Dinâmica de Sistemas Navais e Oceânicos

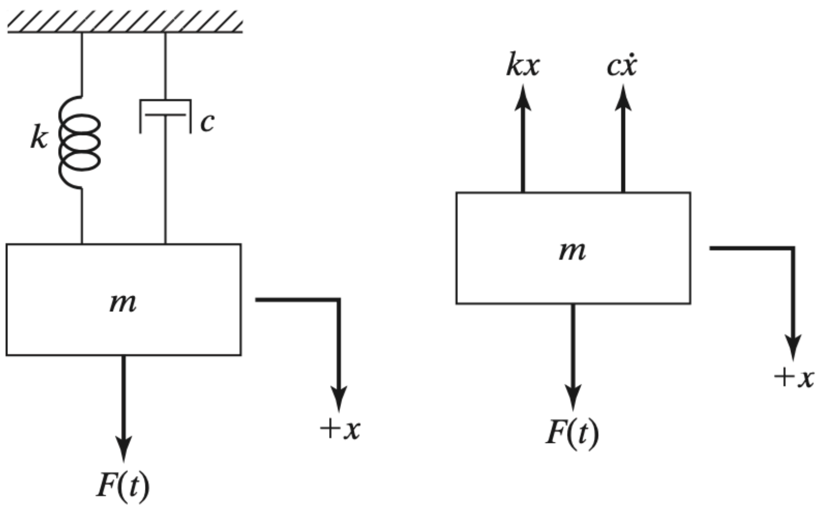
PNV3314 Dinâmica de Sistemas

Aula 10

Vibração amortecida com excitação harmônica

Oscilação forçada amortecida

Excitação externa



(a) (b) Free-body diagram

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

A solução particular é do tipo...

$$x_p(t) = X \cos(\omega t - \phi)$$

...derivando e substituindo:

$$X[(k - m\omega^2)\cos(\omega t - \phi) - c\omega \sin(\omega t - \phi)] = F_0 \cos \omega t$$

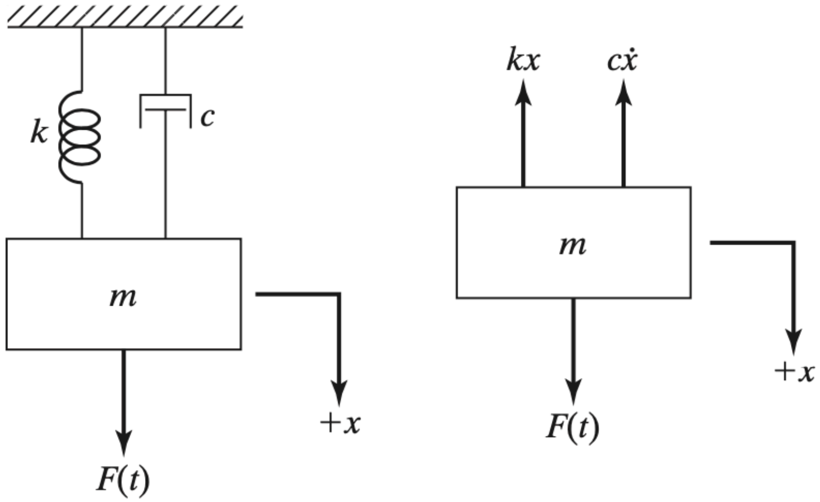
Usando as relações trigonométricas...

$$\cos(\omega t - \phi) = \cos \omega t \cos \phi + \sin \omega t \sin \phi$$
$$\sin(\omega t - \phi) = \sin \omega t \cos \phi - \cos \omega t \sin \phi$$

...e agrupando senos e cossenos

$$X[(k - m\omega^2)\cos \phi + c\omega \sin \phi] = F_0$$
$$X[(k - m\omega^2)\sin \phi - c\omega \cos \phi] = 0$$

Excitação externa



(a) (b) Free-body diagram

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

Descobrimos

$$X = \frac{F_0}{[(k - m\omega^2)^2 + c^2\omega^2]^{1/2}}$$

$$\phi = \tan^{-1}\left(\frac{c\omega}{k - m\omega^2}\right)$$

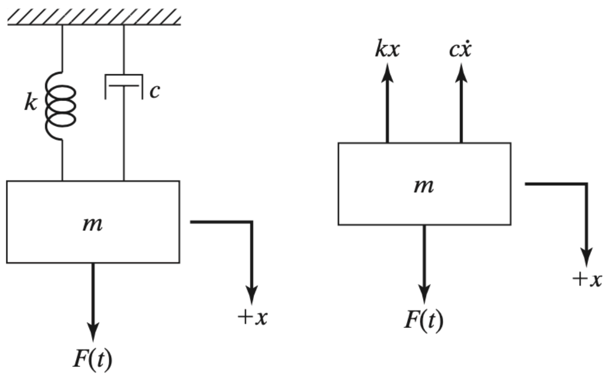
Relembrando os termos...

