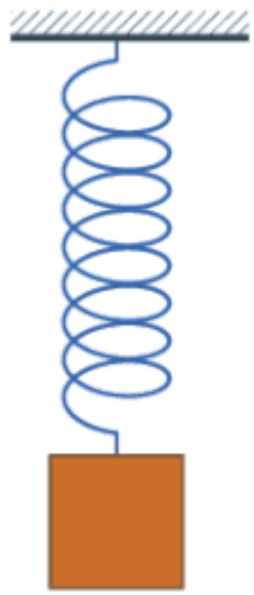
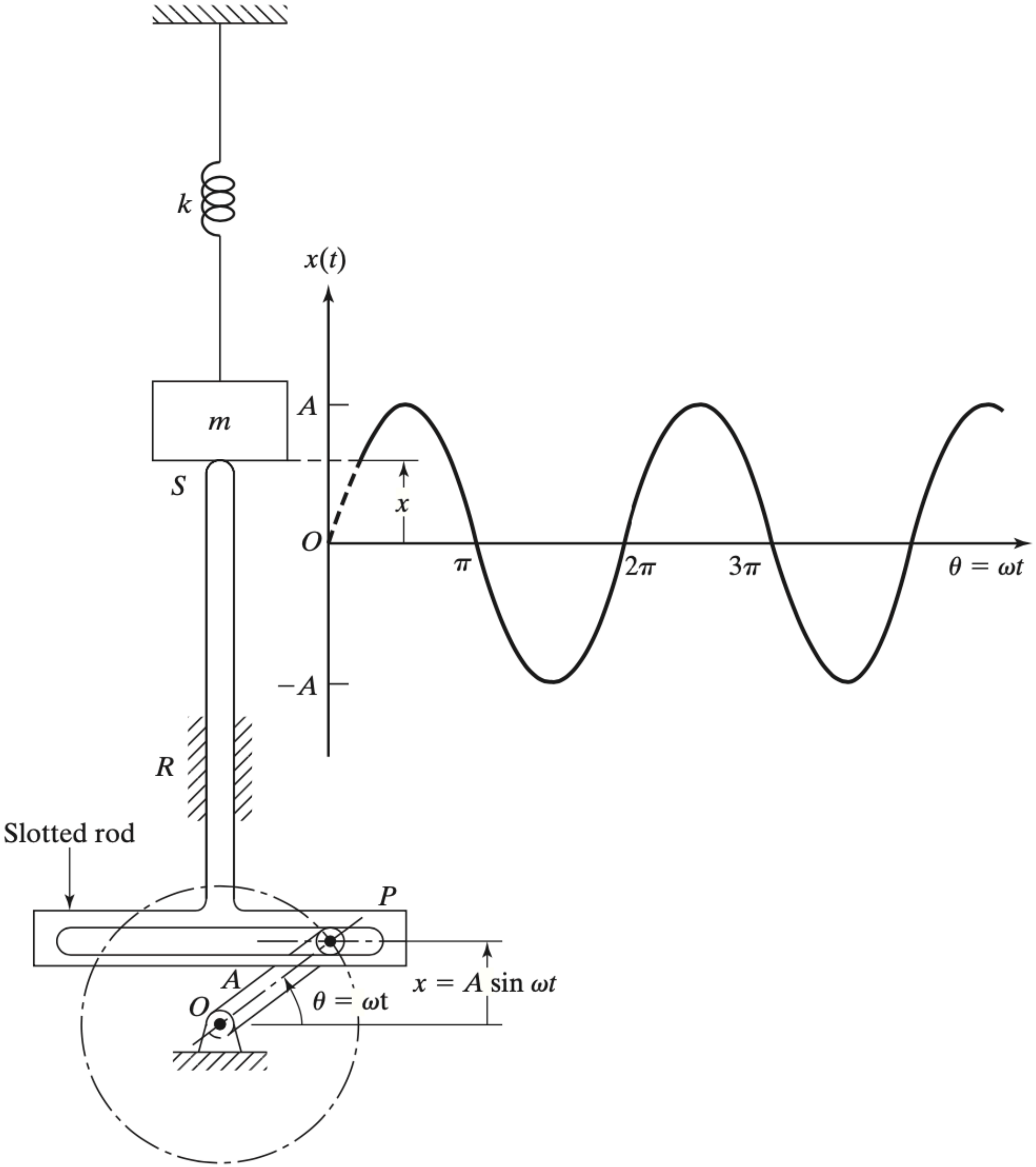
Dinâmica de Sistemas Navais e Oceânicos

