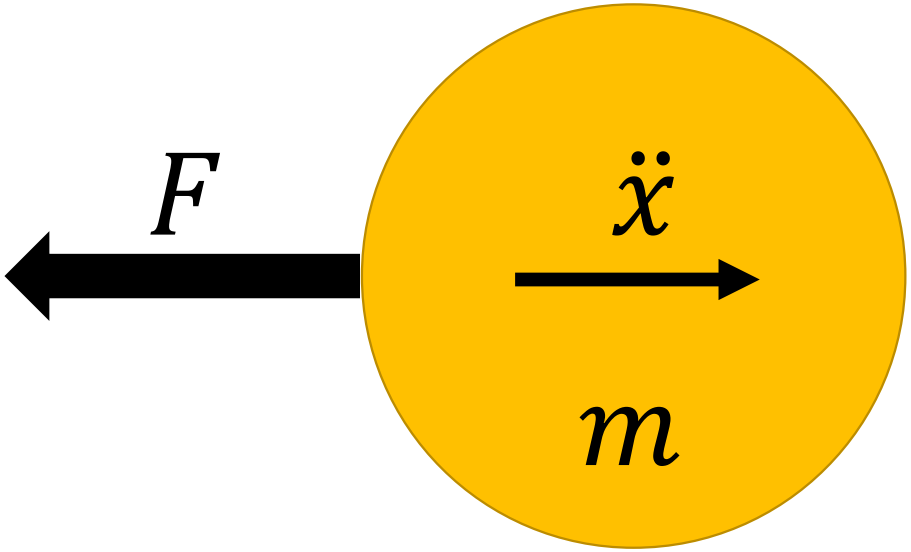
Dinâmica de Sistemas Navais e Oceânicos

PNV3314 Dinâmica de Sistemas

Aula 13

Massa adicional

Inércia adicional



$$F = m\ddot{x}$$

NO VÁCUO

Inércia adicional

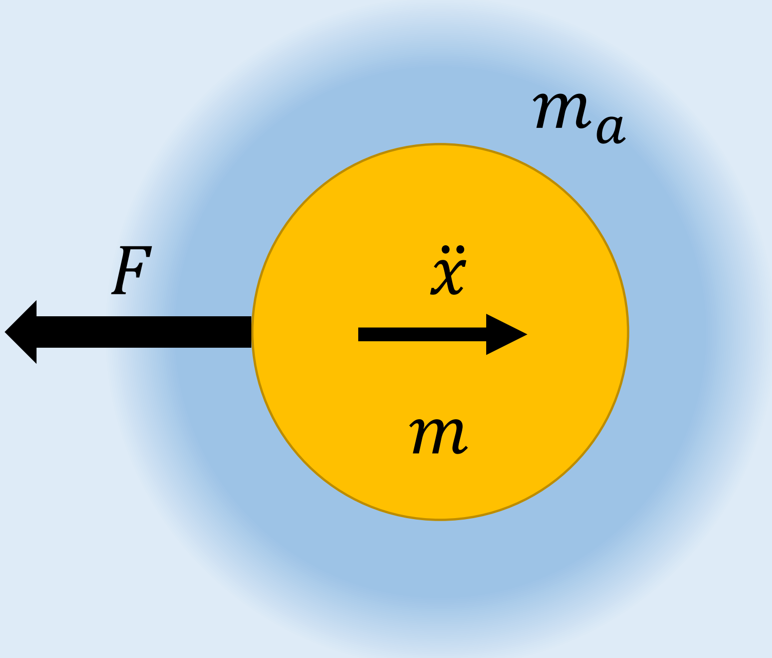
Massa adicional

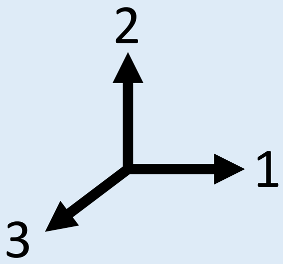
$$F = m\ddot{x} + m_a\ddot{x} = (m + m_a)\ddot{x}$$

Representada pelo coeficiente de massa adicional

$$m_a = \rho \forall C_a$$

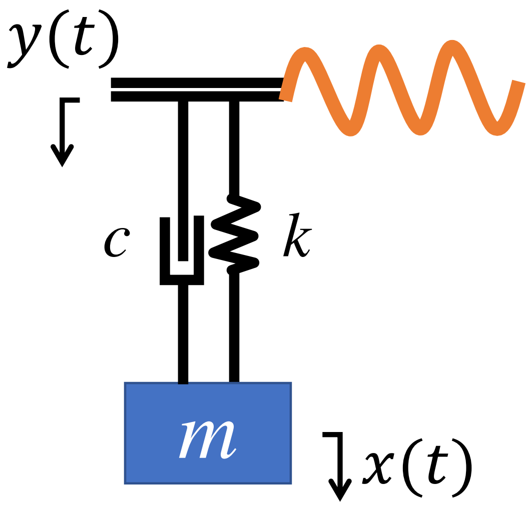
Depende da geometria do corpo e das condições do escoamento externo (amplitude e frequência).





$$C_a = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

NA ÁGUA

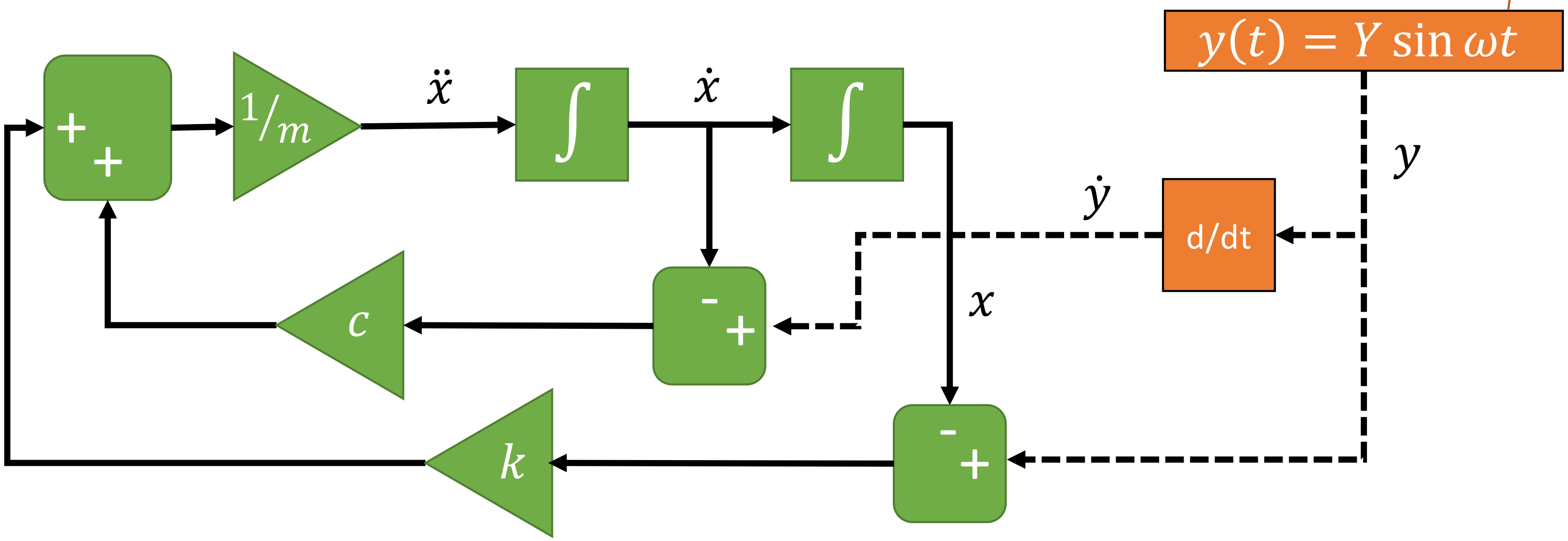


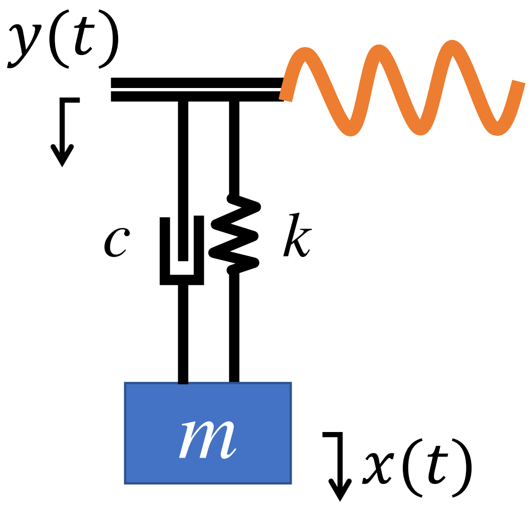
$$m\,\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$y(t) = Y \sin \omega t$$

$$\ddot{x} = \frac{1}{m}[c(\dot{y} - \dot{x}) + k(y - x)]$$

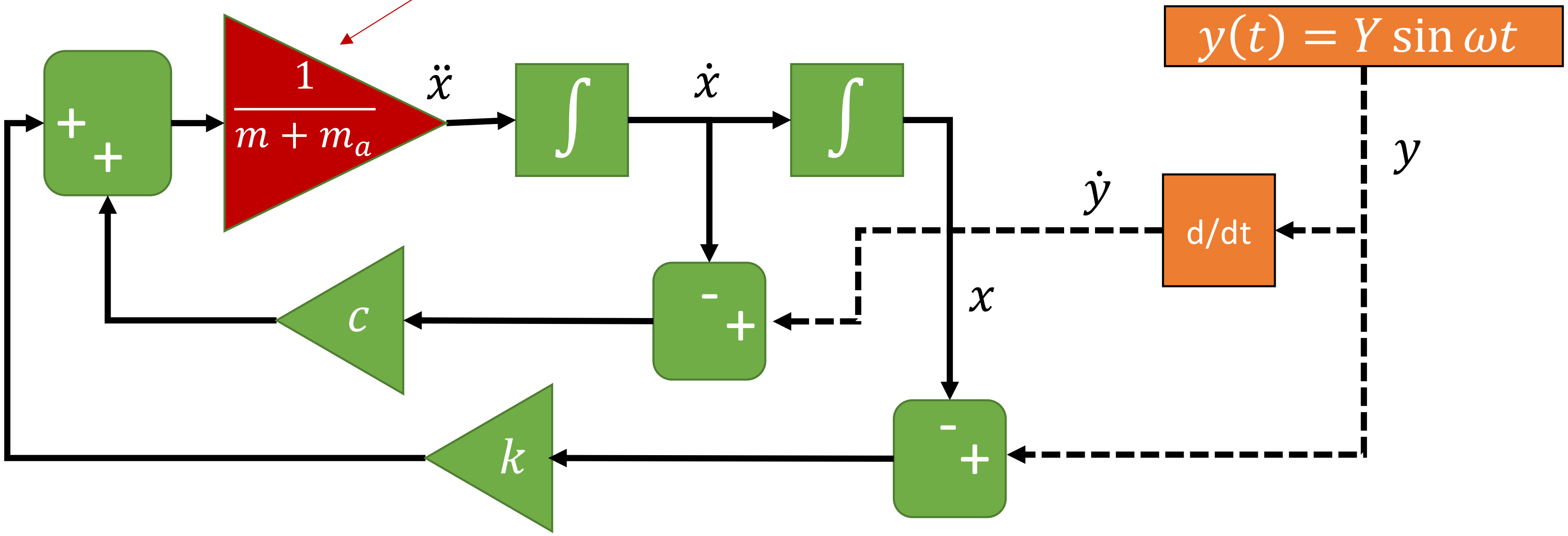
Movimento imposto na base





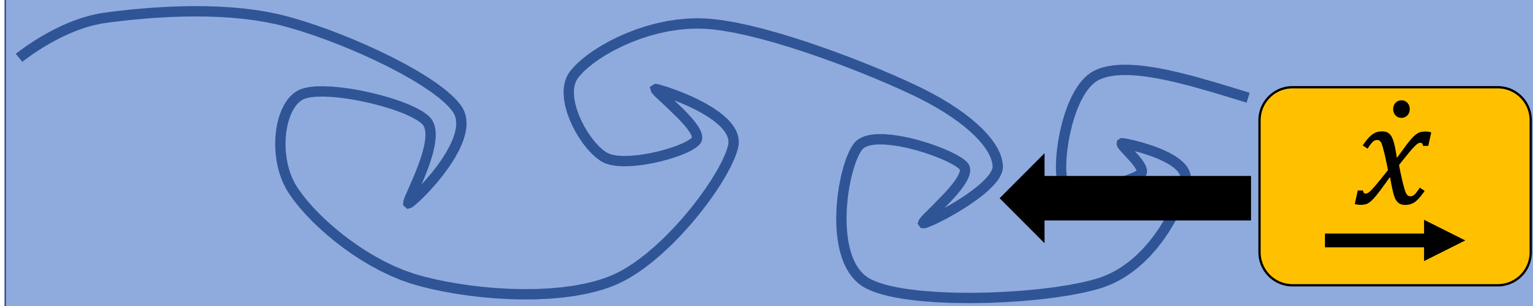
$$(m + \rho\forall C_a)\,\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$\ddot{x} = \frac{1}{m + \rho\forall C_a}\left[c(\dot{y} - \dot{x}) + k(y - x)\right]$$

