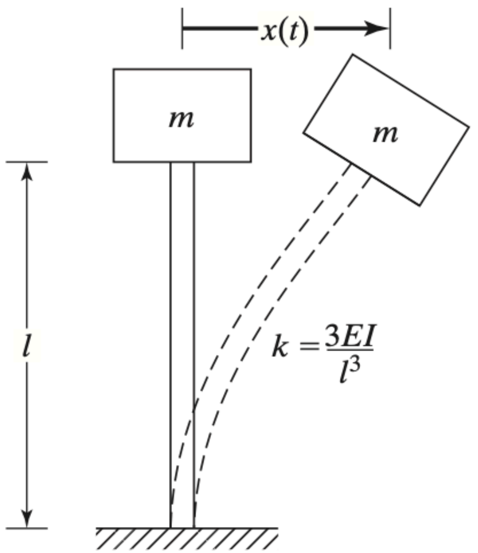
Dinâmica de Sistemas Navais e Oceânicos

PNV3314 Dinâmica de Sistemas

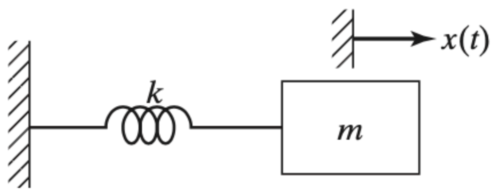
Aula 6

Vibração livre não amortecida

Sistema m-k com 1 grau de liberdade



(a) Idealization of the tall structure



(b) Equivalent spring-mass system

Segunda Lei de Newton

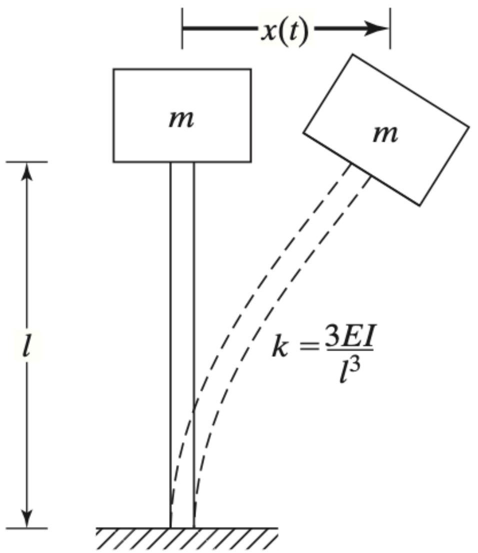
$$\vec{F}(t) = \frac{d}{dt}\left(m\frac{d\vec{x}(t)}{dt}\right)$$

