The lowest tier includes all of the tetrachords whose prime forms begin 01; the middle tier includes all of the tetrachords whose prime forms begin 02; the highest tier contains all of the tetrachords whose prime forms begin 03, along with the redundant set-class 4–19 [=0348]. Within this tetrachordal stack, parsimonious connections are made in four ways, as indicated by the schematic diagram at the top of Figure 7: within a row, within a column, vertically into an adjacent tier, and diagonally into an adjacent tier. These four possibilities correspond to motion by each of the four pitch classes in each set-class.

As with the maps of dyads and trichords, the tetrachordal map in Figure 7 positions 4–1 [0123] (the maximally chromatic tetrachord) and 4–28 [0369] (the maximally even tetrachord) at opposite extremes. Generally speaking, as one moves from left to right in a row, from top to bottom in a column, and upward from tier to tier, the set-classes become less chromatic and more even. Also as with the dyads and trichords, the voice-leading potential of each set-class to move parsimoniously onto different set-classes is apparent from the map. Set-classes 4–1 [0123], 4–9 [0167], and 4–28 [0369] are isolated at corners of the map—each communicates with only one other set-class. By contrast, 4–22 [0247] has parsimonious connections with eight different set-classes, the theoretical maximum for tetrachords.[12]

By virtue of splitting and fusing, the singleton, dyads, and trichords from Figure 6 can be combined with the tetrachords from Figure 7 into the more comprehensive space of Figure 8. Because of the relatively small number of trichords compared to tetrachords, the trichordal tier has to be expanded beyond its relatively pruned appearance in Figure 4, and additional (redundant) positions must be found for many of the trichords. This redundancy permits the map to display all of the parsimonious connections within and between dyads, trichords, and tetrachords.

In reading intercardinality maps, and in tracing the shortest pathways between set-classes, there are two rules to follow: (1) When moving between set-classes of the same size, the path may not involve set-classes of any other size. For example, the distance from 4–1 [0123] to 4–16 [0157] is 7; a shortcut through the trichords, to 3–8 [=046] at a distance of 4, and then a single diagonal step to 4–16 [0157], is not permissible. (2) When moving between set-classes of different sizes, split as early as possible (when moving from small to large) and fuse as late as possible (when moving from large to small). In other words, the path has to move as much as possible through the larger set-classes. For example, the minimal distance between 4–11 [0135] and 3–12 [048] is 5, and one pathway of that size is 0135–0145–0146–0147–0148–048 (fusing down to a trichord only at the last moment). Taking a shortcut via the trichords (0135–035–036–037–048) is not permissible.

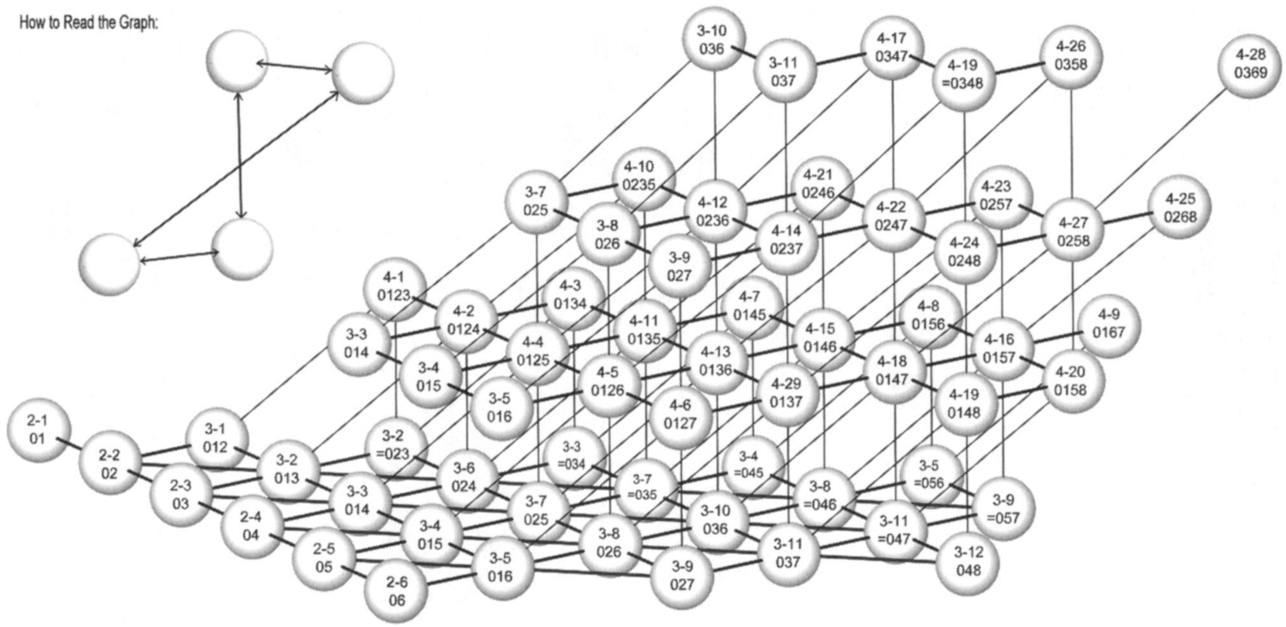


Figure 8. A parsimonious voice-leading space for dyad, trichord, and tetrachord classes

The necessity for interpretive rules of this kind suggests that these maps need to be understood more as game boards than as geometrical models. Indeed, even the single-cardinality maps are not topographically accurate, in that the parsimonious connections between two set-classes may be represented by lines of different length. A parsimonious connection is defined as one permissible move on the board, irrespective of the actual, literal distance of the move.

Figure 9 presents a parsimonious voice-leading space for the pentachord classes. The pentachords can be understood as occupying two separate stacks, one containing location vectors (usually prime forms) beginning 01, and the other containing location vectors (usually prime forms) beginning 02. In the map in Figure 9, these two stacks are conflated, with the result that the nodes contain two slots: the upper position for set-classes from the first stack and the lower position for set-classes from the second. There are now five kinds of connections, corresponding to the five notes of a pentachord: within the rows and columns of each tier; between the tiers either straight up or diagonally; and within each node. In Figure 9, as in preceding examples, the permissible moves are summarized by the schematic drawing in the upper left of the example.

The closer the set-class is to the center of the row, column, tier, or stack, the more likely it is to be able to make all of the different moves and actually find a partner in the new location. Conversely, the closer it lies to an edge of the space, the fewer the number of parsimonious connections to different set-classes it will be able to make. As with the smaller sets, the pentachordal space is organized so that the most chromatic pentachord, 5–1 [01234], is at one extreme while the maximally even pentachord, 5–35 [02479] and [=02579], is at the other.

In the pentachordal space of Figure 9, fifteen set-classes appear in two locations (as in preceding examples, the set-class in a location other than the one specified by the prime form is preceded by an equal sign). Ten of these, indicated with an asterisk, are self-mapping: a shift of a single pitch class by semitone produces another member of the same set-class. As with the four self-mapping trichords ([013], [025], [016], [037]), these ten self-mapping pentachords—[01245], [01237], [01367], [01378], [01458], [02357], [02358], [01369], [01469], and [02479]—all involve inversionally related sets.[13] Five more are unavoidable redundancies—they cannot be pruned away without thereby omitting parsimonious connections.[14]

Figure 10 merges the tetrachordal stack from Figure 7 with the pentachordal stack from Figure 9 to create a parsimonious voice-leading space for tetrachords and pentachords. Because of space limitations, each set-class is now identified only by its Forte name (readers may refer to preceding examples for the corresponding prime forms). Each node now has three positions: the top position for a tetrachord, the middle position for a pentachord from the first of the pentachordal stacks, and the bottom posi-

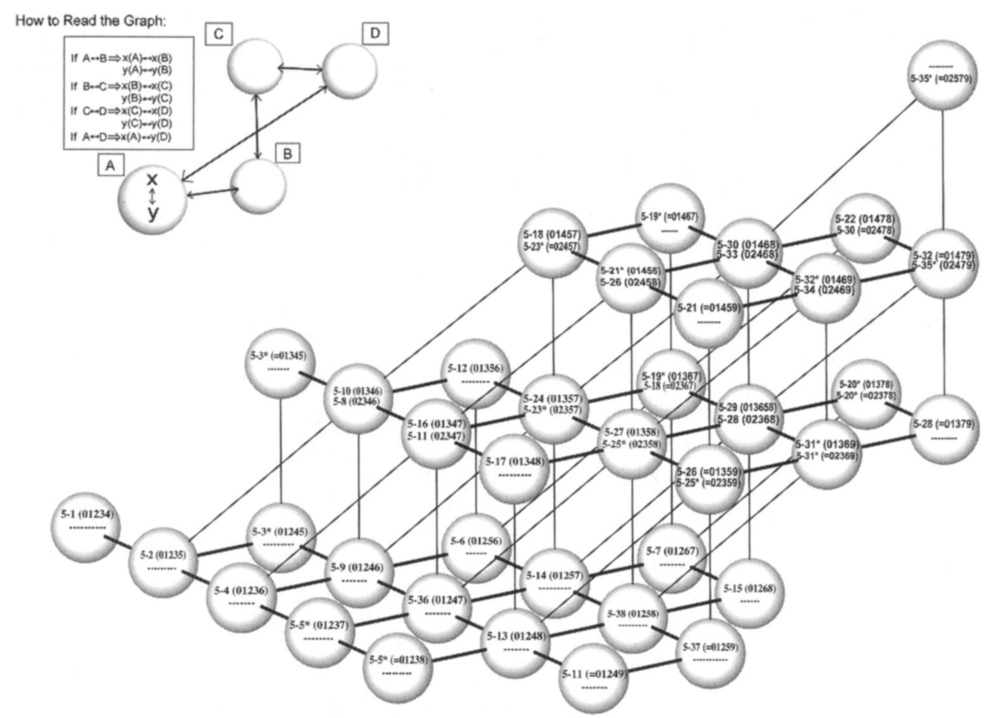


Figure 9. A parsimonious voice-leading space for pentachord classes

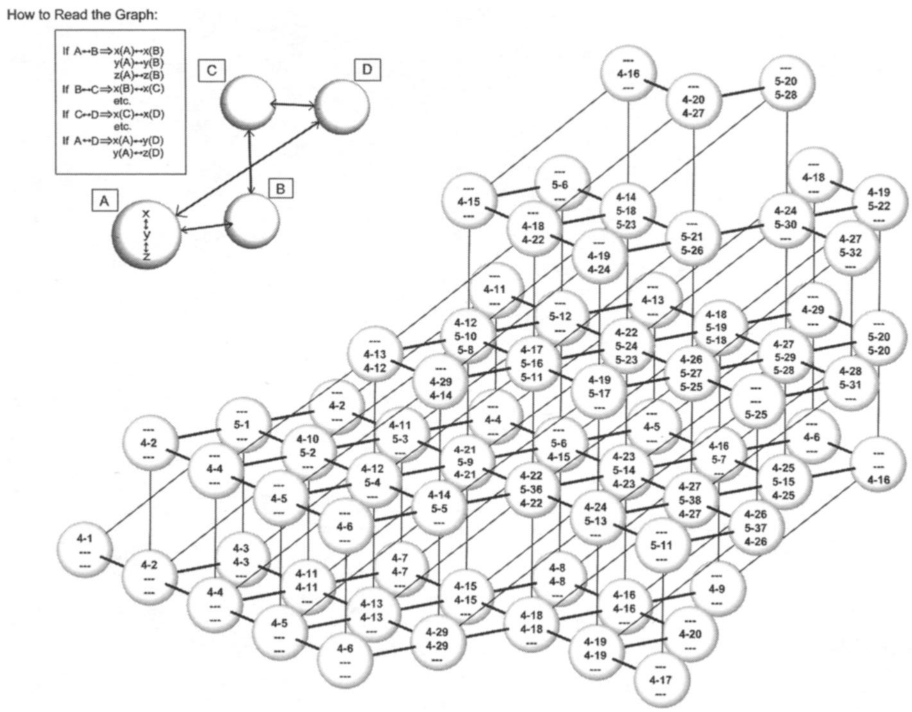


Figure 10. A parsimonious voice-leading space for tetrachord and pentachord classes

tion for a pentachord from the second of the pentachordal stacks (tetrachords occasionally appear in the lower slots, as well). Parsimonious moves within and between nodes are specified at the upper left of the example. Some of the intrapentachordal connections are omitted for the sake of legibility, but all of the connections among the tetrachords and between tetrachords and pentachords are included. To produce these connections, a number of tetrachords now appear in more than one location.[15]

A parsimonious voice-leading space for hexachords is shown in Figures 11 and 12, one with location vectors (mostly prime forms) and the other with Forte names. There are five slots on each node, corresponding to the five stacks in which the set-class originates (with location vectors beginning 012, 013, 014, 023, or 024), although only a few of the nodes actually contain five hexachords. Within a node, parsimonious connections are made up or down within a column or directly across a row. There are also the usual connections from node to node, for a total of six possible connections, corresponding to the six notes of a hexachord. As

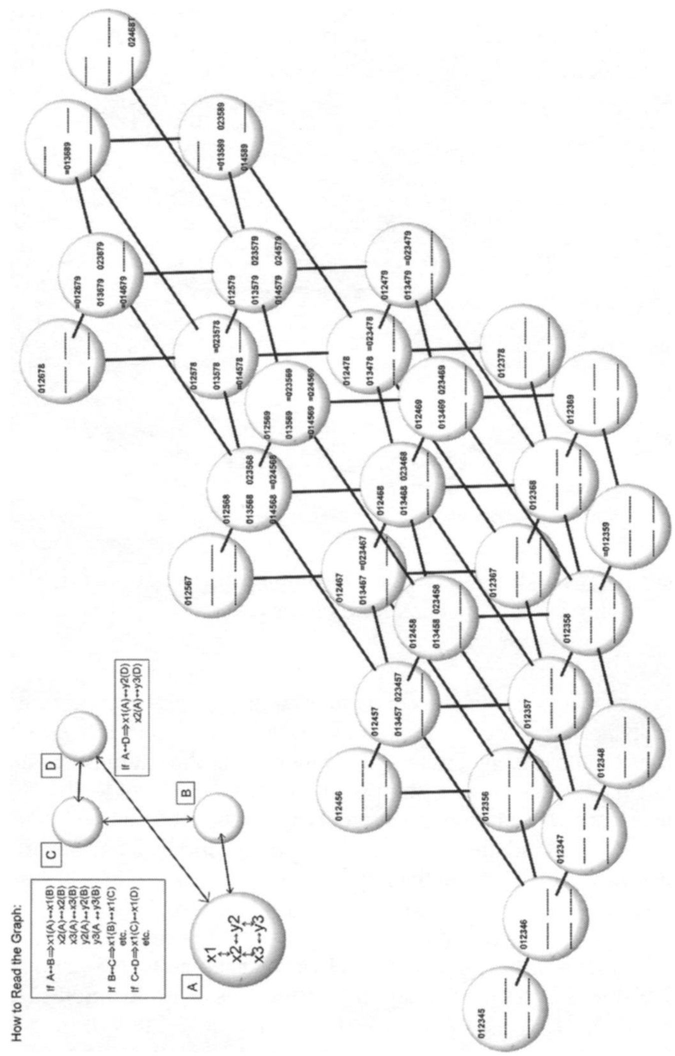


Figure 11. A parsimonious voice-leading space for hexachord classes (location vectors only)