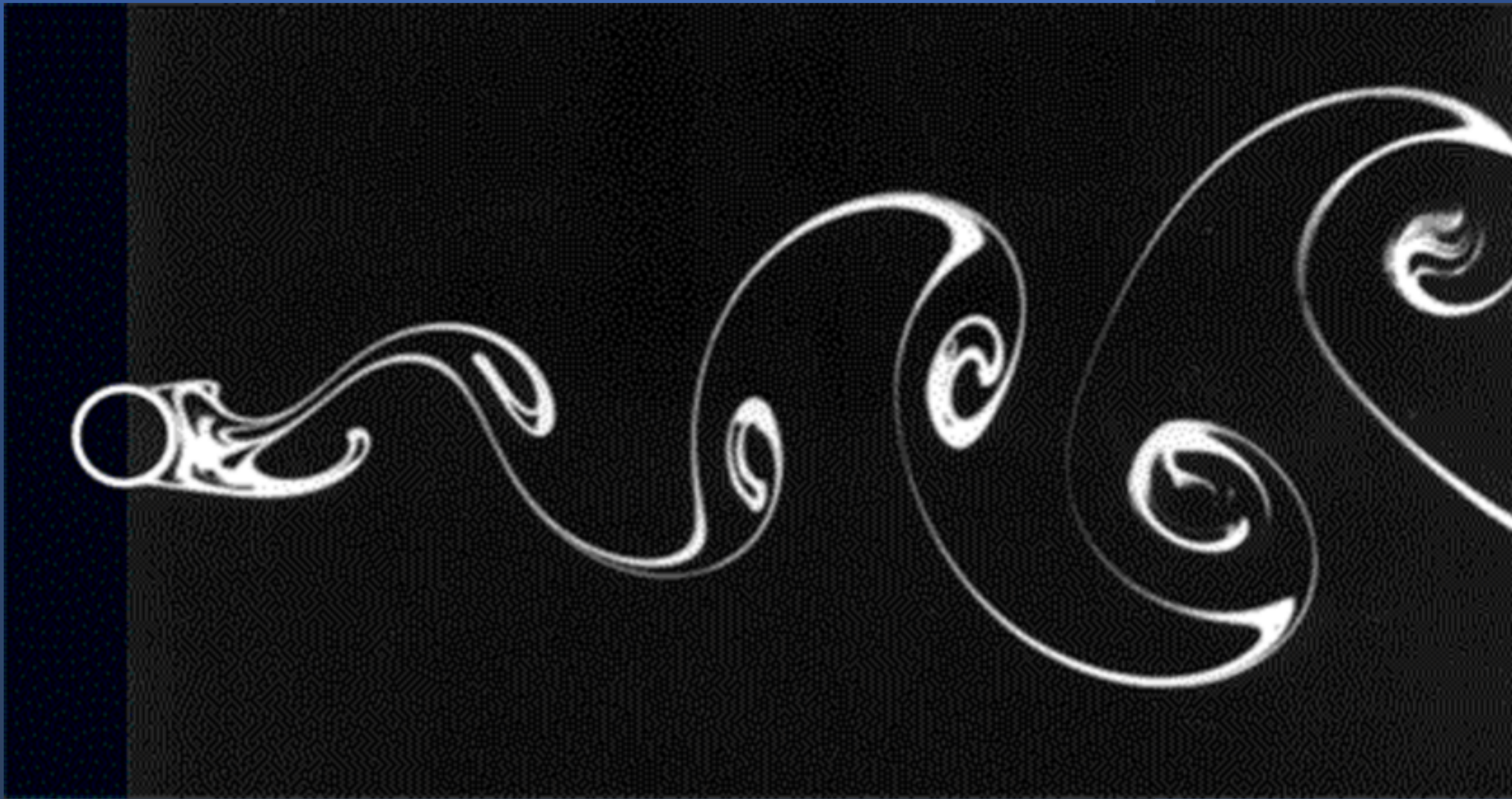


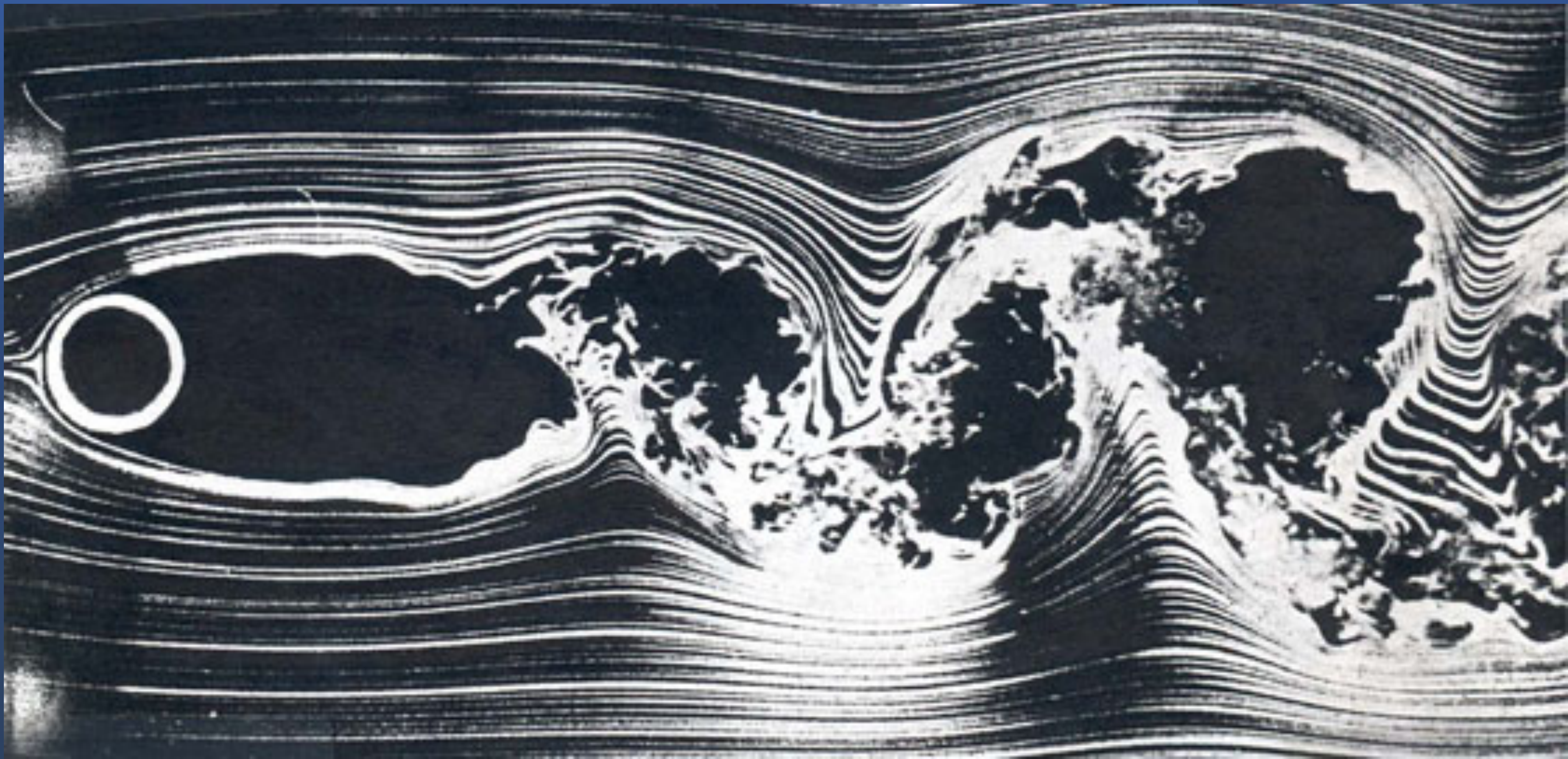
Amortecimento quadrático

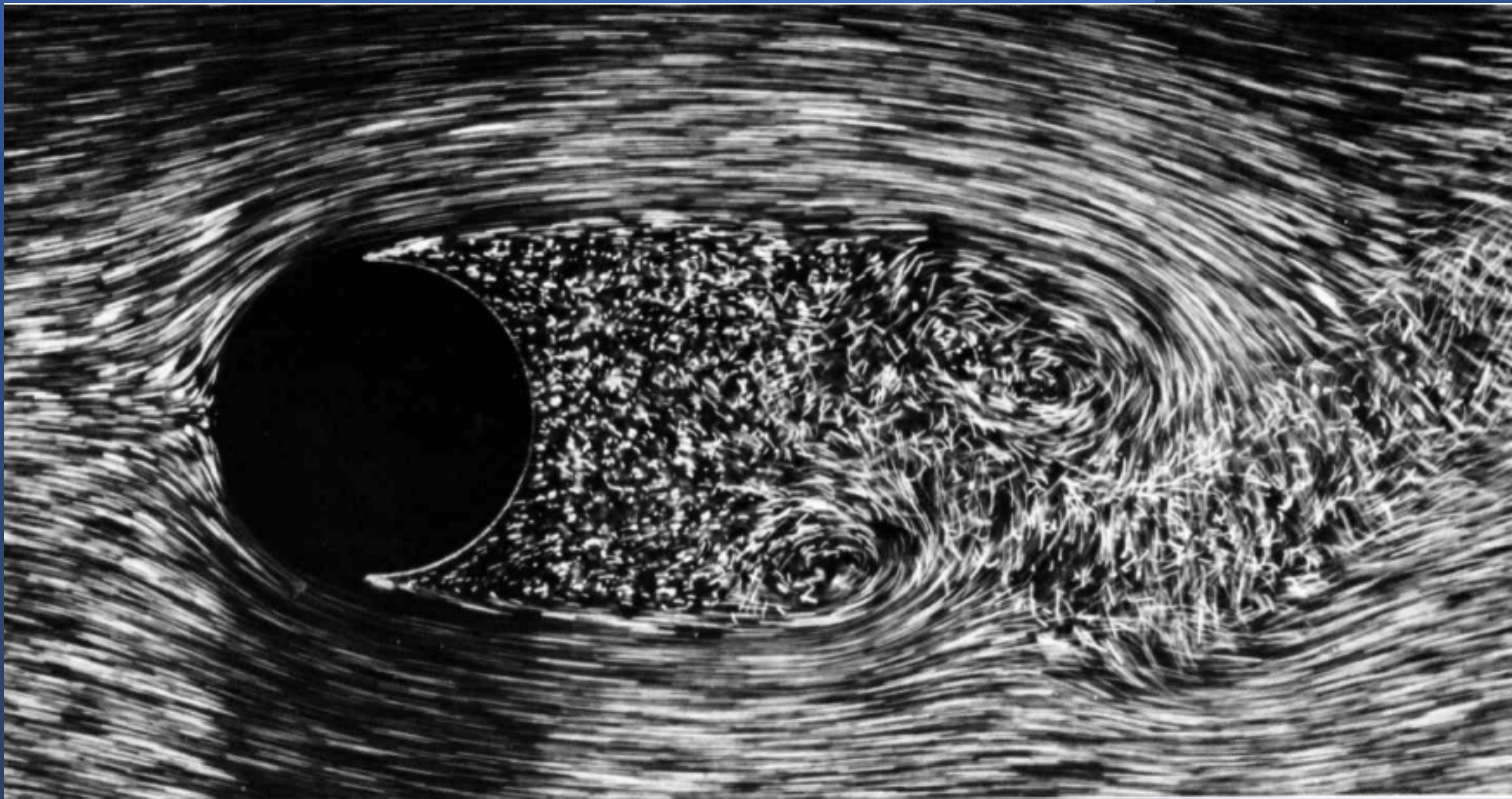
Amortecimento: arrasto hidrodinâmico

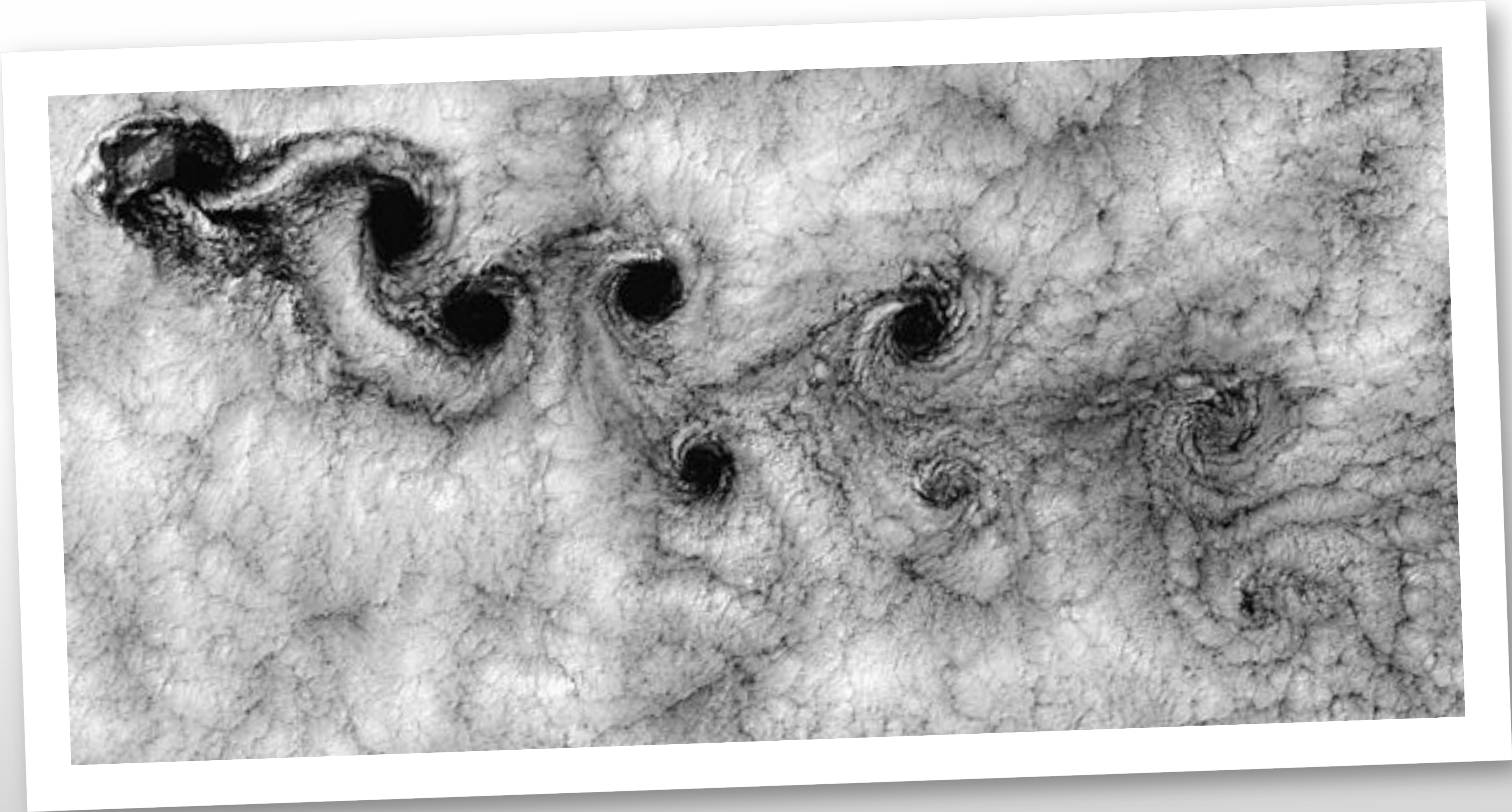


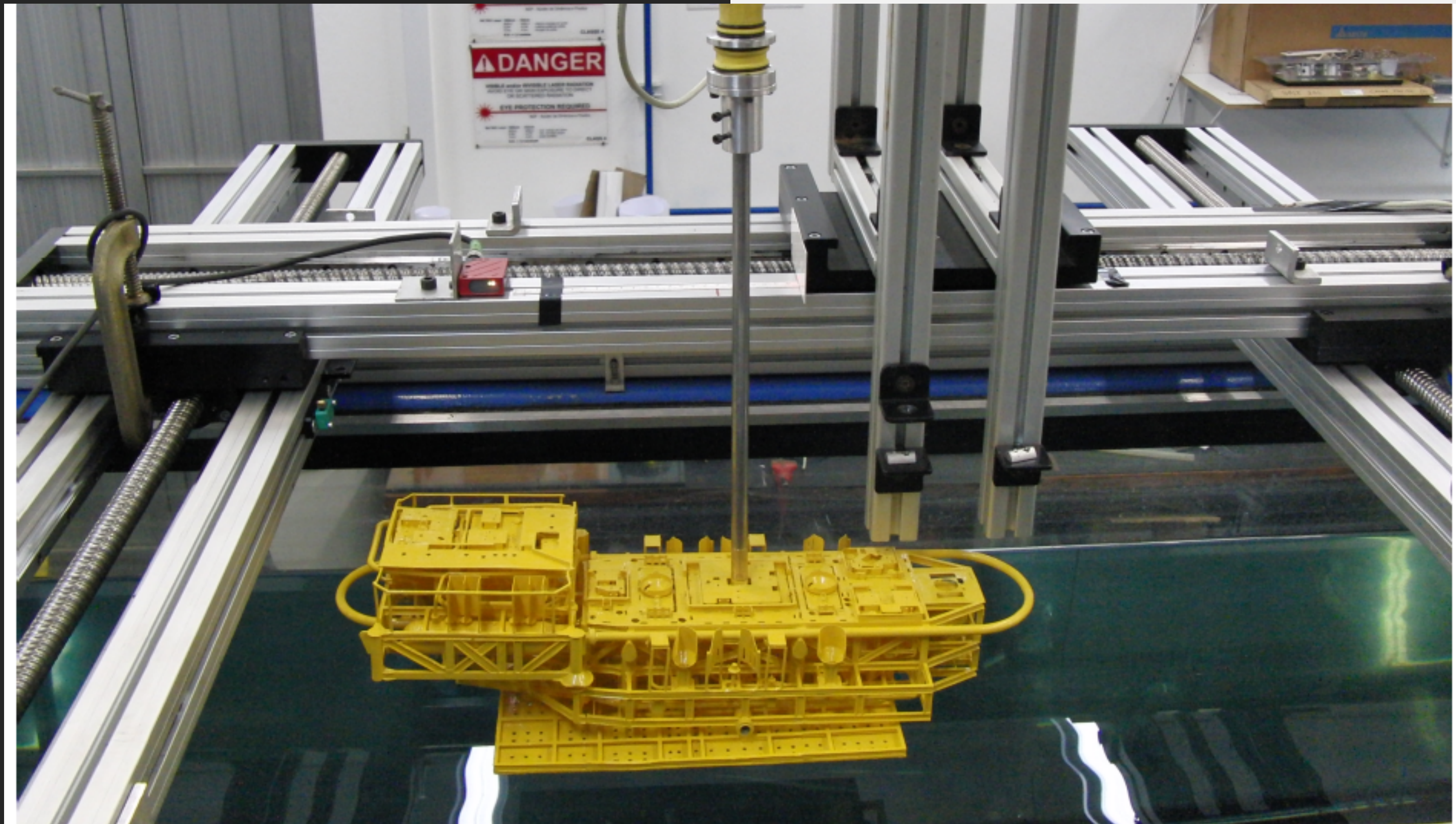