PNV3314 Dinâmica de Sistemas

Aula 5

Movimento harmônico

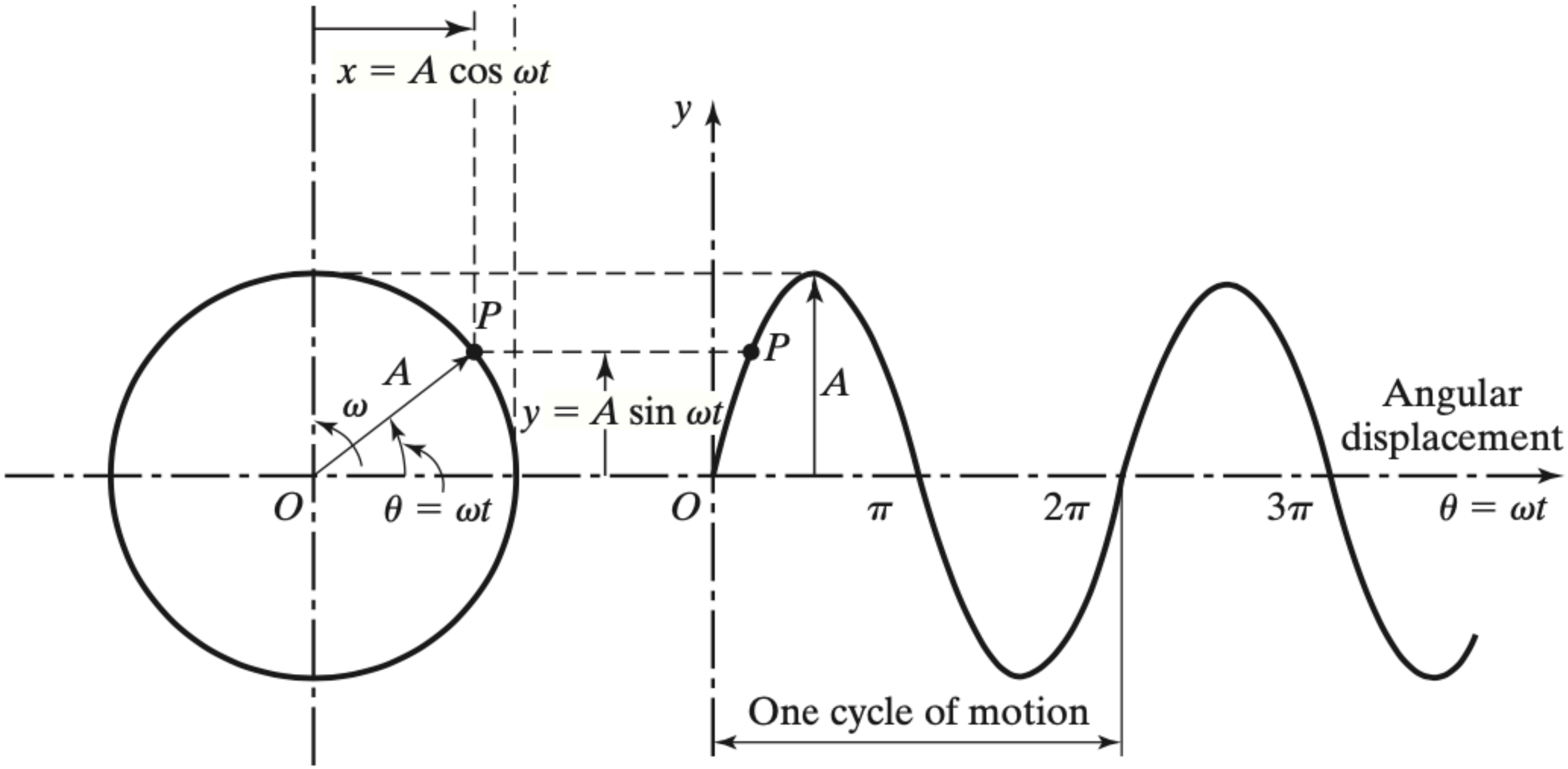




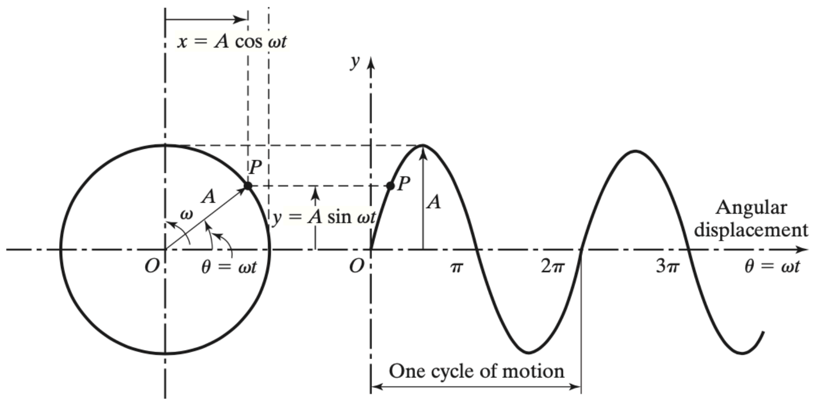
$$x = A \sin \theta = A \sin \omega t$$

$$\frac{dx}{dt} = \omega A \cos \omega t$$