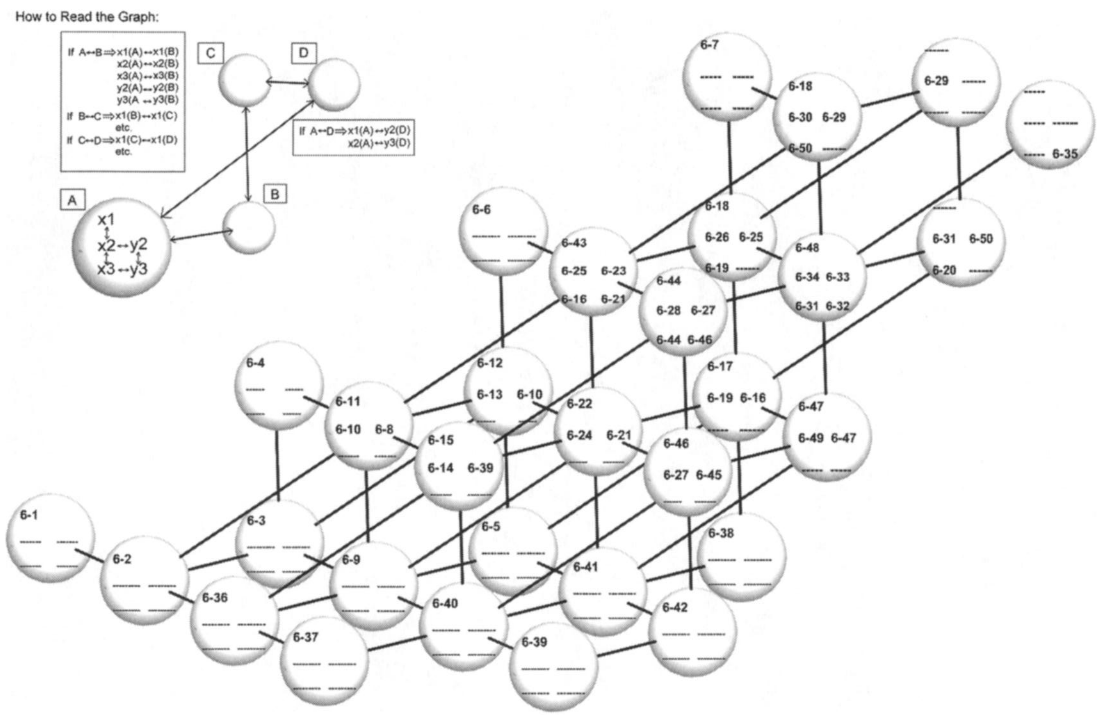


Figure 12. A parsimonious voice-leading space for hexachord classes (Forte names only)

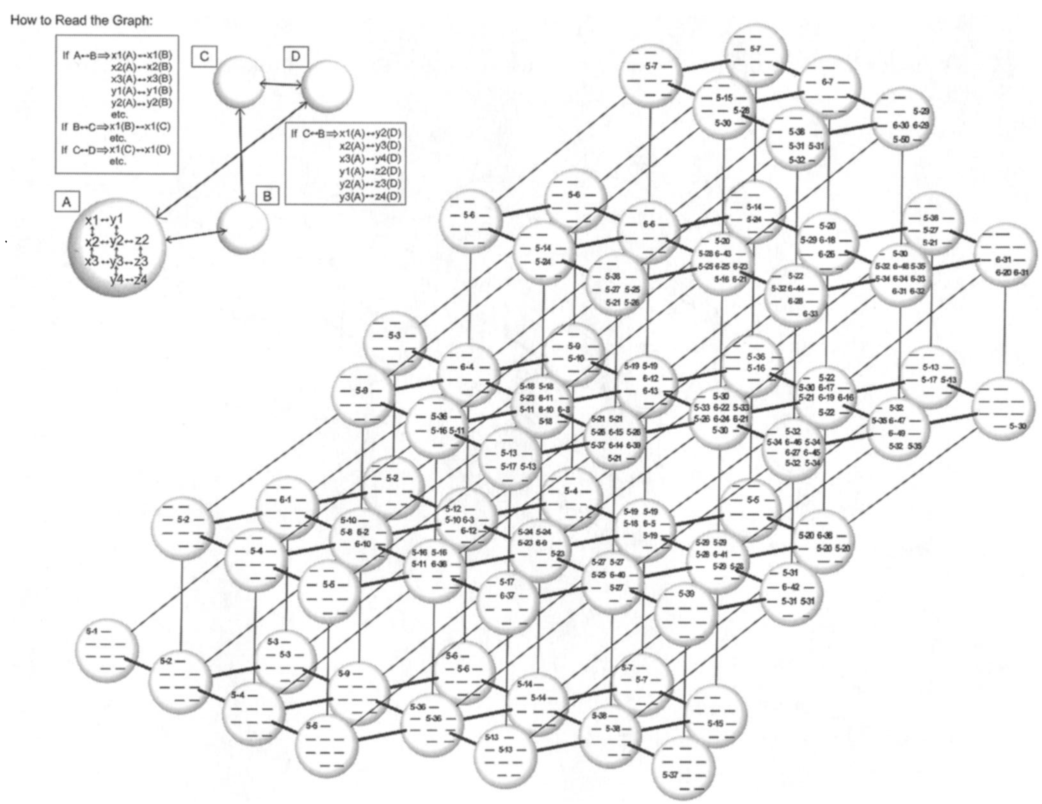


Figure 13. A parsimonious voice-leading space for pentachord and hexachord classes

with the smaller set-classes, the hexachordal space positions the maximally chromatic 6–1 [012345] and the maximally even 6–35 [02468T] at opposite extremes. Furthermore, these are the only two hexachords that connect parsimoniously with only one other. As one moves toward the center of the space, the set-classes have the potential for parsimonious connections with a greater variety of different set-classes. Of the fifty hexachords, fourteen are represented at two locations in order to ensure that all of the parsimonious connections are included.

I conclude this survey of the parsimonious voice-leading space for set-classes with Figure 13, which conflates the spaces for pentachords and hexachords. To accommodate the greater number of hexachords and the large number of parsimonious connections between pentachords and hexachords, many of the pentachords are now found in more than one location, and the map uses Forte names only (readers may refer to preceding examples for the relevant prime forms). Each node contains ten slots and moves within and between nodes specified in the schematic diagram at the upper left of the example.

The parsimonious voice-leading space for all set-classes has now been surveyed in its entirety. This master space contains all set-classes of cardinality 1 through 6 and connects them with optimal, single-semitone-displacement voice leading. Larger set-classes are understood to occupy the same positions in the space as their smaller complements.[16] Within this integrated space, it is possible to measure the distance between any two set-classes based on semitonal offset.

I would like to use this integrated, multicardinality master space, with its foundational interest in connecting harmonies through minimal distance voice leading, as the basis for a significant generalization of Schoenberg's familiar "Law of the Shortest Way."[17] Schoenberg's law operates primarily in the pitch space of diatonic, tonal music. It asserts that the pitches in one harmony should move through minimal distances to the pitches in the next. I propose a corresponding *Law of Atonal Voice Leading* for the set-class space described here. The tonal harmonies (triads and seventh chords) of Schoenberg's law become the full range of set-classes, and his motions by diatonic or chromatic step in pitch space become the abstract optimal voice leadings of set-class space. The generalized, abstract Law of Atonal Voice Leading states that, in Schoenberg's words, "only that be done which is absolutely necessary for connecting the chords." Translated into set-class space, this law specifies a preference for small voice-leading distances: from any set-class, the law specifies a preference for a move to a set-class that is adjacent or nearby within the space, separated by a relatively small voice-leading offset. Both Schoenberg's law and my law engage familiar ideas of musical effort and energy: both imagine that it is easier to move someplace near than someplace far, that traversing a longer distance requires more effort. Both are thus laws of conservation of musical energy.

Who is affected by such laws? Schoenberg's Law of the Shortest Way is usually understood to apply to composers, listeners, and (to some extent) to the musical tones themselves. Composers (particularly student composers engaged in part-writing) are expected to compose in accordance with the law, moving pitches by minimal distances. Listeners are expected to construe music in accordance with the law, interpreting disjunct surfaces with respect to relatively smooth underlying norms. And, in an anthropomorphization of the tones that is among our most persistent "analytical fictions" (this term comes from Guck 1994), the tones themselves are understood as possessing a will, as having inherent tendencies, including the will and tendency to move by minimal distances.

My generalized Law of Atonal Voice Leading applies also to composers, listeners, and the tones themselves, but in slightly different ways. The generalized law obviously exerts less constraint on post-tonal composers than the original law did on tonal composers. As an empirical matter, tonal composers generally obey the Law of the Shortest Way, while post-tonal composers probably do not generally obey the Law of Atonal Voice Leading. The extent to which post-tonal composers prefer to move among set-classes that are situated in relative proximity to each other is a matter for future research (the brief analyses at the end of this article can merely suggest some possible avenues). It may turn out that compliance with this law varies from composer to composer and is a significant marker of stylistic difference (I suspect, e.g., that Schoenberg is a "smoother" composer than Webern in this sense, and that both are less "smooth" than their more conservative, neoclassically oriented contemporaries). It also may turn out that smoothness of voice leading among set-classes is more likely in certain musical situations (e.g., at cadences) or in certain musical textures (e.g., a homophonic texture).

But whether or not relatively smooth voice leading in set-class space is characteristic of or statistically predominant in atonal practice, I would argue that the generalized law nonetheless may have an impact on compositional choices. Even when not observed in practice, it may nonetheless be experienced as an underlying tendency, one that may either be followed or resisted. Compositional choices, either to obey or disobey, may be understood in relation to the underlying law, even if the choice to disobey is made with equal or even greater frequency.

One might even construe a choice to disobey in historical terms, as part of the deliberate flouting of musical convention characteristic of musical modernism in general. A preference for smooth voice leading (among pitches, pitch classes, and set-classes) clearly shapes traditional practice. In denying that preference for or, in more abstract terms, in flouting the Law of Atonal Voice Leading, modernist composers may be giving expression to an aesthetic iconoclasm, asserting their independence of the claims of the tonal tradition.

For listeners, similarly, the Law of Atonal Voice Leading may describe an underlying awareness of voice-leading distance. That is, listeners may experience voice leading among set-classes as either in conformity with or in tension with the law, as involving either shorter or longer distances. This claim is fully testable. Although we may not be able to verify the law by creating a statistical profile of atonal progressions, we could falsify it by testing listeners to see to what extent they experience progressions among set-classes as near or far, with those distances precisely predicted by the model.[18] In the brief analyses that conclude this article, I provide such a test on a sample of one, through introspection.

As for the tones themselves, I find the notion of inertia an intuitively appealing metaphor for one aspect of the behavior of tones (pitches, pitch classes, and the idealized set-class members under discussion here). Tones seek to conserve their energy; when the harmony changes, and they are compelled to move, they do so by as small a distance as possible. In the analyses at the end of this article, I will suggest the interpretive value of this metaphor.

The Quality of Set-Classes (Dissonance and Consonance, Tension and Relaxation, Chromaticness and Evenness)

I noted above that the parsimonious voice-leading space for each cardinality arranged the set-classes between two extremes. At one edge of the space, we find the maximally chromatic set-classes ([01], [012], [0123], [01234], and [012345]), while at the opposite edge we find the maximally even set-classes ([06], [048], [0369], [02479], and [02468T]). We might conceive these as two opposing extremes of harmonic density—the extent to which the notes of a harmony are either packed together or dispersed through whatever space they occupy—and each extreme may be taken as defining a particular harmonic quality. The first quality is that of compactness, denseness, *chromaticness*. The second quality is that of dispersion, spaciousness, *evenness*.[19]

For each cardinality, there is one set-class that is maximally chromatic and one that is maximally even. Figure 14 reprints the dyadic map from Figure 3, now also measuring each set-class's degree of offset from 2–1 [01], the most chromatic, and 2–6 [06], the most even. These degrees of offset are always complementary—they sum to 5, which is the total distance between the two extremes. As one moves down the chart, the set-classes become increasingly spacious; as one moves up through the chart, the set-classes become increasingly dense.

Figure 15 explores the relative chromaticness and evenness of the trichords. The version at the left of the figure shows degrees of offset

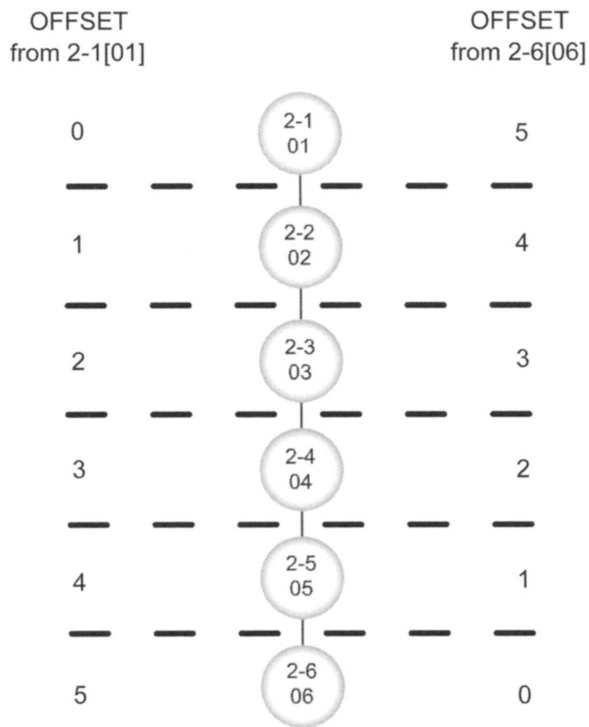


Figure 14. Measuring degrees of offset from the maximally chromatic and maximally even dyads

from 3–1 [012] (the most compact set-class), and the version at the right shows degrees of offset from 3–12 [048] (the maximally even set-class for this cardinality). For the most part, these two measures are complementary, mod 6: the more chromatic the set-class, the less spacious, and vice versa. But there are two exceptions, which I refer to as *rogues*: 3–9 [027] is offset by two from 3–12 [048], but by five from 3–1 [012]; similarly, 3–5 [016] is offset by three from 3–12 [048], but by four from 3–1 [012]. In both cases, these set-classes (which occupy a cul-de-sac on the map) defy the simple complementarity of the qualities of compactness and dispersion.

There are four such rogues among the tetrachords: 4–8 [0156], 4–9 [0167], 4–16 [0157], and 4–25 [0268] do not occupy complementary positions with respect to the maximally chromatic and maximally even extremes. Set-classes 4–9 and 4–25 are particularly roguish: they are maximally uncompact (they are both offset by eight semitones from 4–1), but

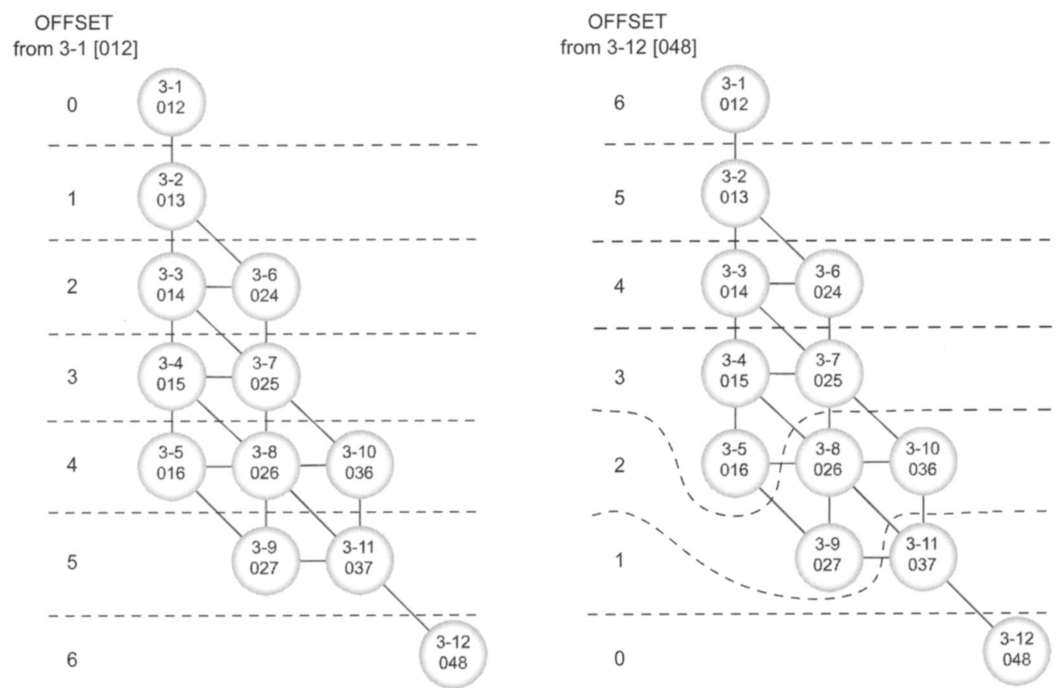


Figure 15. Measuring degrees of offset from the maximally chromatic and maximally even trichords