$$F_D = C_D \frac{1}{2} \rho \dot{x}^2 D L$$



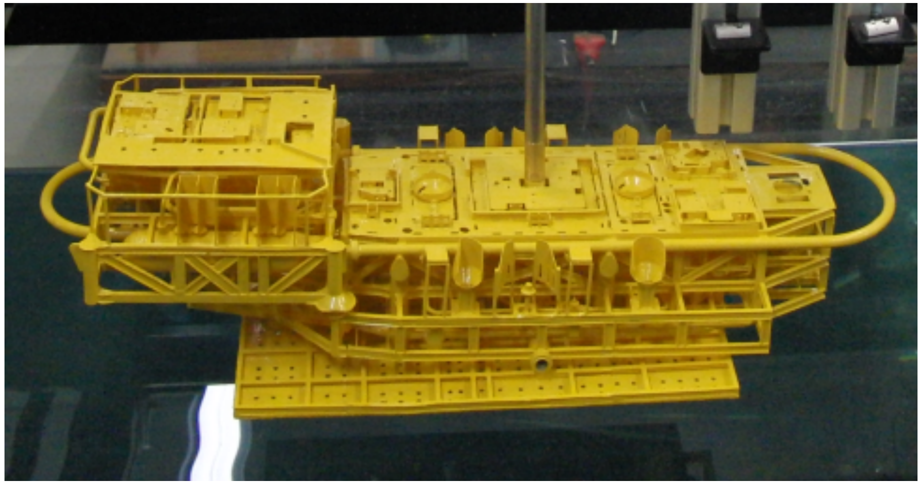




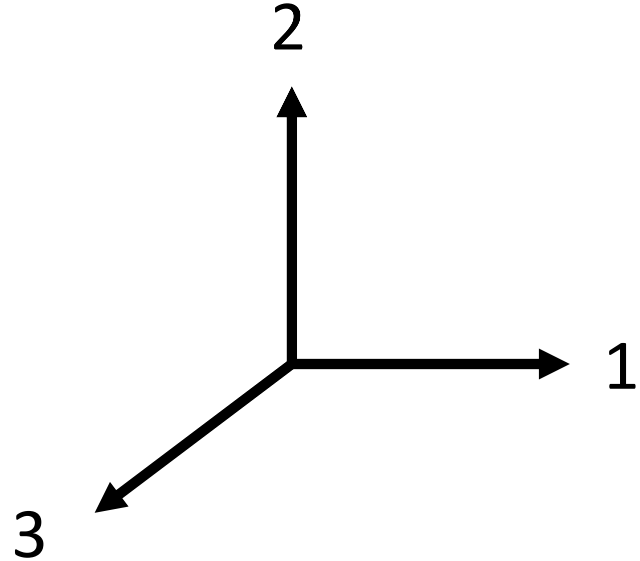



DANGER

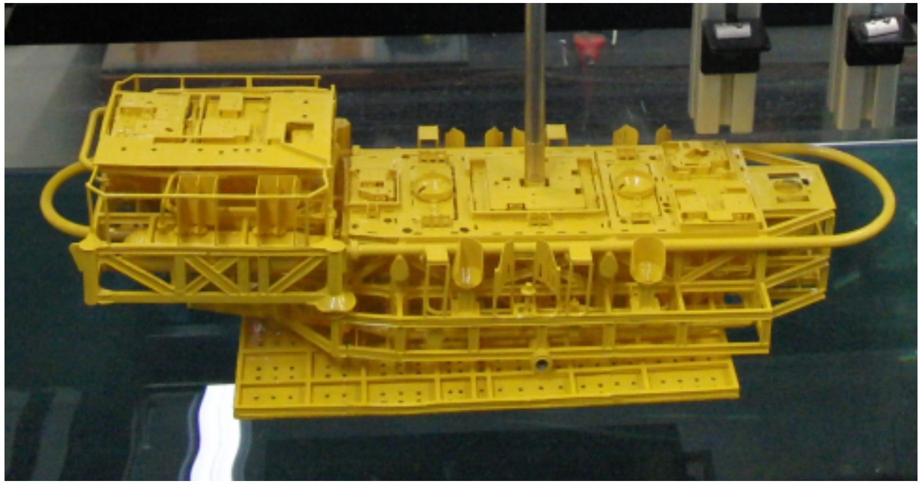
$$F_D = C_D \frac{1}{2} \rho \dot{x}^2 D L$$



