The Perception of Rhythm in Non-Tonal Music: Rhythmic Contours in the Music of Edgard Varèse

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To this point in its relatively brief history, the systematic study of structure in non-tonal music has undergone tremendous changes.[1] Yet its emphasis throughout has been upon pitch and pitch-class structure over rhythmic structure, in spite of the striking rhythmic innovations that have occurred in Western music during this century. Early writings on rhythm in non-tonal music were primarily the work of composers, such as Olivier Messiaen, Milton Babbitt, and Karlheinz Stockhausen, whose contributions detailed the authors' compositional systems and aesthetics.[2] Only in the last decade have publications by the music-theoretical community begun to focus attention consistently upon rhythmic structure in non-tonal music: for example, in articles by Allen Forte, Christopher Hasty, Martha Hyde, and David Lewin, and in the recent books of Lewin, Jonathan Kramer, and Robert Morris.[3]

[1]Publications regarding non-tonal pitch and pitch-class structure now span more than half a century, but few sources survey developments in the field chronologically. The opening section of Janet Schmalfeldt's *Berg's "Wozzeck": Harmonic Language and Dramatic Design* (New Haven and London: Yale University Press, 1983), entitled "Pitch-Class Set Theory: Historical Perspective," gives one account of early developments in the field. It also provides a succinct overview of Allen Forte's theories, as well as definitions for technical terms commonly used in the literature. See also Elizabeth West Marvin, "A Generalized Theory of Musical Contour: Its Application to Melodic and Rhythmic Analysis of Non-Tonal Music and its Perceptual and Pedagogical Implications" (Ph.D. dissertation: University of Rochester, 1988), which also begins with a historical overview (pp. 2–31) and which supplements Schmalfeldt's account by including more current contributions to the field.

[2]Olivier Messiaen, *The Technique of My Musical Language* (1944), trans. John Satterfield (Paris: A. Leduc, 1956); Milton Babbitt, "Twelve-Tone Rhythmic Structure and the Electronic Medium," *Perspectives of New Music* 1/1 (1962), 49–79, reprinted in *Perspectives on Contemporary Music Theory*, ed. Benjamin Boretz and Edward T. Cone (New York: Norton, 1972), 148–179; Karlheinz Stockhausen, ". . . how time passes. . . .," trans. Cornelius Cardew, *Die Reihe* 3 (1959), 10–40; Stockhausen, "Structure and Experiential Time," trans. Leo Black, *Die Reihe* 2 (1959), 64–74. See also Charles Wuorinen, *Simple Composition* (New York: Longman, 1979), particularly Chapters 10 and 12, for a discussion of Babbitt's time-point system.

[3]Representative articles include Allen Forte, "Aspects of Rhythm in Webern's Atonal Music," *Music Theory Spectrum* 2 (1980), 90–109; Forte, "Foreground Rhythm in Early Twentieth-Century Music," *Music Analysis* 2 (1983), 239–268; Christopher Hasty, "Rhythm in Post-Tonal Music: Preliminary Questions of Duration and Motion," *Journal of Music Theory* 25 (1981), 183–216; Martha Hyde, "A Theory of Twelve-Tone Meter," *Music Theory Spectrum* 6 (1984), 14–51; David Lewin, "Some Investigations into Foreground Rhythmic and Metric Patterning," in *Music Theory: Special Topics*, ed. Richmond Browne (New York, Academics Press, 1981), 101–137. Other published investigations into the rhythmic structure of non-tonal music may be found in Kramer's very thorough "Studies of Time and Music: A Bibliography," *Music Theory Spectrum* 7 (1985), 72–106. Recent books

A fundamental distinction must be made in theories of musical time between those that posit an underlying system of equally spaced time points and those that do not.[4] These time points may take the form of a perceived beat, or may represent a small beat subdivision used as an analytical tool for measuring durations. Theories that assume equally spaced time points are more common by far, since the rhythm of the music that these theories model—virtually all Western tonal music and a great deal of non-tonal music as well—has some basic pulse as its foundation. Two of the publications cited above have dealt in some detail with nonbeat-based musical time: Kramer, in his examination of nonlinear time and perception of durational proportions; and Morris, in his model of sequential time, in which the "durations between s-time points are undefined," and thus are not assumed to be equally spaced.[5] This study proposes a theory for analysis of nonbeat-based rhythms, one which differs from most previous work in that it models relative, rather than absolute, measured durations. These patterns of relative durations, here termed *rhythmic contours*, are based upon perceptual strategies that listeners use in the absence of a beat framework.

dealing with this issue include Lewin, *Generalized Musical Intervals and Transformations* (New Haven and London: Yale University Press, 1987); Kramer, *The Time of Music, New Meanings, New Temporalities, New Listening Strategies* (New York: Schirmer Books, 1988); Robert Morris, *Composition with Pitch Classes: A Theory of Compositional Design* (New Haven and London: Yale University Press, 1987).

[4]It is assumed here that the presence of equally spaced time points is required for music to be metric. Meter will be understood, following Maury Yeston's definition (*The Stratification of Musical Rhythm* [New Haven: Yale University Press, 1976], 151–152), as a consonant relationship between two hierarchical levels of equally spaced pulses, requiring both a faster- and a slower-paced pulse. Rhythm will be understood here simply as a succession of durations that may or may not be metrical, and thus may or may not contain a perceived beat.

[5]Morris, 299.

OVERVIEW OF MUSIC-PSYCHOLOGICAL STUDIES

Almost without exception, psychologists agree that listeners familiar with Western tonal music perceive musical rhythms in relation to equally spaced, internally generated beats whenever possible.[6] Dirk-Jan Povel and Peter Essens, for example, describe their beat-based model by contrasting three hypothetical "perceptual clocks": first, an absolute clock, pulsing at a single fixed rate; second, a clock that pulses at a rate derived from the smallest time unit of a given rhythmic sequence; and third, a hierarchical beat-based clock.[7] The authors reject the first two of these clock hypotheses on the basis of their experimental results. The absolute clock, for example, is "unable to explain why a temporal pattern presented at a different tempo will be recognized as structurally identical." Further, "such a model would imply that all sequences having the same number of [temporal] intervals will be equally well perceived and reproduced regardless of the durations of the intervals."[8] Re-

[6]This is not to say that such a perceptual model is universal. Listeners native or acculturated to Arabic or Indian music hear rhythms as additive rather than divisive. Thus, one would expect that the pattern eighth-quarter-quarter would be as easily structured cognitively as the pattern eighth-eighth-quarter to these listeners, despite the fact that the former does not conform to a beat-based model. The psychological studies cited here are generally biased toward Western tradition by virtue of the musical backgrounds of the listeners who participate in these experiments. However, it is interesting to note that the duration-space classes to be posited below model certain aspects of Indian rhythmic practice, since some rhythmic *talas* considered to be variations of each other belong to the same d-space segment class (see note 30 and its accompanying discussion, below).

