R_0 relations with the other sets of cardinal 5, another indication of the distinctiveness of stratum 2 when compared with stratum 1 and 2.

Among the hexachords, again we find R_2 and R_p relations, here between 6-15 and the Z-correspondents 6-Z19/44, all belonging to stratum 1. In the table for sets of cardinal 4, pc set 8-14 is noteworthy for the large number of similarity relations, again of type R_2 and R_p. This set occurs both in stratum 1 and in stratum 3. Pc set 8-21, the long-range pattern discussed above, is maximally dissimilar to both 8-14 and 4-19.

159. Schoenberg, Five Pieces for Orchestra Op. 16/3
Set-complex relations

Stratum 1

	8-8	4-14	4-20									
5-5	K	Kh										
5-11	K	Kh	K									
5-13	K	K	K									
5-16	K		K									
5-Z17	K	Kh	K									
5-Z18	K	Kh	K									
5-20	Kh	Kh	Kh									
5-21	K	K	Kh									
5-24		K										
5-27		Kh	Kh									
5-29	K	Kh	K	5-11	5-13	5-16	5-Z17	5-Z18	5-20	5-21	5-27	5-29
6-15			Kh		Kh					Kh		
6-Z19	Kh	K	Kh			K	K	K	K	Kh		
6-Z24		Kh	K	K*			K*				K*	K*

Stratum 2

	5-24	5-35
6-33	Kh	Kh

Stratum 3

	8-14	4-19
7-3	K	K
5-Z37	K	Kh

Before giving attention to some further aspects of set-complex relations, it should be pointed out that the set-complex structure of the movement is not connected, reflecting the disassociation of strata 1 and 2. On the other hand, as shown in example 159, the set-complex structure for each stratum taken separately is connected.

For stratum 1 the main nexus sets may be regarded as 6-Z19/44, while 4-14 and 4-20 provide connections to the excluded sets. All the nexus sets are prominent in various configurations. Pc set 6-33, the nexus set of stratum 2, is not especially prominent in the music, but is strongly represented by its components, 5-35/7-35 and 5-24/7-24. For stratum 3, surprisingly enough, the nexus set is 5-Z37, the Z-correspondent of the chord of stratum 1.

The strata are, however, linked in very specific ways. Let us consider, first, the connections between strata 1 and 3. The derivation of 4-19 (in stratum 3)

from 5-Z17 of stratum 1 has already been suggested. This derivation is made explicit in the music during the last statement of stratum 3 in measure 260, in the following way: if 4-19 in stratum 3, with fixed pcs [7,8,11,3] is derived from 5-Z17 as it is stated at the outset, then 4-19 in stratum 3 is a transposition of one of the representatives of 4-19 in 5-Z17, with t = 11. And this is exactly the transposition of 5-Z17 that occurs in measure 260 together with the truncated statement of stratum 3. This is shown by dotted lines in example 152.

Of the two 3-note motives in stratum 3, 3-1 has already been discussed with respect to the canonic motive of stratum 1. The corresponding pitches, E, F, F-sharp, occur in stratum 1 only in measure 249, in the voice immediately below the upper voice. The other 3-note motive, 3-4, is not as explicitly related to stratum 1. At each occurrence, however, the two forms of 3-4 in stratum 3 occur while 6-Z19 is being formed by the union of 5-Z17 and 5-16 in stratum 1, and 6-Z19 contains 3-4 four times. Also, it will be recalled that the total content of stratum 1 is pc set 9-4. Finally, note that two of the nexus sets of stratum 1, 4-14 and 4-20, are represented by their complements in stratum 3. In the case of 4-20, the connection is far more specific: the composite segment 8-20 beginning at measure 232 in stratum 1 is duplicated in the composite vertical formed in measure 240 between the two strata.