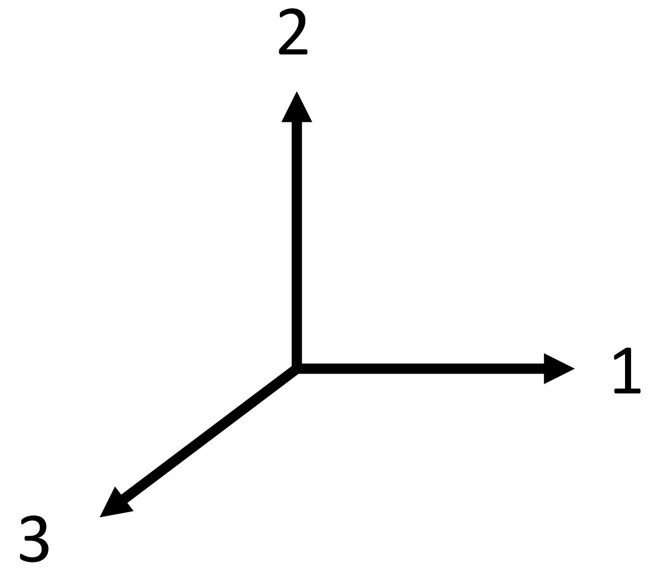
$$C_D = \begin{bmatrix} C_{11} & & \\ & C_{22} & \\ & & C_{33} \end{bmatrix}$$



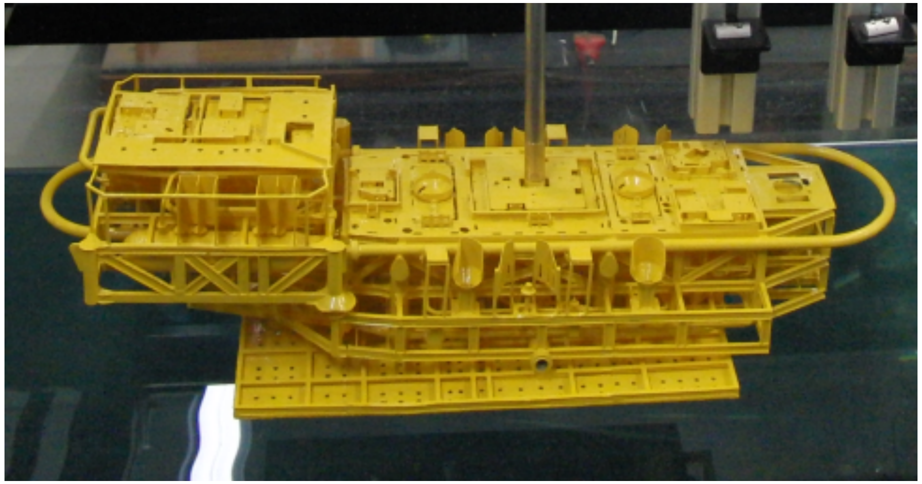
$$F_L = C_L \frac{1}{2} \rho \dot{x}^2 D L$$



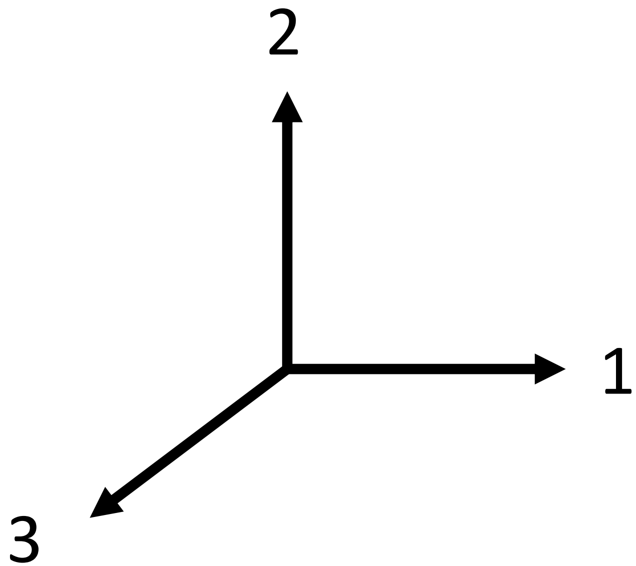
$$C_L = \begin{bmatrix} & C_{12} & C_{13} \\ C_{21} & & C_{23} \\ C_{31} & C_{32} & \end{bmatrix}$$

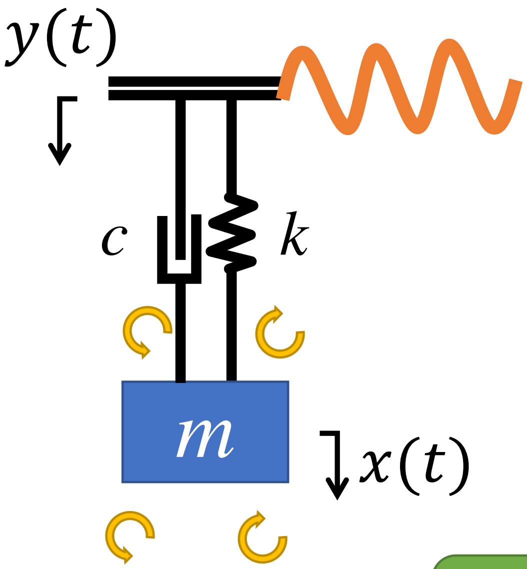


$$F_h = C_h \frac{1}{2} \rho \dot{x}^2 DL$$



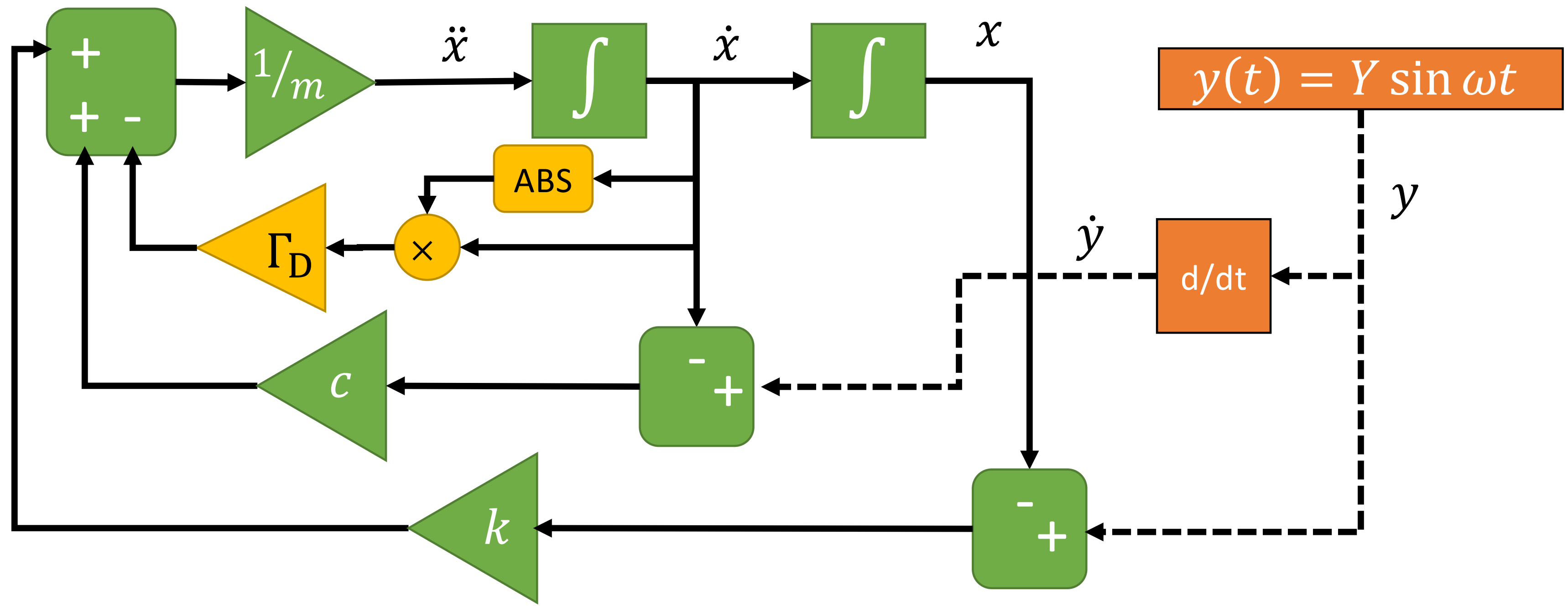
$$C_h = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$



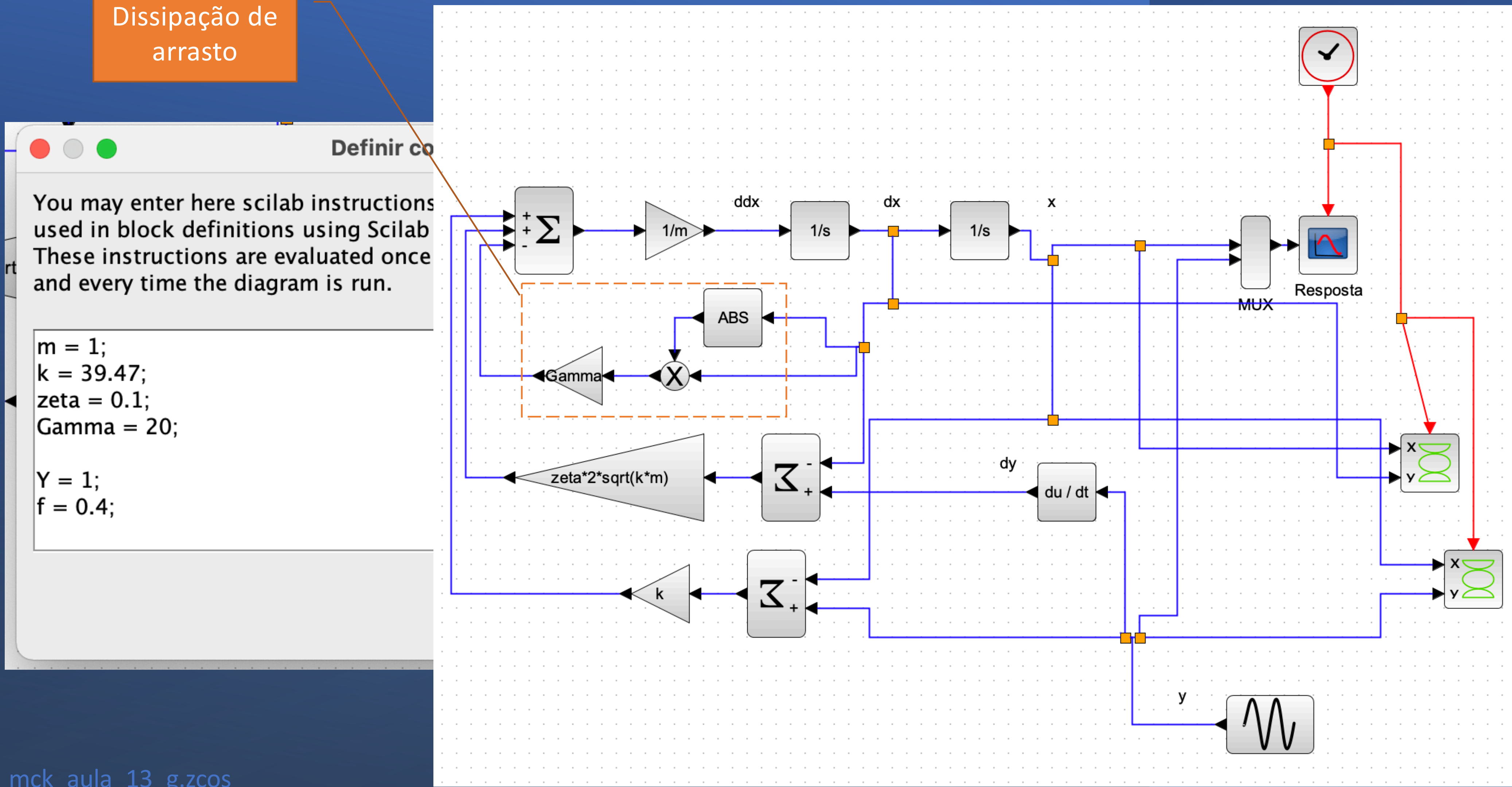


$$m\,\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = C_D\,\frac{1}{2}\rho\dot{x}^2 DL = \Gamma_{\mathrm{D}}|\dot{x}|\dot{x}$$

$$\ddot{x} = \frac{1}{m}\left[c(\dot{y} - \dot{x}) - \Gamma_{\mathrm{D}}|\dot{x}|\dot{x} + k(y - x)\right]$$