[7]Dirk-Jan Povel and Peter J. Essens, "The Perception of Temporal Patterns," *Music Perception* 2 (1985), 411–440 (see 413–414).

[8]Ibid, 413. For example, "the sequence 200 200 400 [msecs.] and 200 400 400 . . . should both be equally well reproduced. In fact, however, subjects reproduce the first sequence perfectly, but the second poorly." In musical notation, this would compare the sequence eighth–eighth–quarter with eighth–quarter–quarter.

garding the second type of clock, Essens and Povel have undertaken further experimentation to determine "whether subjects can use the smallest interval in a temporal pattern as a basic unit in representing other (longer) intervals in the same pattern."[9] According to this hypothesis, duration successions in which intervals relate as 3:1 or 4:1, exact multiples of the basic duration unit, should be reproduced more accurately than ratios of 2.5:1 or 3.5:1. Their results do not support a distinction between such patterns, however, and they conclude that the smallest interval is not used in specifying the time structure of such patterns. The implication of this work is that rhythmic theories based upon tallied multiples of a composition's smallest durational value do not model aural perception. That is not to say that such theories cannot reveal important aspects of a work's rhythmic structure,[10] particularly in compositions where serialized rhythm is directly linked to pitch structure and in certain non-Western musics.[11] Povel and Essens conclude, however, that listeners use a hierarchical clock, in which equally spaced pulses of medium duration are subdivided or concatenated by the listener to structure a duration succession as it is heard.

In a beat-based hierarchical system, each of the durational units may also be subdivided. It is unclear, however, exactly how the listener perceives this subdivision. Experimentation has shown that the cognitive structure for perception of rhythmic subdivisions is far from precise. Eric F. Clarke proposes a perceptual model with two components: equally spaced metrical markers on one level, and a system of untimed procedures organized around these markers at another, "specifying subdivisions in terms of equal and unequal time spans, the unequal subdivisions using a simple distinction between long and short."[12] This categorization of unequal subdivisions simply into longs and shorts is an imprecise measurement of relative duration, and explains the common misperception of the dotted eighth-sixteenth beat subdivision for a triplet's quarter-eighth subdivision, since

[9]Peter J. Essens and Dirk-Jan Povel, "Metrical and Nonmetrical Representations of Temporal Patterns," *Perception and Psychophysics* 37 (1985), 3.

[10]See, for example, Forte, "Aspects of Rhythm in Webern's Atonal Music." His proportional graph is designed in precisely this way: "The integer value 1 is assigned to the smallest durational value in the work (movement). The largest value is the least common multiple of all the other values. . . . The result is a depiction of a precise calibration of component durations, so that any temporal span or pattern can be compared with any other" (p. 91).

[11]In "African Rhythm: A Reassessment," (*Ethnomusicology* 24 [1980], 393–415), Robert Kauffman expands upon various current theories of rhythmic structure in African music. Among these is analytical use of the "density referent," a concept defined by Mantle Hood in *The Ethnomusicologist* (New York: McGraw-Hill, 1971), that refers to the fastest regularly recurring event. Kauffman notes that the density referent "can be used to study and understand temporal elements that would be rendered ambiguous by reference to more subjective concepts of beat. For example, a beat of MM60 can also be perceived as two beats at 120. Density referent, being faster than beat, is not subject to such ambiguities" (p. 396). While density referent is a useful analytical tool, Kauffman suspects that it is not commonly used by African performers or listeners to structure rhythmic patterns perceptually. After observing a teacher instructing African drumming students, Kauffman notes that "Ayitee did not ask his students to count out eight fast pulses. Instead he wanted them to respond to the *gestalt* of the two drum parts. This would seem to suggest that density referent is only one level of a larger metrical organization" (p. 396).

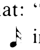
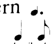
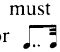
[12]Eric F. Clarke, "Structure and Expression in Rhythmic Performance," in *Musical Structure and Cognition*, ed. Peter Howell, Ian Cross, and Robert West (London and New York: Academic Press, 1984), 225–226. Clarke cites W. Jay Dowling's "Scale and Contour: Two Components of a Theory of Memory for Melodies," *Psychological Review* 85 (1978), 342–354.

both are heard as long-short.[13] In Clarke's "two-component system," the perceived pattern of relative longs and shorts is overlaid upon a metrical matrix that represents beat structure and subdivision. Clarke notes that this theory has certain similarities to W. Jay Dowling's two-component model of melodic contour overlaid upon a diatonic scale framework.[14] Dowling and Dane L. Harwood note the similarity between theories as well, citing Monahan's suggestion that

> rhythmic subdivision patterns are laid on the beat framework in a way analogous to the way melodic pitch contours are laid on the scale framework. . . . Rhythmic subdivisions can thus be said to be encoded in rhythmic contours of relative, not absolute, temporal relationships. Rhythmic contours are like melodic contours in being able to stretch to fit different frameworks (as with change of tempo) and in being able to slide along a given framework (as in displacement of rhythmic accent).[15]

Thus rhythmic contours may be understood as analogous to melodic contours: they represent relative durations in much the same way that melodic contours represent relative pitch height, without a precise calibration of the intervals spanned.

Povel hypothesizes that listeners have at least two possible ways of understanding temporal sequences, and that it is the nature of the rhythm itself that determines which method will be used: those that do not fit a beat-based coding are internally represented as rather unstructured groups of tones.[16] He notes that for nonbeat-based rhythms, an "alternative coding, called 'figural coding' by Bamberger (1978), capitalizes on the perceptual grouping of events . . . [and] detailed information about the relative durations of intervals would seem to be left uncoded."[17] This hypothesis has been substantiated more recently by Jeffrey Summers, Simon Hawkins, and Helen Mayers, who also describe two perceptual models of temporal organization: (1) Gestalt-like groupings; (2) beat-based hierarchies.[18] They note that the first is used to interpret non-metrical rhythms, while the second is used for metrical ones. It is this type of non-metrical rhythmic processing that the theory to be discussed here attempts to model—a musical conception without the "signposts" that beats provide, a conception marked by perceptual grouping according to temporal proximity, and retention of relative rather than absolute measured durations. Nonbeat-based rhythms such as these abound in non-Western musics and in Western music of this century—in some electronic compositions, in serialized rhythmic designs,

[13]This type of error has also been discussed with reference to African drumming performance. In discussing applications of the "density referent" concept, Kauffman notes that: "A performer of the pattern  must be aware of the density referent  in order to avoid the errors  or , but he will ultimately respond to the larger gestalt of each beat or of the entire measure. Thus it would also seem that African musicians respond to some type of metrical organization, which may include various combinations of the density referent" ("African Rhythm: A Reassessment," 396–397). The rhythms that Kauffman cites are all unequal beat divisions which may be categorized as long-short, and which are therefore easily confused.