$$\omega_n = \sqrt{\frac{k}{m}} \qquad \zeta = \frac{c}{c_c} = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{mk}}; \qquad \frac{c}{m} = 2\zeta\omega_n, \qquad \delta_{st} = \frac{F_0}{k} \qquad r = \frac{\omega}{\omega_n}$$

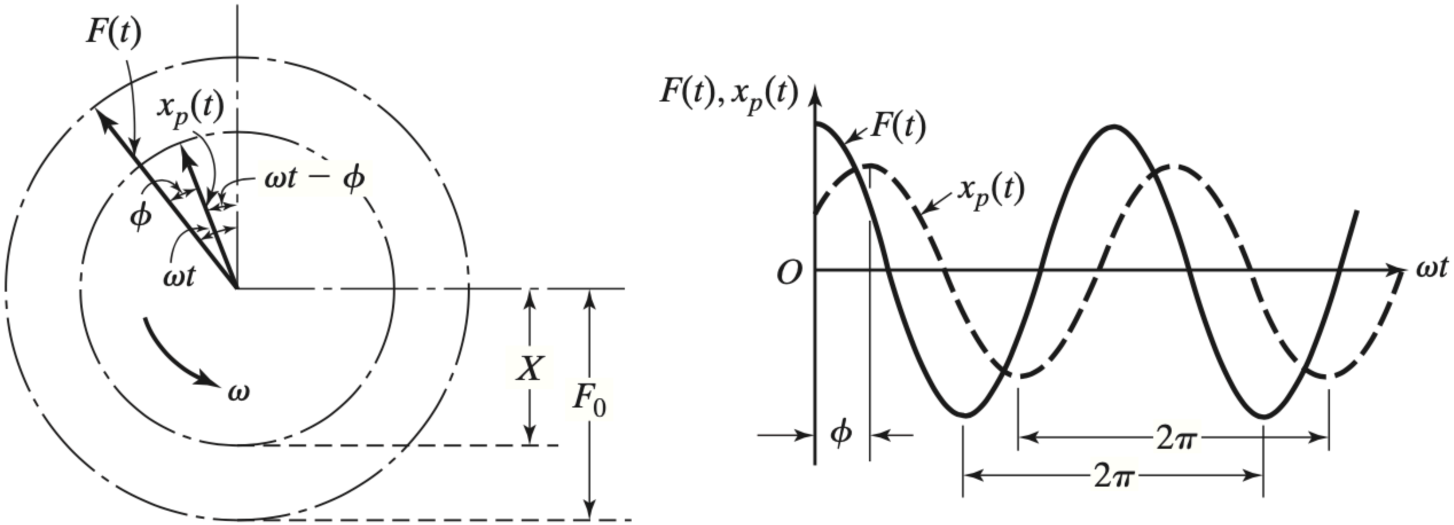
...obtemos

$$\frac{X}{\delta_{st}} = \frac{1}{\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2\right\}^{1/2}} = \frac{1}{\sqrt{(1 - r^2)^2 + (2\zeta r)^2}}$$

