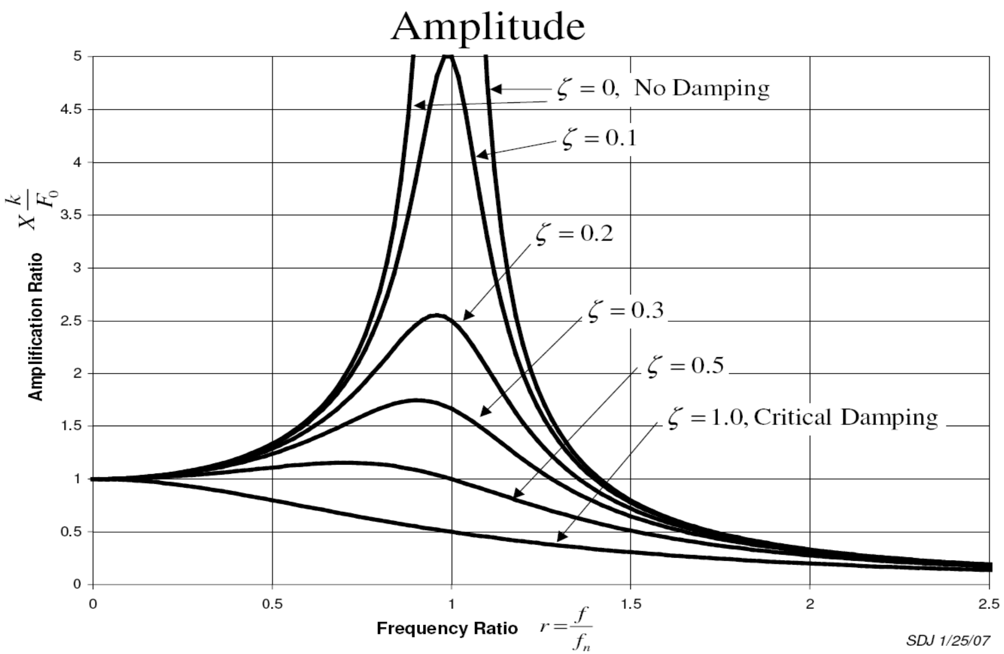
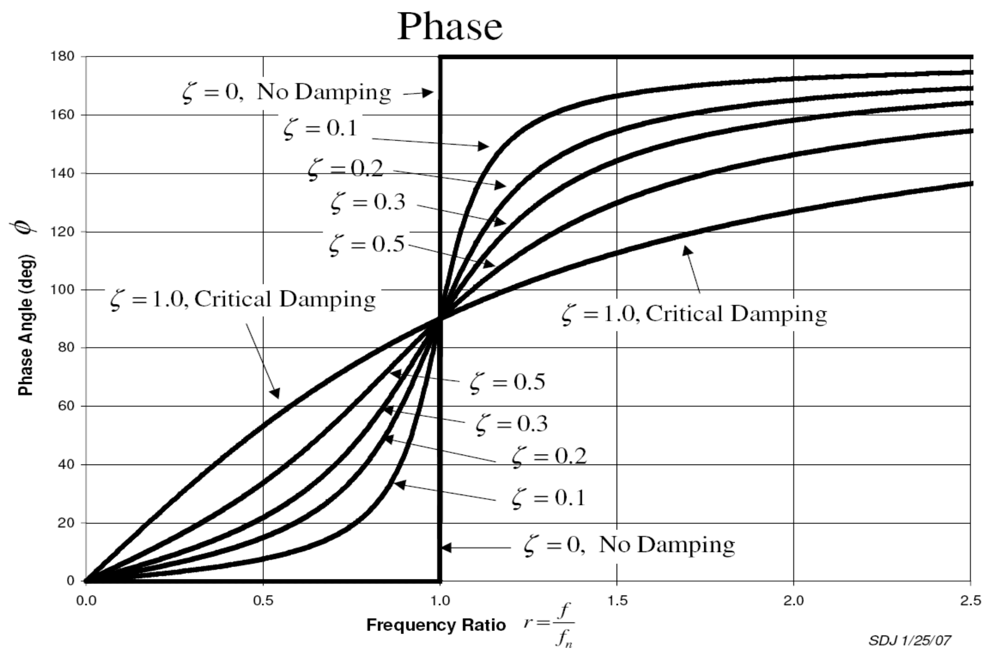
Dinâmica de Sistemas Navais e Oceânicos

PNV3314 Dinâmica de Sistemas

Aula 11



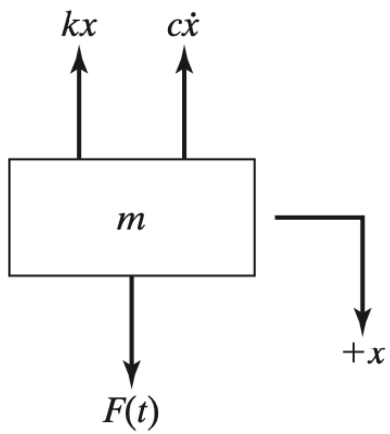


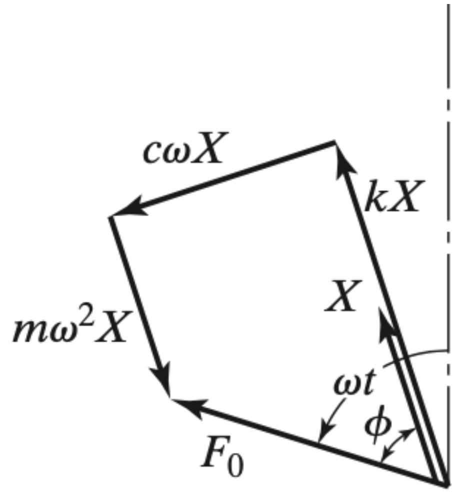
$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

$$x(t) = X\cos(\omega t - \phi)$$

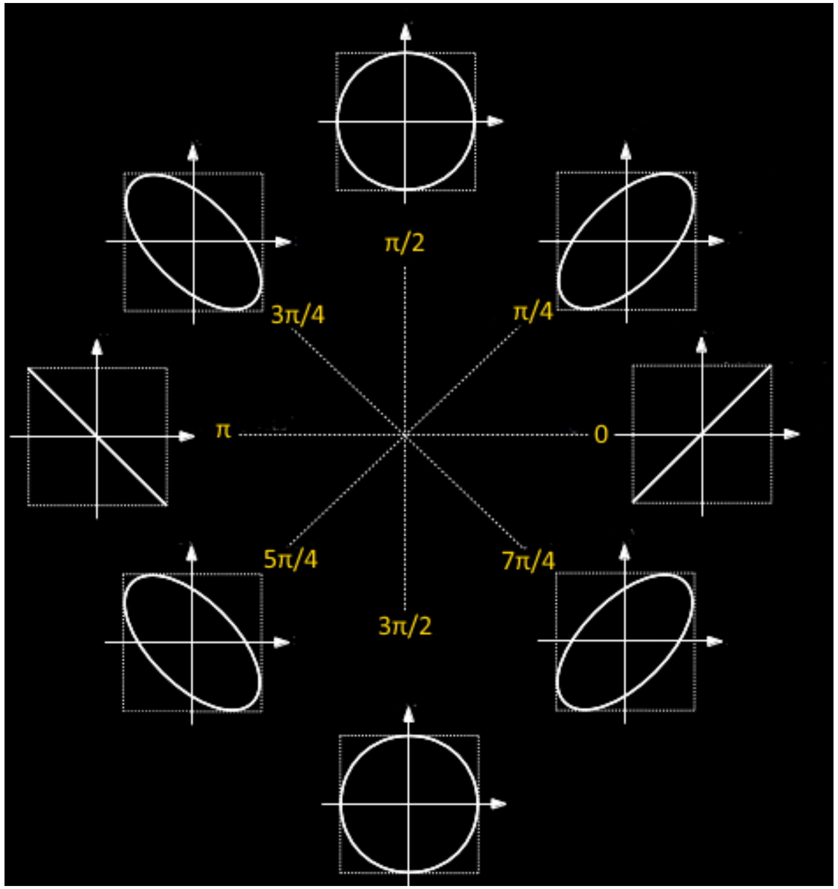
$$\dot{x}(t) = -\omega X \sin(\omega t - \phi)$$