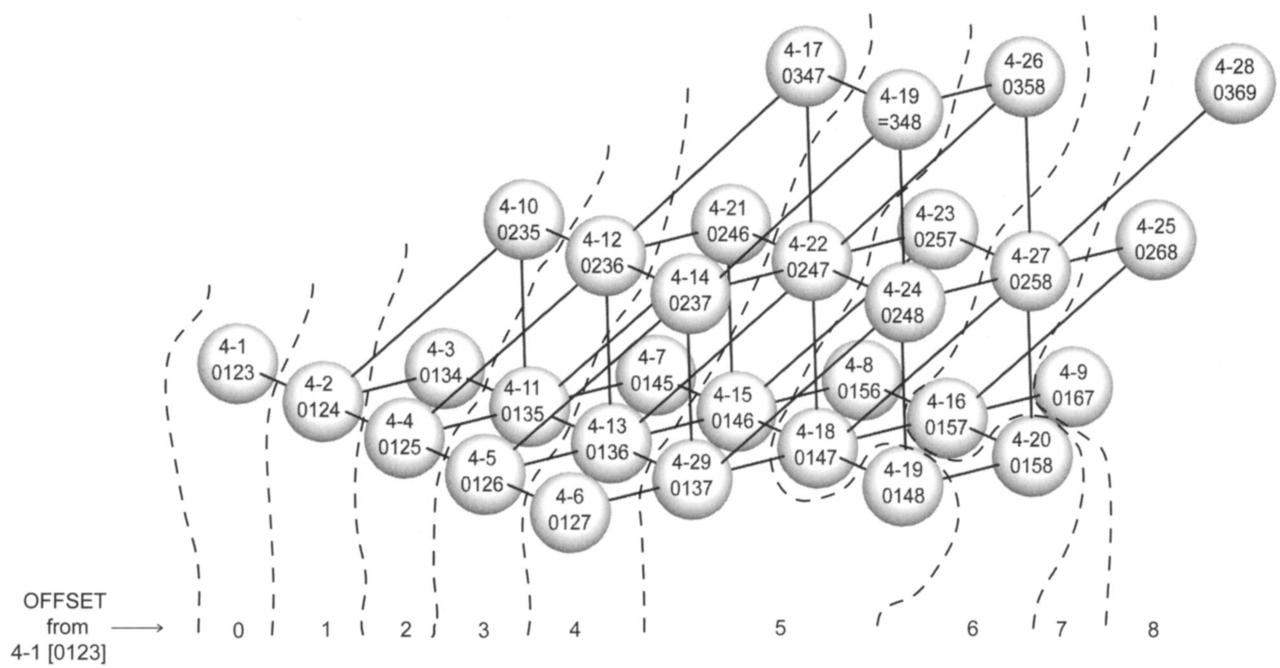


Figure 16. Measuring degrees of offset from the maximally chromatic tetrachord

neither is maximally even or even close to it (4–25 is offset by two and 4–9 by four from the maximally even 4–28). Nonetheless, for the remaining twenty-five tetrachords, the complementarity and opposition of compactness and dispersion operate in a simple, straightforward way: the more compact the set-class, the less even it is, and vice versa.

But roguishness nonetheless remains a persistent problem for the proposed complementarity of chromaticness and evenness. In addition to the two trichordal and four tetrachordal rogues, there are three pentachordal and ten hexachordal rogues.[20] Furthermore, the quality of evenness is somewhat diffuse, particularly compared with the quality of chromaticness. Set-classes that lie in close proximity to the maximally chromatic set-classes tend to share intervallic and subset content and would normally be judged as relatively similar by any of the existing measures of set-class similarity. Set-classes that lie in close proximity to the maximally even set-classes, however, tend to be much more varied in their sonic quality. As a result, I focus here primarily on the quality of chromaticness, while continuing to maintain casually that this quality is understood in part in opposition to the less well-defined quality of evenness.

Accordingly, Figure 16 identifies for the tetrachords their degree of offset from the maximally chromatic 4–1 [0123]. The tetrachordal space is reproduced from Figure 7. Onto that space, I have imposed offset measures, designed deliberately to mimic the contour lines on a topographical map. I mean to suggest that the voice-leading space has a varied terrain, with the set-classes that compose it shifting gradually in character between the extremes of chromaticness and unchromaticness. One interesting feature of the tetrachordal space (shared with other even cardinalities) is that set-classes at the same level of chromaticness are never connected by parsimonious voice leading. To put it the other way around: a parsimonious move between tetrachords always involves a change in degree of chromaticness. Every move in the tetrachordal space thus involves not only a voyage through a measurable distance, but also a change in orientation in the space, either toward or away from maximal chromaticness.

Relative chromaticness can also be measured in the intercardinality spaces. In Figure 17, trichords and tetrachords are evaluated according to their distance from 4–1 [0123], the maximally chromatic set-class for the largest cardinality in the space.[21]

Graphic representation of pentachords and hexachords and of the intercardinality spaces that involve them is relatively complex and is not attempted here. Rather, I deal with those subspaces in an ad hoc basis as the analytical need arises. For all of these subspaces, and for the master space of all cardinalities, however, the same principle applies: the relative chromaticness of set-classes can be meaningfully evaluated by measuring, in semitones of offset, their distance from some maximally chromatic set-class.

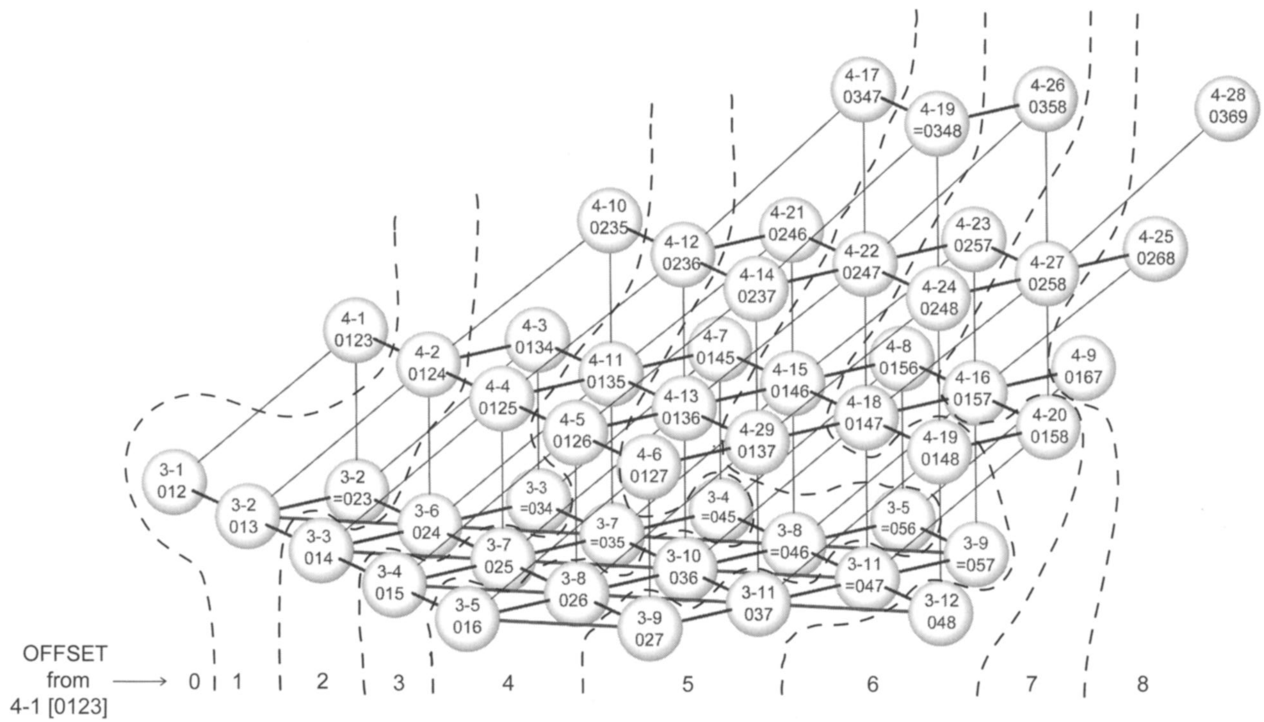


Figure 17. Measuring degrees of chromaticness in the intercardinality space of trichords and tetrachords

Semitonal offset from maximally chromatic set-classes is an effective measure of the quality of chromaticness. As Quinn 2001 has demonstrated, the many different existing measures of set-class similarity are in broad accord about the qualities of set-classes, including their chromaticness. Table 1 selects a number of similarity measures and lists their similarity values for all trichords and tetrachords in comparison to maximally chromatic set-classes.[22]

Table 1 also lists the degrees of voice-leading offset, measured in semitones, from the relevant maximally chromatic set-classes, and these are highly correlated with the similarity measures. Semitonal offset is thus an excellent proxy for similarity measures when it comes to measuring the quality of chromaticness.[23] And semitonal offset has the great advantage not only of producing simple integer values, but also of using the familiar and musically meaningful semitone as its unit of measure.

Table 2 offers a similar comparison of semitonal offset with Quinn's **F**(12, 1) (see Quinn 2006–7). This is Quinn's measure of the quality of chromaticness, which uses a unit of measure called the "lewin" to measure the degree of "chromatic force": the higher the number of "lewins," the more closely a given set-class is understood as sharing the qualities of the prototypes of the chromatic genus.[24]

Compared to the similarity measures, there is an even higher correlation between Quinn's ascriptions of chromatic quality and the same quality as measured by semitonal offset.[25] This confirms the value of semitonal offset from the maximally chromatic set-classes as a measure of the quality of chromaticness, with the added advantage of measuring that quality in simple semitones.

Traditional notions of consonance and dissonance relate in suggestive ways to the qualities of set-class evenness and chromaticness described here. The traditionally dissonant harmonies tend to be the most chromatic ones, and the chromatic harmonies would traditionally be understood as dissonant. The traditionally consonant or stable harmonies are always among the least chromatic (and most even). The reverse is also true, although to a lesser degree: the least chromatic (and most even) harmonies tend to be relatively consonant and stable. Among the dyads, for example, the arrangement in Figure 14 corresponds pretty well with traditional ascriptions of consonance and dissonance: the semitone (set-class 2–1 [01]) is the sharpest dissonance, and the dyads become increasingly consonant as they become increasingly unchromatic (or increasingly even). The maximally even dyad, 2–6 [06], is obviously not a traditional consonance. But the traditionally most consonant dyad, 2–5 [05], is only one degree of offset away—a minimum displacement of the maximally even structure for this cardinality. And the other traditional consonances lie relatively close to the even end of the spectrum while the

Table 1. Comparing offset and similarity as measures of chromaticness

	Offset	IcVSIM	PSATSIM	SATSIM	REL	ATMEMB	ASIM
3-1 [012]	**0**	0.000	0.000	0.000	1.000	1.000	0.000
3-2 [013]	**1**	0.577	0.167	0.167	0.604	0.625	0.333
3-3 [014]	**2**	0.816	0.306	0.333	0.354	0.375	0.667
3-4 [015]	**3**	0.816	0.306	0.333	0.354	0.375	0.667
3-5 [016]	**4**	0.816	0.417	0.333	0.354	0.375	0.667
3-6 [024]	**2**	1.000	0.306	0.333	0.354	0.375	0.667
3-7 [025]	**3**	1.000	0.333	0.333	0.250	0.250	0.667
3-8 [026]	**4**	1.000	0.389	0.333	0.250	0.250	0.667
3-9 [027]	**5**	1.155	0.333	0.333	0.250	0.250	0.667
3-10 [036]	**4**	1.291	0.583	0.500	0.000	0.000	1.000
3-11 [037]	**5**	1.155	0.472	0.500	0.000	0.000	1.000
3-12 [048]	**6**	1.528	0.417	0.500	0.000	0.000	1.000
Correlations:		**0.876**	**0.802**	**0.846**	**–0.871**	**–0.878**	**0.846**
4-1 [0123]	**0**	0.000	0.000	0.000	1.000	1.000	0.000
4-2 [0124]	**1**	0.577	0.111	0.111	0.753	0.773	0.167
4-3 [0134]	**2**	0.816	0.208	0.222	0.662	0.682	0.333
4-4 [0125]	**2**	0.816	0.222	0.222	0.571	0.591	0.333
4-5 [0126]	**3**	1.000	0.347	0.333	0.480	0.500	0.500
4-6 [0127]	**4**	1.155	0.347	0.333	0.480	0.500	0.500
4-7 [0145]	**4**	1.291	0.333	0.333	0.314	0.318	0.500
4-8 [0156]	**6**	1.414	0.458	0.444	0.223	0.227	0.667
4-9 [0167]	**8**	1.528	0.486	0.444	0.223	0.227	0.667
4-10 [0235]	**2**	1.000	0.208	0.222	0.650	0.682	0.333
4-11 [0135]	**3**	1.000	0.222	0.222	0.559	0.591	0.333
4-12 [0236]	**3**	1.155	0.347	0.333	0.543	0.591	0.500
4-13 [0136]	**4**	1.155	0.347	0.333	0.543	0.591	0.500
4-14 [0237]	**4**	1.291	0.333	0.333	0.505	0.545	0.500
4-15 [0146]	**5**	1.155	0.361	0.333	0.377	0.409	0.500
4-16 [0157]	**7**	1.414	0.458	0.444	0.286	0.318	0.667
4-17 [0347]	**4**	1.528	0.431	0.444	0.286	0.318	0.667
4-18 [0147]	**6**	1.414	0.458	0.444	0.286	0.318	0.667
4-19 [0148]	**5**	1.732	0.444	0.444	0.248	0.273	0.667
4-20 [0158]	**6**	1.633	0.444	0.444	0.248	0.273	0.667
4-21 [0246]	**4**	1.633	0.458	0.444	0.223	0.227	0.667
4-22 [0247]	**5**	1.528	0.333	0.333	0.273	0.273	0.500
4-23 [0257]	**6**	1.732	0.333	0.333	0.273	0.273	0.500
4-24 [0248]	**6**	1.826	0.458	0.444	0.182	0.182	0.667
4-25 [0268]	**8**	1.732	0.486	0.444	0.182	0.182	0.667
4-26 [0358]	**6**	1.633	0.431	0.444	0.257	0.273	0.667
4-27 [0258]	**7**	1.528	0.458	0.444	0.257	0.273	0.667
4-28 [0369]	**8**	2.082	0.569	0.556	0.182	0.227	0.833
4-29 [0137]	**5**	1.155	0.361	0.333	0.505	0.545	0.500
Correlations:		**0.856**	**0.896**	**0.874**	**–0.881**	**–0.873**	**0.874**

3-1 [012]	**1**	0.500	0.069	0.067	0.835	0.733	0.333
3-2 [013]	**1**	0.764	0.153	0.133	0.879	0.800	0.333
3-3 [014]	**2**	1.118	0.292	0.300	0.432	0.400	0.556
3-4 [015]	**3**	1.258	0.375	0.400	0.274	0.267	0.778
3-5 [016]	**4**	1.258	0.486	0.433	0.274	0.267	0.778
3-6 [024]	**2**	1.258	0.319	0.367	0.316	0.267	0.556
3-7 [025]	**3**	1.258	0.319	0.333	0.382	0.333	0.556
3-8 [026]	**4**	1.384	0.458	0.400	0.224	0.200	0.778
3-9 [027]	**5**	1.500	0.403	0.400	0.224	0.200	0.778
3-10 [036]	**4**	1.500	0.569	0.500	0.224	0.200	0.778
3-11 [037]	**5**	1.500	0.458	0.500	0.158	0.133	0.778
3-12 [048]	**6**	1.893	0.486	0.533	0.000	0.000	1.000
4-1 [0123]	**0**	0.000	0.000	0.000	1.000	1.000	0.000
4-2 [0124]	**1**	0.577	0.111	0.111	0.753	0.773	0.167
4-3 [0134]	**2**	0.816	0.208	0.222	0.662	0.682	0.333
4-4 [0125]	**2**	0.816	0.222	0.222	0.571	0.591	0.333
4-5 [0126]	**3**	1.000	0.347	0.333	0.480	0.500	0.500
4-6 [0127]	**4**	1.155	0.347	0.333	0.480	0.500	0.500
4-7 [0145]	**4**	1.291	0.333	0.333	0.314	0.318	0.500
4-8 [0156]	**6**	1.414	0.458	0.444	0.223	0.227	0.667
4-9 [0167]	**8**	1.528	0.486	0.444	0.223	0.227	0.667
4-10 [0235]	**2**	1.000	0.208	0.222	0.650	0.682	0.333
4-11 [0135]	**3**	1.000	0.222	0.222	0.559	0.591	0.333
4-12 [0236]	**3**	1.155	0.347	0.333	0.543	0.591	0.500
4-13 [0136]	**4**	1.155	0.347	0.333	0.543	0.591	0.500
4-14 [0237]	**4**	1.291	0.333	0.333	0.505	0.545	0.500
4-15 [0146]	**5**	1.155	0.361	0.333	0.377	0.409	0.500
4-16 [0157]	**7**	1.414	0.458	0.444	0.286	0.318	0.667
4-17 [0347]	**4**	1.528	0.431	0.444	0.286	0.318	0.667
4-18 [0147]	**6**	1.414	0.458	0.444	0.286	0.318	0.667
4-19 [0148]	**5**	1.732	0.444	0.444	0.248	0.273	0.667
4-20 [0158]	**6**	1.633	0.444	0.444	0.248	0.273	0.667
4-21 [0246]	**4**	1.633	0.458	0.444	0.223	0.227	0.667
4-22 [0247]	**5**	1.528	0.333	0.333	0.273	0.273	0.500
4-23 [0257]	**6**	1.732	0.333	0.333	0.273	0.273	0.500
4-24 [0248]	**6**	1.826	0.458	0.444	0.182	0.182	0.667
4-25 [0268]	**8**	1.732	0.486	0.444	0.182	0.182	0.667
4-26 [0358]	**6**	1.633	0.431	0.444	0.257	0.273	0.667
4-27 [0258]	**7**	1.528	0.458	0.444	0.257	0.273	0.667
4-28 [0369]	**8**	2.082	0.569	0.556	0.182	0.227	0.833
4-29 [0137]	**5**	1.155	0.361	0.333	0.505	0.545	0.500
Correlations:		**0.842**	**0.825**	**0.808**	**–0.790**	**–0.743**	**0.697**