[14]Clarke mentions this analogy in "Some Aspects of Rhythm and Expression," 324–325, and in "Structure and Expression in Rhythmic Performance," 226.

[15]W. Jay Dowling and Dane L. Harwood, *Music Cognition* (New York: Academic Press, 1986), 187–188. The authors cite, in particular, Chapter 5 of Monahan's dissertation, "Parallels between Pitch and Time: The Determinants of Musical Space" (Ph.D. dissertation, University of California, Los Angeles, 1984).

[16]Povel, "Internal Representation," 16.

[17]Povel and Essens, "The Perception of Temporal Patterns," 437. The article the authors cite is Jeanne Bamberger, "Intuitive and Formal Musical Knowing: Parables of Cognitive Dissonance," in *The Arts, Cognition, and Basic Skills*, ed. S. S. Madeja (New Brunswick, N.J.: Transaction Books, 1978).

[18]Jeffrey J. Summers, Simon R. Hawkins, and Helen Mayers, "Imitation and Production of Interval Ratios," *Perception & Psychophysics* 39 (1986), 437.

and in rhythmically dissonant passages of works by Carter and Stockhausen, for example.

RHYTHMIC CONTOURS IN DURATION SPACE

Two recent music-theoretical publications have discussed structural parallels between pitch spaces and temporal spaces; they form an important point of departure for the theory of rhythmic contours that follows. Morris's *Composition with Pitch Classes* defines three types of temporal spaces, the structures of which are isomorphic with three pitch spaces: "sequential time" with contour space, "measured time" with pitch space, and "modular time" with pitch-class space. Segments in Morris's sequential time and in contour space are represented numerically by integers from 0 to n–1, where n equals the cardinality of the segment. In both sequential time and contour space, the precise interval between the successive elements of a segment is not calibrated; the integers simply model the concepts of "earlier/later" or "lower/higher" without a precise measurement of how much earlier or how much higher. Morris's measured time is a temporal pitch-space analogy: both are spaces divided into equal measured units numbered with positive and negative integers on either side of some midpoint. Lower pitches or earlier time points are modeled by increasingly positive integers. Finally, modular time is the temporal analogue to pitch-class space, derived from measured time and from pitch space by reduction mod n; its elements are numbered from 0 to cardinality n–1. Lewin's formulation, in *Generalized Musical Intervals and Transformations*, contains six temporal spaces, two of which correspond to Morris's measured time and modular time. In addition, he defines four types of temporal spaces based not on sequential time points, but upon durations. Two of these duration spaces differ only in the way intervallic spans are measured: in the first case as quotients and in the second as differences. The remaining two temporal spaces are modular reductions of these two systems.[19]

An additional type of temporal space is proposed here: a duration space, analogous to contour space, that models relative duration in the same way that contour space models relative pitch height. However, unlike melodic contours in contour space, which can be recognized aurally with some accuracy regardless of context (tonal or non-tonal), rhythmic contours in duration space are very much altered in listeners' perception by their metric contexts. Thus the works to be studied here are non-metrical; they are, in fact, works where a consistent and uniform beat is hard to discern. It is in this type of context, where no consistent beat unit can be perceived, that rhythmic contours of relative shorts and longs best model the listener's perception.[20] At least one composer concurs with the premise that listeners' temporal understanding of nonbeat-based music is based upon perception of relative durations; Gérard Grisey states that

> without a reference pulse we are no longer talking of rhythm but of durations. Each duration is perceived quantitatively by its relationship to preceding and successive durations. This is the case in the rhythmic writing of Messiaen and of the serialist school. In fact,

[19]Morris's temporal spaces are discussed in *Composition with Pitch Classes*, 299–301; Lewin's are defined in *Generalized Musical Intervals*, 22–25.

[20]As Povel noted in the experiments discussed previously, a beat emerges in duration successions where successive equally-spaced shorter durations may be heard as subdivisions of a longer duration. By extension, in beat-based rhythms, strings of equally spaced durations are common (two eighths or four sixteenths, for example). However, in duration successions where no two durations are of equal length, a beat is difficult to hear unless supplied in an accompanying line. Musically-trained listeners may try to "impose" a beat to structure their listening, but if this strategy fails they too rely upon a perceived rhythmic contour of relative shorts and longs.

a micro-pulse allows the performer or conductor to count and execute these durations, but it only exists as a way of working and has no perceptual reality. The more complex the durations . . . the more our appreciation of them is only relative (longer or shorter than . . .).[21]

Duration space (or d-space) is defined here as a type of temporal space consisting of elements arranged from short to long. Elements in d-space are termed durations (durs) and, as mentioned previously, are numbered in order from short to long, beginning with 0 up to (n–1), where n equals the number of elements in the segment and where the precise, calibrated duration of each dur is ignored and left undefined.[22] A d-segment (dseg) is defined as an ordered set of durations in d-space. Just as a contour-space cseg can be realized in pitch space in an infinite number of ways, so can a dseg be realized in measured time by an infinite number of rhythms. Example 1 shows several realizations of the contour-space segment <0 1 2 3> realized in pitch space. Segments in contour space are comprised of c-pitches numbered in order from low to high, thus <0 1 2 3> represents a continuously rising melody, <3 2 1 0> a continuously descending line, and <2 3 0 1> a more angular melody with two changes of direction. Note that the theory is general enough to be applicable to Bach as well as Bartók and that the intervals spanned between contour pitches may vary; thus the stepwise motion of the Bach excerpt is equivalent in contour space to the arpeggiation of the Beethoven example. Example 2 shows <0 1 2 3> as a duration-space segment realized in measured time.[23] Examples (a) through (c) show different possible metrical interpretations, while examples (d) through (f) show some non-metrical realizations drawn from Varèse's *Octandre*.

Example 1. Multiple realizations of <0 1 2 3> in contour space

J. S. Bach, *The Musical Offering*, BWV 1079, X. Canon a 2. Quaerendo invenietis, mm. 1–2

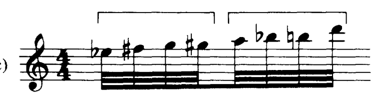
(a)

Beethoven, Piano Sonata in F Minor, Op. 2 No. 1, I, mm. 1–2

(b)

Bartók, *Mikrokosmos* No. 144, "Minor Seconds, Major Sevenths," m. 63

(c)



Stravinsky, *L'Histoire du soldat*, "Marche du soldat," mm. 57–58, trumpet in A

(d)

In numbering durs from short to long, the determination is made from the onset of one dur to the onset of the next, regardless of whether the pitch in question extends through the entire temporal interval spanned or is interrupted by a rest. Thus, dseg (b) of Example 2 still represents <0 1 2 3>

[21]Gérard Grisey, "*Tempus ex Machina:* A Composer's Reflections on Musical Time," *Contemporary Music Review* 2 (1987), 240.