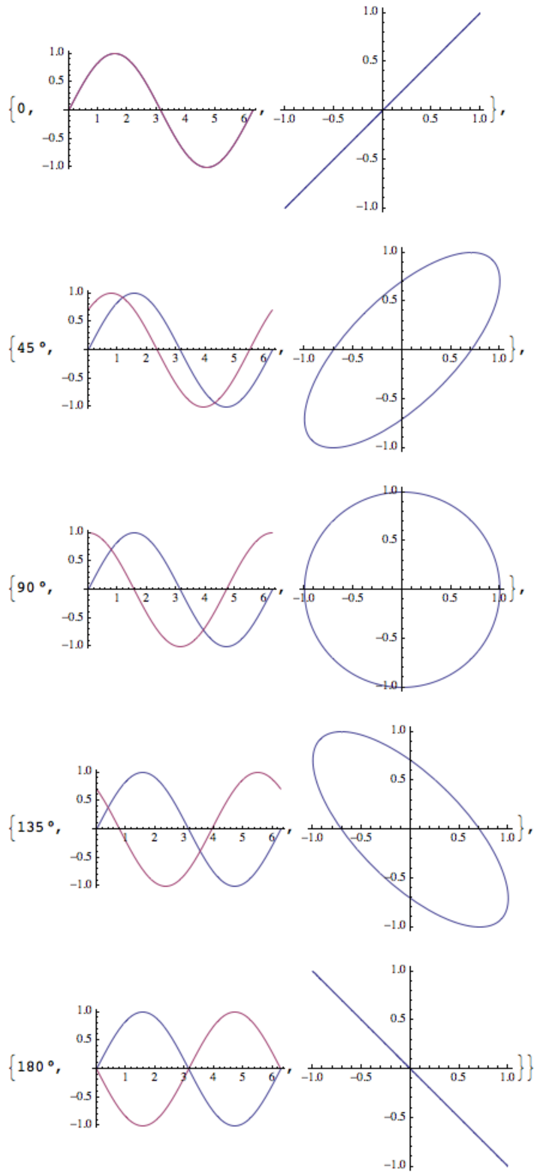
$$\ddot{x}(t) = -\omega^2 X \cos(\omega t - \phi)$$





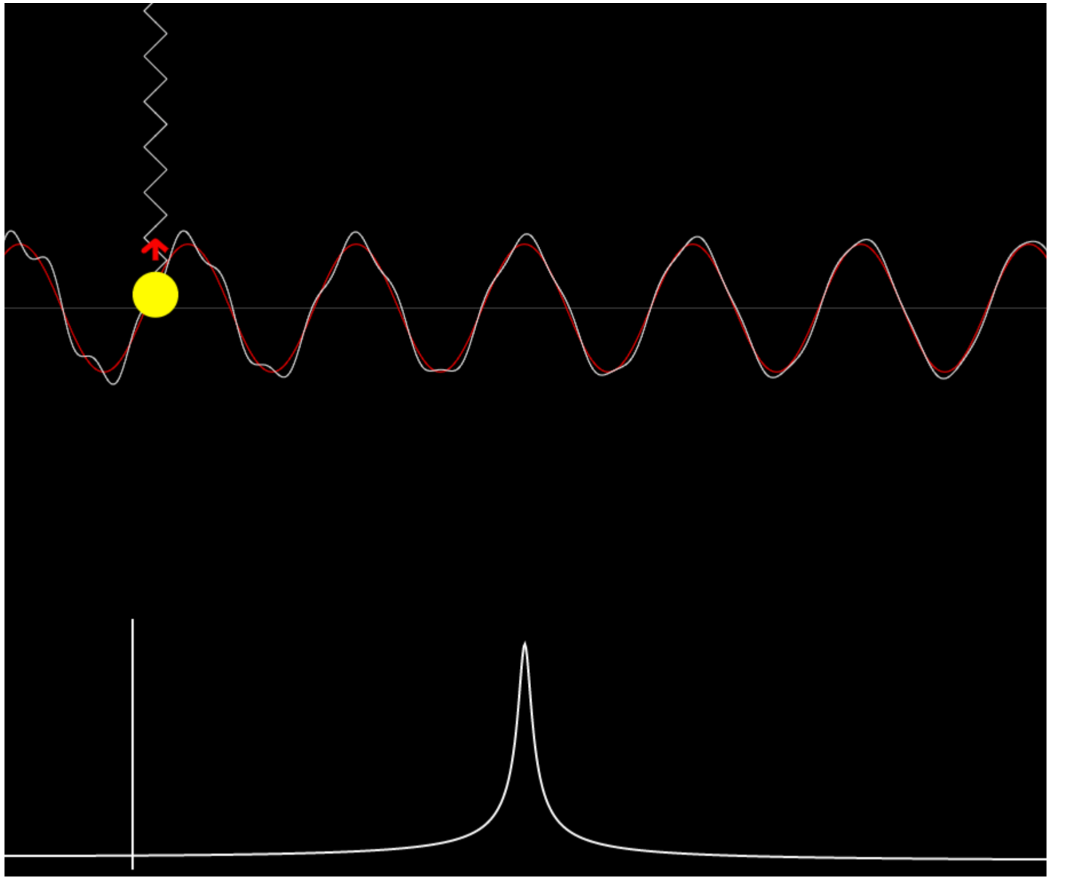
Figuras de Lissajous



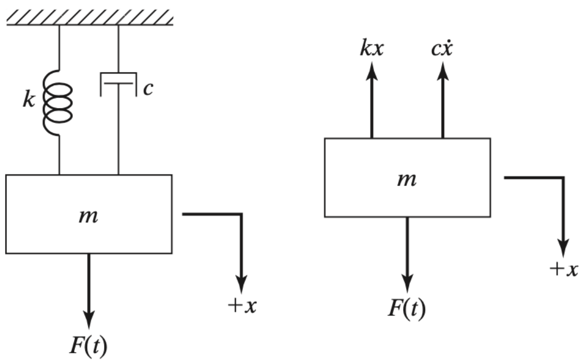


Visualização:

https://www.falstad.com/harmonicosc



Força transmitida para a base



(a) (b) Free-body diagram

Em regime permanente...