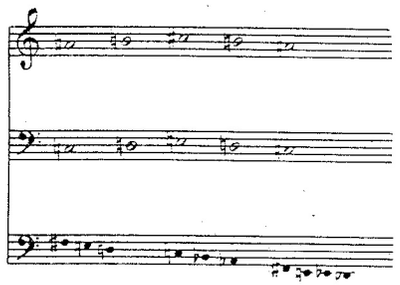
Since the links between strata 2 and 3 have already been discussed in connection with the formation of composite segments, it only remains to examine the links between strata 1 and 2 in somewhat more detail (see ex. 154). First, and perhaps obviously, the vertical statements of 3-9 which make up the initial appearance of 5-35 (measure 229) derive from 3-9 in 5-Z17, specifically from the three upper notes of the chord, by inversion, with t = 11. If we let 6-Z19 represent stratum 1 and 6-33 represent stratum 2, it is evident from the table, example 156, that the only set they share of cardinal greater than 3 is 4-14. There is no convincing link established by means of that set, from which we conclude, as indeed has been mentioned several times above, that the two strata are essentially detached from one another.

Strata 1, 2, and 3 are combined only in measure 251, at the beginning of the final section of the music. Composite segments resulting from this combination are shown in example 152. Again, as in the combination of strata 1 and 3, pc set 8-20 is formed, but now through the agency of the figure D-E, which belongs to stratum 2. Moreover, the figure D-E combines with 5-Z17 in stratum 1 to form 6-Z24, the set that closes the combined strata 1 and 2, as remarked above, and combines with 4-19 of stratum 3 to form 6-Z44, the complement of 6-Z19.

Finally, within pc set 8-20 we find 7-21, the complement of which occurs as a chord in stratum 1 (in measure 249) and, perhaps even more importantly, as the sonority that occupies the last three beats in measure 227.* This single measure (251) therefore synthesizes much of the music represented *in extenso* in the individual strata, and is an appropriate signal for the approaching close.

We come now to the large-scale linear progressions mentioned above. As a result of the successive transpositions of 5-Z17 (ex. 153), the controlling chord of stratum 1, the outer voice pattern formed by 5-Z17 over the span of the entire work consists of a motion traversing ic4. This is shown in example 153 by stemmed notes. A corresponding long pattern is formed by the lowest part of stratum 2, and this is shown in example 154. This pattern is the linear projection of 8-21, the 8-note complement of the "whole-tone" tetrachord. Thus, both the outer voices of stratum 1 and the lower voice of stratum 2 feature ic2. An abstraction of this combination is shown in example 157.†

157.



*Pc set 5-21 is the first theme of the first piece. See example 10.

†Long patterns of this kind are to be found elsewhere in the literature, of course. For example, the bass line of Berg's Op. 6/2 (*Reigen*) consists of an ascending chromatic pattern.

Perhaps the most extraordinary of the linear patterns, however, occurs in measures 248-250 with the descending line that begins on G (shown on the next to lowest stave in example 152). The rhythmic grouping of this line is such that the accented notes at the head of each group form 7-Z38. This, in turn, decomposes into two interlocking hexachords, 6-Z24 and 6-Z19, as indicated in example 152, and contains, as well, 5-Z17 as a contiguous subsegment. Thus, in this rapidly shifting texture, the line serves to unify stratum 1 and stratum 2.

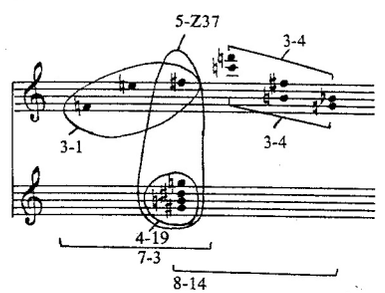
The set complex table, example 156, reflects the associations as well as disassociations of the three strata, many of which have been discussed above.

158. Schoenberg, Five Pieces for Orchestra Op. 16/3
Similarity relations

8-8	8-8												
8-14		8-14											
4-19			4-19										
4-20		R_2	R_2										
7-3	7-3												
5-5		5-5											
5-11	R_2		5-11										
5-13				5-13									
5-16					5-16								
5-Z17			R_2			5-Z17							
5-Z18			R_2		R_2	R_2	5-Z18						
5-20					R_1		R_2	5-20					
5-21			R_2			R_2			5-21				
5-24										5-24			
5-27	R_1		R_2						R_2		5-27		
5-29												5-29	
5-35				R_0	R_0			R_0					5-35
5-Z37			R_2				R_2						5-Z37
7-Z38			R_2		R_2	R_1							R_2
6-15	6-15												
6-Z19	R_2		6-Z19										
6-Z24				6-Z24									
6-33					6-33								
6-Z44	R_2												

A further dimension is provided by the similarity relations displayed in the tables of example 158. Of the 5-note chords in stratum 1, 5-5, 5-13, and 5-24 are bereft of similarity relations, while each of the others has at least one such relation. Notice especially the multiple relations of 5-11, all of type R_2 and R_p. Another special aspect of the similarity relations among the 5-element sets is the number of transitive triples of types R_2 and R_p. Of the five possible triples, three are represented here. Pc set 5-35 (stratum 2) has only

155.