[22]Formulation of these definitions and those that follow are indebted to those for contour space in Morris, cited above. Note that this application of the COM-matrix differs somewhat from Morris's temporal applications of contour theory.

[23]Note in Example 2 that the fourth dur of dseg (d) is ornamented by D♯–E grace notes. The grace notes will not be considered two separate (very short) durs, but rather ornaments (like a trill), belonging to the D♯ and lengthening its duration slightly.

Example 2. Multiple realizations of dseg <0 1 2 3> in measured time:

overlaid upon three possible "temporal grids"

(a)

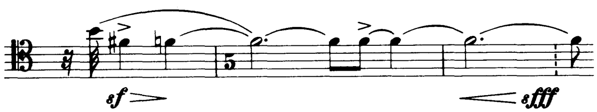
(b)

(c)

from Varèse, *Octandre*, I

(d) m. 2, oboe

(e) mm. 13–15, bassoon



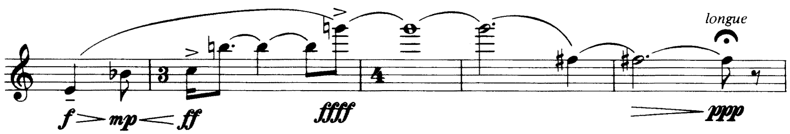
(f) mm. 17–18, oboe

even though it contains a rest. Music psychologists call this temporal span the *inter-onset interval*. As Eric Clarke notes, "This is the most significant measure as far as the rhythmic function of the note is concerned since the other possible measures (onset to offset or offset to onset) refer mainly to the articulation properties of the note."[24] This requirement is one of the features that distinguishes contour space from duration space, since c-space compares points in space while d-space compares pairs of points (onset to onset). This presents certain analytical difficulties in determining the duration of the final note of a succession, because there is no following onset by which to measure the length of that final duration. In the abstract and in the case of certain non-sustaining instruments, the theory might be formulated to exclude the duration of the final note, restricting its role so that it serves only to define the length of the penultimate duration. However, in musical contexts—particularly those involving performance by sustaining instruments, such as voice or wind instruments—the cut-off of the final note has some perceptual validity. It is for this reason that the examples following include the final note's duration. Finally, while a rest that is internal to a duration succession generally adds to the duration of the note preceding, it is also an important criterion for dividing the succession into d-subsegments.

A d-subsegment (dsubseg) is defined as any ordered subgrouping of a given dseg. Example 3 illustrates dsubsegs drawn from a prominent oboe melody in the first movement of *Octandre* (mm. 8–12). Dsubsegs may be compared more easily by renumbering the segments through "translation,"

[24]Eric F. Clarke, "Structure and Expression in Rhythmic Performance," 212.

Example 3. Dsegs and dsubsegs in duration space:
Varèse, *Octandre*, I, oboe, mm. 8–12

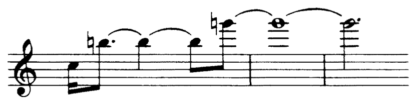


<210354>

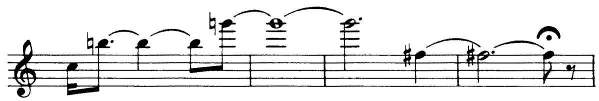
Dsubsegs:

(a) 

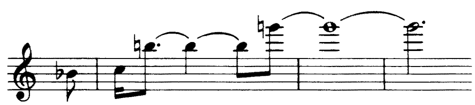
<210>

(b) 

<035> = <012>*

(c) 

<0354> = <0132>*

(d) 

<1035> = <1023>*

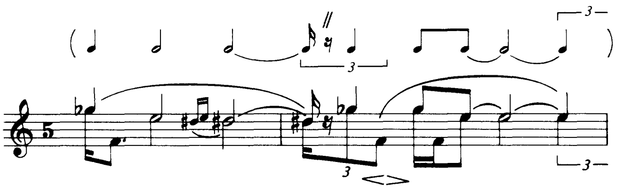
* = by translation

as Example 3 shows. Translation is an operation through which a dsubseg of n distinct durations, not numbered from 0 to (n–1), is renumbered from 0 for the shortest dur to (n–1) for the highest. Dsubsegs in duration space are assumed to be contiguous subgroupings, unlike csubsegs in contour space. In contour space, the listener may group noncontiguous high cps aurally by their close proximity in pitch height, for example, whereas the temporal nature of d-space prevents the listener from grouping all long durs together simply by virtue of their length. Only in the case where melodic contour interacts with perception of rhythmic contour might a case for noncontiguous dsubsegs be made. In compound melody, factors such as pitch proximity might cause the listener to perceive the higher or lower voice (or both) as an independent duration stream. Example 4 shows such an instance. Here, the reiterated low F might be heard as a separate stream, resulting in a noncontiguous dsubseg heard in the upper voice (indicated by stems up). Noncontiguous dsubsegs are clearly a special case; thus the term dsubseg will generally refer only to contiguous dsubsegs unless otherwise specified.

A precise profile of the structure of a duration succession in d-space is provided by Morris's comparison-matrix (or COM-matrix), shown in Example 5. This matrix is a two-dimensional array that displays the results of the comparison function, COM(a,b). In this case, a and b represent any two durs in d-space. If b is longer than a, the function returns "+"; if b is the same length as a, it returns "0"; if b is shorter than a, COM(a,b) returns "–." Each of these matrices has symmetrical properties in which the diagonal of zeros from the upper left-hand to lower right-hand corner forms an axis of symmetry. Each value in the upper right-hand triangle is mirrored on the other side of this diagonal by its inverse. This symmetrical structure is a natural consequence of the fact that the COM-matrix, as used here, compares a dseg with itself.

Two types of equivalence relations are posited for dsegs in duration space, based in part upon the COM-matrix. First, equivalent dsegs are those that generate identical matrices. This definition asserts equivalence for any two duration successions related as those in Example 5a. Measured in terms of the smallest durational unit (the sixteenth note), succession (1) may be represented as <3 1 4> and (2) as <6 2 8>. Succession (2) is an augmentation of (1) in measured time, a relationship that may be shown numerically by multiplying the durational values of (1) by 2. The two successions generate identical matrices, and in d-space are equivalent rep-

Example 4. Non-contiguous subsegs:
Varèse, *Octandre*, I, oboe, mm. 2–3



stems down: <0123> <11002>
stems up:* <012> <102>

*noncontiguous dsubsegs

resentatives of <1 0 2>. Dseg equivalence may also explain why rhythms such as those of Example 5b are often confused by students in early stages of aural skills training, since the two are equivalent in duration space. Finally, Example 5c illustrates an additional instance of dseg equivalence; here the durations of the two successions in measured time are not related by any precise mathematical relationship. Yet succession (4) is a free augmentation of (3); in d-space the two are equivalent representations of <0 1 3 2> and produce identical matrices as shown. Succession (4) is one of the dsubsegs from *Octandre* cited previously in Example 3, and is numbered as in that example to show the translation operation; succession (3) is comprised of the first four notes of the movement. The two successions have clear aural associations—both are prominent solo oboe lines, and the melody of succession (4) represents a rhythmic expansion, or development, of the melody with which the movement opened.