$$x_p(t) = X\cos(\omega t - \phi)$$

$$= X(\cos\omega t\cos\phi + \sin\omega t\sin\phi) = A\cos\omega t + B\sin\omega t$$

$$A = X\cos\phi \qquad B = X\sin\phi$$

A força transmitida para a base será a soma da força da mola e do amortecedor

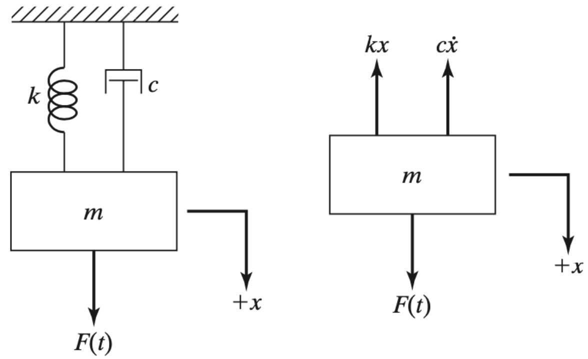
$$\boxed{f_T(t) = k\,x(t) + c\dot{x}(t)} = kX\cos(\omega t - \phi) - c\omega X\sin(\omega t - \phi)$$

$$f_T(t) = kA\cos\omega t + kB\sin\omega t + c\omega B\cos\omega t - c\omega A\sin\omega t$$

$$= (kA + c\omega B)\cos\omega t + (kB - c\omega A)\sin\omega t$$

$$f_T(t) = kA\cos\omega t + kB\sin\omega t + c\omega B\cos\omega t - c\omega A\sin\omega t$$

$$= (kA + c\omega B)\cos\omega t + (kB - c\omega A)\sin\omega t$$



(a) (b) Free-body diagram

Denotando...

$$(kA + c\omega B) = F_T\cos\phi_T \qquad (kB - c\omega A) = F_T\sin\phi_T$$

Escrevemos...

$$f_T(t) = F_T\cos\omega t\cos\phi_T + F_T\sin\omega t\sin\phi_T = F_T\cos(\omega t - \phi_T)$$

Como...

$$F_T = \left[(F_T\cos\phi_T)^2 + (F_T\sin\phi_T)^2\right]^{\frac{1}{2}} = \left[(kA + c\omega B)^2 + (kB - c\omega A)^2\right]^{\frac{1}{2}}$$

$$= \left[(kX\cos\phi + c\omega X\sin\phi)^2 + (kX\sin\phi - c\omega X\cos\phi)^2\right]^{\frac{1}{2}}$$

Resulta em...

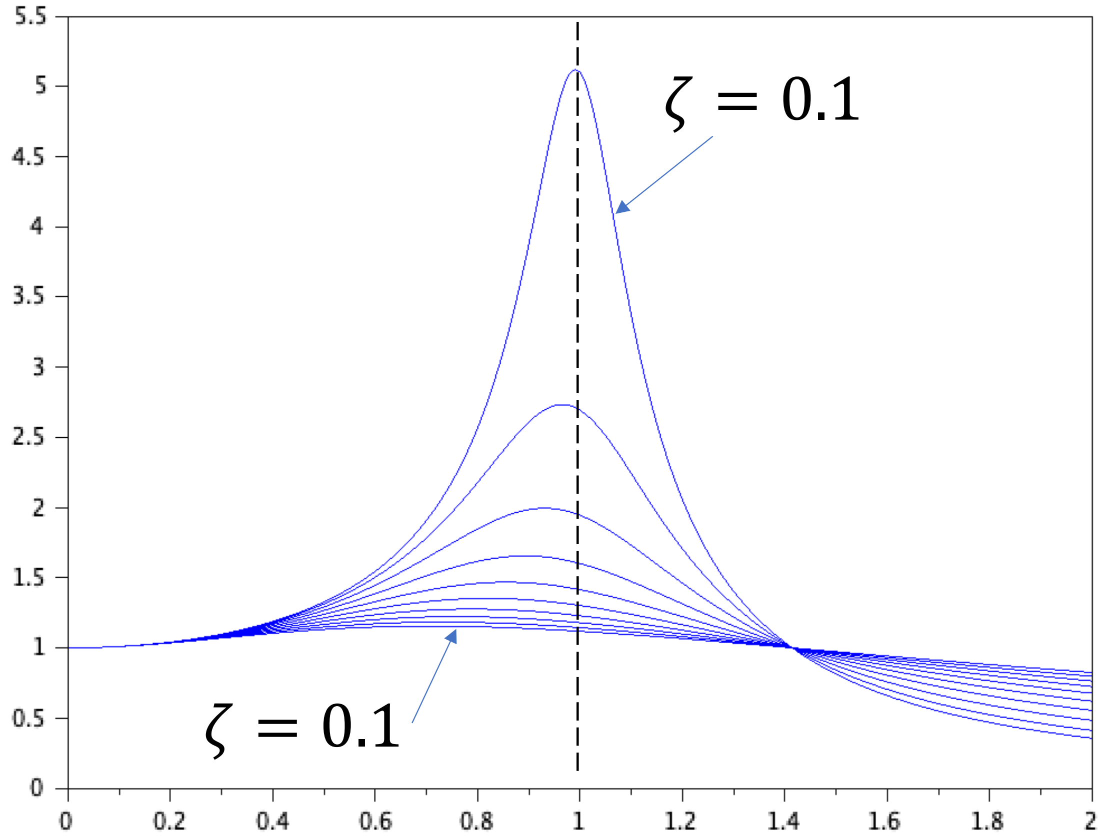
Resulta em...

$$F_T = X\left(k^2 + c^2\omega^2\right)^{\frac{1}{2}} = F_0 \frac{\left(k^2 + c^2\omega^2\right)^{\frac{1}{2}}}{\left[\left(k - m\omega^2\right)^2 + c^2\omega^2\right]} = F_0 \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

$$\tan\phi_T = \frac{kB - c\omega A}{kA + c\omega B} = \frac{kX\sin\phi - c\omega X\cos\phi}{kX\cos\phi + c\omega X\sin\phi} = \frac{\tan\phi - \frac{c\omega}{k}}{1 + \frac{c\omega}{k}\tan\phi}$$

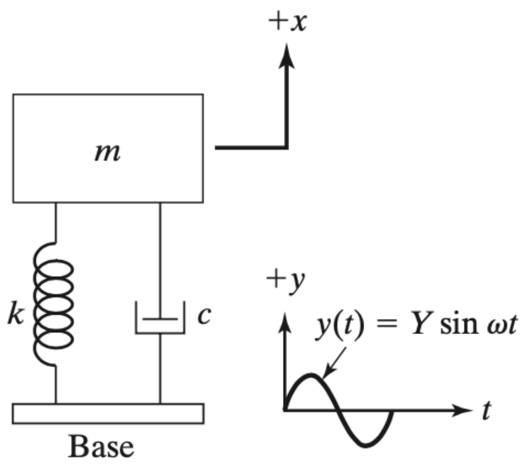
$$\phi_T = \tan^{-1}\left(\frac{2\zeta r^3}{1 - r^2 + (2\zeta r)^2}\right)$$

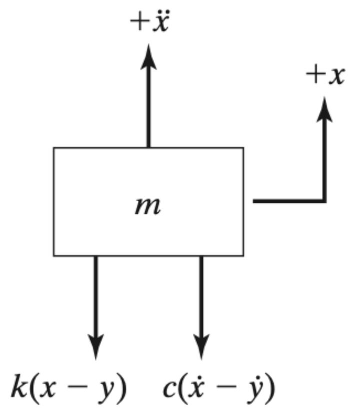
$$\frac{F_T}{F_0} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$



Vibração amortecida com movimento da base

Oscilação amortecida com movimento imposto na base





$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$y(t) = Y \sin \omega t$$

$$m\ddot{x} + c\dot{x} + kx = ky + c\dot{y} = kY \sin \omega t + c\omega Y \cos \omega t$$
$$= A \sin(\omega t - \alpha)$$

$$A = Y\sqrt{k^2 + (c\omega)^2} \qquad \alpha = \tan^{-1}\left[-\frac{c\omega}{k}\right]$$

Movimentar a base é equivalente a aplicar
uma força de magnitude A na massa...

Como vimos antes...