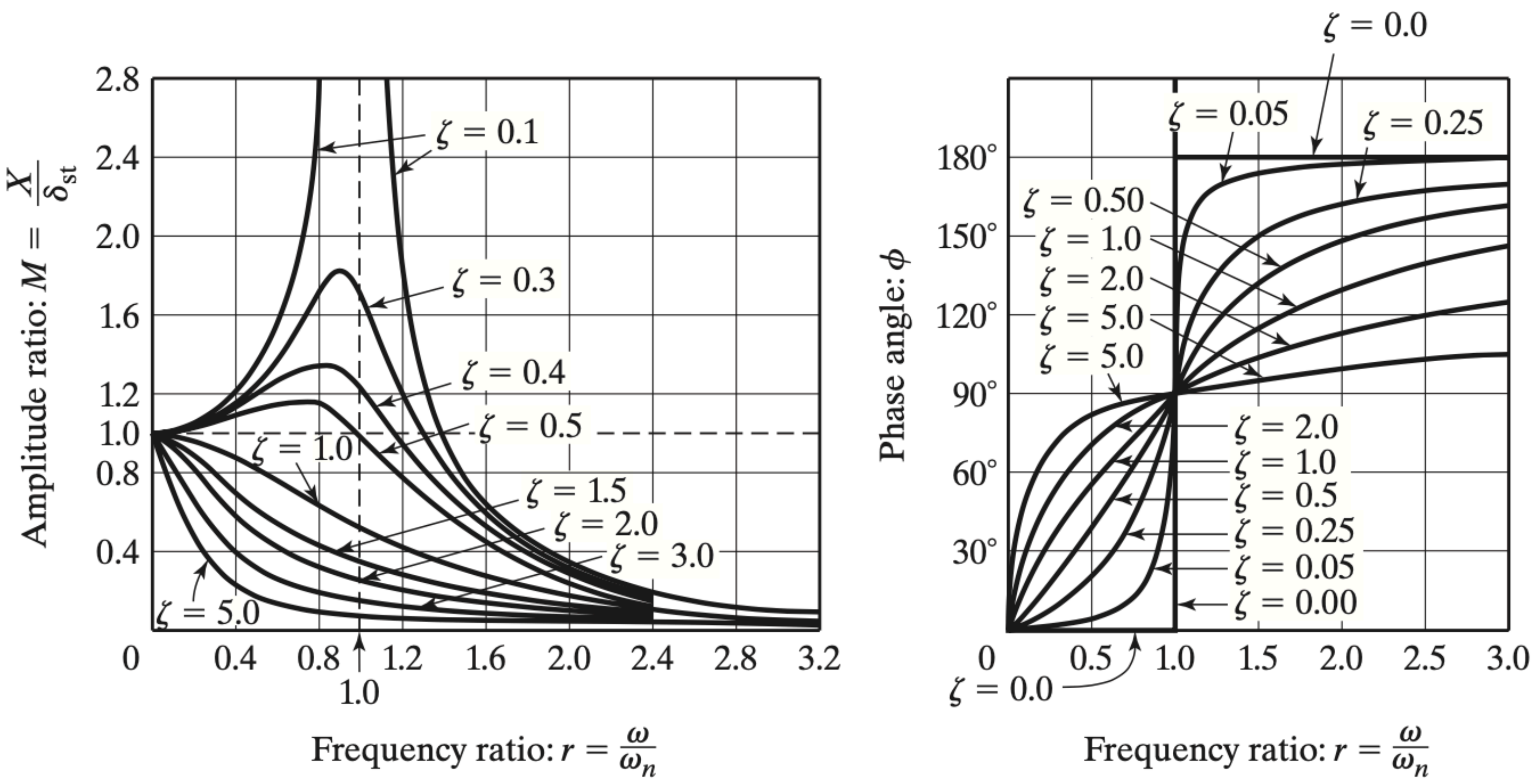
$$\phi = \tan^{-1}\left\{\frac{2\zeta\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right\} = \tan^{-1}\left(\frac{2\zeta r}{1 - r^2}\right)$$