Dissipação de arrasto



mck_aula_13_g.zcos

Amortecimento equivalente

$$F_d = \pm a(\dot{x})^2$$

A energia dissipada em um ciclo harmônico é...

$$\Delta W = 2\int_{-x}^{x} a(\dot{x})^2\,dx = 2X^3\int_{-\pi/2}^{\pi/2} a\omega^2\cos^3\omega t\,d(\omega t) = \frac{8}{3}\omega^2 a X^3$$

Como a energia dissipada em um ciclo por um amortecedor viscoso é...

$$\Delta W = \pi c_{\text{eq}}\omega X^2$$

Resulta em um amortecedor equivalente:

$$c_{\text{eq}} = \frac{8}{3\pi}a\omega X$$

Que produz uma resposta com amplitude

$$\frac{X}{\delta_{st}} = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta_{eq} r)^2}}$$

Ou...

$$X = \frac{3\pi m}{8ar^2}\left[-\frac{(1 - r^2)^2}{2} + \sqrt{\frac{(1 - r^2)^4}{4} + \left(\frac{8ar^2\delta_{st}}{3\pi m}\right)^2}\,\right]^{1/2}$$

No caso da força de arrasto...

$$F_D = C_D \frac{1}{2} \rho \dot{x}^2 DL \qquad\qquad a = C_D \frac{1}{2} \rho DL$$

Que produz...

$$c_{eq} = \frac{8}{3\pi} C_D \frac{1}{2} \rho DL\omega X$$

Que representa a constante de amortecimento equivalente à dissipação da força de arrasto de um corpo sob escoamento.

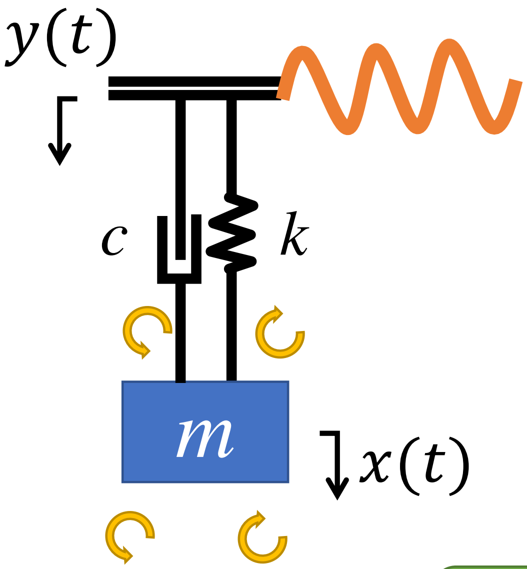
No caso da força de arrasto...

$$F_D = C_D \frac{1}{2} \rho \dot{x}^2 DL \qquad\qquad a = C_D \frac{1}{2} \rho DL$$

Que produz...

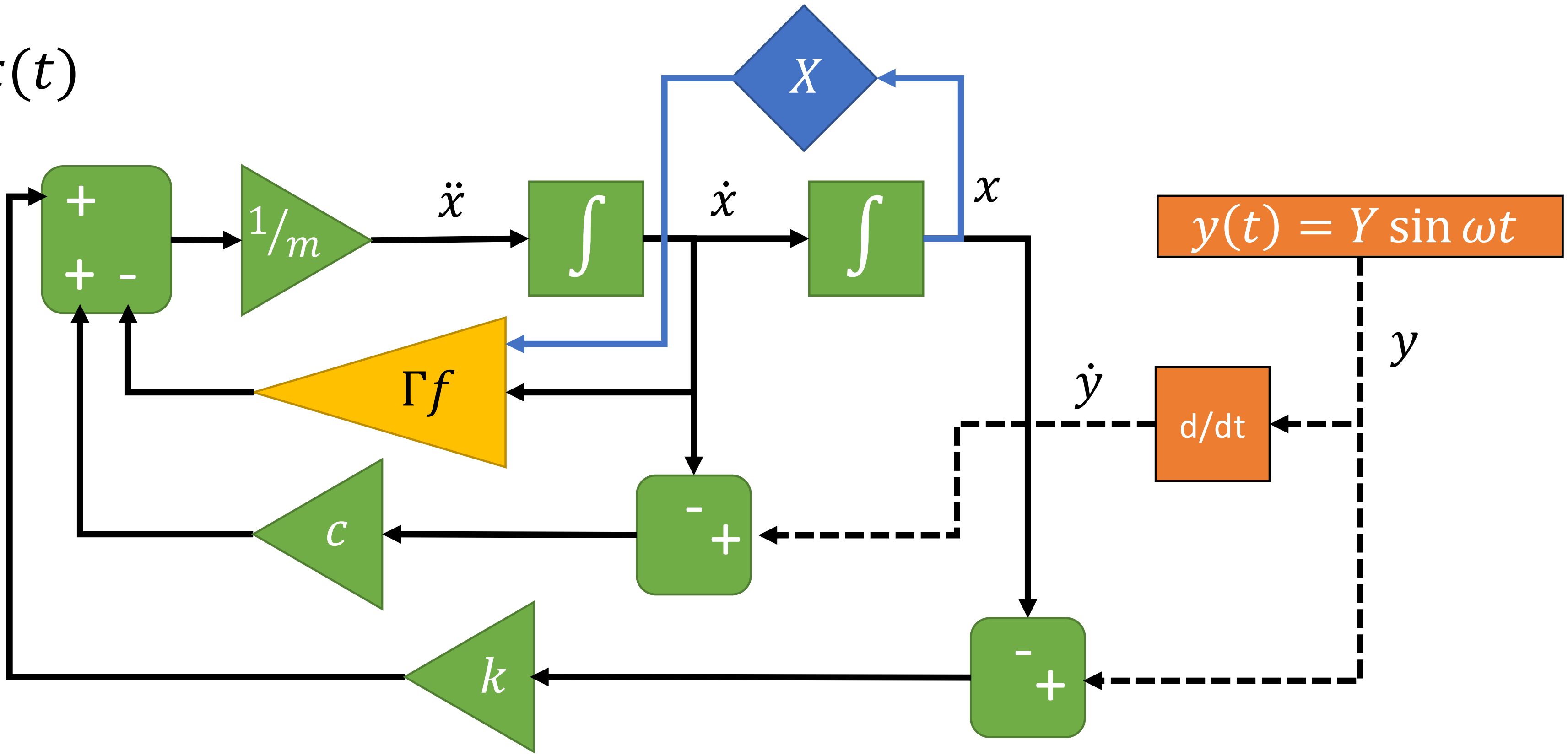
$$c_{eq} = \frac{8}{3\pi} C_D \frac{1}{2} \rho DL\omega X = \frac{8}{3} C_D \rho DLfX = \Gamma f X$$

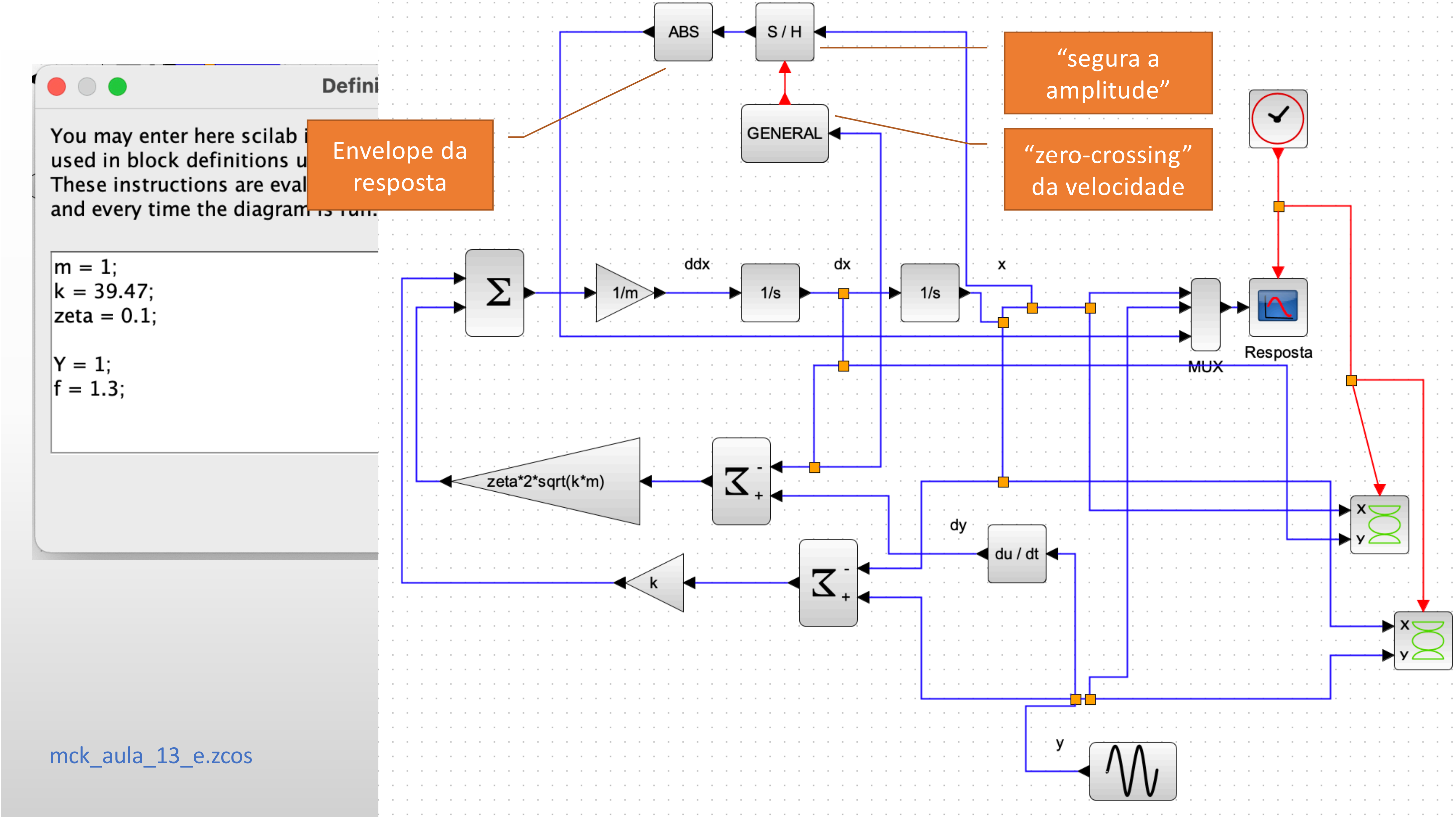
Que representa a constante de amortecimento equivalente à dissipação da força de arrasto de um corpo sob escoamento.