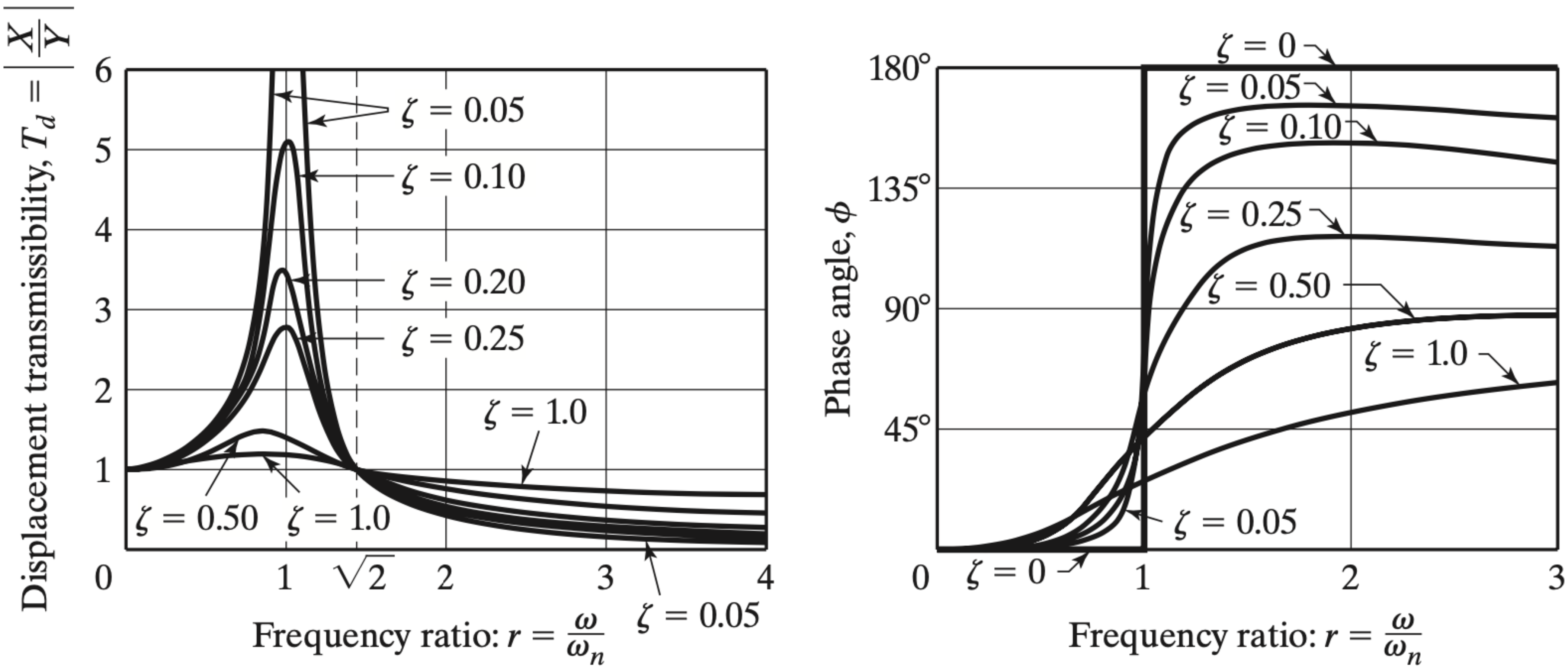
$$x_p(t) = \frac{Y\sqrt{k^2 + (c\omega)^2}}{\left[(k - m\omega^2)^2 + (c\omega)^2\right]^{1/2}} \sin(\omega t - \phi_1 - \alpha) \qquad \phi_1 = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right)$$

Que pode ser reescrita como $x_p(t) = X \sin(\omega t - \phi)$

$$\frac{X}{Y} = \left[\frac{k^2 + (c\omega)^2}{(k - m\omega^2)^2 + (c\omega)^2}\right]^{1/2} = \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}\right]^{1/2}$$

$$\phi = \tan^{-1}\left[\frac{mc\omega^3}{k(k - m\omega^2) + (\omega c)^2}\right] = \tan^{-1}\left[\frac{2\zeta r^3}{1 + (4\zeta^2 - 1)r^2}\right]$$