The second equivalence relation, the duration-space segment class (dsegclass), is defined as an equivalence class made up of all dsegs related by identity, translation, retrograde, inversion, and retrograde inversion. The inversion of a dseg S of n distinct durs is written IS, and may be found

Example 5. Dseg equivalence

(a)

(1)

<3 1 4> in sixteenth-note durations = <1 0 2> in d-space

(2)

<6 2 8> in sixteenth-note durations = <1 0 2> in d-space

matrices:

(1)

	3	1	4
3	0	-	+
1	+	0	+
4	-	-	0

(2)

	6	2	8
6	0	-	+
2	+	0	+
8	-	-	0

(b)

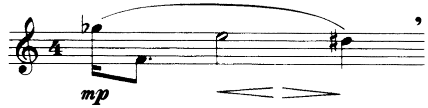
<1 0 2> in d-space

<1 0 2> in d-space

(c)

Varèse, *Octandre*, I, oboe, mm. 9–12

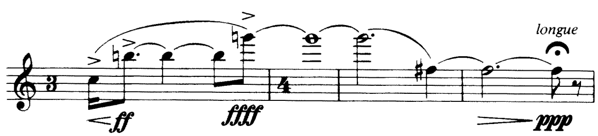
(3)



<0 1 3 2>

Varèse, *Octandre*, I, oboe, m. 1

(4)



<0 3 5 4> = <0 1 3 2>

matrices:

(3)

	0	1	3	2
0	0	+	+	+
1	-	0	+	+
3	-	-	0	-
2	-	-	+	0

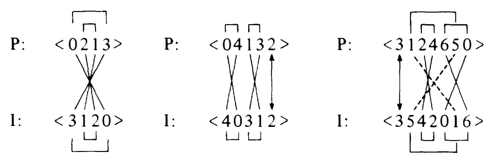
(4)

	0	3	5	4
0	0	+	+	+
3	-	0	+	+
5	-	-	0	-
4	-	-	+	0

by subtracting each dur from (n–1), where n represents the cardinality of the segment. In effect, this results in durations "swapping" positions within the segment. Given an odd value of (n–1), the longest and shortest durations swap positions, the next-to-longest and next-to-shortest swap positions, and so on. If (n–1) is even, the same holds true except that $\frac{n-1}{2}$ retains its position (see Fig. 1.). A comparison of the P and I forms in Example 6, for example, shows that 0 and 3 (or the sixteenth and half) swap positions, as do 1 and 2 (or dotted eighth and quarter). This algorithm for finding the inversion of a rhythmic segment is precisely the one used by Milton Babbitt in his *Three Compositions for Piano.*[25] The retrograde (RS) or retrograde inversion (RIS) may be found by listing the elements of S or IS in reverse order. Thus, in Example 3, dsegs (a) and (b) are R-related, while (c) and (d) are RI-related. Example 6 summarizes these relationships for one representative of dsegclass <0 1 3 2>, representing S and its R, I, and RI transformations and the

[25]Charles Burkhart describes this algorithm in some detail in his introduction to Babbitt's composition in *Anthology for Musical Analysis*, 3rd ed. (New York: Holt, Rinehart, and Winston, 1979), 578–584.

Figure 1.



corresponding COM-matrix for each. Because d-space is isomorphic with c-space, the prime form algorithm and csegclass labels developed for contours in c-space, included as an appendix to a previously published article, may be adopted for use in analyzing rhythmic contours.[26] These labels are hyphenated numbers, with the first number representing the cardinality of the segclass, and the second representing its ordinal position on the list. As an alternative to reprinting the segclass table, duration segments are represented here by their prime forms instead of their dsegclass labels.[27]

To return briefly to the data on temporal perception discussed at the outset, listeners discern a beat most easily in rhythmic patterns where successive equally spaced shorter durations may be heard as subdivisions of a longer duration. It is only in nonbeat-based rhythms—by extension, rhythms with fewer instances of repeated equal durations—that listeners perceive a rhythmic contour of relative shorts and longs without a precise notion of their proportional relationships. For this reason, the theory that models these

[26]Elizabeth W. Marvin and Paul A. Laprade, "Relating Musical Contours: Extensions of a Theory for Contour," *Journal of Music Theory* 31 (1987), 225–267 (see 257–262).

[27]A dseg's prime form is a representative form derived by the following algorithm: (1) if necessary, translate the segment so its content consists of integers from 0 to (n–1); (2) if (n–1) minus the last element is less than the first, invert the segment; (3) if the last element is less than the first, retrograde the segment.

Example 6. Dsegclass equivalence

P: <0132> I: <3201> RI: <1023> R: <2310>

	0	1	3	2
0	0	+	+	+
1	-	0	+	+
3	-	-	0	-
2	-	-	+	0

	3	2	0	1
3	0	-	-	-
2	+	0	-	-
0	+	+	0	+
1	+	+	-	0

	1	0	2	3
1	0	-	+	+
0	+	0	+	+
2	-	-	0	+
3	-	-	-	0

	2	3	1	0
2	0	+	-	-
3	-	0	-	-
1	+	+	0	-
0	+	+	+	0

rhythmic contours has not, up to this point, accounted for the instance of repeated equal durations. For the most part, however, the theory can be extended to include such rhythms. As shown in Example 7, a duration succession such as eighth-quarter-half-quarter would be modeled as a "repeated-note contour," <0 1 2 1> in duration space. Such a contour would generate a matrix containing zeros in positions other than along the main diagonal, as shown. Its dsegclass label is a composite, combining the labels of the two dsegs that are most similar to the repeated-note contour. These dsegs for <0 1 2 1> are illustrated in Example 7; the first is generated by replacing all zeros that appear in the upper right-hand triangle of the repeated-note contour with pluses, and the second with minuses.