$$\vec{F}(t) = m\frac{d^2\vec{x}(t)}{dt^2} = m\ddot{\vec{x}}$$

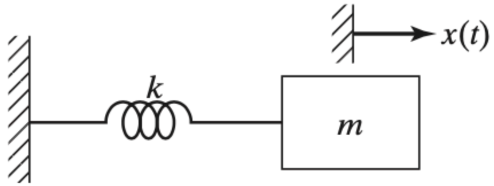
Para sistema em rotação

$$\vec{M}(t) = J\ddot{\vec{\theta}}$$

Sistema m-k com 1 grau de liberdade



(a) Idealization of the tall structure



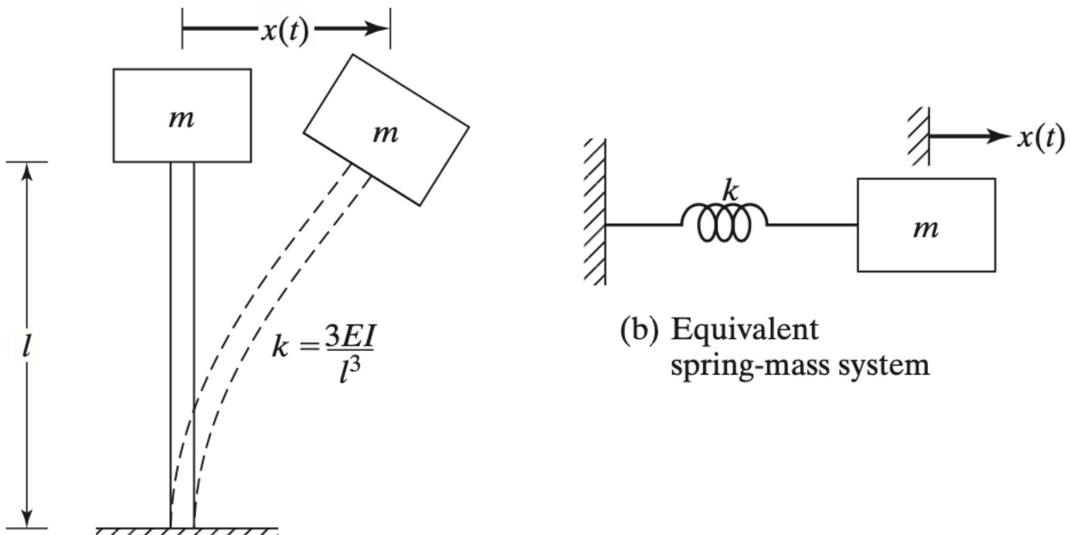
(b) Equivalent spring-mass system

Equação do movimento

$$F(t) = -kx = m\ddot{x}$$

$$m\ddot{x} + kx = 0$$

Sistema m-k com 1 grau de liberdade



(a) Idealization of the tall structure

(b) Equivalent spring-mass system

Equação do movimento

$$m\ddot{x} + kx = 0$$

$$\ddot{x} = -\frac{k}{m}x$$

Considerando movimento harmônico

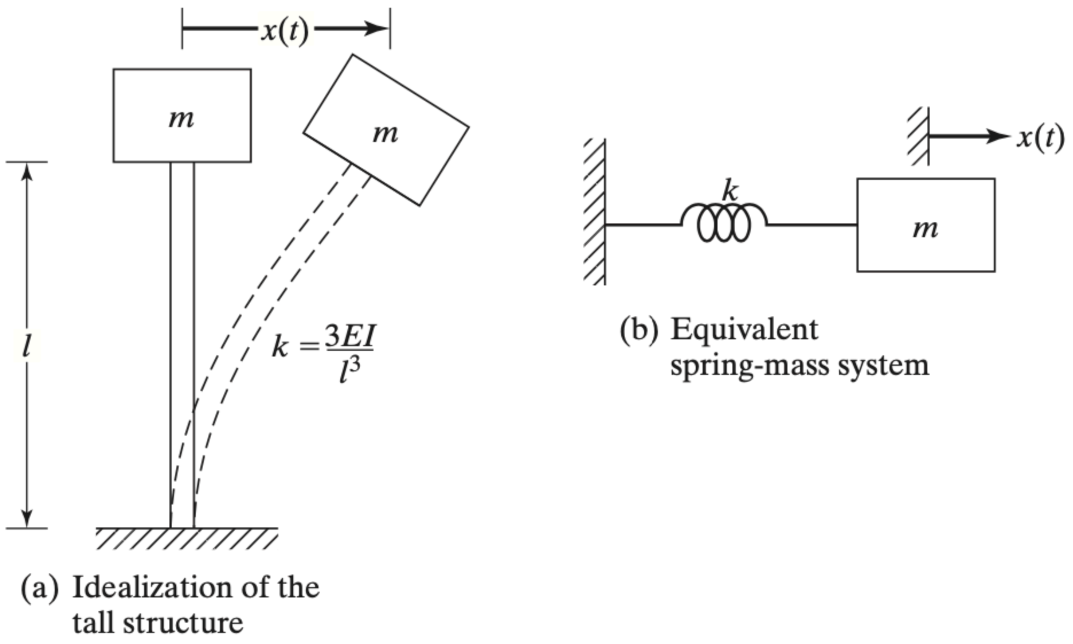
$$x = A\sin\omega t$$

$$\dot{x} = \omega A\cos\omega t$$

$$\ddot{x} = -\omega^2 A\sin\omega t$$

$$\omega^2 A\sin\omega t = \frac{k}{m}A\sin\omega t$$

Sistema m-k com 1 grau de liberdade



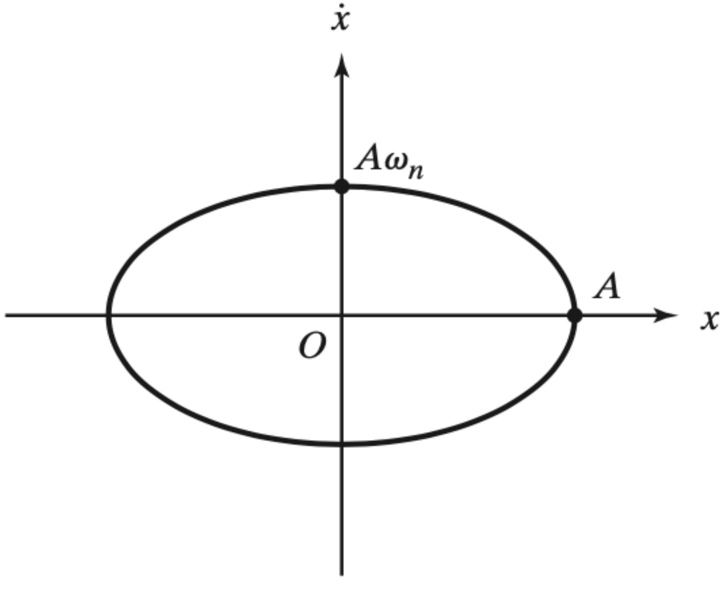
$$\omega^2 A \sin \omega t = \frac{k}{m} A \sin \omega t$$

Frequência natural do sistema harmônico não amortecido

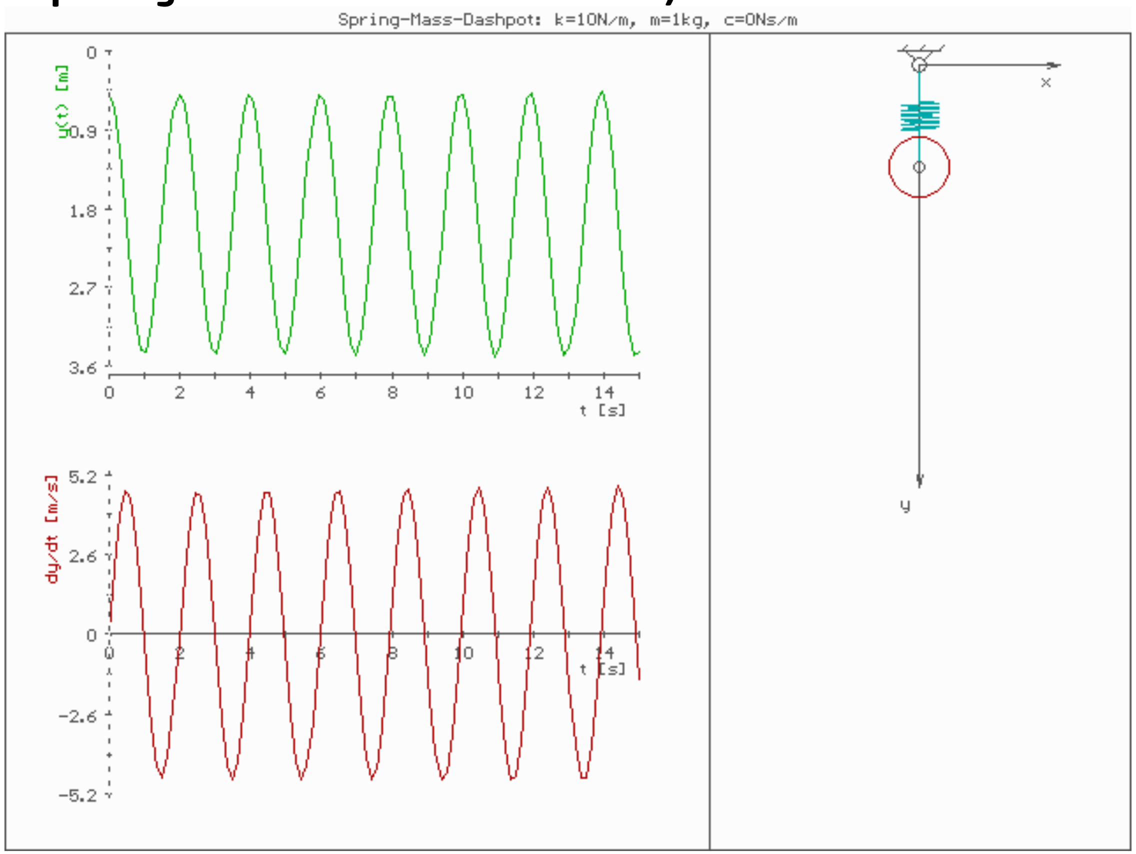
$$\omega = \sqrt{\frac{k}{m}}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Plano de fase (ou espaço de estado)