Transmissibilidade de deslocamento



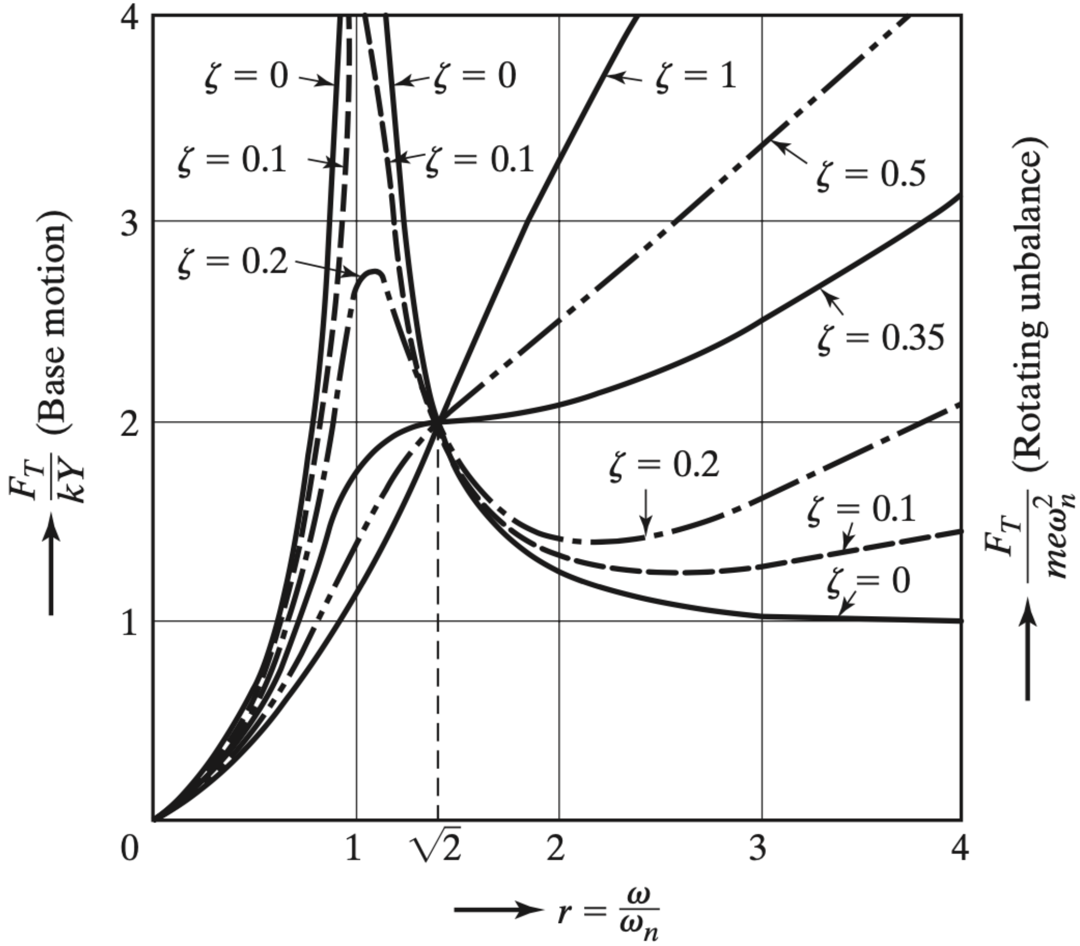
Transmissibilidade de força

$$F = k(x - y) + c(\dot{x} - \dot{y}) = -m\ddot{x}$$

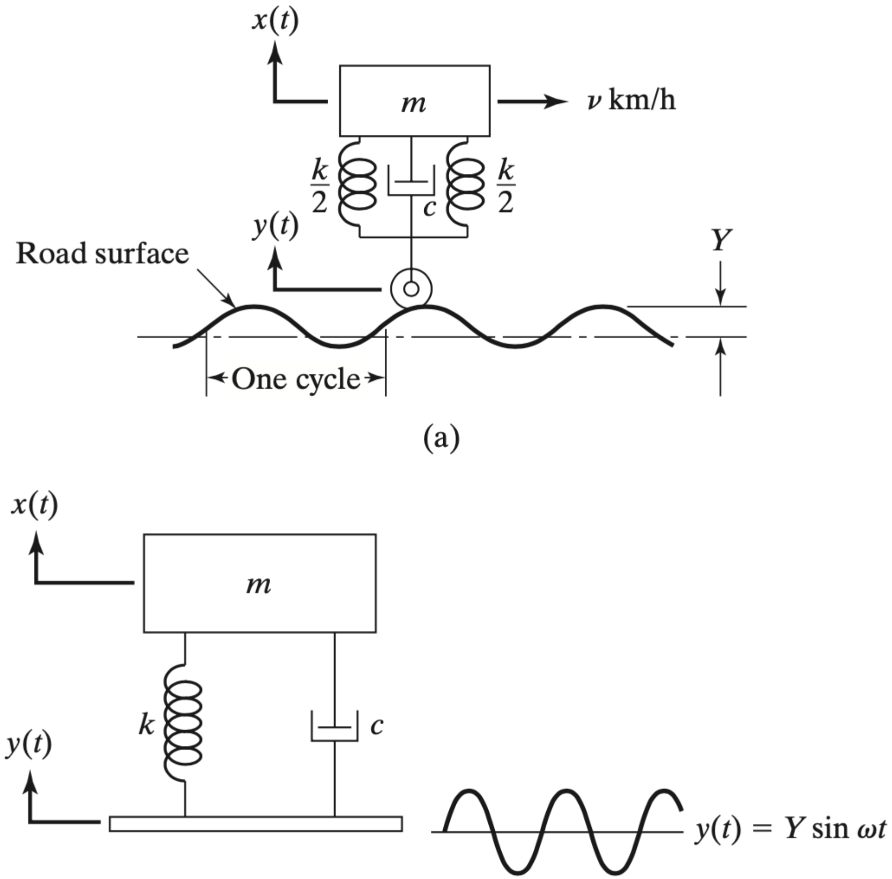
$$F = m\omega^2 X \sin(\omega t - \phi)$$

$$= F_T \sin(\omega t - \phi)$$

$$\frac{F_T}{kY} = r^2 \left[\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2} \right]^{1/2}$$