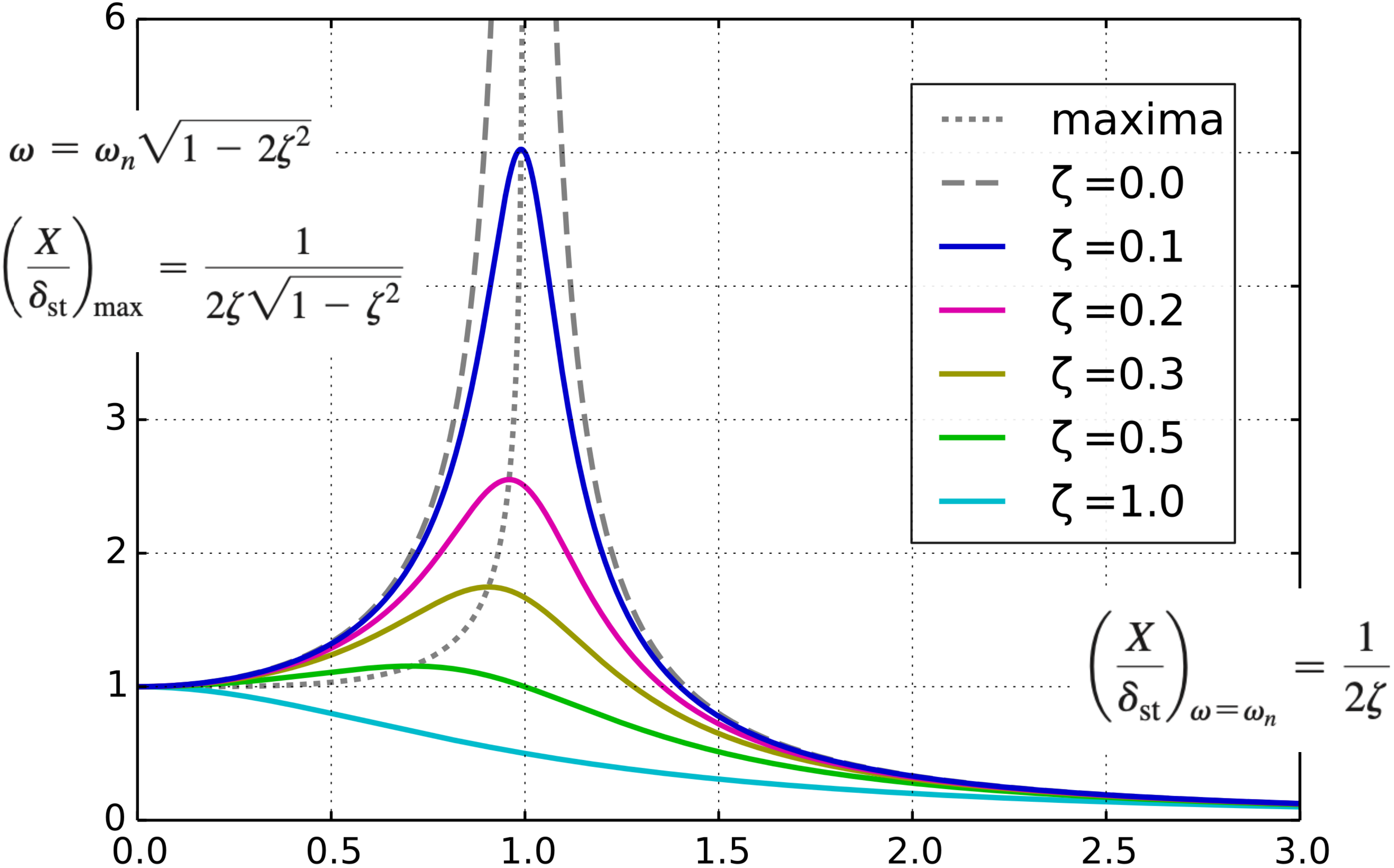
(a) (b) Free-body diagram



(a) Graphical representation (b) Vectorial representation

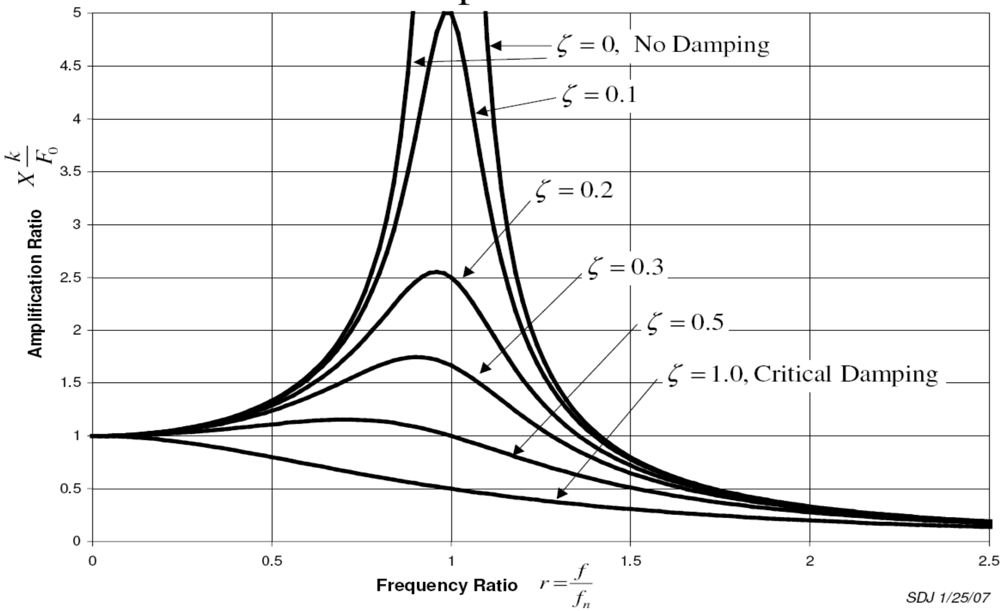


Amplitude ratio: $M = \frac{X}{\delta_{st}}$
Frequency ratio: $r = \frac{\omega}{\omega_n}$
$\zeta = 0.1$
$\zeta = 0.3$
$\zeta = 0.4$
$\zeta = 0.5$
$\zeta = 1.0$
$\zeta = 1.5$
$\zeta = 2.0$
$\zeta = 3.0$
$\zeta = 5.0$
Phase angle: ϕ
$\zeta = 0.0$
$\zeta = 0.05$
$\zeta = 0.25$
$\zeta = 0.50$
$\zeta = 1.0$
$\zeta = 2.0$
$\zeta = 5.0$
$\zeta = 0.00$