The G (see ex. 151) is not part of the motive, but doubles the G at the top of 4-19 (reinforcing the composite segment 6-Z19 formed with 5-Z17 of stratum 1) and thereafter is retained with the F-sharp as a pc reference to both 4-19 and 6-Z19.

Pc set 5-Z37, the Z-correspondent of 5-Z17, plays an important part in the second movement of Op. 16, where it is stated initially as the consequent phrase of the opening theme. It is interesting to note that the antecedent phrase of that theme is 5-3, and that in stratum 3 of the present movement we find 5-Z37 embedded in 7-3.

Some indication of the distinctive structure of each of the three strata has been provided by the foregoing discussion. Let us consider now the composite segments formed between strata. Between strata 1 and 2 these segments do not seem to be significant, in the sense that they do not exhibit the structuring so evident in the strata taken separately. This is especially so in the case of measures 248-250, where the overall texture is so extremely dense. Nevertheless, some of the sets formed by these segments have been included in the set-complex analysis and are shown in example 152. In particular, notice 6-15, the hexachord formed at the onset of stratum 2 in measure 227, and 6-Z24, which closes two of the passages that combine the strata (at measure 250 and measure 260).

Between stratum 1 and stratum 3 there are three composite segments of interest. First, the large segment which forms pc set 8-20 in measure 240 and again in measure 244, and within that, pc set 6-Z19. Both these sets provide links to the opening music of stratum 1, as indicated above. The largest composite segment arising from the two strata is 10-5, and it is interesting to note that this set is the large vertical that ends the first section of the piece. Thus, even in this combination, the effect of stratum 2, with its emphasis on ic5, is made evident.

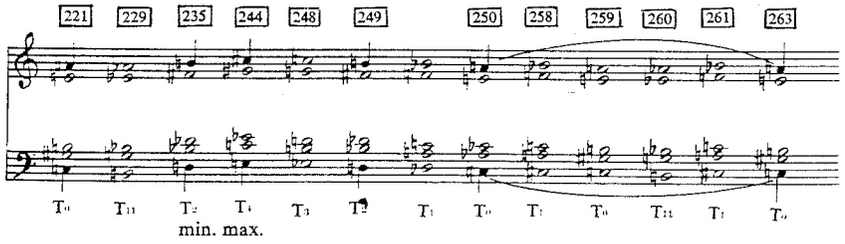
156. Schoenberg, Five Pieces for Orchestra Op. 16/3
Set-complex relations

	8-8	4-14	4-19	4-20	5-13	5-16	5-Z17	5-Z18	5-20	5-21	5-24	5-27	5-29	5-35	5-Z37	7-Z38
7-3		K	K	K												
5-5	K	Kh														
5-11	K	Kh	K	K												
5-13	K	K	Kh	K												
5-16	K		Kh	K												
5-Z17	K	Kh	Kh	K												
5-Z18	K	Kh	K	K												
5-20	Kh	Kh	K	Kh												
5-21	K	K	Kh	Kh												
5-24		K	K													
5-27		Kh	K	Kh												
5-29	K	Kh		K												
5-35	K	K		K												
5-Z37	K	K	Kh	K	5-11	5-13	5-16	5-Z17	5-Z18	5-20	5-21	5-24	5-27	5-29	5-35	5-Z37	7-Z38
7-Z38	K	K	K	Kh	7-3	Kh	Kh	K*	K	K	Kh						Kh
6-15			Kh	Kh	Kh		K	K*	K	Kh			K*	K*		K	K
6-Z19	Kh	K	Kh	Kh				K*			Kh				Kh		K*
6-Z24		Kh	K*	K		K*											
6-33		Kh															

measure 248. In the final section of the piece, A^2, the canonic motive undergoes contour inversion and in measures 235-244 the motive is expanded by repetition of the first two notes, for example, B-C-B-C-B-flat. In the same section the lower voice does not complete the motive, as is always the case in one voice of a traditional canon not of the "infinite" type.