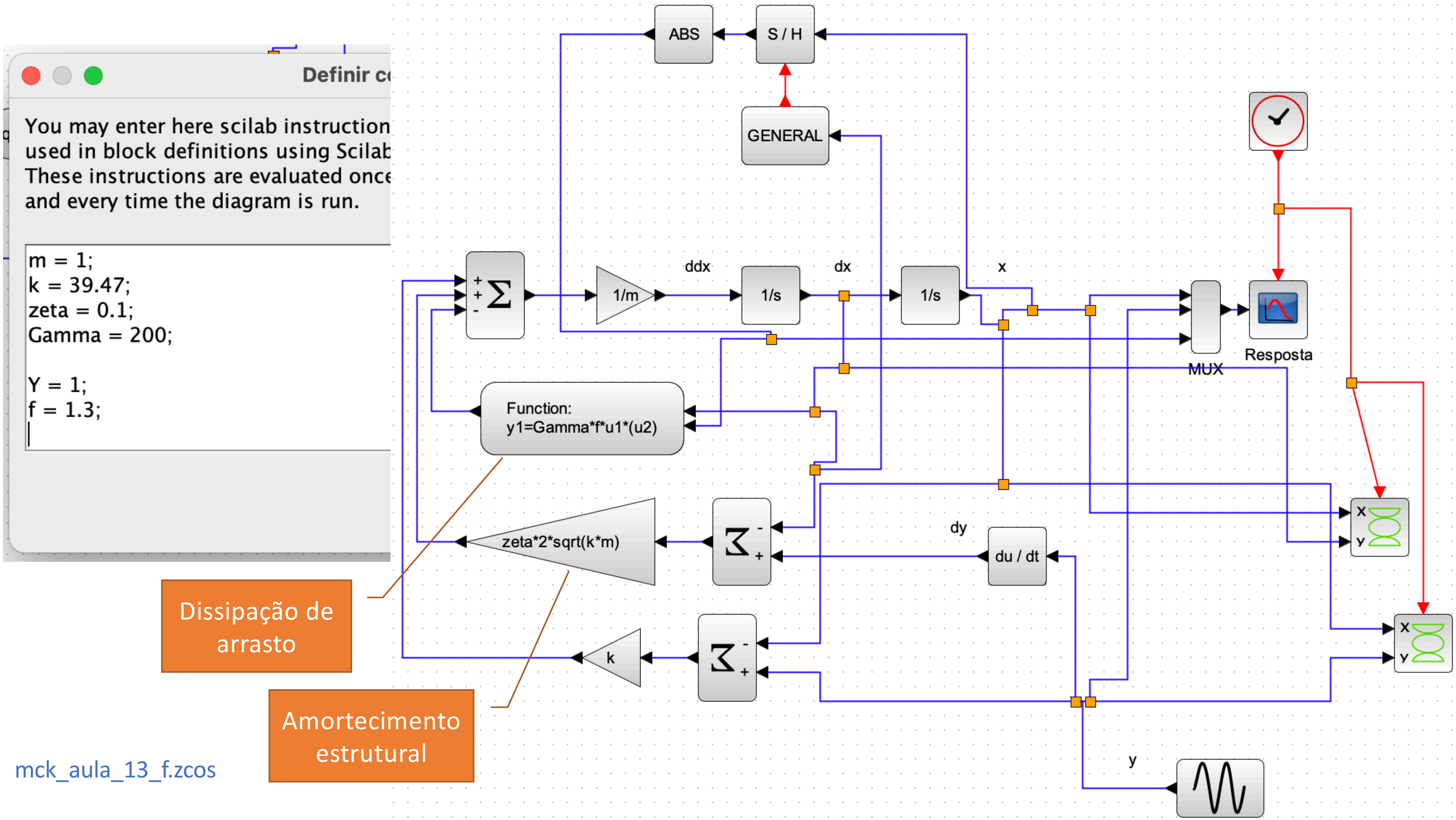


$$m\,\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = C_D\,\frac{1}{2}\rho\dot{x}^2 DL = \Gamma f X\dot{x}$$

$$\ddot{x} = \frac{1}{m}\left[c(\dot{y} - \dot{x}) - \Gamma f X\dot{x} + k(y - x)\right]$$



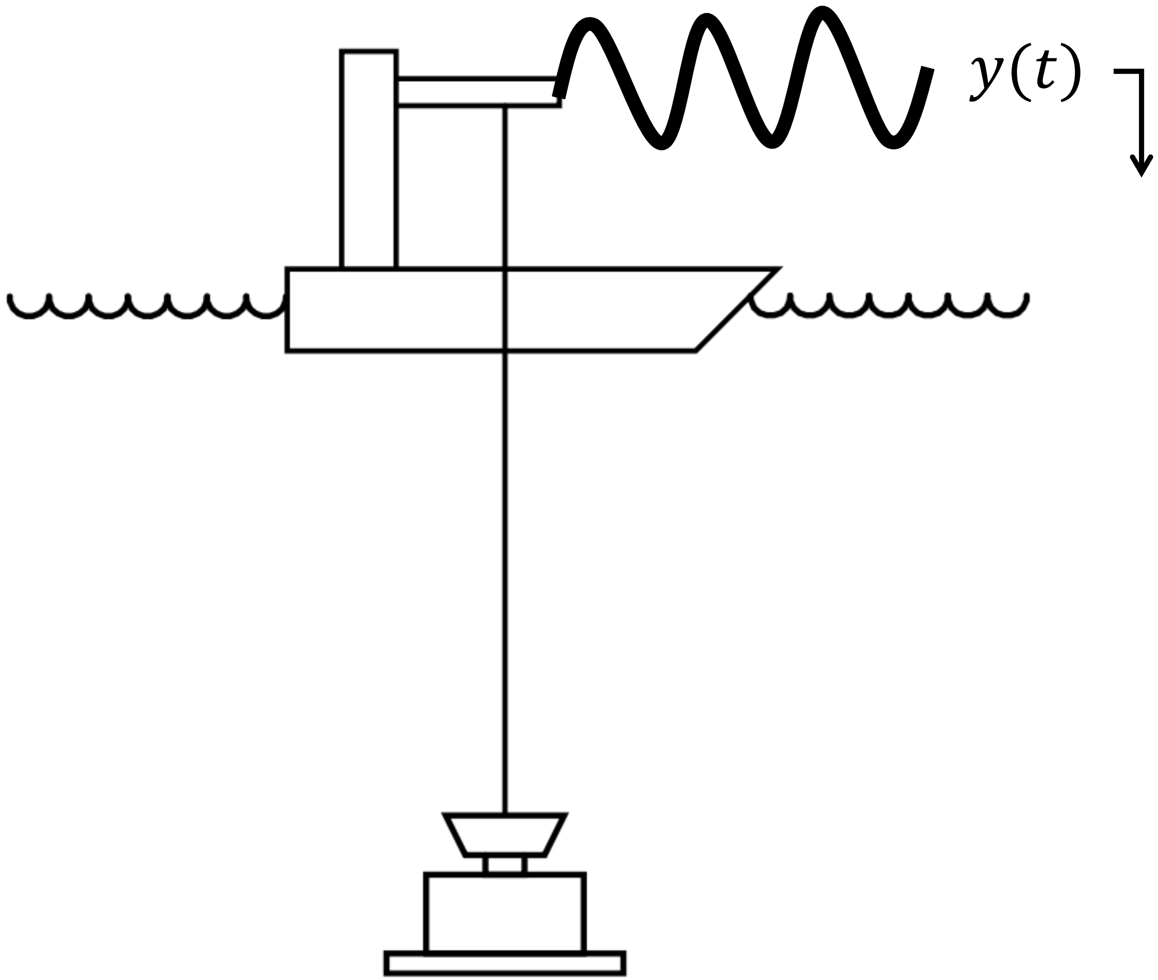

Defini
You may enter here scilab i
used in block definitions u
These instructions are eval
and every time the diagram is run.
m = 1;
k = 39.47;
zeta = 0.1;
Y = 1;
f = 1.3;
mck_aula_13_e.zcos
Envelope da resposta
“segura a amplitude”
“zero-crossing” da velocidade
ABS
S / H
GENERAL
ddx
dx
x
1/m
1/s
1/s
MUX
Resposta
zeta*2*sqrt(k*m)
dy
du / dt
k
y



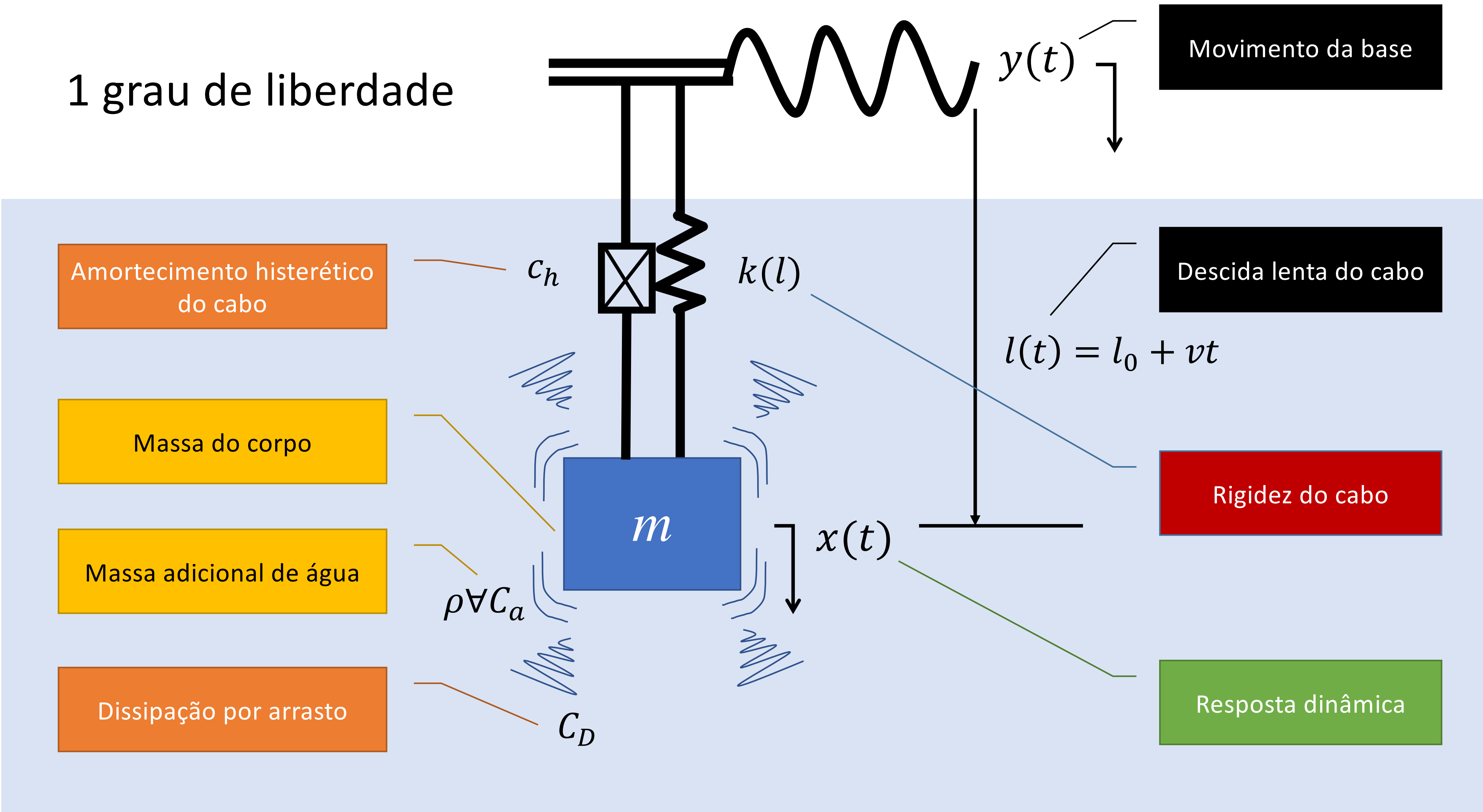
mck_aula_13_f.zcos

Dinâmica do lançamento do manifold submarino (em água)