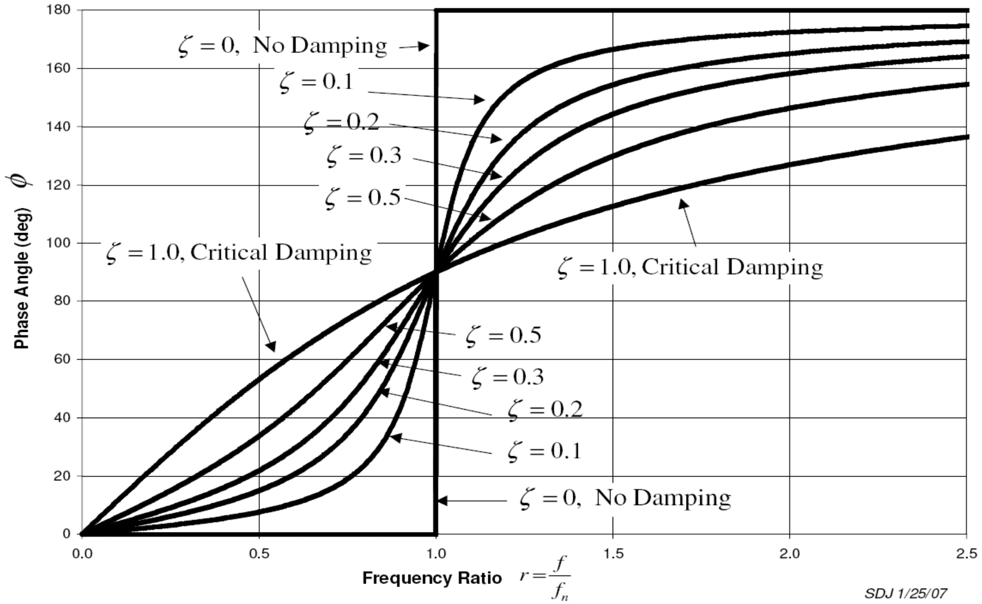


$\omega = \omega_n\sqrt{1 - 2\zeta^2}$
$\left(\frac{X}{\delta_{st}}\right)_{max} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$
$\left(\frac{X}{\delta_{st}}\right)_{\omega=\omega_n} = \frac{1}{2\zeta}$
maxima
ζ =0.0
ζ =0.1
ζ =0.2
ζ =0.3
ζ =0.5
ζ =1.0
6
3
2
1
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Amplitude



Phase

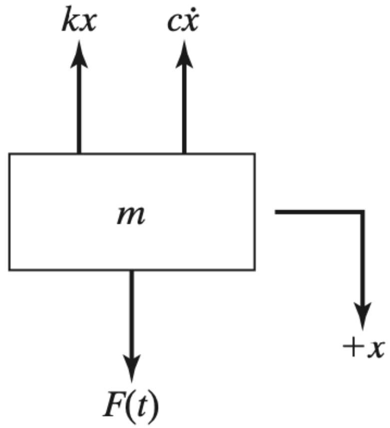


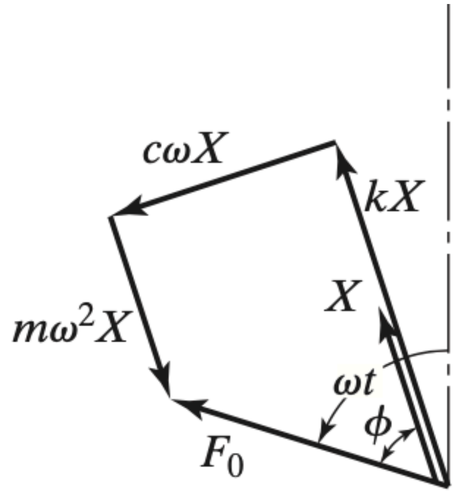
$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