In order to generalize contour theory from c-space to d-space, a number of important differences between the spaces must be acknowledged, some of which have been touched upon above. While these differences do not affect the general applicability of the theory to duration space, they do have important implications for the perception of various transformations that occur in the music to be studied below. First, a fundamental perceptual difference between contour space and duration space is that while melodic contours are easily remembered, and same/different comparisons accurately made by most listeners regardless of context (tonal or

Example 7. Repeated-note dseg <0 1 2 1> and related matrices

<0121>

$$\begin{array}{c|cccc} & 0 & 1 & 2 & 1 \\ \hline 0 & 0 & + & + & + \\ 1 & - & 0 & + & \boxed{0} \\ 2 & - & - & 0 & - \\ 1 & - & 0 & + & 0 \end{array} \qquad \begin{array}{c|cccc} & 0 & 1 & 3 & 2 \\ \hline 0 & 0 & + & + & + \\ 1 & - & 0 & + & \boxed{+} \\ 3 & - & - & 0 & - \\ 2 & - & - & + & 0 \end{array} \qquad \begin{array}{c|cccc} & 0 & 2 & 3 & 1 \\ \hline 0 & 0 & + & + & + \\ 2 & - & 0 & + & \boxed{-} \\ 3 & - & - & 0 & - \\ 1 & - & + & + & 0 \end{array}$$

non-tonal), rhythmic contours may not be recognized by listeners as identical if their underlying metrical structures differ. It is for this reason that compositions chosen for analysis and discussion here are works that do not strongly invoke a perceived beat or meter. A second consideration is that since the elements of d-space are durations and are therefore measured from the onset of one event to the onset of the next, each duration is dependent upon two points for its identity rather than one. Thus, while a point in contour space is immediately perceivable, a duration in d-space is not perceived until the second point of the pair defines its length. Third, segments in c-space or d-space may be divided into subsegments for comparison and analysis. Unlike c-space segments, non-contiguous d-segments are probably perceived by the listener only if their elements are associated by some other musical feature, such as extreme low or high register (as in Ex. 4), by a pattern of accentuation, or by some other mode of articulation. Thus, segmentation of non-contiguous d-segments should be considered analytically only if one of these conditions holds.

Finally, while the operations of inversion and retrograde inversion have a clear perceptual basis in pitch-space and contour-space, application of these operations to successions in duration-space may be more difficult to perceive. Music psychologists have not yet explored the question of whether listeners can perceive R, I, and RI transformations upon duration successions. Retrograde rhythms have been used for centuries in conjunction with pitch retrogrades in musical composition, but it is unclear how well listeners recognize the pitch transformation, much less the rhythmic retrograde. In the case of metric music, rhythmic retrogrades violate such expectations as long notes coinciding with "strong" beats or initiating measures; thus the new metric context of the retrograde succession makes this transformation difficult to recognize aurally. Yet musical experience and intuition suggest that rhythmic retrogrades may be perceivable if their length is not excessive. Certainly the palindromic rhythms of Webern's Variations, Op. 27 and Symphony, Op. 21 can be heard for a short while, if not for their full length.[28] The issue of rhythmic inversion is a more complex one. Few composers before this century attempted to "invert" duration successions, since there was no established procedure as to how inversion might operate in a temporal space. Yet in both contour space and duration space, the operations of identity, inversion, retrograde, and retrograde inversion can be shown to model certain transformations occurring in non-tonal compositions. In contour space, experimentation has shown that these transformations can indeed be perceived;[29] in duration space, however, some questions remain as to the perceptibility of the I and RI operations.

[28]Of course, the pitch palindrome in these cases assists the listener in perceiving the rhythmic palindrome.

[29]See W. J. Dowling and D. S. Fugitani, "Contour, Interval, and Pitch Recognition in Memory for Melodies," *Journal of the Acoustical Society of America* 49 (1971), 524–531; Dowling, "Mental Structures Through Which Music is Perceived," *Documentary Report of the Ann Arbor Symposium: National Symposium on the Application of Psychology to the Teaching and Learning of Music* (Reston, Va.: Music Educators' National Conference, 1981), 141–149.

Qualifications aside, the theory proposed here does indeed model aspects of rhythmic structure in non-metrical music, not only in the Western non-tonal repertory that is considered here, but in certain non-Western musics as well. For example, in their discussion of South Indian rhythmic *talas*, Kanthimathi Kumar and Jean Stackhouse describe the divisions of *tala* in Karnatic music.[30] They list the seven main *talas* in terms of their number and grouping of counts, then note that each of these seven *talas* has five forms. An adaptation of their table showing the five forms (*Jatis*) of the *tala dhruva* is given in Table 1.[31] Note that each of the five variations of *dhruva* has an equivalent rhythmic contour, < 1 0 1 1 >. Each of the other six main *talas' Jatis* shares the same dseg. Thus the seven *talas* are equivalence classes according to the definition of dseg proposed here.

Table 1. Five forms of the *tala dhruva*

	different jatis of dhruva	count distribution	total counts	dseg
1.	tisra	3 + 2 + 3 + 3	11	< 1011 >
2.	chaturasra	4 + 2 + 4 + 4	14	< 1011 >
3.	khanda	5 + 2 + 5 + 5	17	< 1011 >
4.	misra	7 + 2 + 7 + 7	23	< 1011 >
5.	sankeerna	9 + 2 + 9 + 9	29	< 1011 >

AN ANALYTICAL APPLICATION

Generalization of contour theory to the temporal domain enables analysts to address two aspects of musical structure—melodic contour relations and non-metric rhythmic structure—that are too often slighted in analyses of non-tonal compositions. The analysis of Varèse's *Density 21.5* that follows focuses upon these two types of analysis, noting recurring melodic and rhythmic contours that work in conjunction with pitch- and set-class structure to shape the work's formal design. Varèse's music provides an ideal context in which to illustrate analytical applications of the rhythmic theory proposed here since, as Jonathan Bernard notes, the composer's "penchant for rhythmic complexity seems to have been aimed at nearly complete and constant disruption of pulse, of any semblance of regularity in beat pattern. . . . It is difficult to find passages in Varèse where the beat, or even some simple subdivision or compound of it, is literally stressed for more than a couple of measures."[32] Bernard describes here precisely the type of nonbeat-based context that listeners are most likely to perceive in terms of a rhythmic contour.

Density 21.5 is the most often analyzed of Varèse's compositions.[33] Although the work's structure is as much founded upon recurring rhythmic contours as it is upon re-

[30]Kanthimathi Kumar and Jean Stackhouse, *Classical Music of South India: Karnatic Tradition in Western Notation* (Stuyvesant, N.Y.: Pendragon Press, 1988), 21–23.

[31]Ibid., 23.

[32]Jonathan W. Bernard, *The Music of Edgard Varèse* (New Haven and London: Yale University Press, 1987), 133.