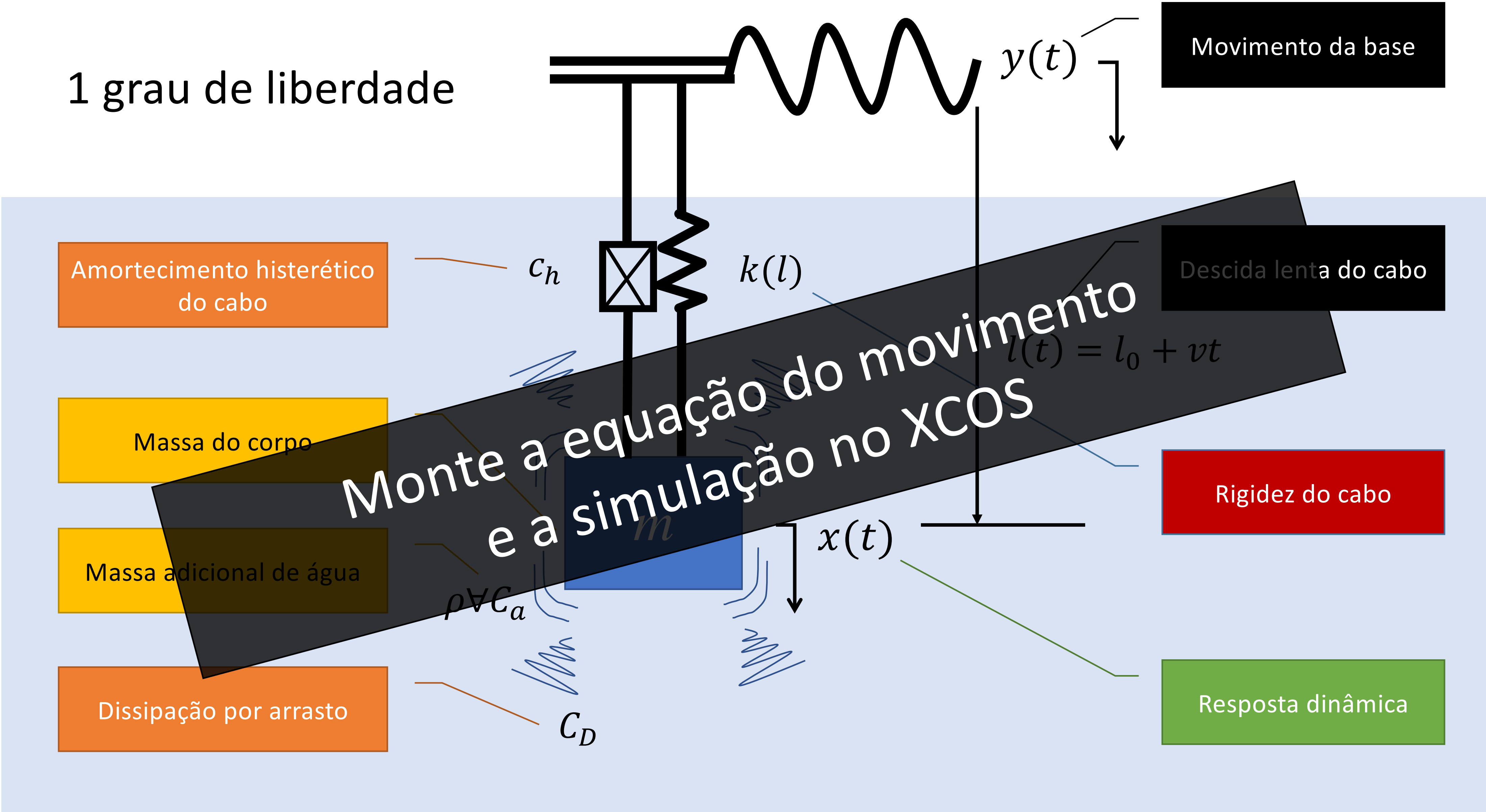
Colocando tudo junto



$y(t)$


1 grau de liberdade
Movimento da base
$y(t)$
Descida lenta do cabo
$l(t) = l_0 + vt$
Amortecimento histerético do cabo
c_h
$k(l)$
Rigidez do cabo
Massa do corpo
m
Massa adicional de água
$\rho\forall C_a$
$x(t)$
Resposta dinâmica
Dissipação por arrasto
C_D

1 grau de liberdade



Instabilidade hidrodinâmica

Instabilidade fluidodinâmica

Sabemos que $c_{eq} > 0$ dissipa energia (amortecimento).

Que é o caso da força de arrasto...

$$F_D = C_D \frac{1}{2} \rho \dot{x}^2 D L$$

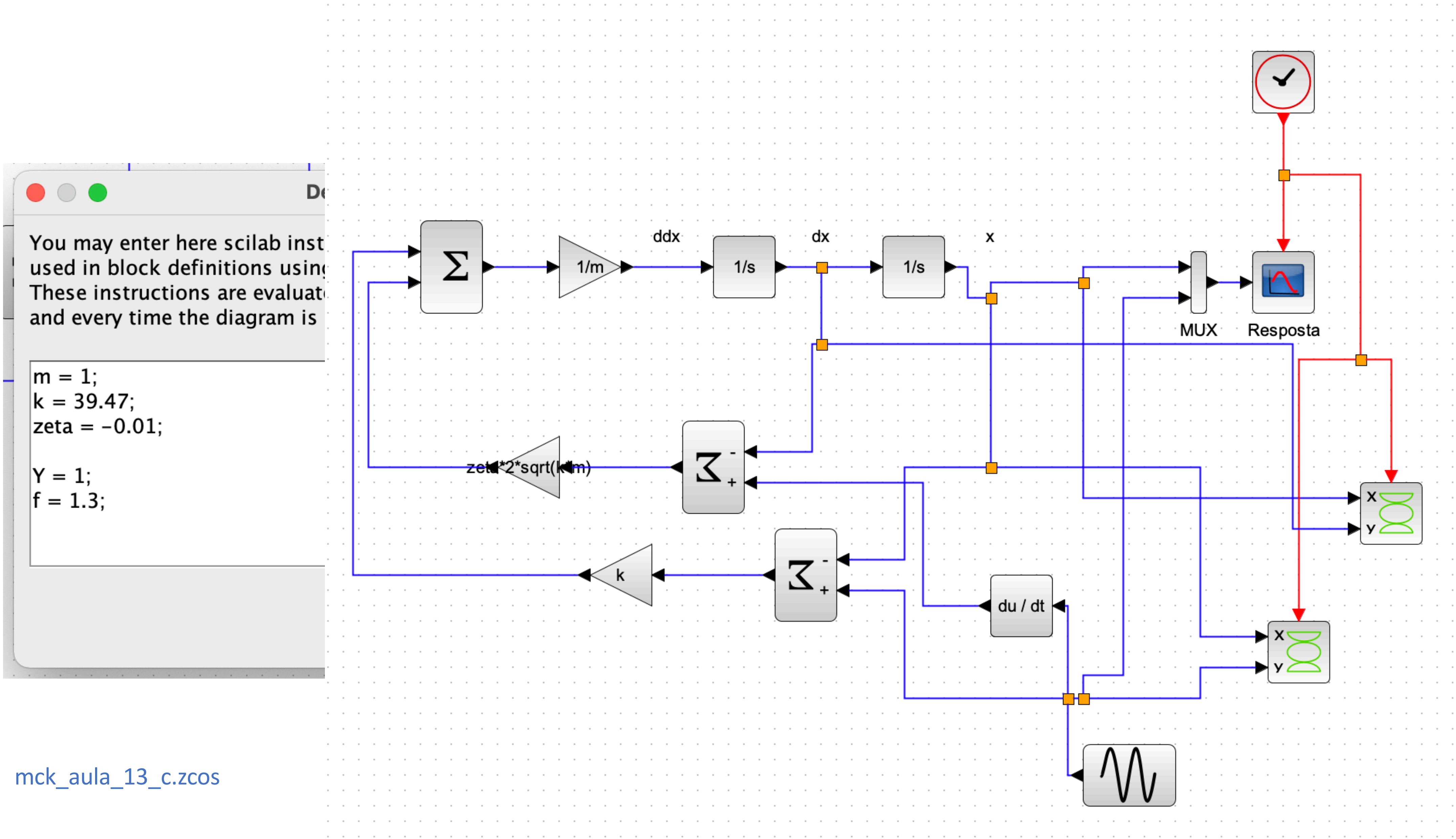
Instabilidade fluidodinâmica

Sabemos que $c_{eq} > 0$ dissipa energia (amortecimento).

Que é o caso da força de arrasto...

$$F_D = C_D \frac{1}{2} \rho \dot{x}^2 D L$$

O que aconteceria se $c_{eq} < 0$?



mck_aula_13_c.zcos

Instabilidade fluidodinâmica

Sabemos que $c_{eq} > 0$ dissipa energia (amortecimento).

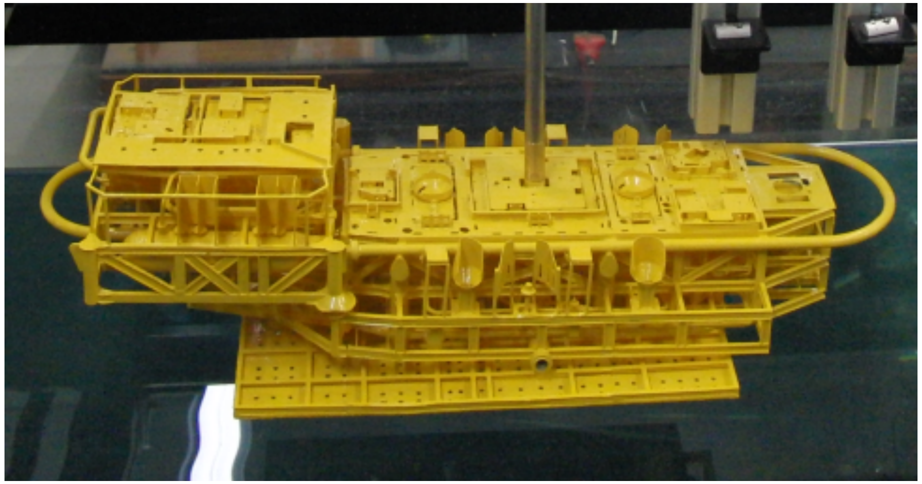
Que é o caso da força de arrasto...

$$F_D = C_D \frac{1}{2} \rho \dot{x}^2 D L$$

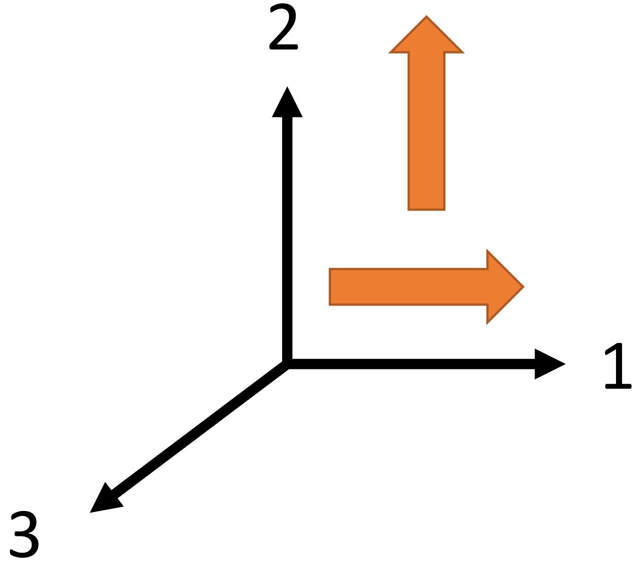
O que aconteceria se $c_{eq} < 0$?