This canonic structure sheds considerable light on the chord succession of stratum 1. That is to say, if all the voices moved in the canonic pattern at the same time, beginning from the initial chord as given, the result would be a succession of verticals composed of the same set, pc set 5-Z17, as in the last measures, 259-263. Thus, with the exception of 5-Z17, the chords here result from the temporal displacement of the canonic motive in the various constituent voices and are not the determinants of motion in the most fundamental sense.* Pc set 5-Z17 therefore plays a basic role in the movement as a whole, and example 153 shows the set as it occurs throughout.

153. Schoenberg, Five Pieces for Orchestra Op. 16/3
Stratum 1



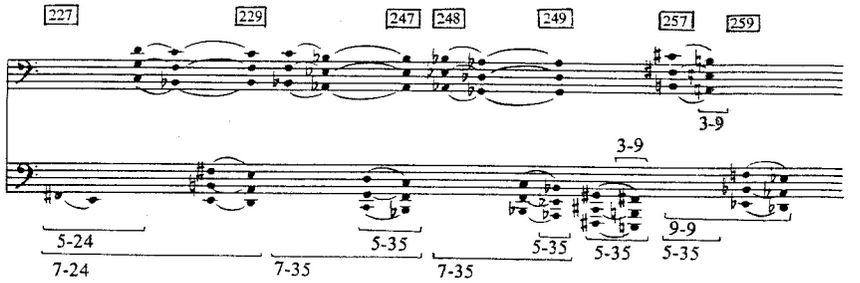
Note that both minimum and maximum invariance result from the values of t. More will be said about this substructure further along.

The verticals that result from the canonic "coloring" of 5-Z17 comprise a

*According to an entry in Schoenberg's diary, quoted in Rufer 1959, p. 14, the title of this movement was to have been *Akkordfärbungen (technisches)*—chord coloring, technical. In this sense, the canon can be regarded as effecting the coloring of the initial sonority, 5-Z17.

more or less ordered sequence—especially in the stretto section, beginning at measure 248. Moreover, the composite segments formed by pairs of verticals within stratum 1 are of significance. In particular, the succession 5-Z17-5-16 yields 6-Z19 in each case, and this is the predominant hexachord in a texture otherwise made up mainly of sets of cardinals 4 and 5. Still larger composite segments within stratum 1 form 8-8 or 8-20 and (over the entire sequence) 9-4. Of these, 8-20 and 9-4 are represented by complements in the music at significant points. It should also be noted that pc set 4-19 (stratum 3) is contained in 9-4 nine times—more than any other tetrachord.