Table 2. Comparing offset and Quinn's F-space as measures of chromaticness

	Offset	"**F**(12,1)"
3-1 [012]	**0**	2.732
3-2 [013]	**1**	2.394
3-3 [014]	**2**	1.932
3-4 [015]	**3**	1.414
3-5 [016]	**4**	1.000
3-6 [024]	**2**	2.000
3-7 [025]	**3**	1.506
3-8 [026]	**4**	1.000
3-9 [027]	**5**	0.732
3-10 [036]	**4**	1.000
3-11 [037]	**5**	0.518
3-12 [048]	**6**	0.000
	Correlation:	**–0.995**
4-1 [0123]	**0**	3.346
4-2 [0124]	**1**	2.909
4-3 [0134]	**2**	2.732
4-4 [0125]	**2**	2.394
4-5 [0126]	**3**	1.932
4-6 [0127]	**4**	1.732
4-7 [0145]	**4**	1.932
4-8 [0156]	**6**	1.000
4-9 [0167]	**8**	0.000
4-10 [0235]	**2**	2.449
4-11 [0135]	**3**	2.236
4-12 [0236]	**3**	1.932
4-13 [0136]	**4**	1.732
4-14 [0237]	**4**	1.506
4-15 [0146]	**5**	1.414
4-16 [0157]	**7**	0.518
4-17 [0347]	**4**	1.414
4-18 [0147]	**6**	1.000
4-19 [0148]	**5**	1.000
4-20 [0158]	**6**	0.518
4-21 [0246]	**4**	1.732
4-22 [0247]	**5**	1.239
4-23 [0257]	**6**	0.897
4-24 [0248]	**6**	1.000
4-25 [0268]	**8**	0.000
4-26 [0358]	**6**	0.732
4-27 [0258]	**7**	0.518
4-28 [0369]	**8**	0.000
4-29 [0137]	**5**	1.414
	Correlation:	**–0.964**

	Offset	"**F**(12,1)"
3-1 [012]	**1**	2.732
3-2 [013]	**1**	2.394
3-3 [014]	**2**	1.932
3-4 [015]	**3**	1.414
3-5 [016]	**4**	1.000
3-6 [024]	**2**	2.000
3-7 [025]	**3**	1.506
3-8 [026]	**4**	1.000
3-9 [027]	**5**	0.732
3-10 [036]	**4**	1.000
3-11 [037]	**5**	0.518
3-12 [048]	**6**	0.000
4-1 [0123]	**0**	3.346
4-2 [0124]	**1**	2.909
4-3 [0134]	**2**	2.732
4-4 [0125]	**2**	2.394
4-5 [0126]	**3**	1.932
4-6 [0127]	**4**	1.732
4-7 [0145]	**4**	1.932
4-8 [0156]	**6**	1.000
4-9 [0167]	**8**	0.000
4-10 [0235]	**2**	2.449
4-11 [0135]	**3**	2.236
4-12 [0236]	**3**	1.932
4-13 [0136]	**4**	1.732
4-14 [0237]	**4**	1.506
4-15 [0146]	**5**	1.414
4-16 [0157]	**7**	0.518
4-17 [0347]	**4**	1.414
4-18 [0147]	**6**	1.000
4-19 [0148]	**5**	1.000
4-20 [0158]	**6**	0.518
4-21 [0246]	**4**	1.732
4-22 [0247]	**5**	1.239
4-23 [0257]	**6**	0.897
4-24 [0248]	**6**	1.000
4-25 [0268]	**8**	0.000
4-26 [0358]	**6**	0.732
4-27 [0258]	**7**	0.518
4-28 [0369]	**8**	0.000
4-29 [0137]	**5**	1.414
	Correlation:	**–0.902**

traditional dissonances such as interval class 1 and interval class 2 lie at or close to the chromatic end of the space.

Among the trichords, similar relationships obtain. The maximally unchromatic (and maximally even) trichord, an augmented triad (set-class 3–12), is not a traditional consonance, although it is considered one of four standard triad types. The consonant major or minor triad (set-class 3–11) is a minimally perturbed version of the maximally even structure for this cardinality. At one more degree of offset toward the chromatic end of the space, we find three trichords that, if not traditionally consonant, nonetheless have a distinct and relatively stable harmonic identity: the diminished triad (set-class 3–10 [036]), the incomplete dominant-seventh chord (set-class 3–8 [026]), and the "fourths chord" (set-class 3–9 [027]). For the trichords, then, like the dyads, the consonant or relatively stable sonorities are clustered toward the even edge of the space, while the most dissonant three-note combinations are the more chromatic ones.

The same is true of the tetrachords: the tetrachords that are traditionally understood as harmonically stable (if not actually consonant) are the least chromatic ones, distant from the maximally chromatic 4–1 [0123] and in close proximity to the maximally even 4–28 [0369]. Within one or two degrees of offset from 4–28 (the diminished seventh chord), we find all of the remaining seventh chords of tonal theory: the dominant-seventh or half-diminished seventh chord (4–27 [0258]), the major-seventh chord (4–20 [0158]), and the minor-seventh chord (4–26 [0358]). Traditional tonal theory does not discuss harmonies that contain more than four notes, but among the smaller sets, the relationship between consonance/dissonance or stability/instability and evenness/compactness is reasonably clear.

During the past century, a number of theorists have attempted to create systems of classification for harmonies, taking into account the wide variety of configurations in contemporary music. Some of these classification systems are entirely neutral as to harmonic quality, but others have tried to characterize harmonies as relatively tense or relaxed, stable or unstable, consonant or dissonant. In this category, the most prominent systems are probably those of Hindemith 1942, Hanson 1960, and Krenek 1940.[26] None of these systems is fully elaborated with respect to this issue. Furthermore, there are contradictions both within and between the approaches of these three theorists. Nonetheless, they do share a general sense that the more compressed harmonies are more likely to be relatively tense and unstable while the more dispersed harmonies are more likely to be relatively relaxed and stable.

Hindemith's approach, concerned as it is with identifying harmonic roots and sensitive as it is to registral arrangement and chordal inversion, does not dovetail well with the set-class–oriented approach taken here.[27]

Hanson's approach is more compatible. Although Hanson does not make the consonance–dissonance distinction a focal point of his theory, he does talk about it in a sustained and interesting way in chapter 11 of his treatise, which is devoted to the hexachord 6–27 [013469].[28] Hanson systematically extracts all of the subsets of this hexachord and ranks them according to "their degree of consonance or dissonance." He then composes a passage that arranges them "in order of their relative dissonance, beginning with the three most consonant triads—major and minor—and moving progressively to the increasingly dissonant sonorities." He argues that there is a sense of "increasing tension" over the course of the progression.

Within each cardinality, Hanson's intuitive, interval-based measure of relative dissonance dovetails reasonably well with my offset-based measure. That is, the harmonies he considers relatively consonant are those that have relatively large degrees of offset from maximal chromaticness, and those he considers relatively dissonant are those with relatively small degrees of offset from maximal chromaticness.

Krenek's approach to the question of consonance and dissonance (what he calls "tension-degrees of chords") is more explicit and systematic, but he discusses only trichords. He observes: "Atonality has neither rules for a special treatment of dissonances nor does it formulate a harmonic theory comparable with that of tonality. The only characteristic of a chord that has to be taken into consideration is the degree of tension that the chord shows by virtue of its constituent intervals" (1940, 19).

Krenek provides two overlapping classification set-class schemes for the "three-tone chords" by which he is able to assess the degree of tension of all twelve trichordal set-classes. In the first scheme (see Table 3), Krenek assigns set-classes to six categories based on the consonant or dissonant quality of the intervals they contain (interval classes 3, 4, and 5 are consonant; interval class 2 is a mild dissonance; interval class 1 is a sharp dissonance; interval class 6 is neutral).

This classification system accounts for eight of the twelve trichordal set-classes. Krenek observes that "in a three-tone chord, the intervals of

Table 3. Krenek's first scheme for assessing the "tension degree" of trichords

Krenek Category	Interval Content	Set-class
1	3 consonant intervals	3-11[037], 3-12[048]
2	2 consonant intervals, 1 mild dissonance	3-7[025]
3	1 consonant interval, 2 mild dissonances	3-6 [024]
4	2 consonances, 1 sharp dissonance	3-3[014], 3-4[015]
5	1 consonance, 1 mild and 1 sharp dissonance	3-2[013]
6	1 mild and 2 sharp dissonances	3-1[012]

Table 4. Krenek's second scheme for assessing the "tension-degree" of trichords

Consonant	Mild	Sharp
3-10[036], 3-11[037]	3-7[025], 3-8[026], 3-9[027]	3-4[015], 3-5[016]

five semitones (perfect fourth) and six semitones (diminished fifth) will assume the character of a consonance or a dissonance depending on the third tone added." On that basis, Krenek creates a second system of classification for the seven trichordal set-classes that contain either interval class 5 or interval class 6: such chords are labeled as consonant, mild, or sharp (see Table 4). This second system accounts for the four set-classes missing from the first classification system, and accounts again in a new way for three previously classified set-classes.

Figure 18 reprints the voice-leading map for trichords and shows both the degree of offset from the maximally chromatic trichord, 3–1 [012], and the results of Krenek's two classification systems. Krenek's intervallic approach and my offset-based approach produce strikingly similar results: the sonorities he considers consonant cluster toward the bottom of the map; those he considers dissonant cluster toward the top. Based on Krenek's systems of classification, one can imagine the chart as tilted on two axes: an implicit progression from dissonant to consonant that runs downhill from the top of the chart to the bottom and, to a lesser extent, from the left side of the chart to the right.

Beyond the speculations of such composers as Hindemith, Hanson, and Krenek, cognitive scientists have long been interested in the perception and cognition of chord qualities, including the qualities of consonance and dissonance. Recently, Huron (1994) has proposed "an index of tonal consonance constructed by amalgamating experimental data from three well-known studies."[29] Two of Huron's explanatory charts are provided in Table 5.

The first chart summarizes the results of previous studies of intervallic consonance or dissonance. From these experimental data, Huron derives the consonance rating for each of the six interval classes, as shown in the second chart. Huron asserts, "This index can be regarded as a rough approximation of the perceived consonance of typically equally tempered interval classes constructed by using complex tones in the central pitch region" (293).

By multiplying these index values by the interval-class vector, Huron is able to create a consonance value for any set-class:

> We might define an *aggregate dyadic consonance* value that is calculated by multiplying the number of intervals of a given size by the associated

consonance values, and summing the results together for all interval classes. . . . Those pitch-class sets that provide many consonant intervals and few dissonant intervals would tend to have higher aggregate dyadic consonance values. Large negative scores would indicate a set that provides many dissonant intervals and relatively few consonant intervals. (294)

Huron's "aggregate dyadic consonance values" correlate to a significant degree with the offset measure proposed here. That is, generally

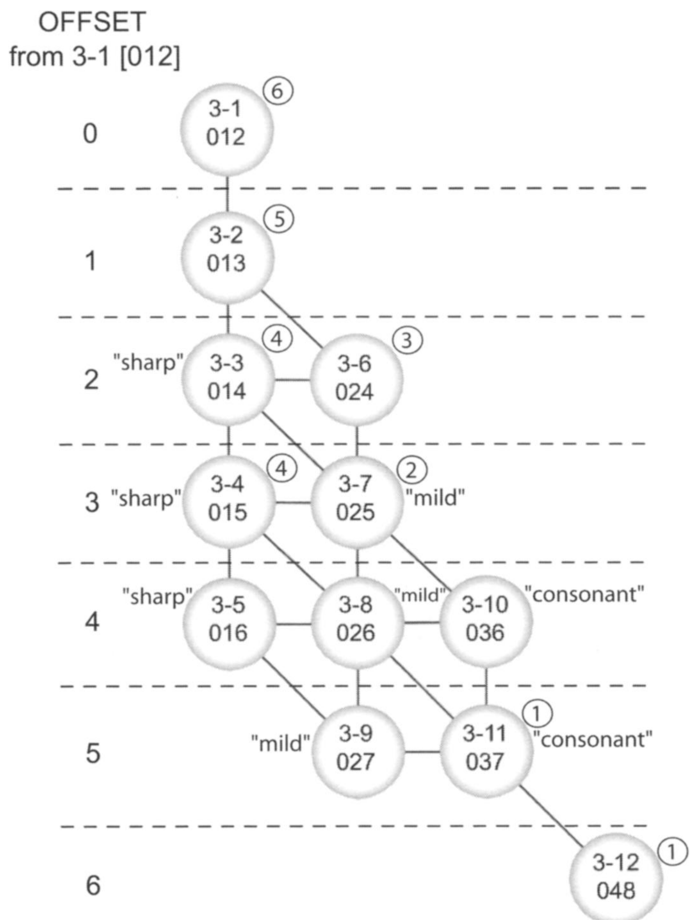


Figure 18. Krenek's two schemes for classifying the "tension degrees" of trichords compared with my offset measure, both superimposed on the parsimonious voice-leading space for trichords

speaking, the less chromatic the set-class, the higher its consonance value, and the more chromatic the set-class, the lower its consonance value. Figure 19 provides a chart that compares Huron's consonance values with my offset values for trichords, tetrachords, and two of the pentachords, 5–1 [01234] and 5–35 [02479]. The set-classes generally cluster along the diagonal line, indicating correspondence between consonance value and unchromaticness. For set-classes above the line, their unchromaticness, as measured by semitonal offset, exceeds their consonance value according to Huron. In general, these set-classes contain at least one tritone, an interval that is maximally unchromatic and yet not consonant. For the set-classes below the line, their chromaticness, as measured by semitonal offset, exceeds their lack of consonance value, according to Huron. In general, these set-classes contain interval class 3, a relatively chromatic interval as measured by semitonal offset and yet one that is relatively consonant according to Huron. Despite these discrepancies, semitonal offset measured from maximally chromatic set-classes and Huron's consonance values for set-classes correlate reasonably well.[30]

Table 5. Two charts from Huron (1994)

Measures of Tonal Consonance and Dissonance for Various Harmonic Intervals

Interval	Malmberg (1918) Consonance	Hutchinson & Knopoff (1979) Dissonance	Kameoka & Kuriyagawa (1969) Dissonance
m2	0.00	.4886	285
M2	1.50	.2690	275
m3	4.35	.1109	255
M3	6.85	.0551	250
P4	7.00	.0451	245
TT	3.85	.0930	265
P5	9.50	.0221	215
m6	6.15	.0843	260
M6	8.00	.0477	230
m7	3.30	.0998	250
M7	1.50	.2312	255

Interval Class Index of Tonal Consonance

Interval Class	Consonance
m2/M7	–1.428
M2/m7	–0.582
m3/M6	+0.594
M3/m6	+0.386
P4/P5	+1.240
A4/d5	–0.453