[33]See, for example, Marc Wilkinson, "An Introduction to the Music of Edgar Varèse," *The Score and I.M.A. Magazine* 19 (1957), 5–18; Martin Gümbel, "Versuch an Varèse *Density 21.5*," *Zeitschrift für Musiktheorie* 1 (1970), 31–38; James Tenney with Larry Polansky, "Temporal Gestalt Perception in Music," *Journal of Music Theory* 24 (1980), 205–241; Jean-Jacques Nattiez, "Varèse's *Density 21.5*: A Study in Semiological Analysis," trans. Anna Barry, *Music Analysis* 1 (1982), 243–340; Carol K. Baron, "Varèse's Explication of Debussy's *Syrinx* in *Density 21.5* and an Analysis of Varèse's Composition: A Secret Model Revealed," *The Music Review* 43 (1982), 121–134. In addition, *Perspectives of New Music* 23 (1984) contains three articles in a Varèse Forum: James Siddons, "On the Nature of Melody in Varèse's *Density 21.5*," 298–316; Jeffrey Kresky, "A Path through Density," 318–333; and Marion Guck, "A Flow of Energy: *Density 21.5*," 334–347. Bernard's analysis concludes *The Music of Edgard Varèse*, 217–232.

curring pitches or set classes, no analysis published to date adequately addresses the issue of rhythmic structure. The work may be divided into three large sections, as illustrated in Example 8. Two of these sections are further subdivided; B contains two contrasting subsections marked B-I and B-II in the example, while A′ contains a return of material from both the A and B sections. The boundary between subsections in each case is marked by a recurring fanfare-like motive labeled "x" in the diagram.

A recurring rhythmic figure, dseg <0 0 1>, initiates most phrases of the A section, as shown in melodies (a) through (f) of Example 9. Segment (d) consists entirely of this duration succession. All of the remaining segments begin with either <0 0 3 2 1> or <0 0 2 3 1>. Although these two successions do not share many embedded subsegments, comparison of their matrices reveals a high degree of similarity; their content is identical in nine out of ten positions. Three of the six segments cited here—labeled (b), (c), and (f)—begin with the rhythmic contour <0 0 2 3 1>, yet their melodic contours and set-class structures differ. Melody (c) may be heard as a variation of (b), since it immediately follows (b) musically and since its rhythmic contour differs only with respect to the final duration. Further, the pitches of melody (c) are a literal subset of those in melody (b) (which, incidentally, are the same pitches as melody (a)—thus (a), (b), and (c) form a kind of "continuous variation"). The melodic contours of the first four notes of (b) and (c) differ a great deal, however: in terms of adjacencies, <– + + > in (b) as opposed to < + – –> in (c). Although melody (f) begins with the same rhythmic contour as segments (b) and (c), it contains no common pitch classes with either segment. Further, segment (f)'s melodic contour differs markedly from the others. This melody contains only three distinct pitches, forming the repeated-note contour <2 1 2 0 1 2 0 1> and set class 3–1 [0,1,2]. The remaining two segments, (a) and

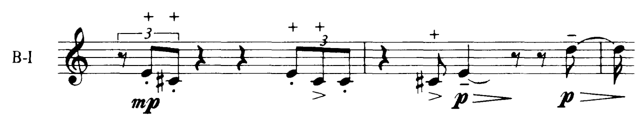
Example 8. Formal design of Varèse, *Density 21.5*

A(mm. 1–23)	B (mm. 24–40)			A′ (mm. 41–61)			
	B-I	"x"	B-II	A′	B-II′	"x"	B-I′
	24–28	29–32	32–40	41–45	46–51	51–53	53–61

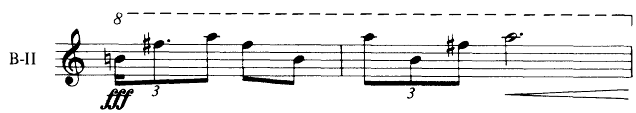
primary thematic material:

(mm. 1–3)

(mm. 24–26)



(mm. 32–33)



(mm. 29–30)

(e), are very similar rhythmically, since the rhythmic contour of the latter, <0 0 3 2 1>, can be embedded contiguously in the former. The first three notes of each forms the 3–1 trichord, as did segment (f). Neither the melodic contour

Example 9. Primary rhythmic contours in *Density 21.5*, A and A′ sections

A:

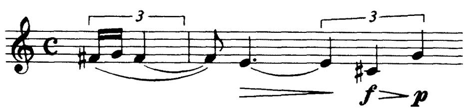
(a) mm. 1-2

dseg = <0042135> cseg = <2130304> sc 5-4
dsubsegs = <0021>,<00321> pcs = {1,4,5,6,7}
by translation

(b) mm. 3-4

dseg = <002312> cseg = <213404> sc 5-4
dsubsegs = <0012>,<00231> pcs = {1,4,5,6,7}

(c) mm. 4-5

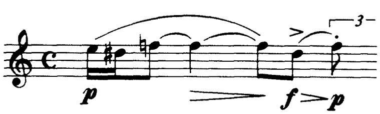


dseg = <002311> cseg = <232103> sc 4-13
dsubsegs = <0012>,<00231> pcs = {1,4,6,7}

(d) m. 9

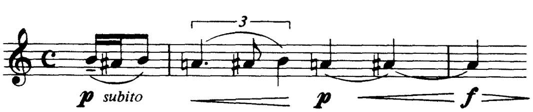
dseg = <001> cseg = <101> sc 2-1
pcs = {0,1}

(e) m. 15



dseg = <00321> cseg =<10202> sc 3-1
dsubseg = <0021> pcs = {3,4,5}

(f) mm. 21-22



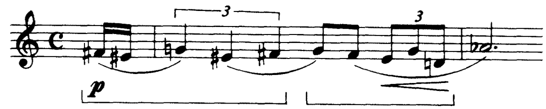
dseg = <00241345> cseg = <21201201> sc 3-1
dsubsegs = <0012>,<00231> pcs = {9,10,11}

A′: <0 0 1>, <0 0 2 1>, and <0 0 1 2> embedded as dsubsegs

(g) mm. 41-42

dseg = <001> cseg = <102> sc 3-1
pcs = {5,6,7}

(h) mm. 42-43



dseg = <00333112224> cseg = <21312321304> sc 5-4
dsubseg = <001> pcs = {2,5,6,7,8}
bracketed dsubsegs: both = <00111>

"x" (based on rhythmic contour of (a)):

(i) mm. 29-30

dseg = <0021> cseg = <3210> sc 3-1
pcs = {5,6,7}

(j) mm. 51-52

dseg = <021> cseg = <210> sc 3-5
pcs = {0,5,6}

(k) m. 52

dseg = <02111> cseg = <21021> sc 3-5
dsubseg = <021> pcs = {0,5,6}

nor the pitch-class content of (e) can be embedded literally in (a), however.

The three-note figure with which the A′ section begins, segment (g) of Example 9, marks a return to the duration succession, melodic contour, and 3–1 set class of the composition's opening, although the precise pitches differ by a semitone. The composer repeats and expands this motive in melody (h), which follows (g) immediately in the score. Segment (h) is a member of set class 5–4 [0,1,2,3,6], as were the opening two melodies of the work. The rhythmic contour of this final reference to the material of the A section differs most strongly from the contours of the rhythms that preceded it. This duration succession, <0 0 3 3 3 1 1 2 2 2 4>, contains four instances of repeated equal durations, and more closely resembles the repeated-duration successions that are featured in the B section than the rhythms of the A section.