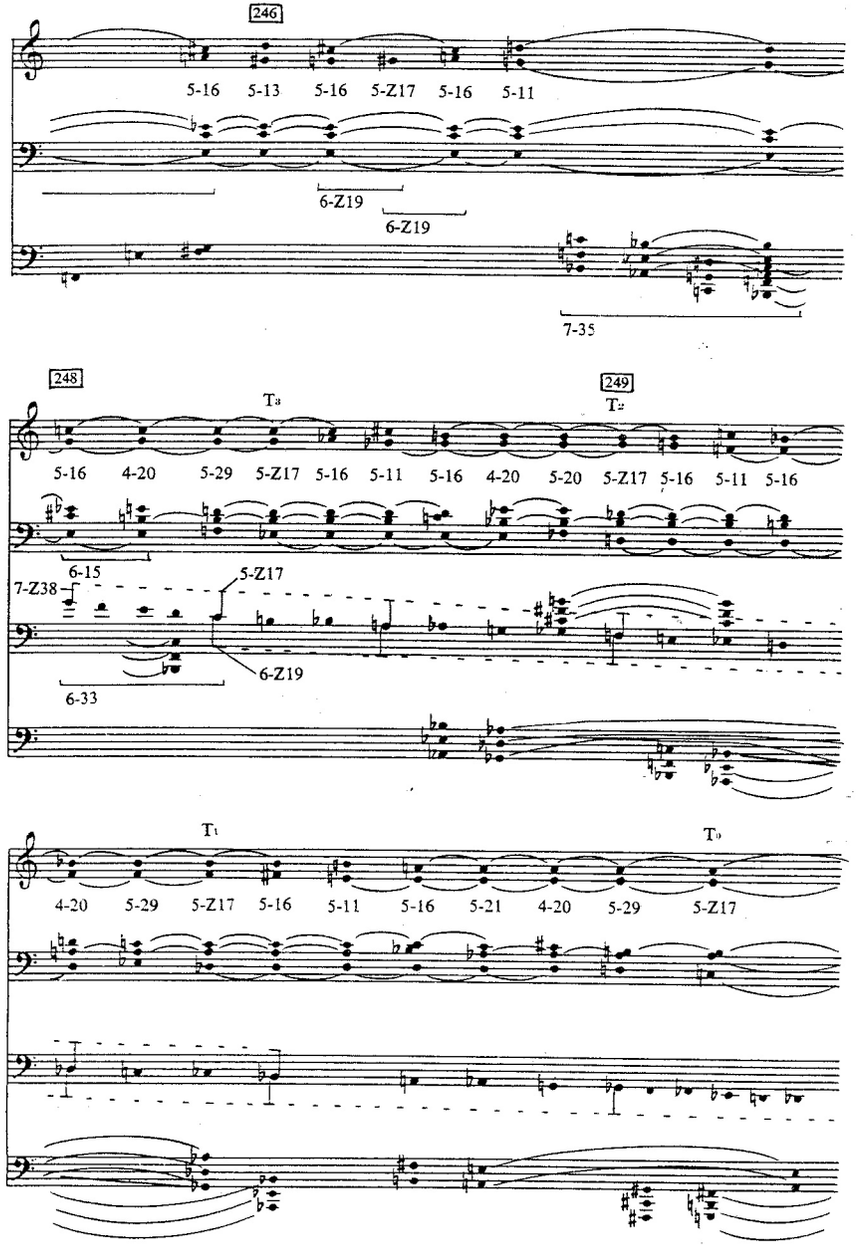
154.