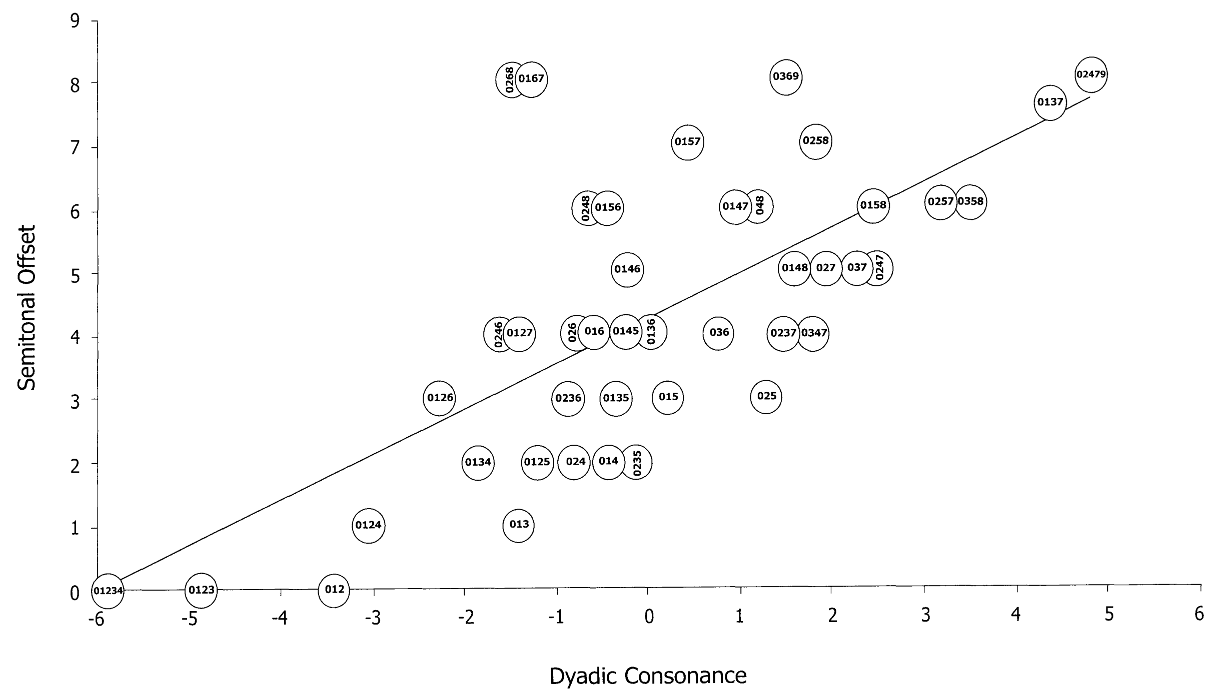


Figure 19. Trichords, tetrachords, and pentachordal extrema charted by offset (measured from the most chromatic set-class within a given cardinality) and Huron's "dyadic consonance"

Based on the speculations of Hindemith, Hanson, and Krenek and on research in music cognition, particularly the work of Huron, I think it is reasonable to imagine that there is a close correspondence between traditional notions of consonance/relaxation/stability versus dissonance/tension/instability and the concepts of evenness/dispersion versus compactness/compression developed here. On that basis, I propose a *Law of Atonal Harmony*: harmonies tend to move away from a state of chromaticness and toward a state of relative dispersion within the available musical space.[31]

Like the Law of Atonal Voice Leading, this Law of Atonal Harmony has a bearing on composers, listeners, and the tones themselves. The extent to which post-tonal composers observe this law is a subject for further research. I would suspect that in a variety of post-tonal styles, especially more conservative styles, harmonic progressions generally do observe the law, beginning and ending on relatively unchromatic harmonies and reserving the most chromatic harmonies for points of relative tension. Observance of the law may be a significant way of differentiating among the compositional styles of different composers and post-tonal repertoires.

Whether or not it is obeyed, however, the law may still have significant bearing on the kinds of harmonic choices composers make. It may be the case that composers make harmonic decisions in light of a conscious or unconscious sense of the relative degrees of chromaticness of the harmonies. If that is the case, then the law may be understood to constrain their decisions even when they decide to disobey it. As with the Law of Atonal Voice Leading, a decision to disobey the Law of Atonal Harmony may have a historical and aesthetic dimension. Resistance to the laws may be construed as part of the larger project of modernism in music.

It may also be the case that listeners interpret harmonic progressions in light of the Law of Atonal Harmony, experiencing harmonies as relatively chromatic or unchromatic. It would be hard to imagine a definitive test of this matter, but it should be possible to determine whether a listener's experience of harmonies is conditioned to some extent by their degree of chromaticness. The analyses at the end of this article will offer some narrow, preliminary judgments on this point.

As for the tones themselves, it can be interpretively suggestive and useful to ascribe motivations to them, as so many music theories do. Tones within a harmony may be construed as having a negative charge that repels other tones—they seek to maximize the distances between them. Harmonies thus seek to move from a state of relative compression to a state of relative dispersion. The value of this metaphor will be tested in the analyses at the end of this article.

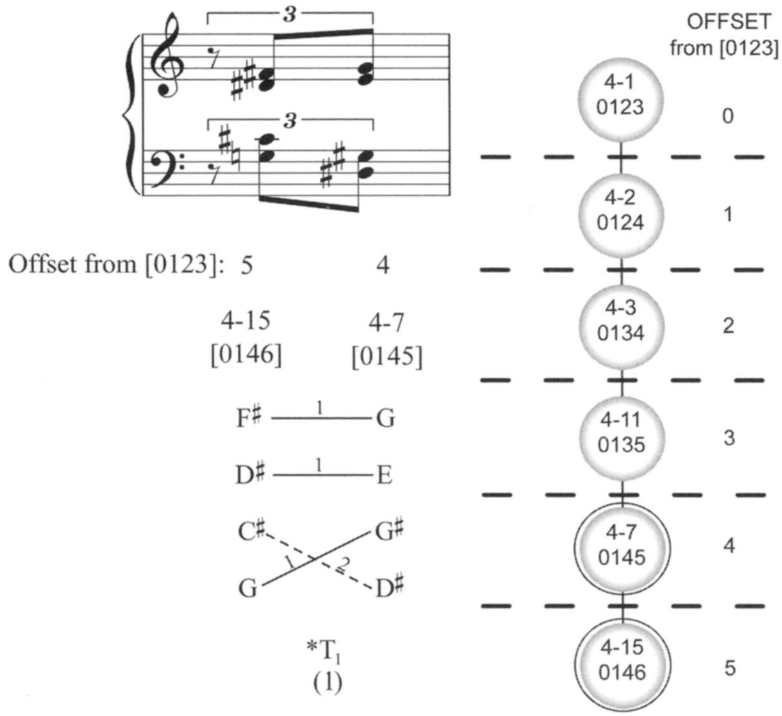
The two laws articulated here—a Law of Atonal Voice Leading that invokes a preference for smooth movement in set-class space and a Law of Atonal Harmony that invokes a preference for relatively unchromatic harmonies—are complementary to each other. Within a harmony, the notes seek to maximize their distance from each other, as the harmony seeks to become more spacious. Between harmonies, the notes seek to minimize their voice-leading distances. Furthermore, voice leading (parsimonious or not) is the means by which the tones of a harmony adjust their relationship to each other, either moving closer together (expending energy to overcome the underlying tendency away from chromaticness) or moving farther apart (relaxing by moving away from a state of compactness). The desire of tones to move apart from each other motivates voice leading, which occurs as the means by which tones readjust their harmonic relationships, thus producing harmonic progression. Harmonies generate voice leading, which in turn produces new harmonies.

Analyses

Example 1 focuses on a pair of chords, played by the celesta, in the first measure of the first of Webern's Six Pieces for Orchestra op. 6. In the right hand, a minor third (D♯–F♯) moves up a semitone (to E–G). The motion in the left hand is in the opposite direction but is not strictly parallel: the upper note of the tritone G–C♯ descends five semitones while the lower note descends four semitones, moving onto a perfect fourth. These large descending leaps produce a registrally distinct ascending semitone, G3–G♯3, that can be heard as reinforcing the more explicit ascending semitones in the right hand.

If we think about the progression transformationally, we might imagine the chords as connected by $*T_1$, with three of the four voices actually ascending by semitone in pitch space. The fourth voice is offset by one semitone from this prevailing motion: it ascends two semitones (in pitch-class space).[32] The progression is thus maximally uniform, approaching as closely as possible an actual, crisp pitch-class transposition at T_1. At the level of the set-class, the motion is thus smooth, involving the displacement of a single function by a single semitone.

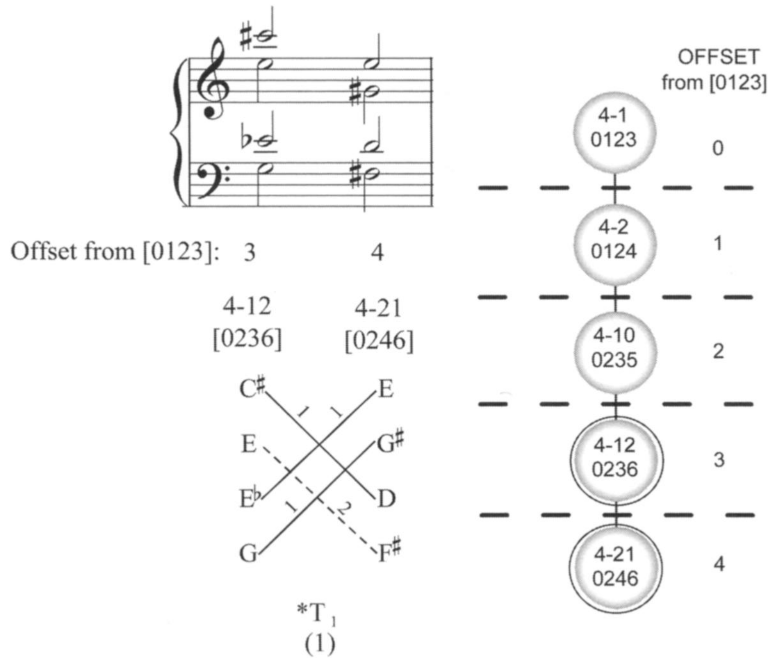
The two set-classes involved, 4–15 [0146] and 4–7 [0145], are thus adjacent to each other on the tetrachordal map.[33] Furthermore, [0145] lies one click closer to the maximally chromatic 4–1 [0123] than does [0146] (Example 1 contains just the sector of the tetrachordal map that includes both [0145] and [0146] as well as a shortest path from [0145] to [0123]). As a result, this progression involves a slight tensing, moving from a relatively spacious harmony to a relatively chromatic one. As the two chords are disposed in pitch space, however, the reverse is true: the second chord is more widely spaced than the first. But a countervailing sense of compression and increasing chromaticness is also apparent in the relatively dissonant quality of the second chord, with its two representatives



Example 1. Webern, Six Pieces for Orchestra op. 6/1, m. 1

of interval class 1.[34] The theoretical model suggests that this progression is a smooth move (in accordance with the Law of Atonal Voice Leading) in the direction of greater chromaticness (pushing against the Law of Atonal Harmony), and the musical surface bears that out to a significant degree.

Example 2 explores the cadential piano chords that conclude one of the first phrases in the first movement of Crawford's Violin Sonata (the chords are arpeggiated in the actual music). As arranged in pitch space, the chords are arranged in two registral pairs, as in the progression discussed in Example 1. The two lower lines both descend by semitone, while the two upper lines descend by large leap, eight semitones in the alto and nine in the soprano.[35] It is also possible, however, to interpret the progression as a single transformational gesture, with the minor sixth in the left hand in the first chord moving up one pitch-class semitone onto the minor sixth in the right hand in the second chord, while the major sixth in the right hand in the first chord contracts slightly as it moves onto



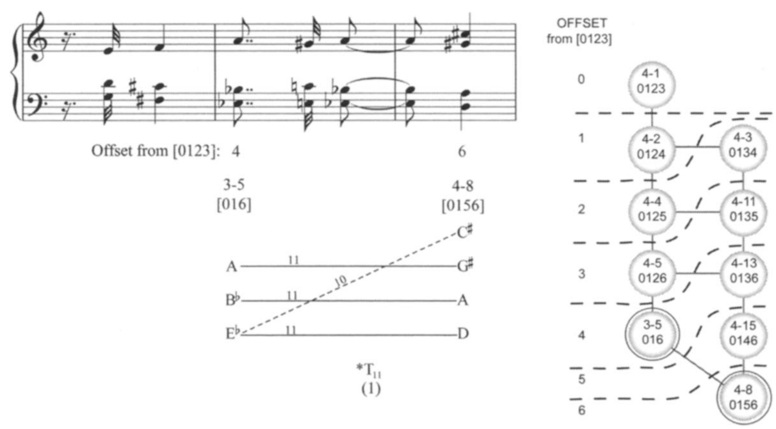
Example 2. Crawford (Seeger), Violin Sonata, first movement, m. 14

the minor sixth in the left hand of the second chord, with one voice moving up one pitch-class semitone while the other deviates slightly, moving up two pitch-class semitones (see Example 2). Viewing the progression as a whole, we might say that the first chord maps onto the second at $*T_1$, with an offset of one semitone.[36] That is, three of the four transformational voices move at actual, crisp T_1, while the fourth deviates by one semitone from that prevailing motion. The progression is thus maximally uniform.

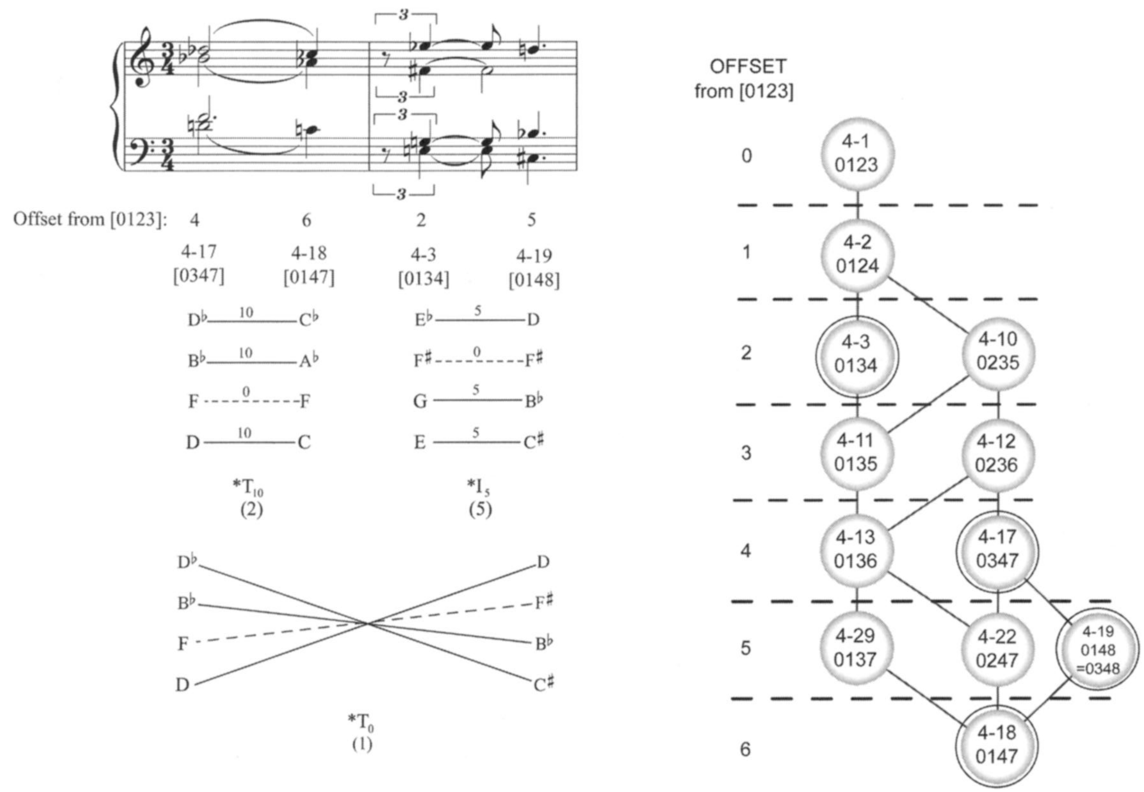
In set-class space, the progression involves a smooth move from 4–12 [0236] to 4–21 [0246].[37] Appropriately for a cadential situation, the progression also involves a slight harmonic relaxation, with the second chord one click less chromatic than the first.[38] As in the Webern progression of Example 1, the actual pitch disposition of the chords contradicts this underlying potential for diminished chromaticness: the second chord is far more compact than the first in pitch space. Nonetheless, the relative spaciousness of the second chord in pitch-class space is apparent enough in the lack of interval class 1 in the second chord. One might imagine that while an actual transposition would have preserved the chordal semitone, the fuzzy transposition has the effect of purging it—by deviating from the prevailing T_1, that one recalcitrant voice creates a harmony that is slightly less chromatic. This progression thus conforms to both laws—it is a smooth move toward a less chromatic harmony.