$$\frac{d^2x}{dt^2} = -\omega^2 A \sin \omega t = -\omega^2 x$$



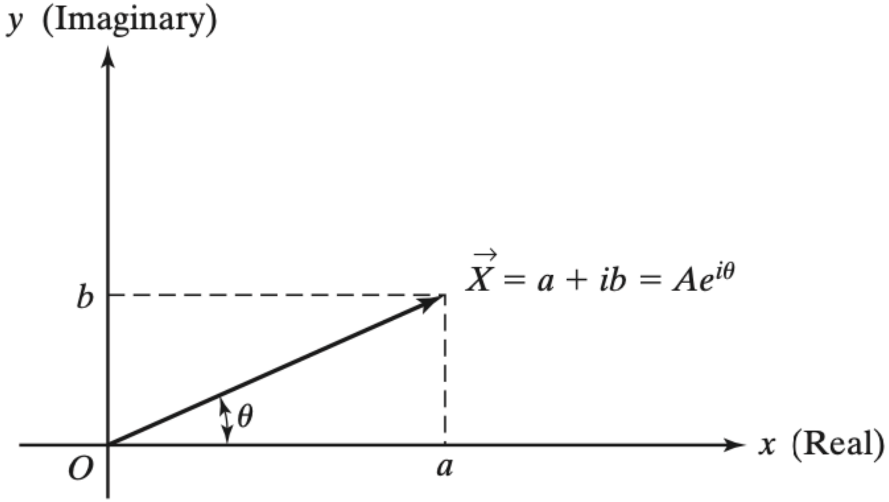
$x = A \cos \omega t$
y
P
P
A
A
ω
$y = A \sin \omega t$
Angular displacement
O
$\theta = \omega t$
O
π
2π
3π
$\theta = \omega t$
One cycle of motion