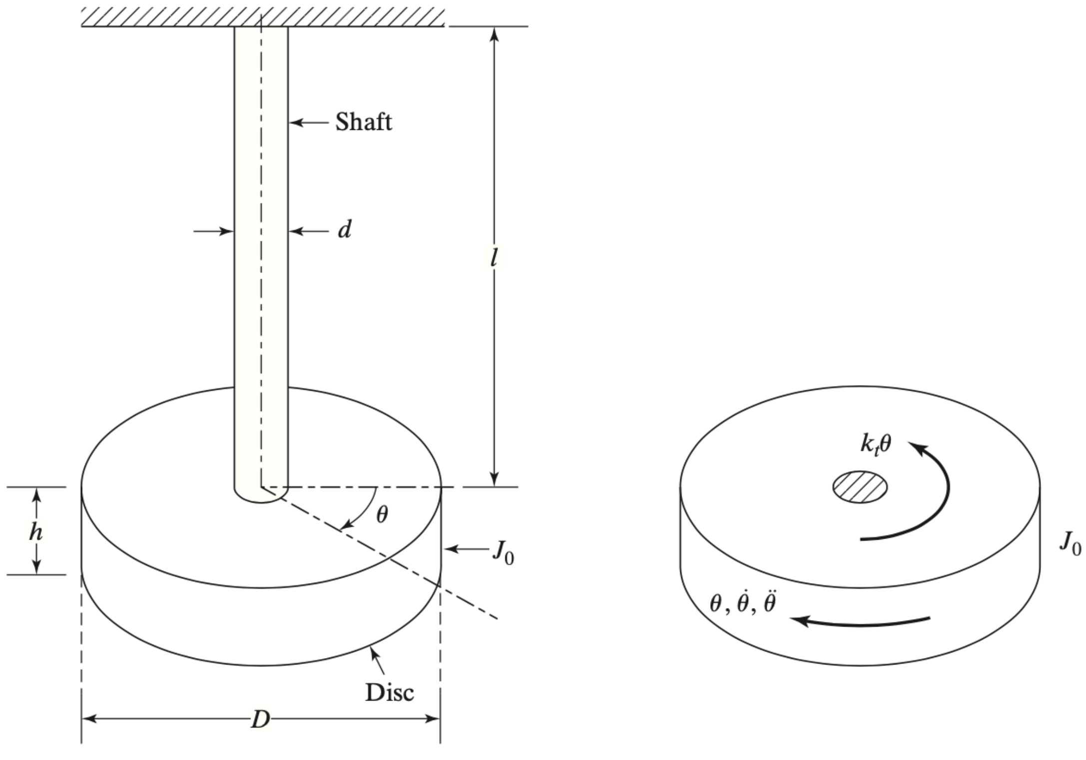
(b)



Sistema torsional

$$J_0\ddot{\theta} + k_t\theta = 0$$

$$\omega_n = \left(\frac{k_t}{J_0}\right)^{1/2}$$

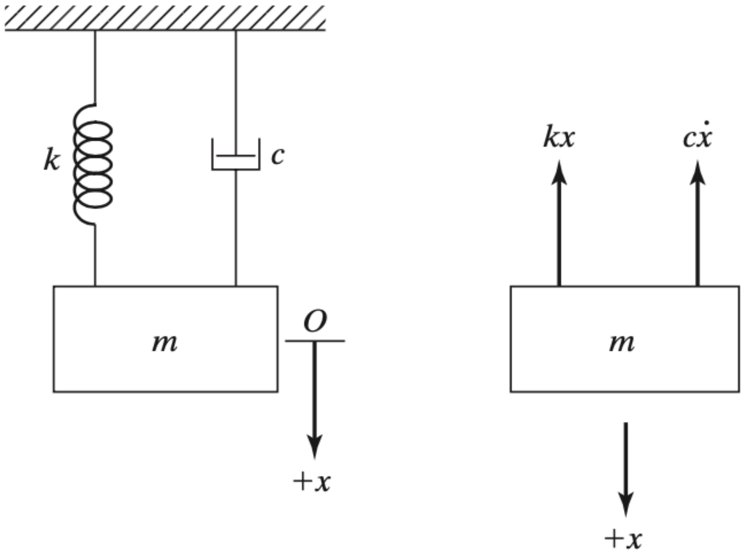


Visualização

- https://www.math.ksu.edu/~albin/apps/mass_spring.html
- https://www.myphysicslab.com/springs/single-spring-en.html

Vibração livre amortecida

Sistema m-c-k com 1 grau de liberdade



System

(a)

Free-body diagram

(b)