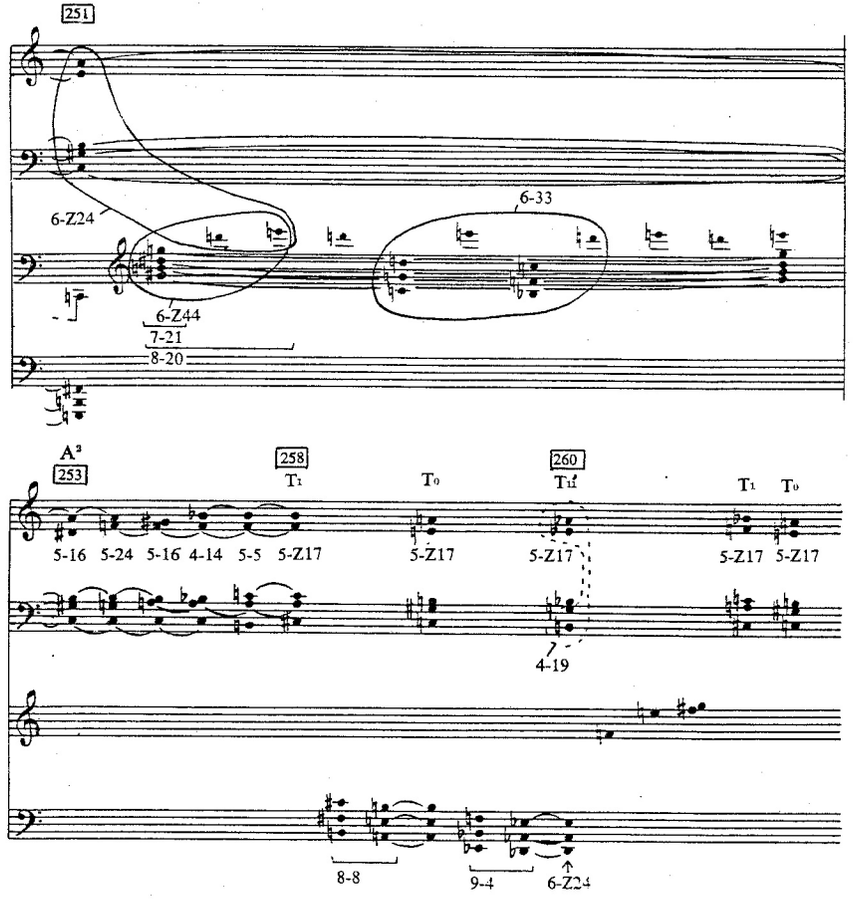


Turning now to stratum 2, the reader's attention is directed to example 154, which summarizes the motion of that component over the entire movement. The initial motive, F-sharp-E, derives of course from the last dyad of the canonic motive. Composite segments formed by this stratum are shown in example 154. Sets of three types are featured: 7-24, 7-35, and 3-9. In each case complement embedding occurs. Intervals of ic2 and ic5 are characteristically prominent in all these sets. Discussion of the long-range pattern shown in example 154 is postponed for the present.

Unlike strata 1 and 2, stratum 3 does not exhibit a long-range linear pattern, but its subparts remain unchanged at each occurrence. The components of stratum 3 are shown in detail in example 155. Note that the large set, 8-14, formed by the composite segment, as indicated in example 152, is the complement of the tetrachord at measure 233 in stratum 1. This is not the only link between the two strata. Pc set 4-19 is contained twice in 5-Z17 and is one of the three sets represented as subsets of 5-Z17 (4-14 being another, of course). Moreover, pc set 3-4 is the complement of stratum 1 over the entire first section of the music. And finally, pc set 3-1 here is the contour inversion (with octave displacement of the last two notes) of the canonic motive of stratum 1, just as it occurs in A^2. This detail has been misrepresented, perhaps understandably so.* The motive is not F-E-G, but F-E-F-sharp.

*Cf. Craft 1968. Webern made the same mistake.


246
5-16 5-13 5-16 5-Z17 5-16 5-11
6-Z19
6-Z19
7-35
248
T3
249
T2
5-16 4-20 5-29 5-Z17 5-16 5-11 5-16 4-20 5-20 5-Z17 5-16 5-11 5-16
6-15
5-Z17
7-Z38
6-Z19
6-33
T1
T0
4-20 5-29 5-Z17 5-16 5-11 5-16 5-21 4-20 5-29 5-Z17


251
6-Z24
6-33
6-Z44
7-21
8-20
A3
253
258
T1
T0
260
T11
T1 T0
5-16 5-24 5-16 4-14 5-5 5-Z17 5-Z17 5-Z17 5-Z17 5-Z17
4-19
8-8
9-4
6-Z24

er structure of the music. But the sets which they display point to the strong link between similarity relations and the set-complex structure over the large span of music which has been examined.

The dearth of strongly represented R_p is analogous to the paucity of invariants among equivalent sets. From this we can conclude, perhaps trivially, that interval content, not pc content, is of prime importance for this music, and that similarity relations based upon interval content have a primary role within the set-complex structure.

A condensed score of the third of Schoenberg's Five Pieces for Orchestra Op. 16 is given in example 151. Although this is a correct representation of the pitch content of the music it does not show the orchestration. A good deal has been said about the latter, which is purportedly an actualization of the composer's *Klangfarben* idea, as set forth at the end of his *Harmonielehre* (1911).*

The present version of the score (prepared by the author), by omitting the complexities of orchestration, reveals clearly certain fundamental structural features of the composition which are not especially evident in the full score.† In particular, this version shows important linear progressions of large scale which are concealed by the complicated orchestral tapestry. Most of these have gone quite unrecognized, if one can judge from the published literature on the work.

Example 152 is a sketch of the entire movement, and this will be the main illustration for the discussion that follows.