Motive "x," dividing both the B and A′ sections, provides contrast to the material that surrounds it by virtue of its sudden change of register and dynamic, but the contour segments and duration successions used here are not new to the work. Segment (i) of Example 9 contains three short statements of "x"—the first and last forming dseg <0 0 2 1> (which was heard previously in segments (a) and (e)) and the central statement forming dseg <0 0 1>, the segment that has been heard repeatedly in the work as a kind of rhythmic "head motive" of every melody discussed thus far. In this case, however, the motive is initiated with equal-duration thirty-second notes rather than the sixteenths of most previous statements. The melodies of segments (i), (j), and the initial notes of (k) are the continuously descending melodic contours <2 1 0> and <3 2 1 0>; these contours recur regularly throughout the piece as do their inversions, to be discussed in Example 10 following.

This analysis of melodic and rhythmic contours in *Density 21.5* closes with a discussion of two compositional techniques used in this work, the analysis and discussion of which are greatly enhanced by the precise language of contour theory. The first of these involves the composer's development of melodic material by registral expansion—that is, by varying the pitch-space realization of a reiterated c-segment. The second involves his use of contour equivalence spanning both the pitch-registral and temporal domains. Example 10 shows three instances of pitch-space expansion within recurring equivalent contours, a technique that plays an important role in linking the A and A′ sections, as well as the B and B′ sections. The first example of contour expansion occurs within the A section in mm. 3–4 and mm. 13–14. Here, the cseg <1 0 2 3> is expanded from a total pitch compass of a minor third to one of a perfect twelfth. Likewise, the rhythm of mm. 13–14 represents a contour expansion of the first in sequential time; both are d-space statements of <0 0 1 2>. Second, mm. 9–10 of the A section contain a long melody that for two measures oscillates up and down a semitone, creating the cseg <1 0 1 0 1 0 1 0 1 0>. This contour recurs in the A′ section, mm. 46–48, this time as a minor-third oscillation. In the third instance, the melody beginning in the second measure of the B section (m. 22) forms cseg <0 1 3 0 3 0>. It returns in the B′ section expanded in register by a semitone, and with one additional cp.[34]

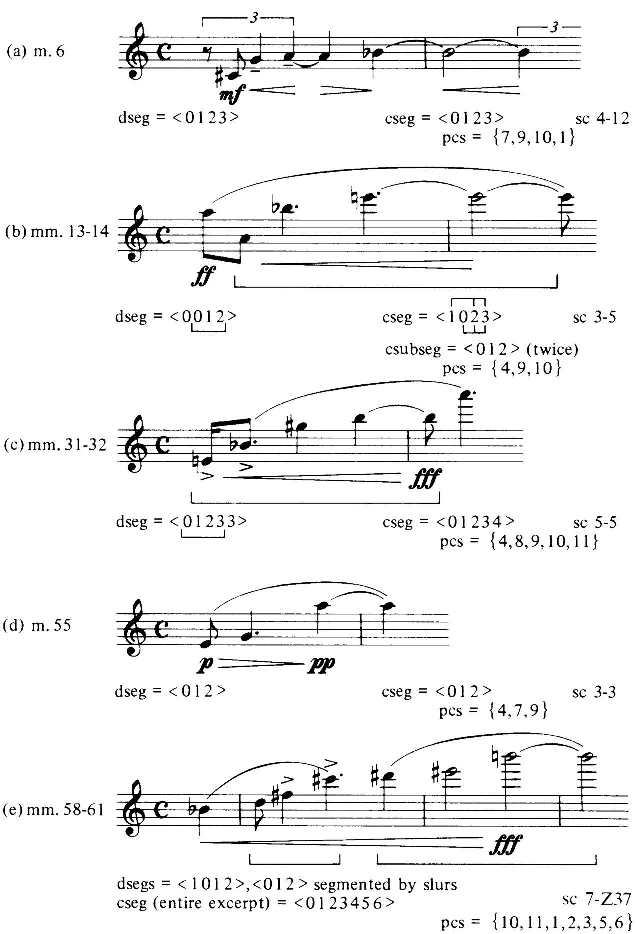
A striking feature of this work is the occurrence of several segments that are equivalent in contour space and duration space. Melody (a) of Example 11 is one such example. This melody is structured such that each successive pitch is both higher and longer than the one that preceded it; thus both facets of its structure can be represented by the succession <0 1 2 3>. Although melody (b) begins with a repeated duration, the rhythmic and melodic contours of the last three

[34]Bernard notes this relationship as well (p. 230).

Example 10. Contour expansion in *Density 21.5*

notes can be represented by the succession <0 1 2>. In addition, if the melody is aurally segmented by register, the non-contiguous dsubseg formed by the three highest pitches is also <0 1 2>. Melody (c) also contains one repeated duration (this time at the end of the melody) but, disregarding this repetition, its initial contour may be represented in both the registral and temporal domains as <0 1 2 3>. A case might be made for the function of the continuously ascending melody as a cadential gesture in this work, since melody (c) serves this function immediately before the B-II section begins, and the ascending melody of mm. 44–45 (not shown in Ex. 11) concludes the first subsection of the A′ section.

Example 11. A recurring melodic/rhythmic contour in *Density 21.5*



Further, the melody that concludes the entire composition, melody (e) of Example 11, is the longest continuously rising contour of the work, composed of contour pitches <0 1 2 3 4 5 6>. Although this duration succession does not represent continuously longer durations, the subsegments (as indicated by the composer's slurring) contain two embedded statements of dsegment <0 1 2>. Thus, the final melody of the work has been prepared aurally by similar gestures, with both its contour and rhythmic structure heard previously in other cadential contexts. Finally, segments (b), (c), and (e) also feature a continuous crescendo. Each pitch in the bracketed successions is higher, longer, and louder than the previous. Thus <0 1 2 3> is also represented in a "dynamic space," measured from soft to loud.

In summary, generalization of contour theory into other domains enables the analyst to compare diverse facets of musical structure along a single sequential scale. The analytical examples drawn from *Octandre* and *Density 21.5* have shown ways in which analysis of duration successions as rhythmic contours may clarify some aspects of one composer's musical language.

ABSTRACT

This paper develops a theory that models nonbeat-based rhythms as "rhythmic contours" of relative longs and shorts, drawing upon discussions of temporal spaces appearing in recent work of Robert Morris and David Lewin and upon various music-psychological investigations of rhythmic perception. A new type of temporal space is proposed: a duration space (d-space) analogous to Morris's contour space, in which elements are ordered sequentially from short to long. After developing equivalence relations for d-space segments, illustrated by excerpts from Edgard Varèse's *Octandre*, the paper concludes with an analysis of *Density 21.5* that focuses upon relationships among rhythmic contours.