The succession of chords in Example 3 consists of two separate gestures, each supporting a melody that ascends ⟨+1, +4⟩.[39] The first melodic unit, E–F–A, is transposed up four semitones onto the second, G♯–A–C♯. The accompanying dyads, however, generally move downward, in contrary motion.[40] The chord that concludes the first gesture, E♭–B♭–A, is transposed down a semitone to become the bottom three notes of the four-note chord that ends the phrase, D–A–G♯–C♯. The C♯ atop the cadential chord, creating the first four-note chord in the passage, arises as part of the ascending melodic sequence. In voice-leading terms, however, it can be thought of as a slightly fuzzy $*T_{11}$ from the bass E♭ in the preceding chord. In this sense, the bass E♭ splits, simultaneously moving down a semitone onto D (in parallel motion with the tenor and alto voices) and down two (pitch-class) semitones onto C♯. Alternatively, one might imagine the C♯5 of the final chord as emerging from an implied E♭5 atop the previous chord. In either interpretation, the progression is $*T_{11}$ with an offset of 1.

In pitch-class space, the progression is maximally uniform, deviating from a straight transposition by only one semitone. In set-class space, the progression thus represents a smooth move from set-class 3–5 [016] to set-class 4–8 [0156], in conformity with the Law of Atonal Voice Leading. Because of the vagaries of intercardinality space, this single semitone displacement involves two semitones of greater offset from the maximally chromatic tetrachord, [0123]. The progression is thus a smooth (set-class)



Example 3. Schoenberg, Five Orchestral Pieces op. 16/1, mm. 1–3



Example 4. Ruggles, “Lilacs,” mm. 1–2

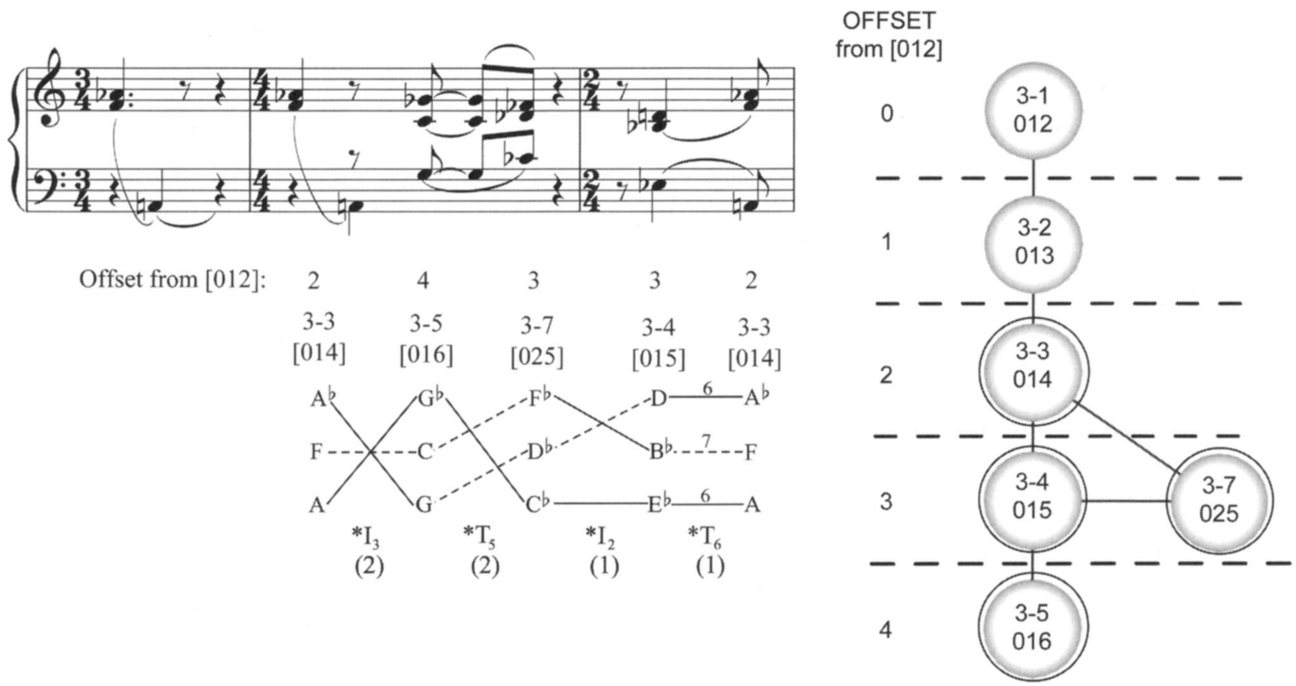
move in the direction of less chromaticness, a move therefore toward greater relaxation. That [0156] is interpreted as less chromatic than [016] may seem counterintuitive: the higher cardinality and additional semitone of [0156] would seem to speak for greater rather than lesser chromaticness. But the additional note in the final chord, the C♯, not only brings an additional semitone into the chord but also brings an additional interval class 4 (with A) and interval class 5 (with G♯). According to the theoretical model, the presence of these larger intervals more than compensates for the additional semitone.[41] Indeed, the final chord does give the impression of greater stability and is thus suitable for a cadential gesture, in conformity with the Law of Atonal Harmony.

Example 4 considers the opening harmonic progression in Ruggles's "Lilacs."[42] The progression consists of two pairs of chords separated by a rest. When the first chord moves to the second, three of the voices move down two semitones while the fourth voice holds. Transformationally, that is $*T_{10}$ with an offset of 2. In set-class space, the voice-leading distance is thus 2, and the motion takes the progression from a greater to a lesser degree of chromaticness.[43]

Something similar happens in the second pair of chords, only now the transformational relationship is via inversion, not transposition. Three of the voices move via I_5, while the remaining voice holds (i.e., maps onto itself at I_0) and thus deviates by five semitones from the prevailing inversion. That sense of the chords as related by inversion is felt most vividly in the bass and tenor, which wedge away from each other symmetrically. As with the first pair of chords, this progression involves a harmonic relaxation, moving sharply away from a relatively chromatic chord to a relatively unchromatic one.[44]

We thus have two parallel gestures, each taking us from relative harmonic tension to relative harmonic relaxation, in conformity with the Law of Atonal Harmony. The distance traversed in set-class space in the first gesture is relatively small (in near conformity with the Law of Atonal Voice Leading), and the second is relatively large. The two gestures together bring the music almost back to its starting point: the last chord has three tones in common with the first, and the remaining voice deviates by only semitone. In our transformational language, the first and last chords are related at $*T_0$ with an offset of 1. The first gesture takes us away from our starting point; the second virtually returns us to it. Upon our return, however, we find a chord whose pitches are almost evenly dispersed through a wide pitch space, thus amplifying the slight expansion in pitch-class space from [0347] to [0148].[45] The overall gesture, then, is a smooth move in the direction of chordal relaxation and is thus in conformity with both laws.

Example 5 examines the five trichords that compose the opening phrase of Sessions's Piano Sonata. In twelve-tone terms, this phrase involves a

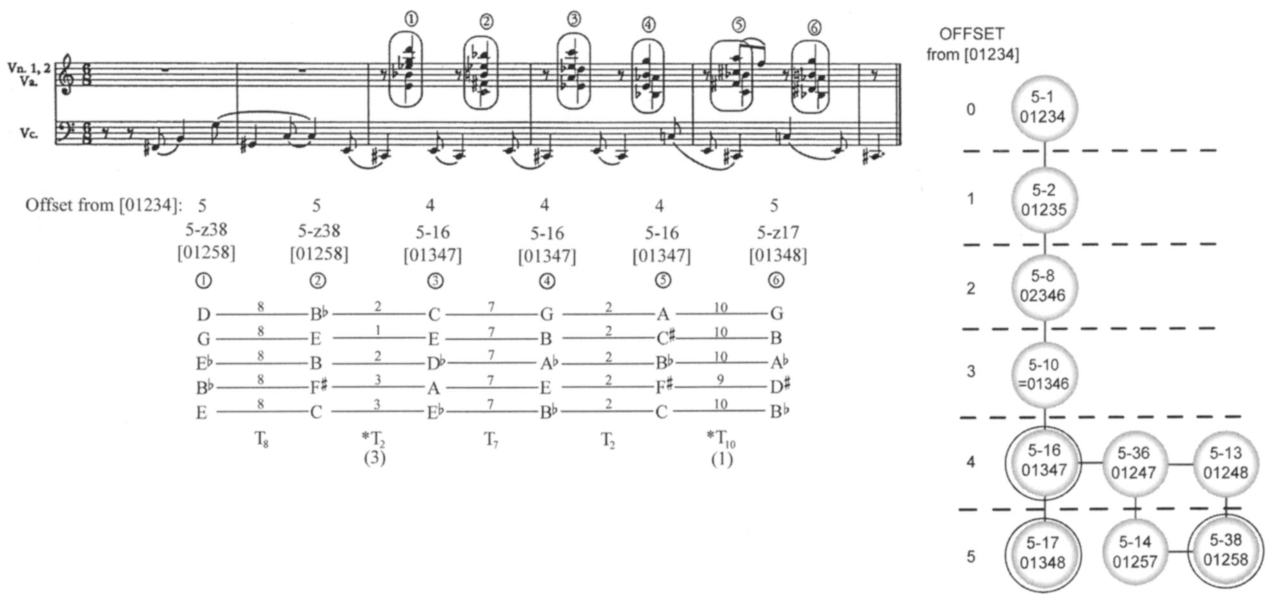


Example 5. Sessions, Piano Sonata, first movement, mm. 1–3

presentation of the four discrete trichords of a series, concluding with a return of its first trichord. The transformational voice leading is somewhat complex, but let us focus on the cadential gesture of the final two chords of the progression. The bass (E♭–A) and soprano (D–A♭) both move by tritone, although in opposite directions in pitch space. The alto nearly follows suit, but leaps one semitone too far (B♭–F, instead of the T_6-conforming B♭–E). The progression is thus $*T_6$ with an offset of one semitone, producing a smooth move in set-class space. Indeed, the five-chord progression as a whole is relatively uniform and balanced, with offset values of only 1 or 2. In set-class space, the cadential gesture involves a slight increase in chromaticness, and the progression as a whole describes an arc, starting from a position of relative chromaticness in the first chord, then a small leap in the direction of relative unchromaticness, followed by a gradual reattainment of the original level and, in fact, of the first chord itself. The progression conforms to the Law of Atonal Voice Leading, with its smooth moves in set-class space, but cuts directly against the Law of Atonal Harmony, beginning and ending in a state of heightened chromaticness.

Example 6 focuses on a progression of six pentachords, representing three pentachord-classes.[46] From Chord 1 to Chord 2 is a simple pitch transposition down four semitones (modeled as T_8 in the example). There is obviously considerable exertion involved, a total expended energy or displacement of twenty semitones, as each of the five voices moves four semitones. At the same time, because of the absolute uniformity of the movement of the voices, there is no offset at all and, in set-class space, no movement at all. The progression from Chord 2 to Chord 3 presents a different situation. In pitch space, less energy is actually expended (eleven semitones of total displacement), and in that immediate sense the progression is smoother. But compared to the progression from Chord 1 to Chord 2, there is a sense of disruption. Instead of all doing the same thing, the five voices move by three different intervals, and compared to a crisp transposition, there is now a deviation of three semitones.

And the disruption is easily heard and felt as such—the relative uniformity of the progression from Chord 1 to Chord 2 compared with the relative nonuniformity of the progression from Chord 2 to Chord 3. The offset of Chord 3 is equivalent to the distance traversed in set-class space. What that offset number signifies is not a greater degree of effort but rather a greater degree of disruption or deviation with respect to T or I. The progression from Chord 2 to Chord 3 not only involves a relatively high degree of disruption but also involves a slight intensification (measured in terms of offset from the maximally chromatic pentachord, 5–1 [01234], Chord 3 is slightly more chromatic than Chords 1 and 2). That slight intensification is reflected in the slight shrinkage of the outer boundary of Chord 3 compared to Chords 1 and 2: the chord is not only more com-



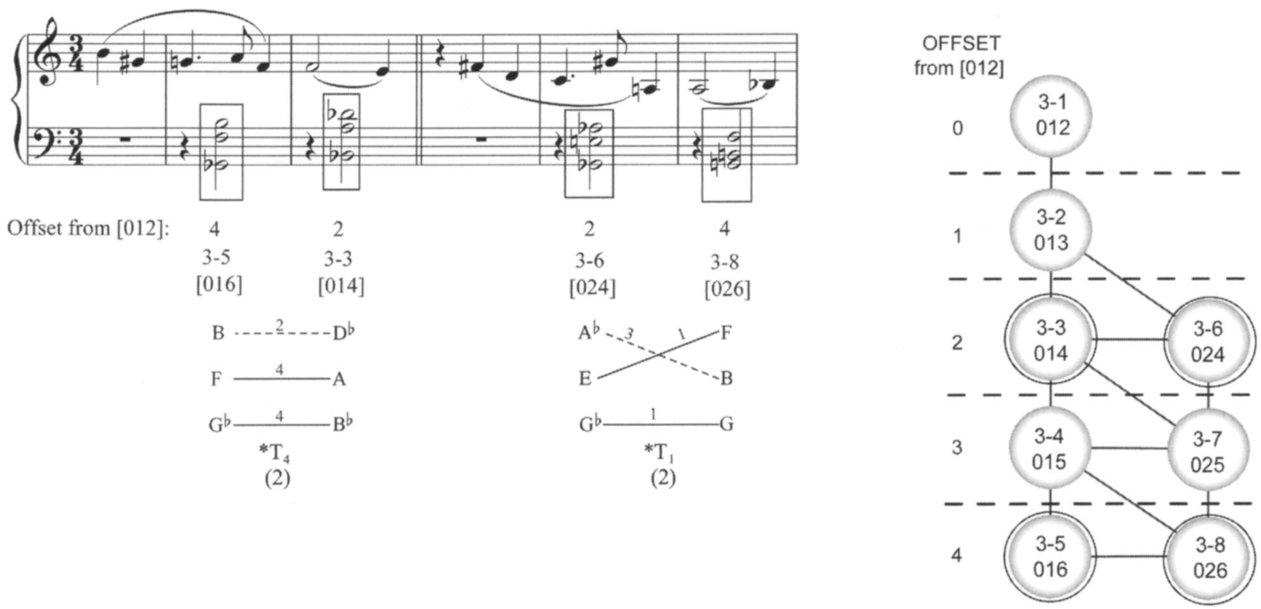
Example 6. Webern, Movements for String Quartet op. 5/5, mm. 1–6

pressed (more chromatic) in set-class space, but literally more compressed in pitch space.

From Chord 3 to Chord 4 and Chord 4 to Chord 5, we return to the normative crisp transposition of the opening. Again, it is not that the voices are not moving, but rather that the moves are undisrupted, normative T (realized as actual pitch transpositions). The progression from Chord 5 to Chord 6 is both a slight disruption of normative T (offset = 1) and a correspondingly slight relaxation back to the original level of chromaticness. And the spacing of the final chord is slightly more spacious, more evenly distributed in pitch space. Within abstract set-class space, the progression appears to involve a relatively significant disruption of voice-leading uniformity toward the beginning (measured as a leap of three within the space), a disruption that produces an increase of tension, and then at the end, a relatively slight disruption of voice-leading uniformity (measured as a step of one within the space), producing a slight relaxation of tension back to the tension level of the opening. In that sense, the progression conforms to both laws, consisting as it does of an initiating gesture that involves a greater voice-leading distance and an increase in tension and an answering gesture that involves a smoother move in set-class space and a corresponding relaxation in tension, bringing the phrase to an end. The working of these laws and the abstract harmonic and voice-leading potentials they embody are realized quite explicitly by the actual chords moving within pitch space.