$$m\ddot{x} = -c\dot{x} - kx$$

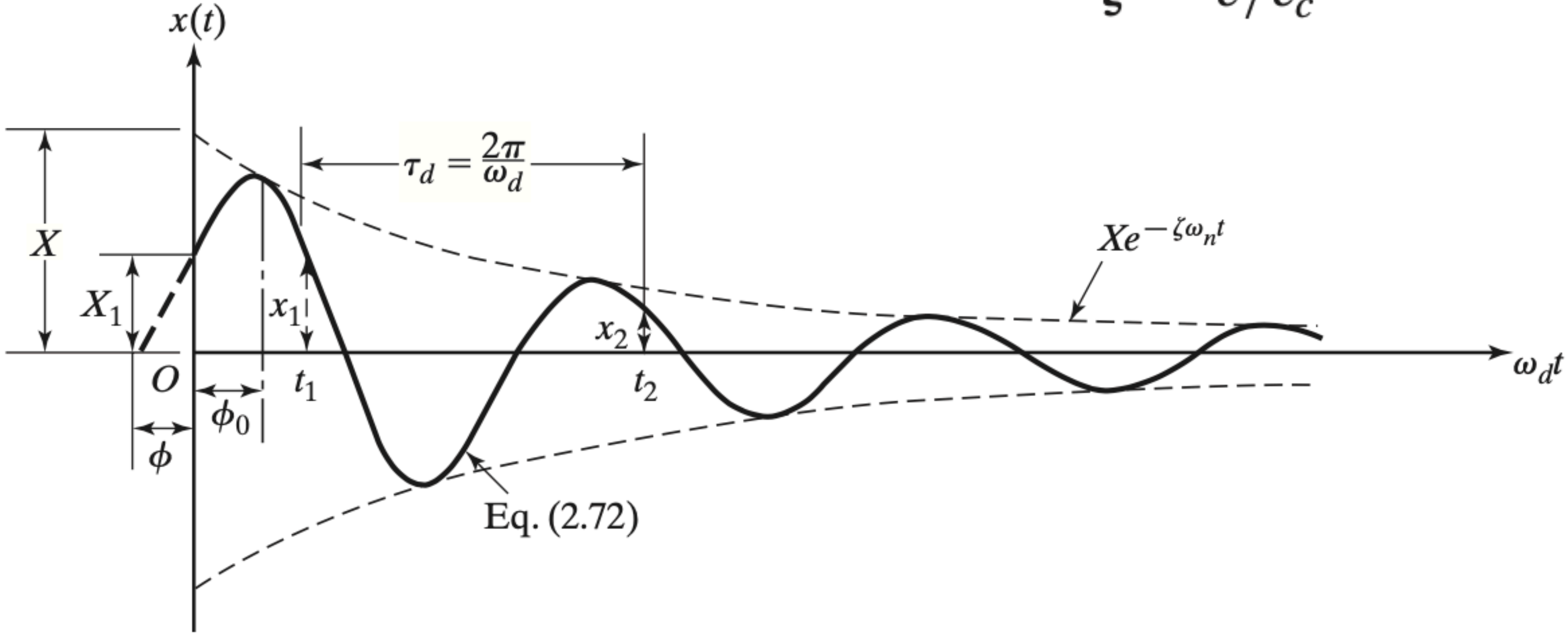
Equação do movimento

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\omega_d = \sqrt{1 - \zeta^2}\,\omega_n$$