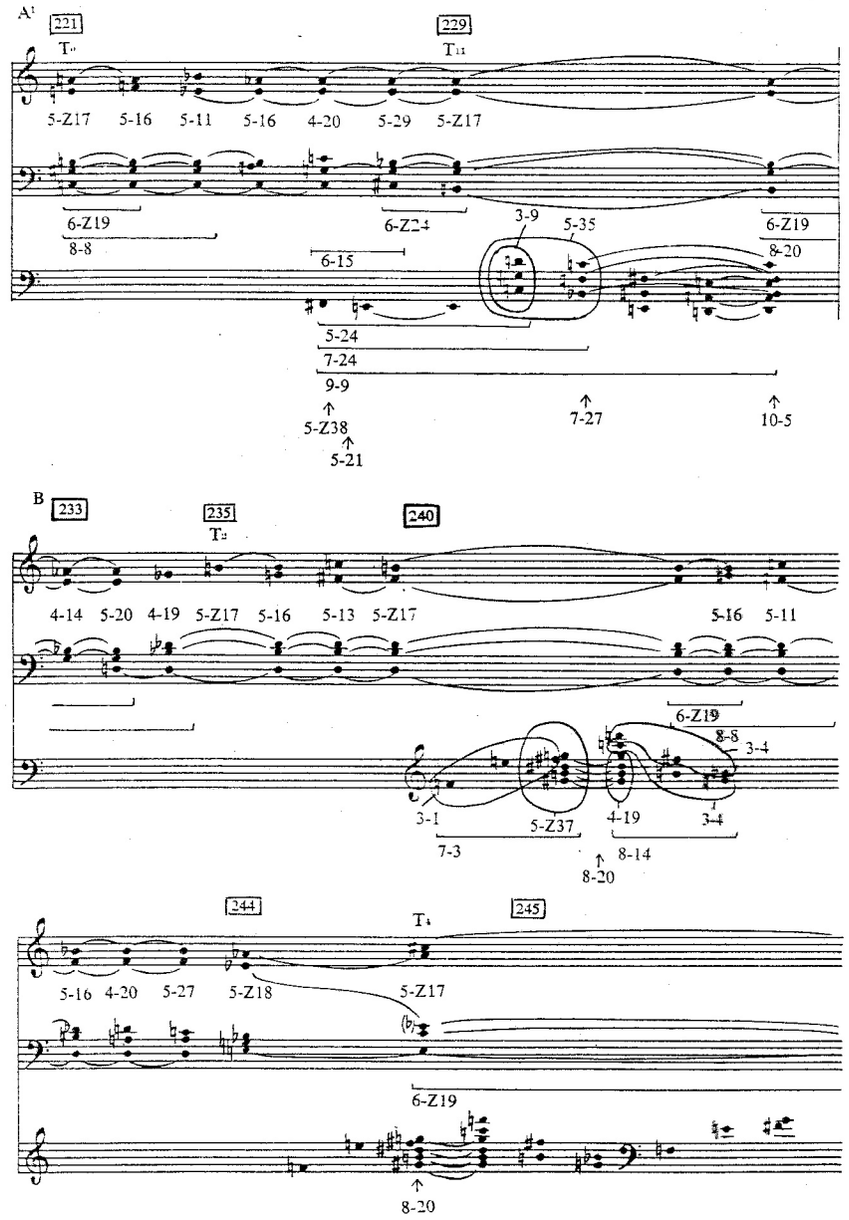
This remarkable work divides into three main sections, as indicated in example 152. In addition, it is organized into three distinct components each consisting of an initial formation and its development, which will be referred to as stratum 1, stratum 2, and stratum 3. Stratum 1 begins with the opening chord succession and forms the basic continuum of the music. Stratum 2 is initiated with the lower register formation beginning in measure 227 and is characterized by superimposed linear statements of the "whole-step" motive. The remaining component, stratum 3, first introduced in measure 240, contains several subparts, of which the most prominent are 4-19, 3-1, and 3-4. Some observations on each of these strata follow.

Stratum 1, ostensibly a succession of chords, exhibits a more primary structural feature: it is canonic. This can be seen in the upper voice motive A-B-flat-A-flat, replicated by the voices below it, in order, as E-F-E-flat, B-C-B-flat, G-sharp-A-G, and C-C-sharp, B. This imitative linear substructure continues throughout the movement at various pitch levels, and is systematically projected in stretto fashion even through the dense passage beginning in

*See, for example, Craft 1968.

†They are also not evident in the two-piano reduction prepared by Webern in 1913 (C. F. Peters Edition 3378).

152. Schoenberg, Five Pieces for Orchestra Op. 16/3



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For Sharland

The Structure of Atonal Music

Allen Forte

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