Example 7 examines four accompanying chords in a familiar passage, the opening of Schoenberg's Piano Piece op. 11/1. The first pair of chords is connected by $*T_4$, realized in pitch space. The bass and the tenor ascend four semitones. The soprano ascends also but falls two semitones short of the goal marked out for it by the bass and tenor—T_4 would have brought it all the way to D♯, but it lands instead on D♭. In the second pair of chords, the bass and tenor again guide the progression. They each move up one semitone. The soprano again is the deviant voice. The prevailing T_1 would have sent it to A. Instead, it not only overshoots its goal, landing instead on B, but also transfers the B to another octave, crossing over the tenor in the process. If it was two semitones too low in the first progression (in pitch space), now in the second progression it is two semitones too high (in pitch-class space).

The errancy of the soprano creates a slight tensing of the harmony in the first progression and a corresponding relaxation in the second. The second chord is two degrees more chromatic than the first, and its greater compression in pitch-class space is confirmed by shrinkage of the registral span of the second chord: the interval between tenor and soprano is now 4 instead of 6. In the second progression, the fourth chord is two degrees less chromatic than the third, although, because of the voice crossing, the registral span of the chord actually shrinks. Nonetheless, the fourth chord



Example 7. Schoenberg, Piano Piece op. 11/1, mm. 1–3 and 9–11

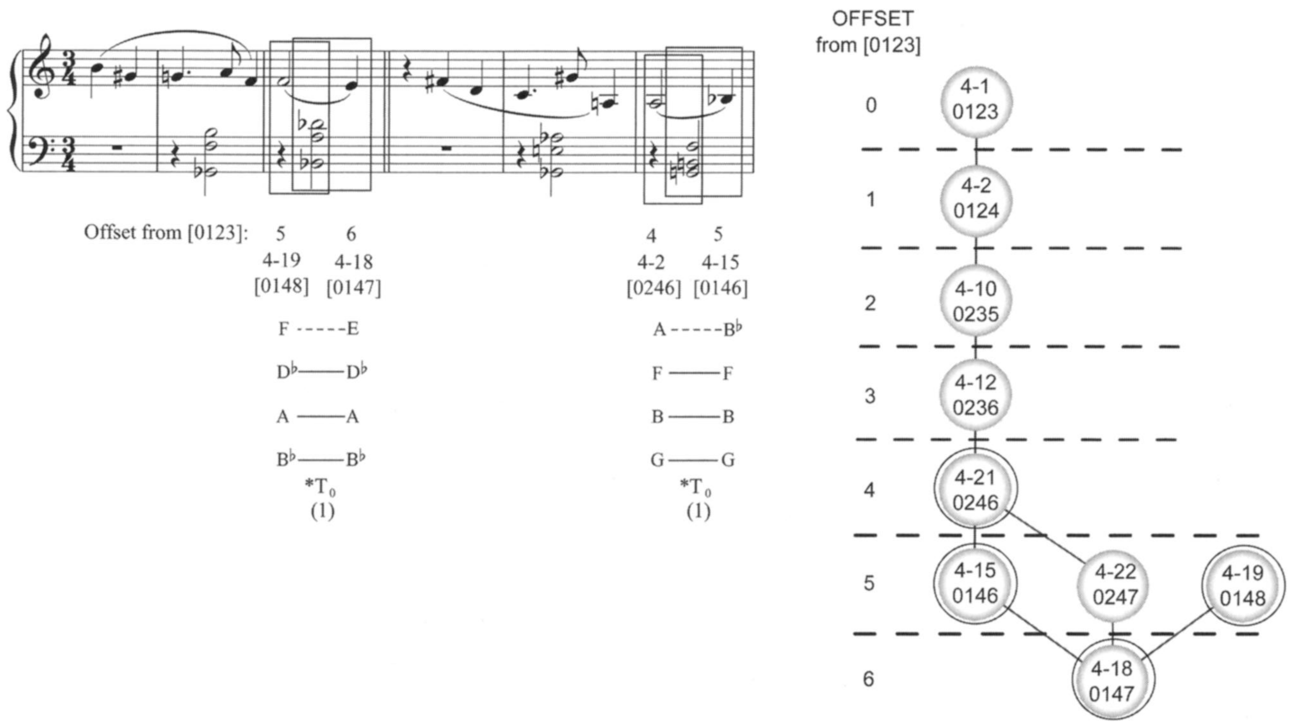
is certainly less "sharp" sounding than the third, to use Krenek's term and classification. The cumulative effect is that of a kind of antecedent and consequent relationship between the two progressions. In the first, the soprano's deviance produces a slight tensing. The second begins at the level of tension at which the first left off, but now the soprano's deviance produces an exactly corresponding relaxation, returning us to the level at which we began. The first progression strains against the Law of Atonal Harmony; the second moves in accordance with it and repairs the violation of the first.

We can gain additional perspective on the same passage by attending to the apparent suspension figures that conclude each of the phrases (see Example 8). The traditional suspension figure, of course, involves resolution from a relatively dissonant harmony to a relatively consonant one. According to the model presented here, that is exactly what Schoenberg has done, as well, although with a variety of different tetrachords. Both figures involve smooth voice leading, as three voices hold while a fourth, the soprano, moves by semitone. The offset is thus 1, realized in the most direct possible way. The voice leading is as smooth as possible, in pitch space, pitch-class space, and set-class space. The first figure creates a progression from 4–19 [0148] to 4–18 [0147], a parsimonious move toward less chromaticness, although somewhat ambiguously so.[47]

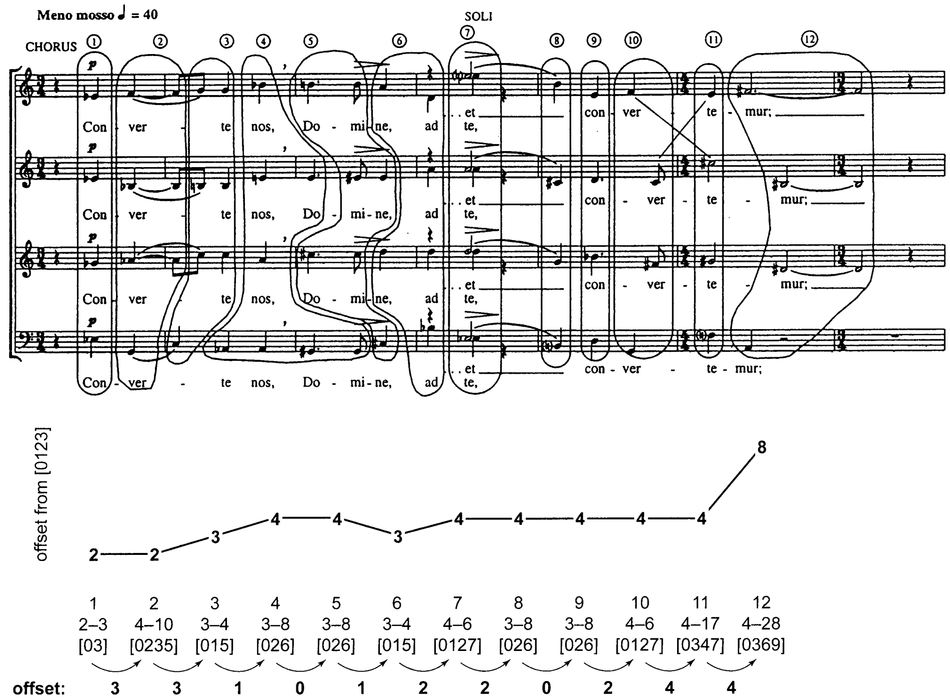
The second figure creates a progression from 4–21 [0246] to 4–15 [0146], again a parsimonious move toward less chromaticness, and in a reasonably straightforward way.[48] Schoenberg's apparent suspension figures thus have a harmonic as well as melodic basis, involving the resolution of a relatively tense harmony to a relatively relaxed one. In that sense, Schoenberg's suspensions observe the laws for both harmony and voice leading: they are smooth moves in the direction of lesser chromaticness.

Example 9 examines one of the twelve-tone chorales so common in Stravinsky's late music.[49] This analytical chart takes no account of the actual realization of these harmonies in pitch or pitch-class space. Rather, it attends just to the distances traversed in set-class space (indicated beneath the chart) and the degrees of harmonic tension involved.[50] The progression begins with relatively large moves in set-class space and then moves relatively smoothly from the third to the tenth chords, at which point it takes its two largest leaps. In terms of the quality of the harmonies, the progression shows a general tendency away from chromaticness (measured as semitonal offset from 4–1 [0123]). In that sense, the progression as a whole strongly corresponds to the Law of Atonal Harmony, beginning with relatively tense, chromatic chords and ending with the maximally unchromatic diminished seventh chord (which is also a maximally even set-class).

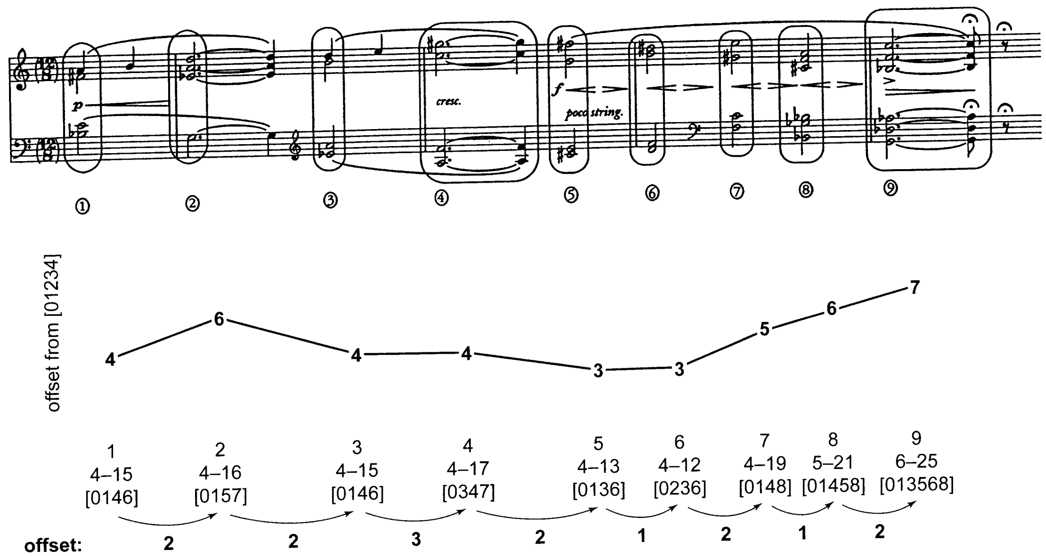
Example 10 provides a similar broad picture of a chorale passage from Schoenberg's Piano Piece op. 11/2.[51] Without attempting any direct



Example 8. Schoenberg, Piano Piece op. 11/ 1, mm. 1–3 and 9–11



Example 9. Stravinsky, *Threni*, mm. 405–17



Example 10. Schoenberg, Piano Piece op. 11/2, mm. 9–13

engagement with the actual spacing of the harmonies, Example 10 gives a general sense of the size of the motions in set-class space and of the quality of the harmonies as it shifts over the course of the passage. In set-class space, the voice leadings are all relatively smooth. In terms of harmonic quality, the passage begins with relatively unchromatic chords, becomes increasingly chromatic in the middle, and then reverses course, culminating in the final chord of the passage, which is also the least chromatic chord in the passage. Taken in this fairly abstract sense, the passage may be said strongly to engage the Law of Atonal Voice Leading, in its distinct preference for relatively smooth moves, and the Law of Atonal Harmony, in its move away from and back to a state of relative relaxation and repose.

* * *

In the first part of this article, I established a way of measuring the distance between set-classes within a parsimonious voice-leading space that contains them. This measure of distance led to a Law of Atonal Voice Leading that embodied a preference for relatively short (smooth) moves within the space. In the second part of this article, I established a way of characterizing the quality of harmonies, based on their relative proximity to a state of maximal chromaticness, which corresponds closely to traditional intuitions about harmonic tension (and relaxation). We were thus led to a Law of Atonal Harmony that embodied a preference for relatively unchromatic (spacious) harmonies.

Whether or not these laws are empirically predictive, whether or not composers are more likely to obey or disobey them, they may provide a useful basis for interpreting atonal harmony and voice leading. The laws may be understood to embody underlying tendencies, and working with or against those tendencies may have an impact on how harmonic progressions operate and how they may be construed.

Theories of tonal music have frequently had recourse to spatial metaphors, understanding pitches, triads, and keys as relatively near or far within a space, and characterizing motions within that space in terms of patterns of tension and relaxation.[52] In the work presented here, a similar framework is extended to the world of atonal harmony and voice leading in an effort to provide reliable measures of distance and direction for set-classes within an intuitively attractive set-class space.

NOTES

I thank a number of friends and colleagues for their helpful criticism and advice, including Clifton Callender, Jason Hooper, Jerry Ianni, Fred Lerdahl, Ian Quinn, Shaugn O'Donnell, and Dmitri Tymoczko. The musical examples were prepared by Mario Mazzoli. Tables 1 and 2 and Figure 19 were prepared by Thomas Robinson. Both Mazzoli and Robinson have been indispensable interlocutors during this project. I am also grateful for the comments I received when I presented earlier versions of this work at meetings of Music Theory Southeast (2005), the Music Theory Society of New York State (2005), and the Society for Music Theory (2005).

1. For discussion of transformations as fuzzy or crisp and related notions of "near-transposition" and "pseudo-transposition," see Straus 1997 and 2003, Lewin 1998, and Quinn 1996.
2. On semitonal offset as a measure of the distance between sets and set-class, see Lewin 1998 and Straus 2003.
3. *Uniformity* represents a fuzzification of transposition—the more uniform a voice leading, the more nearly it approximates actual, crisp transposition. See Straus 2003.
4. *Balance* represents a fuzzification of inversion—the more balanced a voice leading, the more nearly it approximates actual, crisp inversion. See Straus 2003.
5. For a related discussion and some analytical application, see Carter 1997.
6. On compositional spaces generally and voice-leading spaces in particular, see Morris 1995 and 1998. For geometrical modeling of voice-leading spaces based on smooth connections among sets and set-classes, see Callender 2004; Cohn 2003; Quinn 2006–7; Straus 2003; Tymoczko 2006 and forthcoming; and Callender, Quinn, and Tymoczko 2008.
7. In some of the single-cardinality maps and all of the intercardinality maps, some set-classes are found in more than one location. This redundancy is necessary to reveal all of the parsimonious connections among set-classes. It is difficult to generalize about which redundancies are required. See Callender, Quinn, and Tymoczko 2008 for further discussion.
8. Clough and Douthett 1991 is the classic study of evenness in the literature.
9. This is the way in which the trichords have been represented in previous publications—see Straus 2003 and Callender 2004.
10. The terms "split" and "fuse" come from Callender 1998. Callender says a note may split onto two, a semitone above and below (e.g., A splits onto G♯ and B♭), and that two notes a whole tone apart may fuse onto the note that lies between them (e.g., B♭ and G♯ fuse onto A). Straus 2003 and the present article adopt Callender's terms but change the definition slightly, permitting a pitch class to split onto itself and the pitch class a semitone above or below (e.g., A splits onto either A and B♭ or A and G♯) and permitting two notes a semitone apart to fuse onto one (e.g., A and B♭ fuse onto either A or B♭). Ariza 2007 includes charts that list the number of semitones of offset between every pair of set-classes, of both the same and different cardinality. The spatial model presented here is essentially a way of visualizing the information contained in those charts.
11. The inherent three-dimensionality of the tetrachordal space is discussed in Cohn 2003, which represents the tetrachord classes as a tetrahedron. The configuration

of the space into tiers within a stack is more consistent with the present approach and will permit an integration of the tetrachords with the set-classes of other sizes.