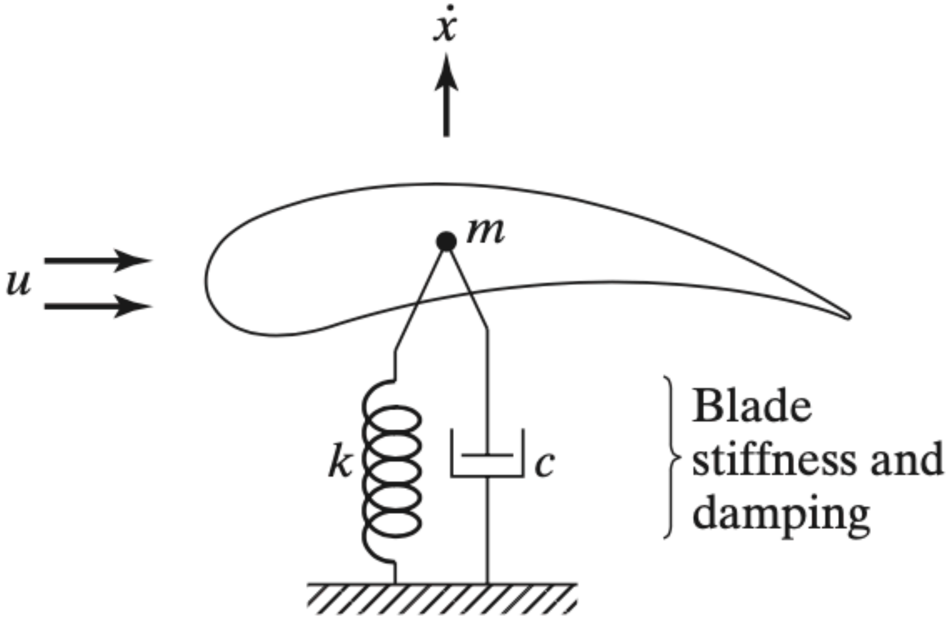
Seria possível "arrasto negativo"?

$$F_h = C_h \frac{1}{2} \rho \dot{x}^2 D L$$

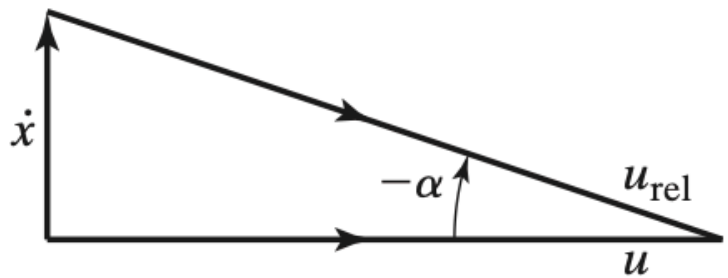


$$C_h = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$





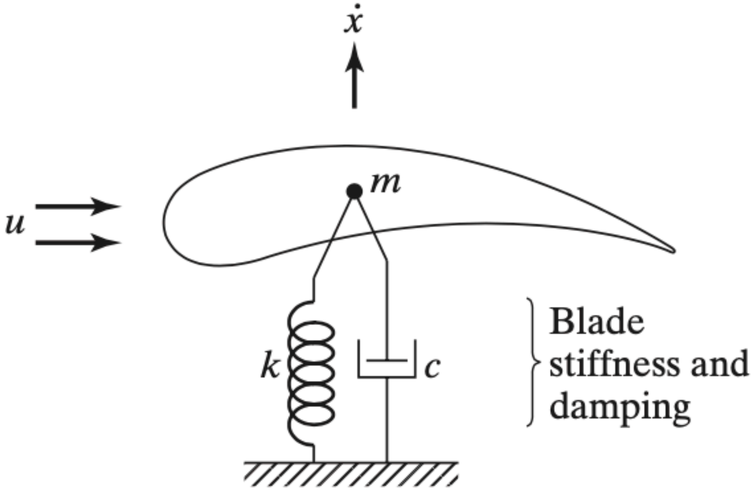
$$\alpha = -\tan^{-1}\left(\frac{\dot{x}}{u}\right)$$

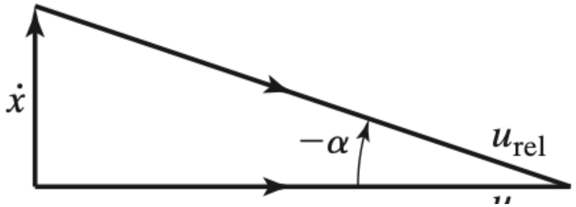


$$F = \frac{1}{2}\rho u^2 D C_x$$

$$C_x = \frac{u_{\text{rel}}^2}{u^2}\left(C_L \cos\alpha + C_D \sin\alpha\right)$$

Força vertical…





$$\alpha = -\tan^{-1}\left(\frac{\dot{x}}{u}\right)$$

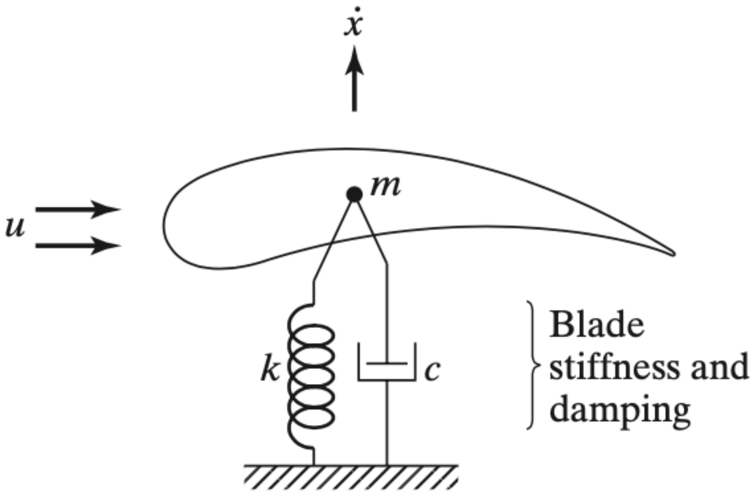
$$C_x = \cancel{\frac{u_{\text{rel}}^2}{u^2}}\left(C_L \cos\alpha + C_D \sin\alpha\right)$$

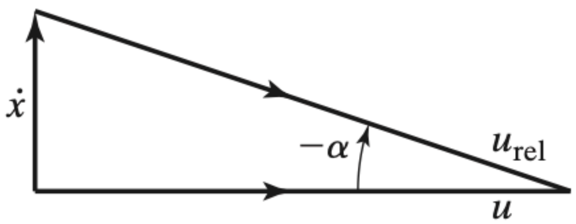
Para α pequeno: $u_{\text{rel}} \simeq u$

$$C_x = C_L \cos\alpha + C_D \sin\alpha$$

Aproximando por série de Taylor…

$$C_x \simeq C_x\Big|_{\alpha=0} + \frac{\partial C_x}{\partial \alpha}\Big|_{\alpha=0} \cdot \alpha$$





$$\alpha = -\tan^{-1}\left(\frac{\dot{x}}{u}\right)$$

Para α pequeno: $\alpha = -\dfrac{\dot{x}}{u}$

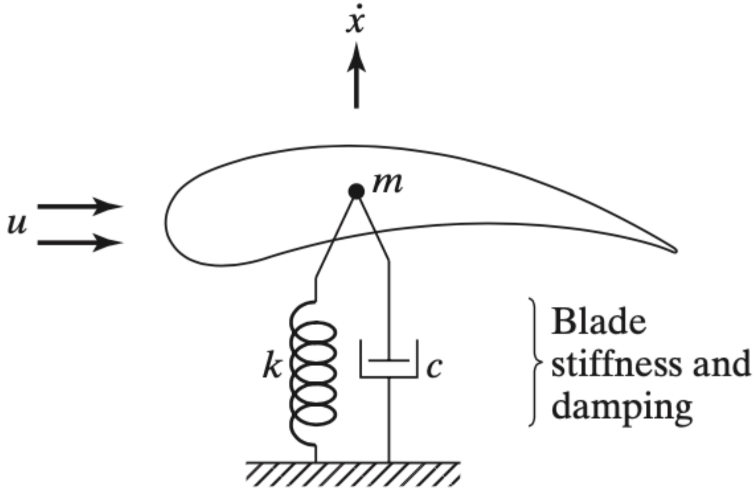
$$C_x \simeq C_x\Big|_{\alpha=0} + \frac{\partial C_x}{\partial \alpha}\Big|_{\alpha=0} \cdot \alpha$$

Que pode ser expandido como...

$$C_x = (C_L \cos\alpha + C_D \sin\alpha)\Big|_{\alpha=0}$$

$$+ \alpha\left[\frac{\partial C_L}{\partial \alpha}\cos\alpha - C_L \sin\alpha + \frac{\partial C_D}{\partial \alpha}\sin\alpha + C_D\cos\alpha\right]\Bigg|_{\alpha=0}$$

$$C_x = C_L\Big|_{\alpha=0} - \frac{\dot{x}}{u}\left\{\frac{\partial C_L}{\partial \alpha}\Big|_{\alpha=0} + C_D\Big|_{\alpha=0}\right\}$$



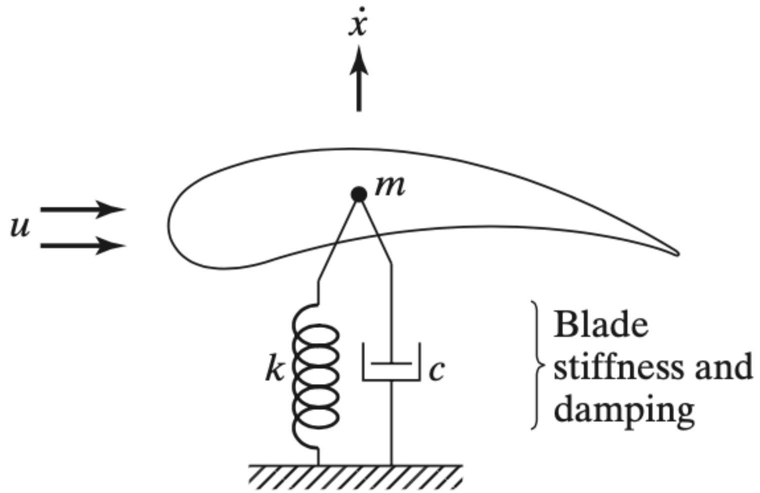
Que produz a força...

$$F = \frac{1}{2}\rho u^2 D C_L\bigg|_{\alpha=0} - \frac{1}{2}\rho u D \frac{\partial C_x}{\partial \alpha}\bigg|_{\alpha=0} \dot{x}$$

...na equação do movimento...

$$m\ddot{x} + c\dot{x} + kx = F = \frac{1}{2}\rho u^2 D C_L\bigg|_{\alpha=0} - \frac{1}{2}\rho u D \frac{\partial C_x}{\partial \alpha}\bigg|_{\alpha=0} \dot{x}$$

$$m\ddot{x} + \underbrace{\left[c + \frac{1}{2}\rho u D \frac{\partial C_x}{\partial \alpha}\bigg|_{\alpha=0}\right]}_{\text{Amortecimento equivalente}} \dot{x} + kx = 0$$



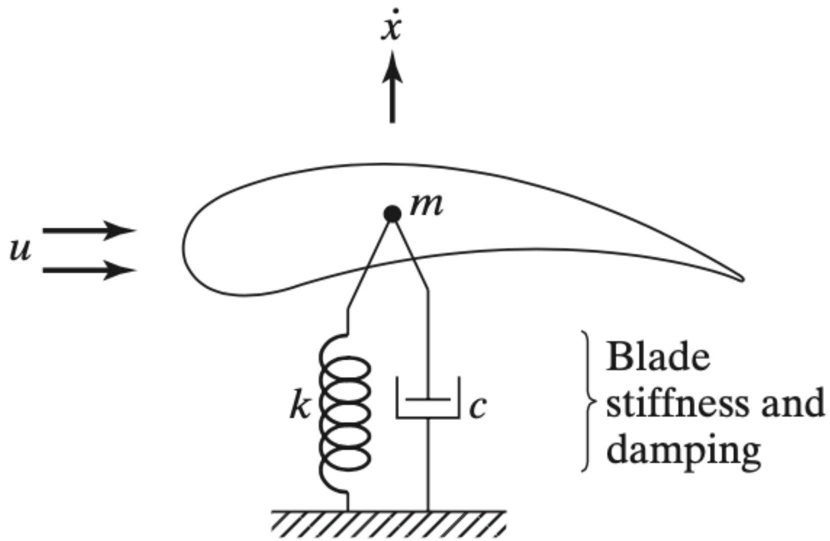
$$m\ddot{x} + \underbrace{\left[c + \frac{1}{2}\rho u D \left.\frac{\partial C_x}{\partial \alpha}\right|_{\alpha=0}\right]}_{\text{Amortecimento equivalente}} \dot{x} + kx = 0$$

CONDIÇÃO DE ESTABILIDADE: $c_{eq} > 0$

Como este amortecimento varia com a velocidade do escoamento incidente…

$$u = -\left[\frac{2c}{\rho D \left.\frac{\partial C_x}{\partial \alpha}\right|_{\alpha=0}}\right]$$

Haverá uma velocidade crítica do escoamento acima da qual o regime é instável (amortecimento equivalente é negativo): fenômeno de *galloping*.



Galloping ocorrerá na direção transversal ao escoamento.

O arrasto sempre será dissipativo.

A sustentação pode induzir à instabilidade.

BEM-VINDOS ÀS VIBRAÇÕES INDUZIDAS PELO ESCOAMENTO
(venha fazer iniciação científica)