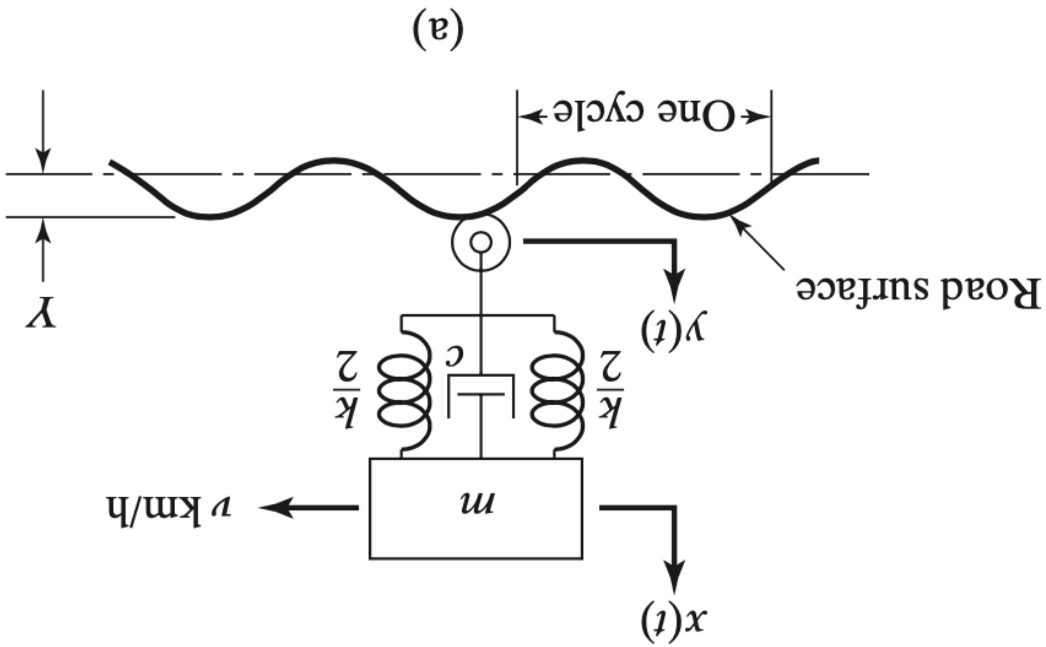
Exemplo: veículo em pista ondulada

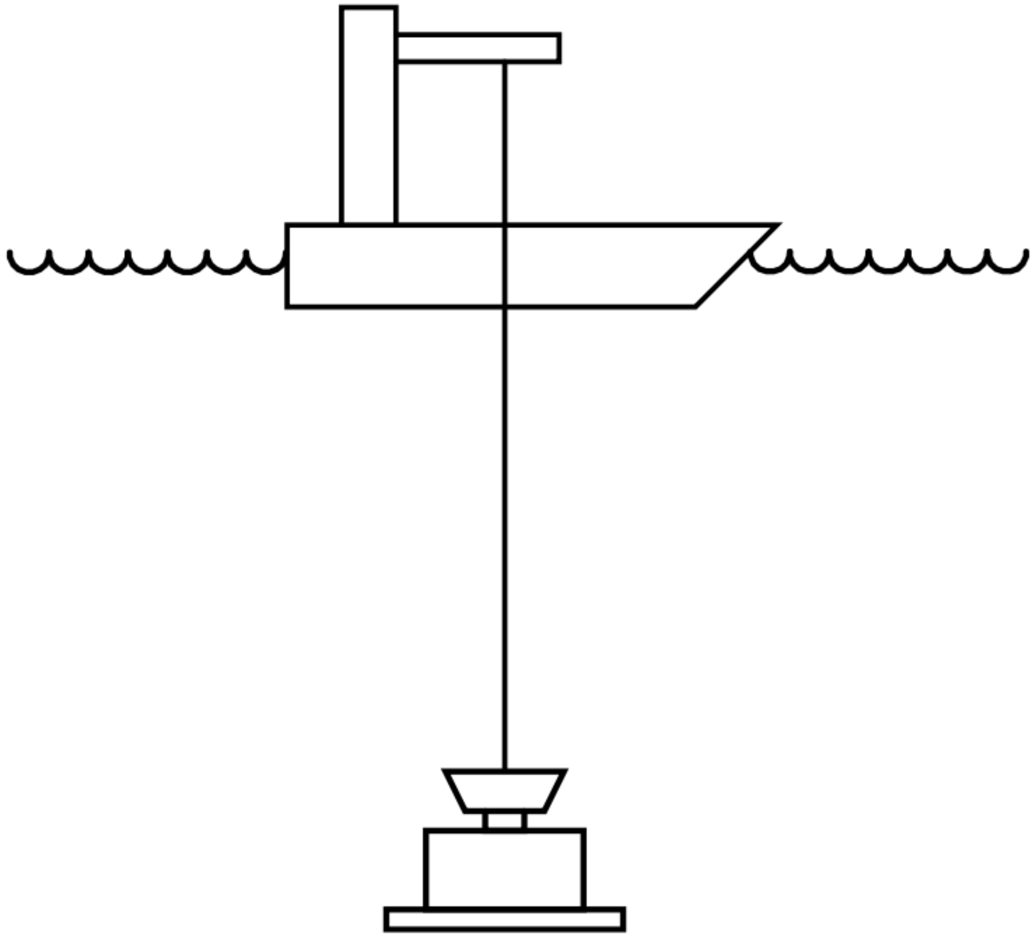


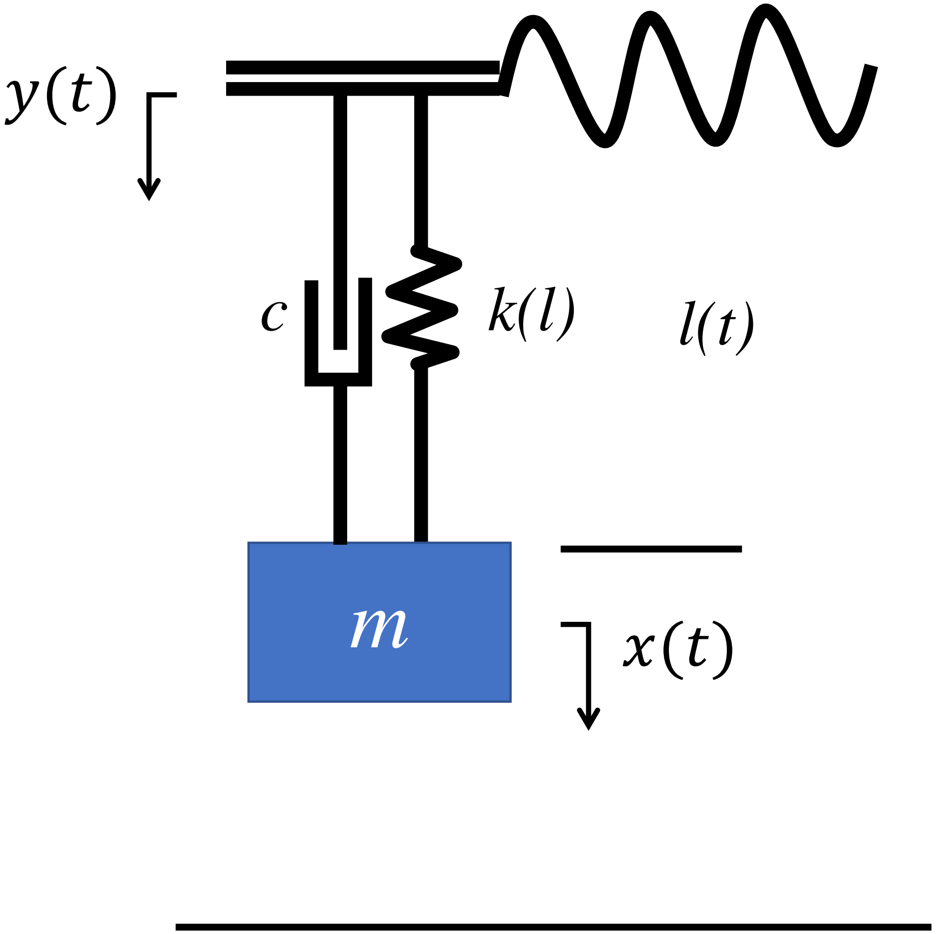
(a)

(b)

Exemplo: “manifold em mar ondulado”