$$c_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{km} = 2m\omega_n$$

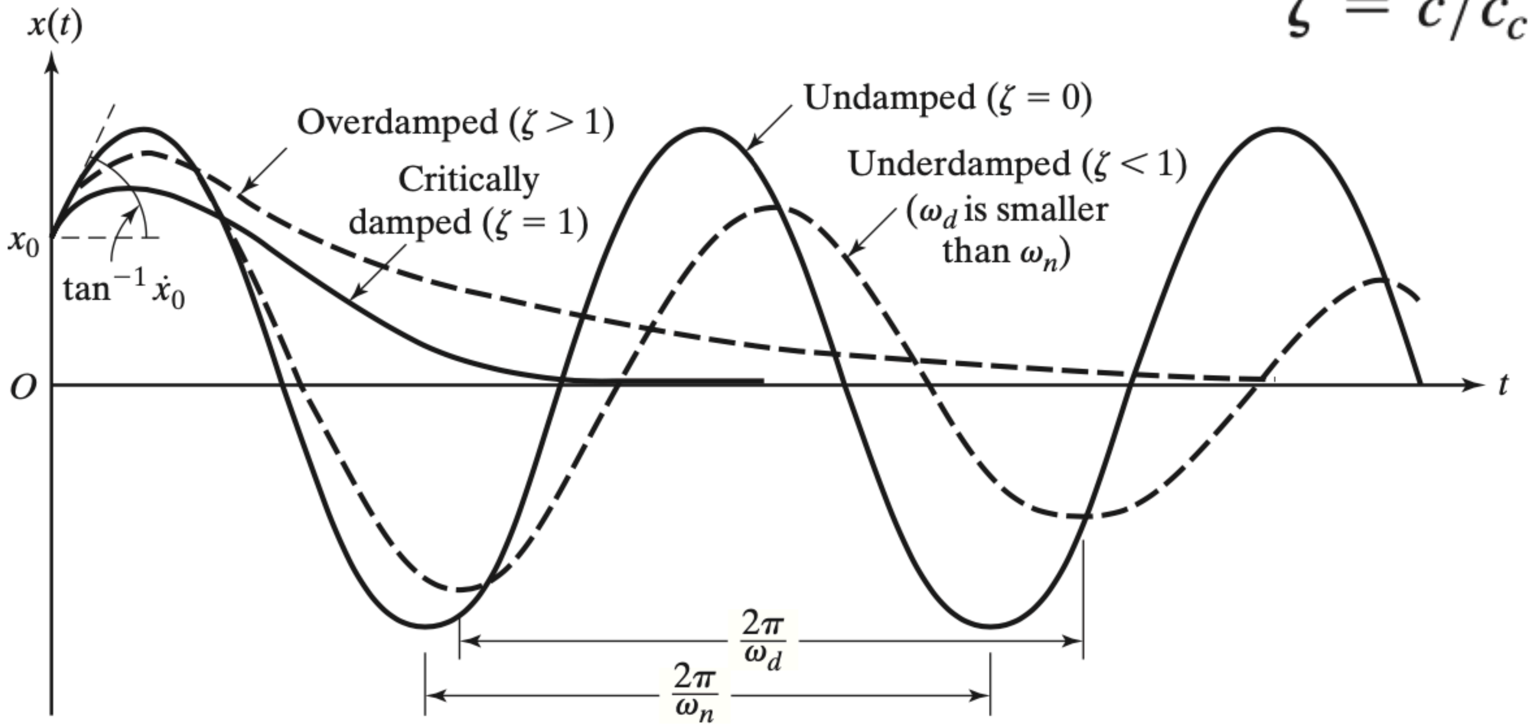
$$\zeta = c/c_c$$



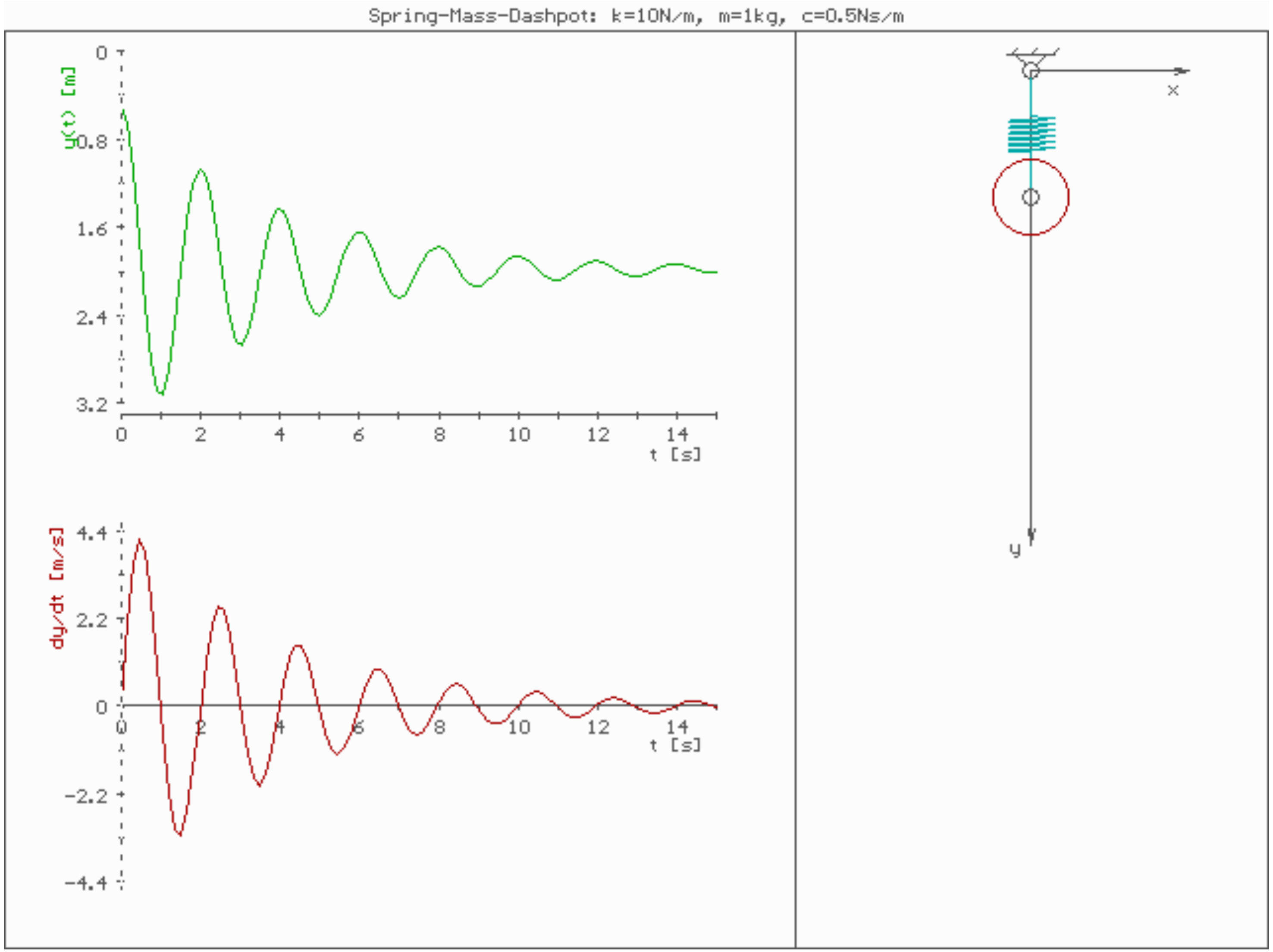
$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n$$