$$x(t) = X \cos(\omega t - \phi)$$

$$\dot{x}(t) = -\omega X \sin(\omega t - \phi)$$

$$\ddot{x}(t) = -\omega^2 X \cos(\omega t - \phi)$$





Solução completa

$$x(t) = x_h(t) + x_p(t)$$

$$\omega_d = \sqrt{1 - \zeta^2}\,\omega_n$$

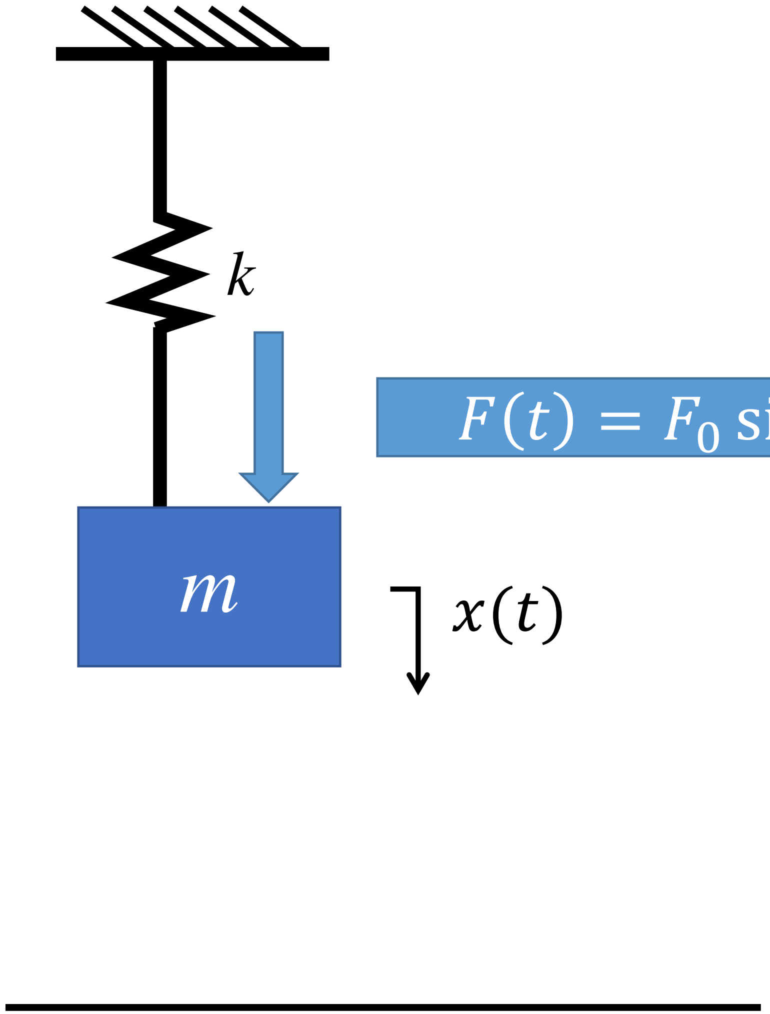
$$x(t) = \underbrace{X_0 e^{-\zeta\omega_n t}\cos(\omega_d t - \phi_0)}_{\text{transitório}} + \underbrace{X\cos(\omega t - \phi)}_{\text{regime permanente}}$$

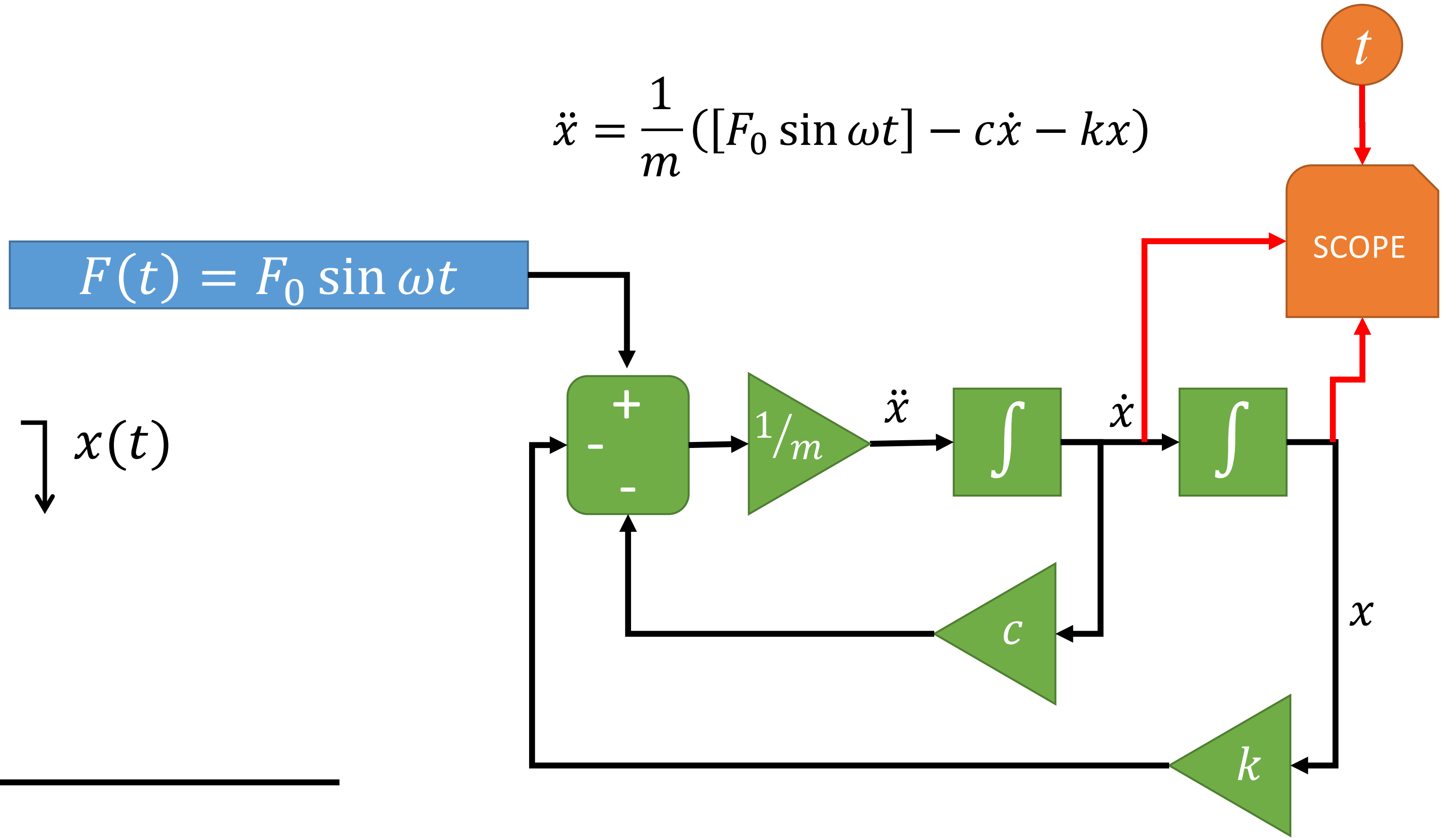
$$\left.\begin{aligned} X_0 &= \left[(x_0 - X\cos\phi)^2 + \frac{1}{\omega_d^2}(\zeta\omega_n x_0 + \dot{x}_0 - \zeta\omega_n X\cos\phi - \omega X\sin\phi)^2\right]^{\frac{1}{2}} \\ \tan\phi_0 &= \frac{\zeta\omega_n x_0 + \dot{x}_0 - \zeta\omega_n X\cos\phi - \omega X\sin\phi}{\omega_d(x_0 - X\cos\phi)} \end{aligned}\right\}$$