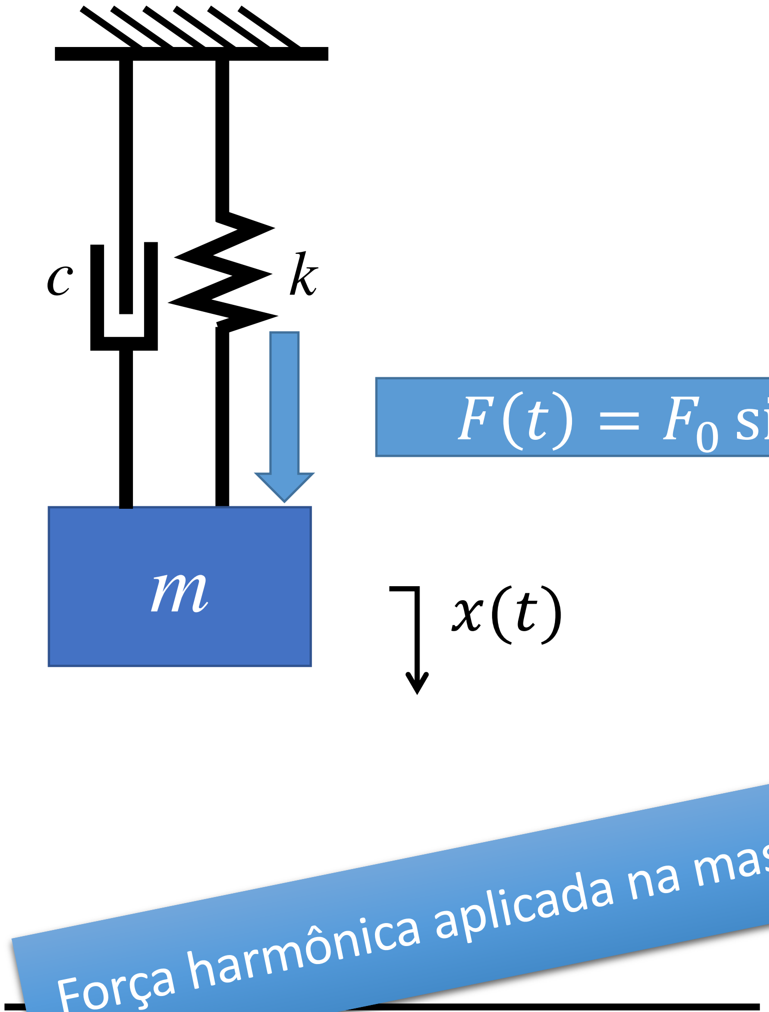


Modelagem do manifold “em ar” com movimento harmônico imposto na embarcação

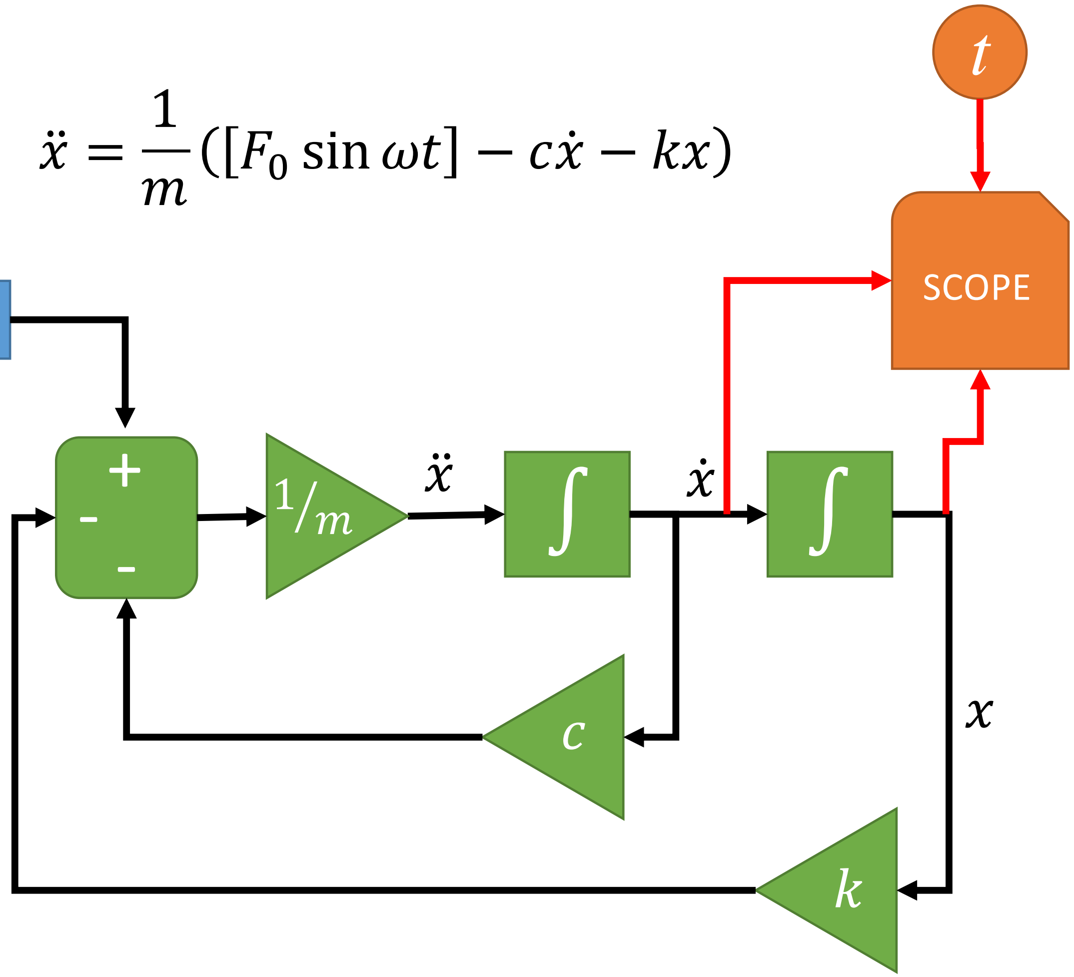
Simulação de sistema dinâmico

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$$

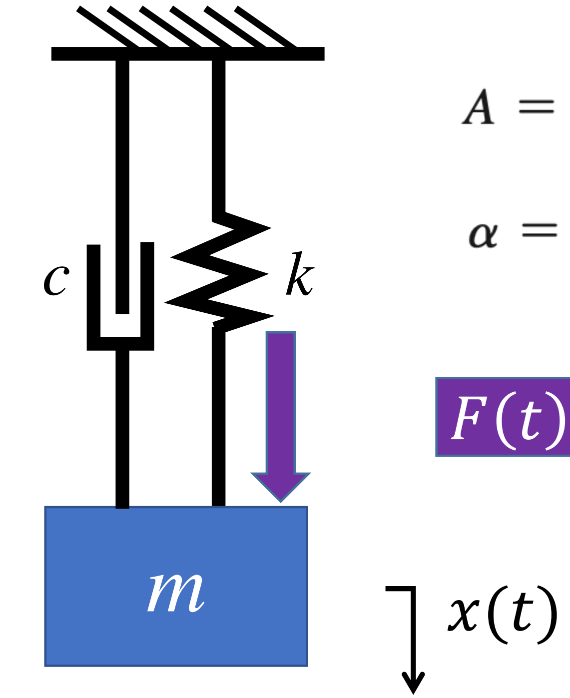
$$\ddot{x} = \frac{1}{m}([F_0 \sin \omega t] - c\dot{x} - kx)$$



$F(t) = F_0 \sin \omega t$



Força harmônica aplicada na massa



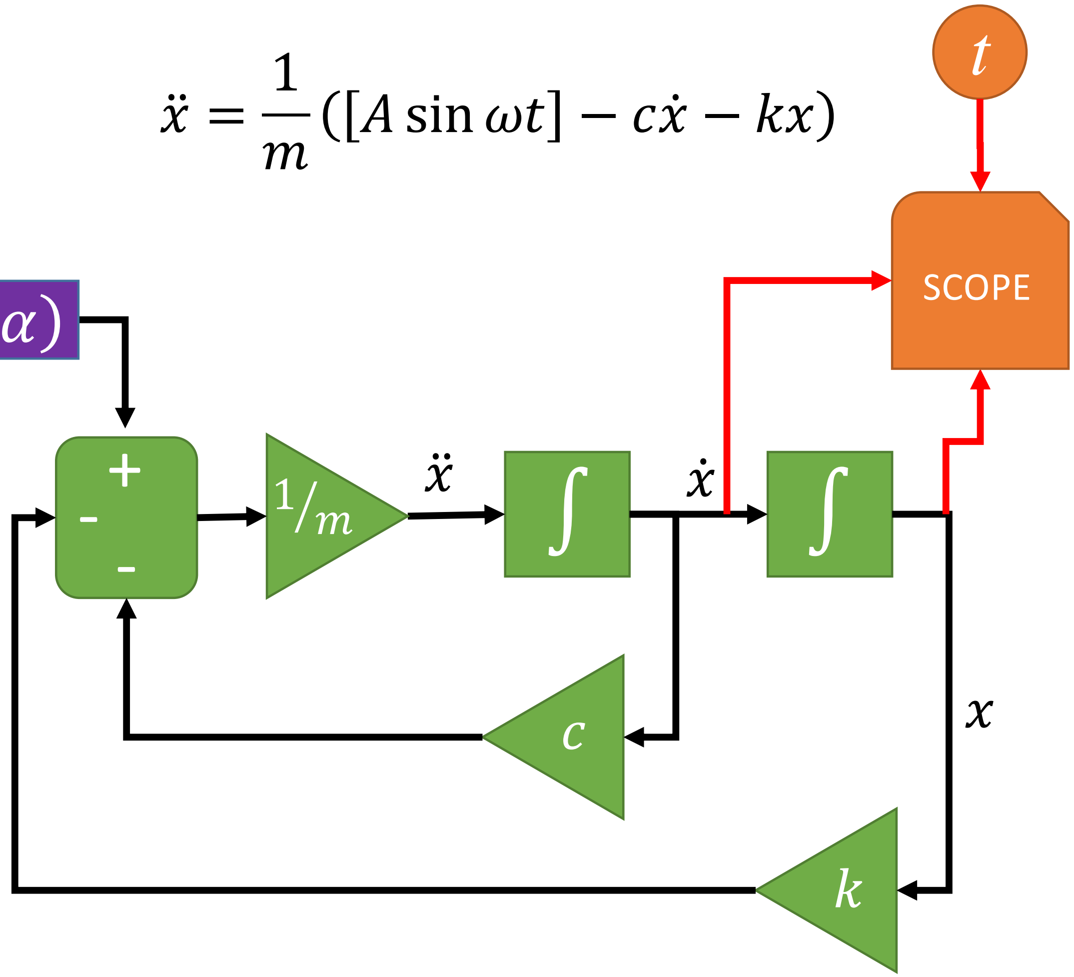
$$A = Y\sqrt{k^2 + (c\omega)^2}$$

$$\alpha = \tan^{-1}\left[-\frac{c\omega}{k}\right]$$

$$m\ddot{x} + c\dot{x} + kx = A\sin(\omega t - \alpha)$$

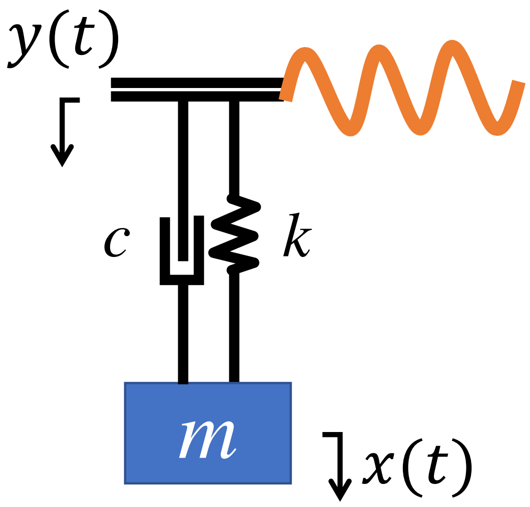
$$\ddot{x} = \frac{1}{m}([A\sin\omega t] - c\dot{x} - kx)$$