12. Cohn 2003 observes this special property of set-class 4–22 [0247], which he designates the "queen bee" of the tetrachords. In this same sense, 3–8 [026] is the queen bee of trichords. There is no comparable queen bee for dyads, pentachords, or hexachords. Nonetheless, the variety of different set-classes to which a given set-class has parsimonious connections remains a deep and interesting aspect of the voice-leading potential of any set-class, one of its characteristic, inherent properties. In general, set-classes that lie at or near the middle of these voice-leading spaces have richer potential for parsimonious connection to other set-classes; set-classes that lie near the edges of the space are relatively impoverished in this regard.
13. Only set-classes of odd cardinality are capable of parsimonious self-mapping. On the pentachordal map, 5–20 is represented by [01378] and [=02378]. The former is the Forte-style prime form and treated as such by me (without =). The Rahn-style prime form [01568] lies off the map.
14. To put it the other way around, to present a parsimonious voice-leading space in which each parsimonious connection is listed at least once, some set-classes will have to appear more than once. The general issue of set-class redundancy in voice-leading spaces for sets and set-classes is addressed in Callender, Quinn, and Tymoczko 2008.
15. The layout of the map produces some apparent self-mapping among tetrachords in the lowest tier—these are artifacts of the layout and should be ignored.
16. This statement is true for the single-cardinality maps; those for dyads, trichords, tetrachords, and pentachords may be taken to stand for maps of decachords, nonachords, octachords, and septachords. Unfortunately, because of the nature of splitting and fusing, the intercardinality maps may *not* be taken to stand for maps of larger complementary set-classes. For example, the parsimonious voice-leading space for trichords and tetrachords in Figure 8 is very similar to, but not identical with, the space for nonachords and octachords. That is because, for a given split that connects two set-classes, there may not be a corresponding fuse to connect their complements, and vice versa. For example, [248] can move parsimoniously to [2348] by splitting the 4 onto 34, and [2348] can move parsimoniously to [248] by fusing the 34 onto 4. But [0135679TE] (the complement of [248]) cannot move parsimoniously onto [015679TE] (the complement of [2348]), because the 3 has no semitone-related pitch class onto which to fuse. Similarly, [015679TE] cannot move parsimoniously onto [0135679TE] because there is no split available to produce the 3. In general, if a split has the effect of filling in a whole-tone, there will be no corresponding fuse for the complementary pitch class. Similarly, if a fuse has the effect of creating a whole-tone, there will be no corresponding split for the complementary pitch class. The practical impact of this on the present study is that, while the single-cardinality maps may be taken to stand for maps of complementary set-classes, the intercardinality maps stand only for themselves. This study presents no maps of cardinalities 6×7, 7×8, 8×9, or 9×10.
17. The first of these directions [for connecting chords] requires that in the voice leading, at first, *only that be done which is absolutely necessary for connecting the chords*. This means each voice will move only when it must; each voice will take the

smallest possible step or leap, and then, moreover, just that smallest step which will allow the other voices also to take small steps. Thus, the voices will follow (as I once heard Bruckner say) the "*law of the shortest way.*" (Schoenberg 1978, 39)

Vaisälä 2004 generalizes Schoenberg's law into what he calls a "proximity principle of voice leading, under which small melodic (=horizontal) intervals, usually semitones and whole-tones, function as voice-leading intervals (melodic connectives with no arpeggiating function) and larger ones function as arpeggiations" (13). Lewin 1998 develops a related concept of "maximally close voice leading," which depends on a concept of "total shift"—a measure of the total distance traveled by all of the voices added together: "[T]he total number of semitones traversed by the . . . voices, as they move from their notes in one chord to their notes in the next chord" (24). Lewin engages familiar metaphors of musical exertion in referring to "melodic husbandry" (30, 31) and the possibility of moving between sets "with as little overall strain as possible" (38).

18. A recent study, Rogers and Callender 2006, offers some preliminary, qualified corroboration for the views expressed here. This study presented subjects with pairs of trichords and asked them to rate the perceived distance on a ten-point scale. The authors conclude: "The sum of individual voice-leading motion was, indeed, shown to approximate listeners' overall perception of distance. At the same time, however, our results suggest that many factors (e.g., displacement size, tuning environment, direction of motion, and relationship of moving voices) interact with one another, contributing to our sense of musical distance in a more complex fashion than had been previously recognized" (1691).
19. The question of harmonic quality is discussed extensively in Quinn 2006–7. Quinn investigates what he calls "quality space," abbreviated as $\mathbf{F}(c, d)$, where c is the size of chromatic universe (normally 12) and d is the cardinality of the maximally even set that serves as the prototype for a particular qualitative genus. Quinn 2006–7 describes six harmonic characters or "genera" that any set might express in a greater or lesser degree, with each quality associated with one of the six maximally even sets up to complementation (part 1, 121). The quality I am describing as "chromaticness" corresponds to the first of Quinn's genera, that is, $\mathbf{F}(12, 1)$. The quality I am calling "evenness" has no direct analogue in Quinn's system.
20. For the pentachords, there are three rogues: 5–7 [01267], 5–20 [01568], and 5–22 [01478]. Of these, 5–22 is particularly roguish: it is maximally distant from 5–1, and thus maximally uncompact, but it is two steps shy of being maximally even. Among the hexachords, fully ten out of fifty are rogues: 6–6 [012567], 6–7 [012678], 6–17 [012478], 6–18 [012578], 6–20 [014589], 6–29 [023679], 6–30 [013679], 6–38 [012378], 6–43 [012568], and 6–50 [014679]. Of these, 6–7 and 6–30 are particularly roguish—they are maximally uncompact, but not maximally even. These ten rogues include three Z-related pairs: 6–6/38, 6–17/43, and 6–29/50. A general discussion of roguishness is beyond the scope of this article, but as a general matter, roguish set-classes are those that can be decomposed into both even or nearly even subsets and chromatic or nearly chromatic subsets (e.g., [0167] as both [01] + [67] and [06] + [17]).
21. In the interest of legibility, a small number of redundant trichords have been omitted from the more complete map of this space in Figure 8. As a result, the follow-

ing parsimonious connections between trichords and tetrachords have been omitted: 014–0124, 015–0125, 016–0126, 025–0235, 026–0236, 027–0237, 036–0236, 037–0237, and 037–0347. All of the remaining connections between trichords and tetrachords, and all of the intracardinality connections (trichords to trichords, tetrachords to tetrachords) are present.

22. The sources of these similarity measures are as follows: IcVSIM from Isaacson 1990, ASIM from Morris 1979–80, REL from Lewin 1979–80, ATMEMB from Rahn 1979–80, and PSATSIM and SATSIM from Buchler 1998.
23. Tables 1 and 2 are derived from Robinson 2006, which demonstrates that "proximity in semitonal offset space does accurately represent similarity [among set-classes] but only with respect to the maximally uneven or most chromatic set class in the cardinality" (2). The correlation between offset and similarity as measures of chromaticness is similarly high for set-classes of larger cardinality, and always higher for set-classes of the same size.
24. Generic prototypicality may be interpreted as maximal *imbalance* on the associated Fourier Balance—at least to the extent that *a generic prototype tips its associated Fourier balance more than any other chord of the same cardinality possibly can.* . . . Our metaphorical Fourier Balances tip when the force of a pitch class is applied to them. . . . On Fourier Balance 1, each pitch-class *n* tips the balance toward *n* o'clock. To model the force it exerts on the balance, then, we can use an arrow of unit length oriented to point toward *n* o'clock. We will name such arrows *a*(*n*), and refer to the unit of force or length that they represent as a **lewin**, abbreviated **Lw**. . . . The quantification of imbalance . . . is enabled by the technique of arrow addition. In particular, each Fourier Balance represents a particular way of associating pcs [pitch classes] with arrows, and the degree to which a chord is imbalanced on that Fourier Balance . . . is proportional to the length of the arrow resulting from the addition of the arrows associated with the constituent pcs of that chord. (Quinn 2006–7, part 3, 41–44)
25. The correlation is nearly perfect for set-classes of a single cardinality, and the correlation is similarly high for the larger set-classes. The correlation is less high for intercardinality comparisons, for example, trichords and tetrachords together. In general, the offset measure considers the smaller set-classes as slightly more chromatic than Quinn's **F**(12, 1).
26. The issue of "chordal tension" is also discussed in Persichetti 1961 (19–23), but the discussion there is largely derivative of Krenek and Hindemith.
27. Hindemith 1942 divides chords into two principal groups: those that contain a tritone and those that do not. Within each of these principal groups, he further divides chords into those that contain seconds or sevenths (and have a determinate root), those that do not (but still have a determinate root), and those that are indeterminate as to root. Hindemith is more concerned with the extent to which chords may be understood to have a determinate root than the extent to which they are relatively consonant or dissonant. So, for example, the chords in his fifth and sixth categories (as far distant as possible from the consonant triad in the first category) are (036), (027), and (048). While these are all symmetrical and thus relatively "indeterminate" with respect to a possible root, they are obviously not relatively tense or dissonant by any measure.
28. I am translating into modern terminology. Hanson generates this hexachord as a cycle of minor thirds (C–E♭–G♭–A) to which the beginning of another cycle of

minor thirds, beginning a perfect fifth higher (G–B♭), is adjoined. In fact, this hexachord type can be formed from the combination of any diminished-seventh chord with any disjunct minor third.

29. The studies Huron cites are Malmberg 1918, Kameoka and Kuriyagawa 1969, and Hutchinson and Knopoff 1978.
30. The Pearson correlations are as follows: for trichords, .831; for tetrachords, .570; for pentachords, .778; for hexachords, .579.
31. Within this article, the space in question is set-class space. The law may also have application in pitch class and pitch space.
32. It would be possible, of course, to model the voice leading between these chords in different ways. One alternative would be to construe it in the manner of O'Donnell 1998 as a "split transposition," with one dyad moving at T_1 and another at a slightly skewed T_5 or T_6. This interpretation would produce voices that coincide with the registral lines. Another alternative would be to invoke Klumpenhouwer networks, in which case the two chords would be related at hyper-T_3 (the details are left to the reader). This K-net interpretation would produce the same four voices as the transformational voice leading proposed here.
33. Lerdahl (2001, 344–46) proposes an alternative measure of atonal chord distance, one that calculates the sum of the interval classes and pitch classes not shared between two chords. According to Lerdahl's measure, the distance between the two chords of Example 1 is 4: the second chord introduces two new pitch classes compared to the first and is missing two interval classes contained in the first. The possible range of values for tetrachords is from 1 (two chords share the same interval content but differ by one pitch class, e.g., C♯–D–E–F and D–E–F–F♯) to 8 (two chords have entirely distinct pitch-class content and the second introduces four interval classes not contained in the first, e.g., C♯–E–G–B♭ and D–D♯–F♯–G♯). According to Lerdahl's measure, then, the two chords in Example 1 are not notably close. Inevitably, a measure of chordal distance that values the presence or absence of common pitch classes will not coordinate well with a measure, like the one proposed here, that operates in set-class space and thus ignores actual pitch-class content.
34. Huron's dyadic consonance measure also judges the first chord as slightly more consonant than the second: the consonance value of 4–15 [0146], –0.243, is slightly higher than that of 4–7 [0145], –0.250. According to Quinn 2006–7, [0146] exerts less chromatic force than [0145] (1.414 rather than 1.932 lewins). The measures of Huron and Quinn thus support the judgment made here, which I think would also conform to the intuitive assessment of most listeners. The standard similarity measures all consider [0145] and [0146] as having comparable degrees of similarity with respect to [0123], but they do not necessarily agree about which is more similar to [0123] (see Table 1).
35. As with Example 1, alternative interpretations involving split transformations or K-nets suggest themselves. One could imagine the progression as involving a dual transposition, with T_{11} in the lower parts and a fuzzy $*T_3$ or $*T_4$ in the upper. The chords could also be represented as negatively isographic K-nets, related by hyper-I_0 (again, the details are left to the interested reader).
36. A different transformational voice leading would be produced by $*I_9$, also with an offset of 1: C♯–G♯, E–E, E♭–F♯, and G–D. This interpretation would have the advantage of maintaining the integrity of the registral pairs, but the disadvantages

associated with the relative difficulty of apprehending the inversion as compared to the transposition of pitch class.

37. Lerdahl's chord distance measure would consider these chords quite far apart: on a scale of 1–8, they are at a distance of 5 (the second chord introduces three new pitch classes and excludes two interval classes contained in the first).
38. This judgment of the relative chromaticness of 4–12 [0236] and 4–21 [0246] is contradicted by Huron's dyadic consonance measure, which punishes [0246] for its three whole-tones and rewards [0236] for its two minor thirds. Huron's value for [0236] is –0.889 and for [0246] is a relatively dissonant –1.427. Quinn, however, rates [0246] as having somewhat less chromatic force than [0236]: 1.732 as opposed to 1.932 lewins, effectively punishing [0236] for its semitone. The traditional similarity measures also confirm the sense developed here that [0236] is closer to the maximally chromatic tetrachord, [0123], than is [0246]—see Table 1.
39. The resulting trichord constitutes a basic motive for Schoenberg's Five Pieces for Orchestra op. 16. Babbitt 1987 (157–58) explores the relationship between the melodic transposition of Example 3 and the structure of the "Farben chord" from the third movement.
40. The contrary motion suggests an interpretation via K-nets. See Stoecker 2002, which, however, omits consideration of the final, four-note chord. Interpreting the third and sixth chords of the progression as K-nets, and relating them via hyper-T_9, would entail the same voice leading (E♭–C♯, E♭–D, E♭–A, and A–G♯) as described here.
41. This judgment is confirmed by Huron's dyadic consonance measure, which rates [0156] (–0.443) as slightly more consonant than [016] (–0.641). Quinn considers [016] and [0156] as having equal chromatic force: 1.000 lewins. The similarity measures generally, but not universally, confirm the judgment made here—see Table 1.
42. See Slottow 2001 and Straus 2003 for related discussions.
43. Huron's dyadic consonance measure contradicts this judgment, assigning 4–17 [0347] a relatively high consonance value (1.772) compared to 4–18 [0147] (0.933). The contradiction is due primarily to the presence in [0147] of a tritone, which I rank as the least chromatic interval, but which Huron classifies as a relative dissonance. Quinn's judgment is more in accord with my own. He assigns greater chromatic force to [0347] (1.414 lewins) than to [0147] (1.000 lewins).
44. Huron concurs, ranking 4–3 [0134] among the most dissonant tetrachords (with a dyadic consonance value of –1.864) and 4–19 [0148] among the most consonant (1.564). Similarly, Quinn judges [0134] as having near-maximum chromatic force (2.732 lewins) while [0148] has correspondingly little chromatic force (1.000 lewins). The similarity measures all agree that [0134] is considerably more like [0123] than is [0148].
45. Huron's dyadic consonance measure considers [0347] slightly more consonant than [0148]. In contrast, Quinn describes [0347] as having slightly more chromatic force than [0148]. As is usually the case, my offset measure is better aligned with Quinn than with Huron.
46. For a related discussion of this passage, see Straus 2003 (323–24).
47. Huron's dyadic consonance measure contradicts this judgment: [0148] receives a value of 1.564, while [0147] is rated at 0.933. Huron's measure is responding to [0148]'s lack of a tritone and its inclusion of additional major thirds, compared to

[0147]. Quinn considers the two tetrachords as exerting the same degree of chromatic force: 1.000 lewins.

48. Huron considers [0146] (with a dyadic consonance value of –0.243) considerably more consonant than [0246] (with a dyadic consonance value of –1.427). The soprano's move from A to B♭ creates a harmonic semitone (between B♭ and B) but simultaneously creates a positively weighted minor third (G–B♭) and perfect fourth (F–B♭). These two relatively consonant intervals outweigh the relatively dissonant semitone. In Quinn's chromatic quality space, [0146] is similarly considered as exerting less chromatic force (1.414 lewins) than does [0246] (1.732 lewins). Set-class 4–21 [0246] tips Quinn's Fourier Balance strongly to the right; substituting B♭ for A (or 1 for 2 in the prime forms) diminishes the force of that tip. Within the semitonal offset space described here, it is easier to compress the pitch class together from a starting position of [0246]—they would converge on 3, in the middle. That is why [0246] is judged as closer to the maximally chromatic [0123]. The judgment of the relative character of [0146] and [0246], shared by Huron, Quinn, and my offset measure, is generally contradicted by the similarity relations, which consider [0146] more similar to [0123] than is [0246]—see Table 1.
49. For the twelve-tone derivation of this and similar passages in late Stravinsky, see Straus 1999.
50. See Hindemith 1942 for related charts of "harmonic fluctuation."
51. For a discussion of this passage in terms of Klumpenhouwer networks, see Lewin 1994. Note that in Example 10, the degree of chromaticness is measured in terms of offset from 5–1 [01234].
52. Lerdahl 2001 surveys and extends that tradition. Lerdahl's commitment to designing reliable measures of distance and tension/relaxation within his tonal pitch space has provided an important intellectual model for the work presented here.

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