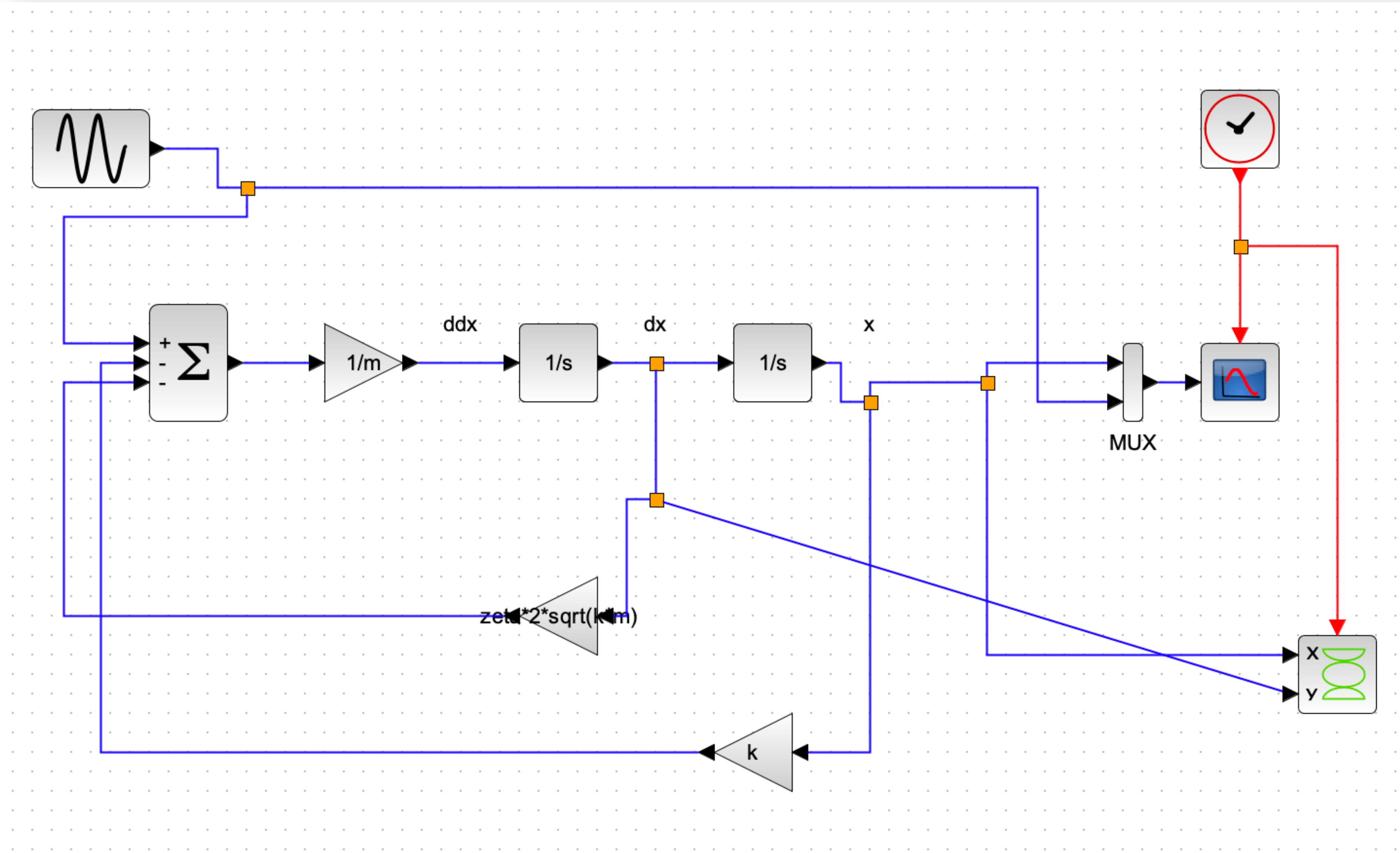
Simulação de sistema dinâmico

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$$

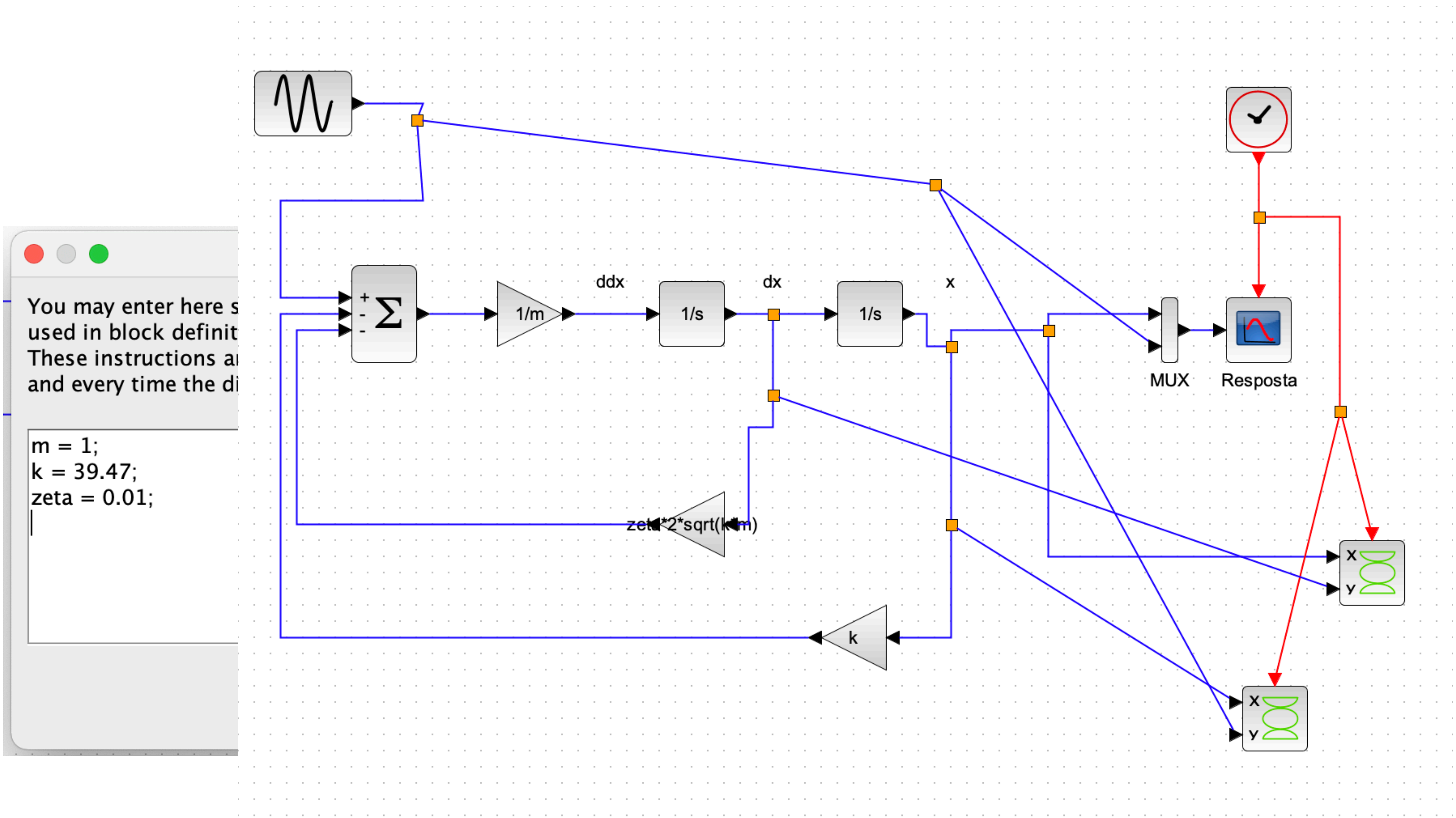
$$\ddot{x} = \frac{1}{m}([F_0 \sin \omega t] - c\dot{x} - kx)$$







ddx
dx
x
1/m
1/s
1/s
MUX
zeta*2*sqrt(k*m)
k
x
y


You may enter here s
used in block definit
These instructions ar
and every time the di
m = 1;
k = 39.47;
zeta = 0.01;
1/m
ddx
1/s
dx
1/s
x
MUX
Resposta
k
x
y
x
y

Figuras de Lissajous