$$c_c = 2m\sqrt{\frac{k}{m}} = 2\sqrt{km} = 2m\omega_n$$

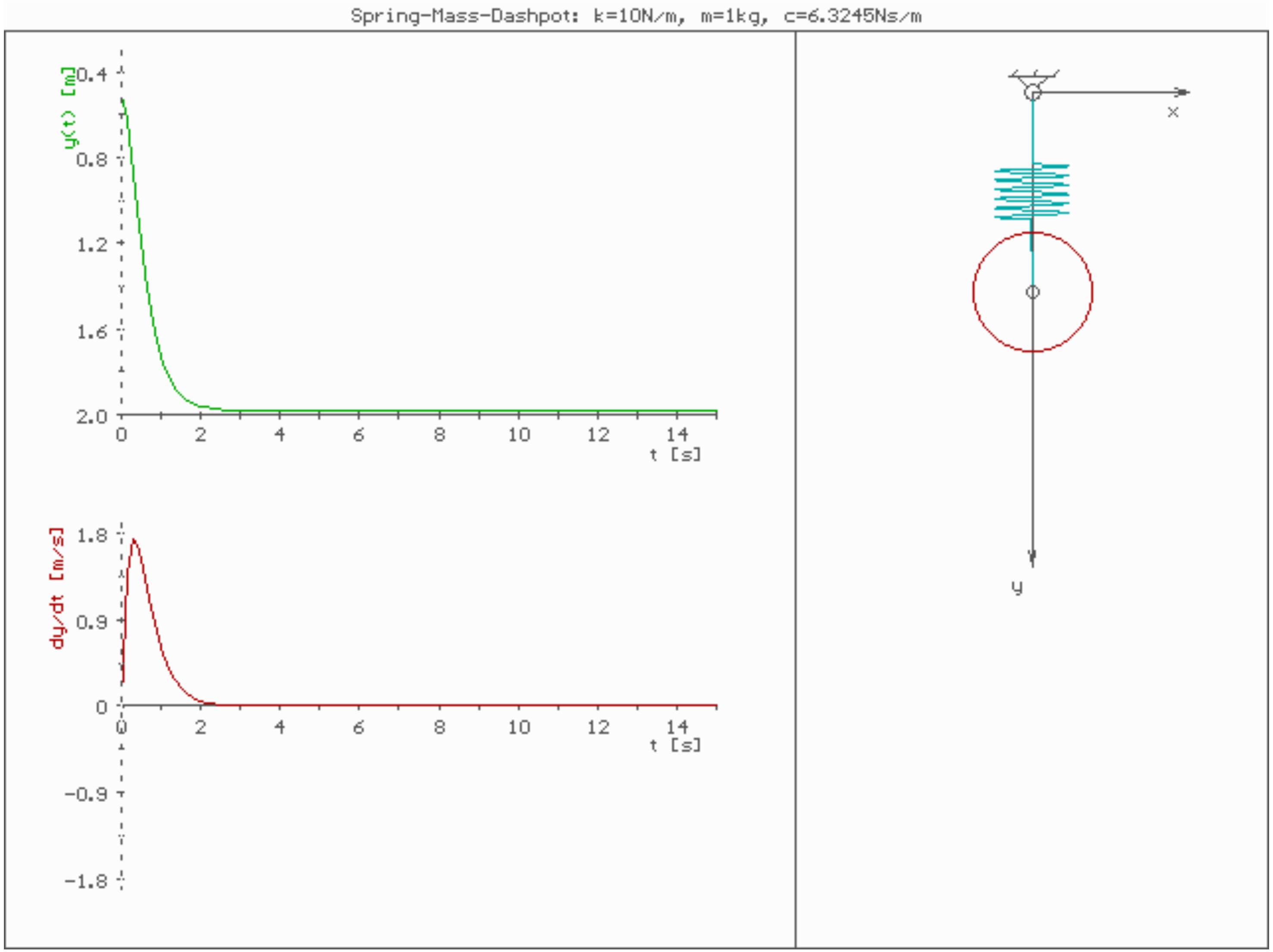
$$\zeta = c/c_c$$



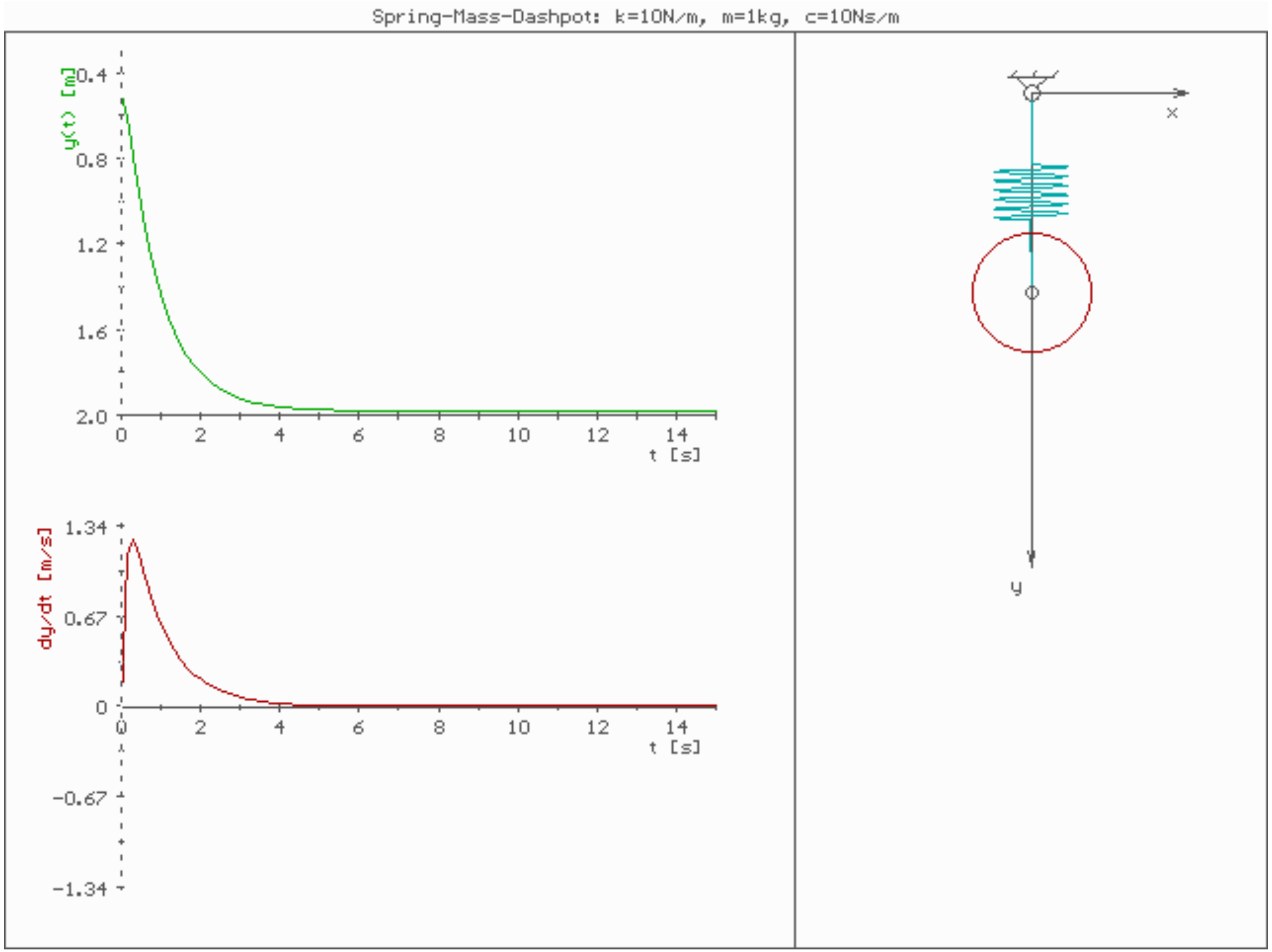
Sub amortecido



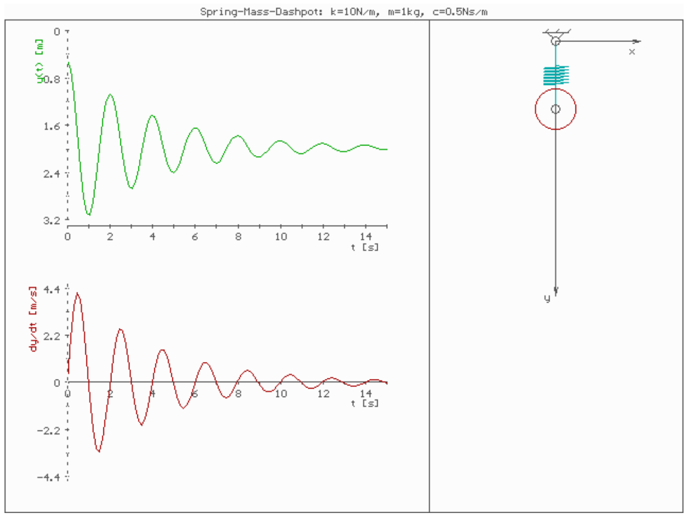
Criticamente amortecido



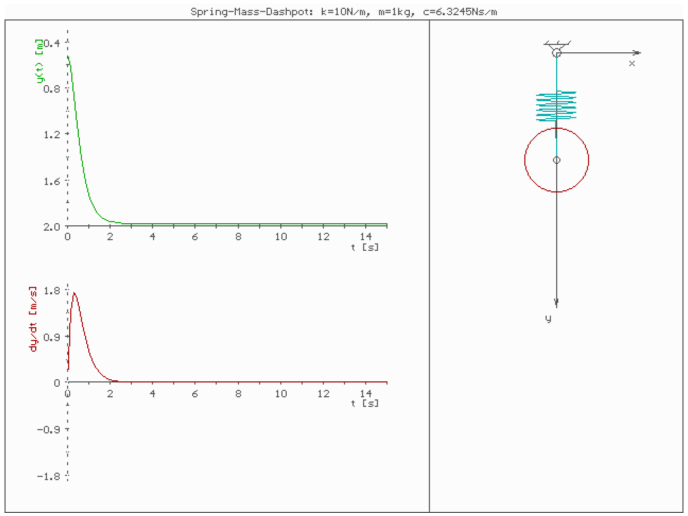
Super amortecido