$F(t) = A\sin(\omega t - \alpha)$



1

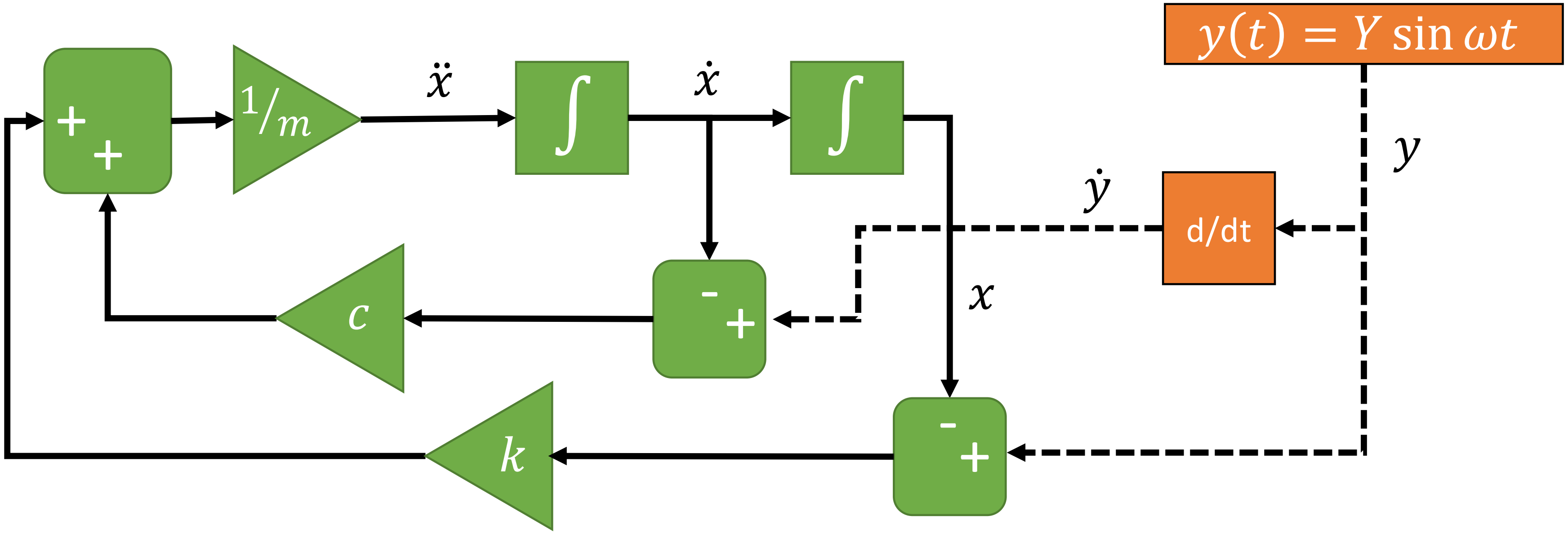
Força harmônica equivalente aplicada na massa



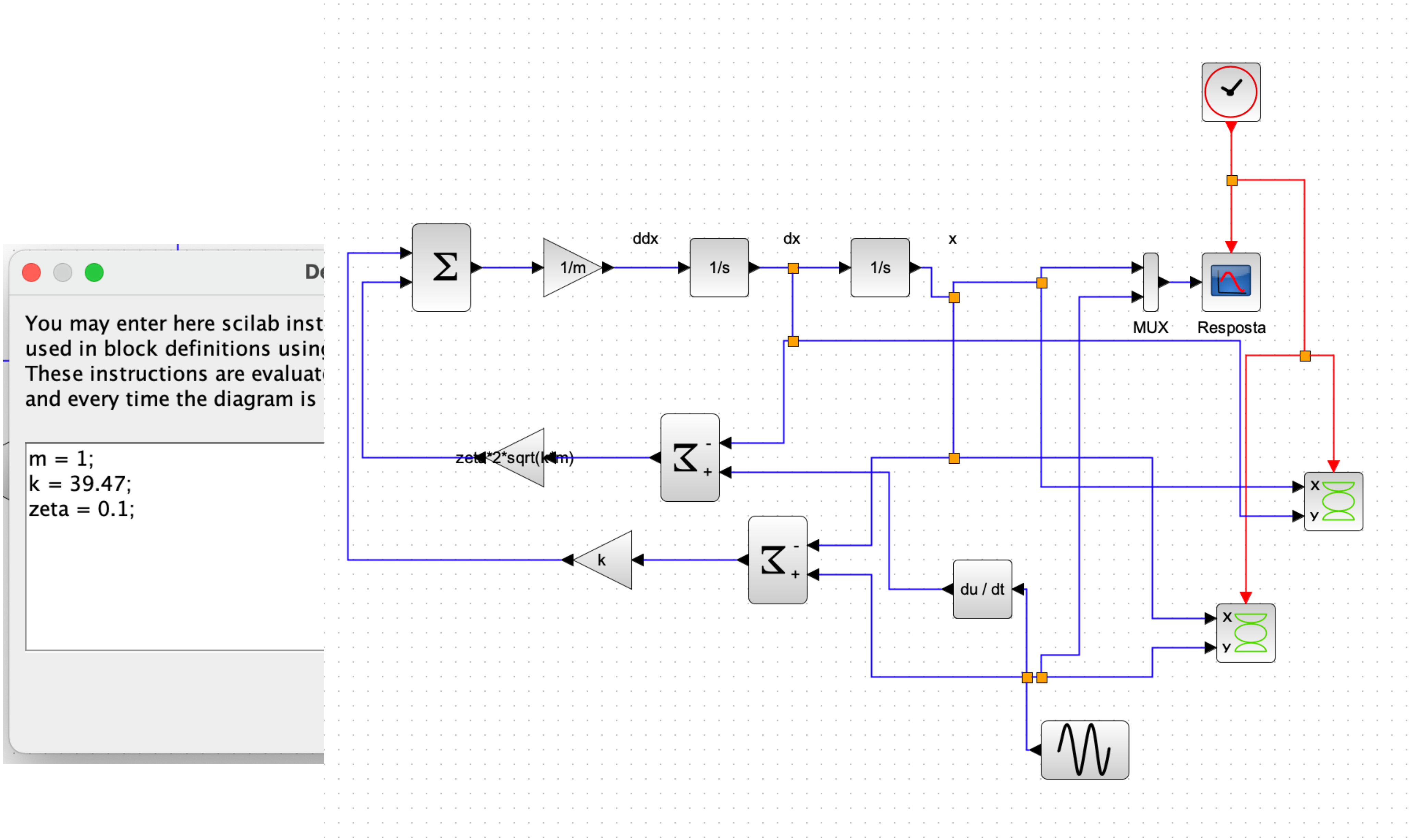
$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$y(t) = Y \sin \omega t$$

$$\ddot{x} = \frac{1}{m}[c(\dot{y} - \dot{x}) + k(y - x)]$$



2


De
You may enter here scilab inst
used in block definitions usin
These instructions are evaluat
and every time the diagram is
m = 1;
k = 39.47;
zeta = 0.1;
1/m
ddx
1/s
dx
1/s
x
MUX
Resposta
zeta*2*sqrt(k*m)
k
du / dt
x
y
x
y