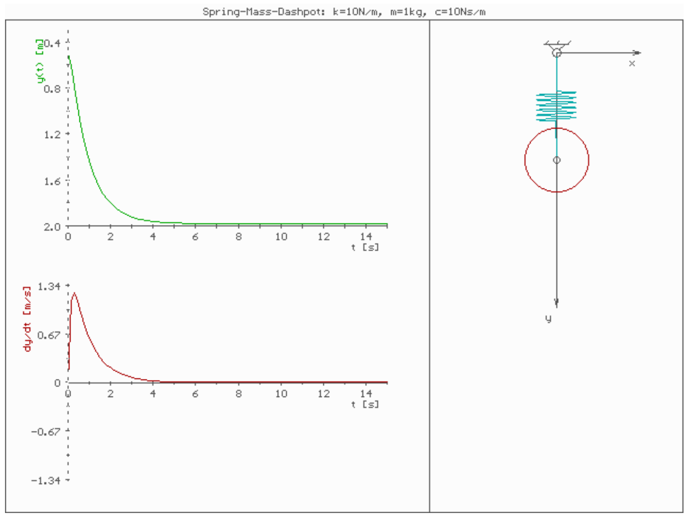
Sub amortecido

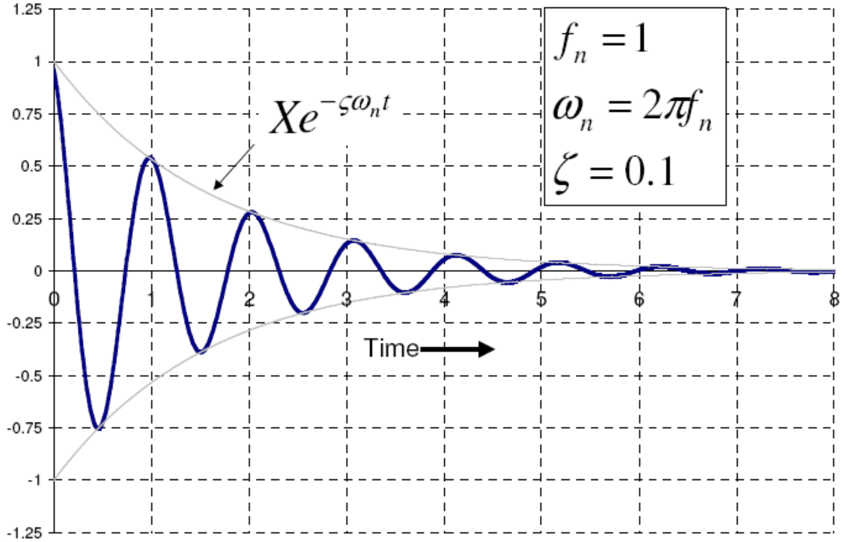


Criticamente amortecido

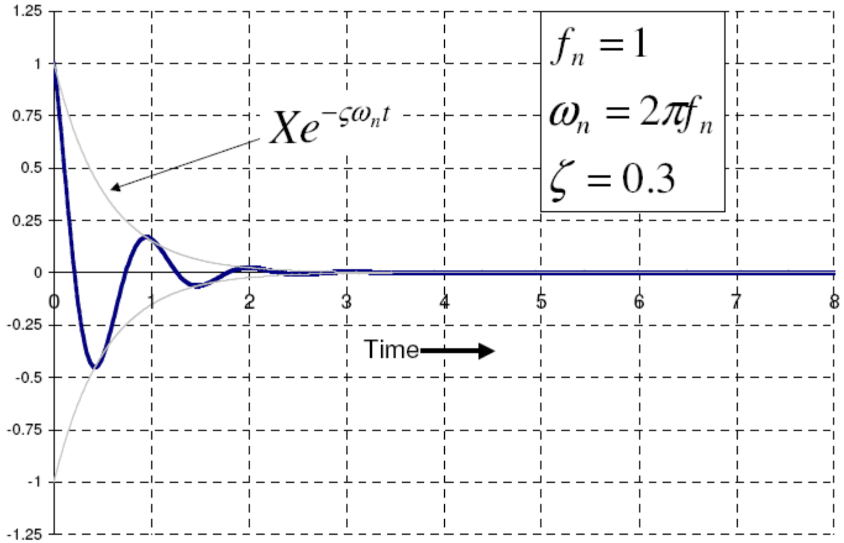


Super amortecido

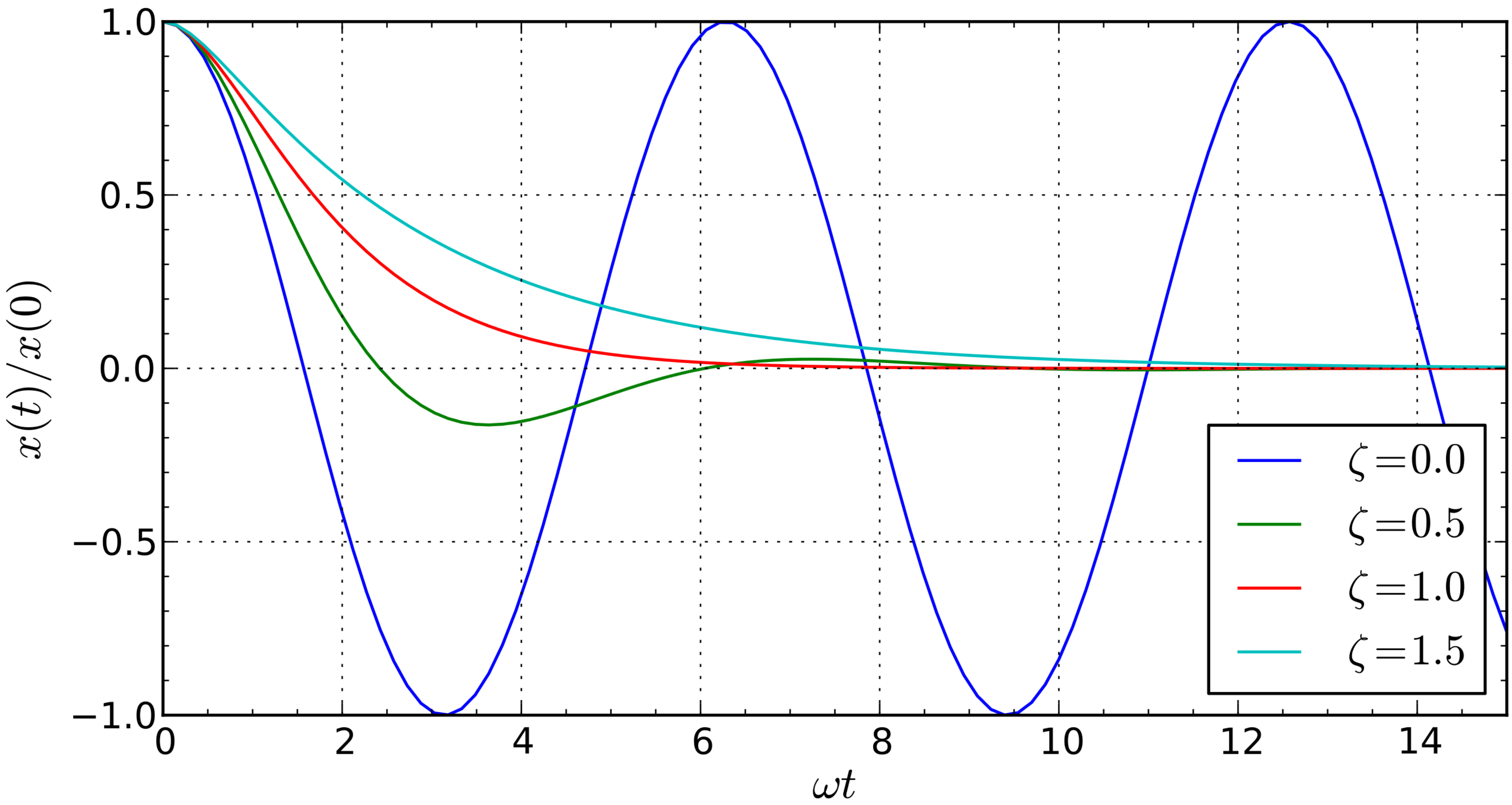




$f_n = 1$
$\omega_n = 2\pi f_n$
$\zeta = 0.1$
$Xe^{-\varsigma\omega_n t}$
Time

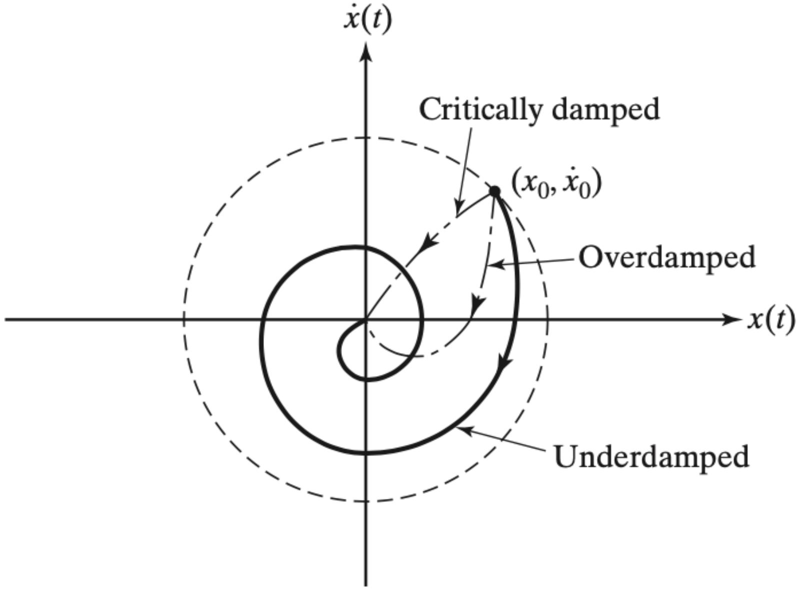


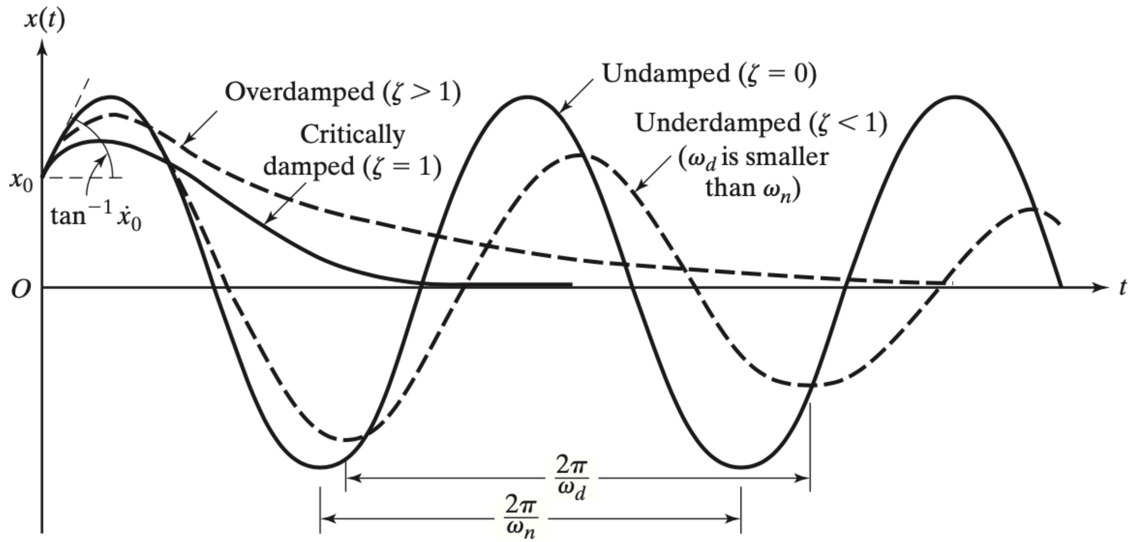
$f_n = 1$
$\omega_n = 2\pi f_n$
$\zeta = 0.3$
$Xe^{-\varsigma\omega_n t}$
Time



1.0
0.5
0.0
−0.5
−1.0
0
2
4
6
8
10
12
14
$x(t)/x(0)$
ωt
$\zeta = 0.0$
$\zeta = 0.5$
$\zeta = 1.0$
$\zeta = 1.5$

Plano de fase





Visualização

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