$$\vec{X} = a + ib$$

$$\vec{X} = A\cos\theta + iA\sin\theta$$

$$\vec{X} = Ae^{i\omega t}$$

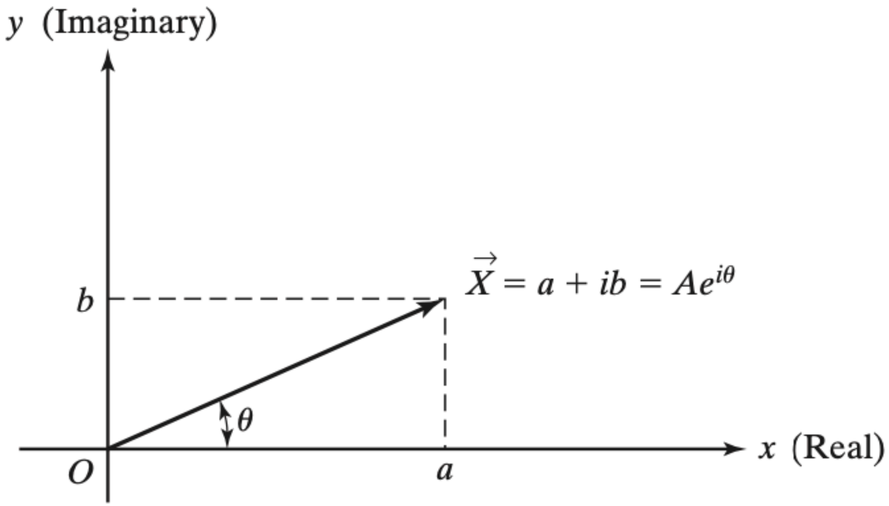


$$\frac{d\vec{X}}{dt} = \frac{d}{dt}\left(Ae^{i\omega t}\right) = i\omega Ae^{i\omega t} = i\omega\vec{X}$$

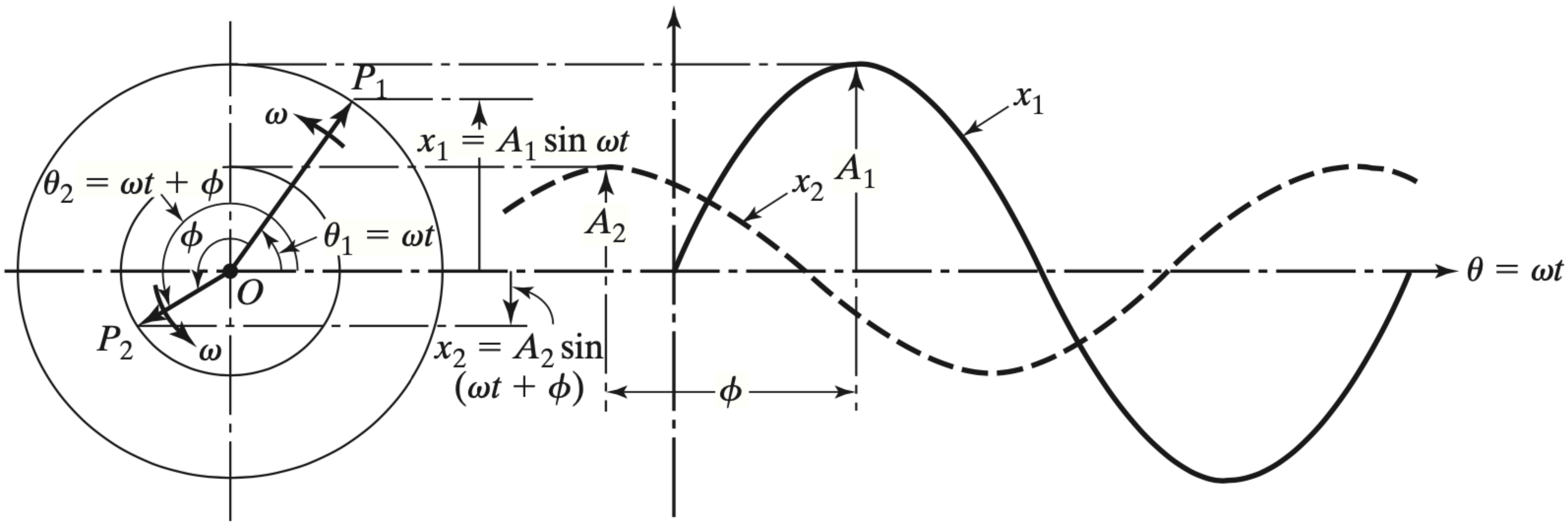
$$\frac{d^2\vec{X}}{dt^2} = \frac{d}{dt}\left(i\omega Ae^{i\omega t}\right) = -\omega^2 Ae^{i\omega t} = -\omega^2\vec{X}$$

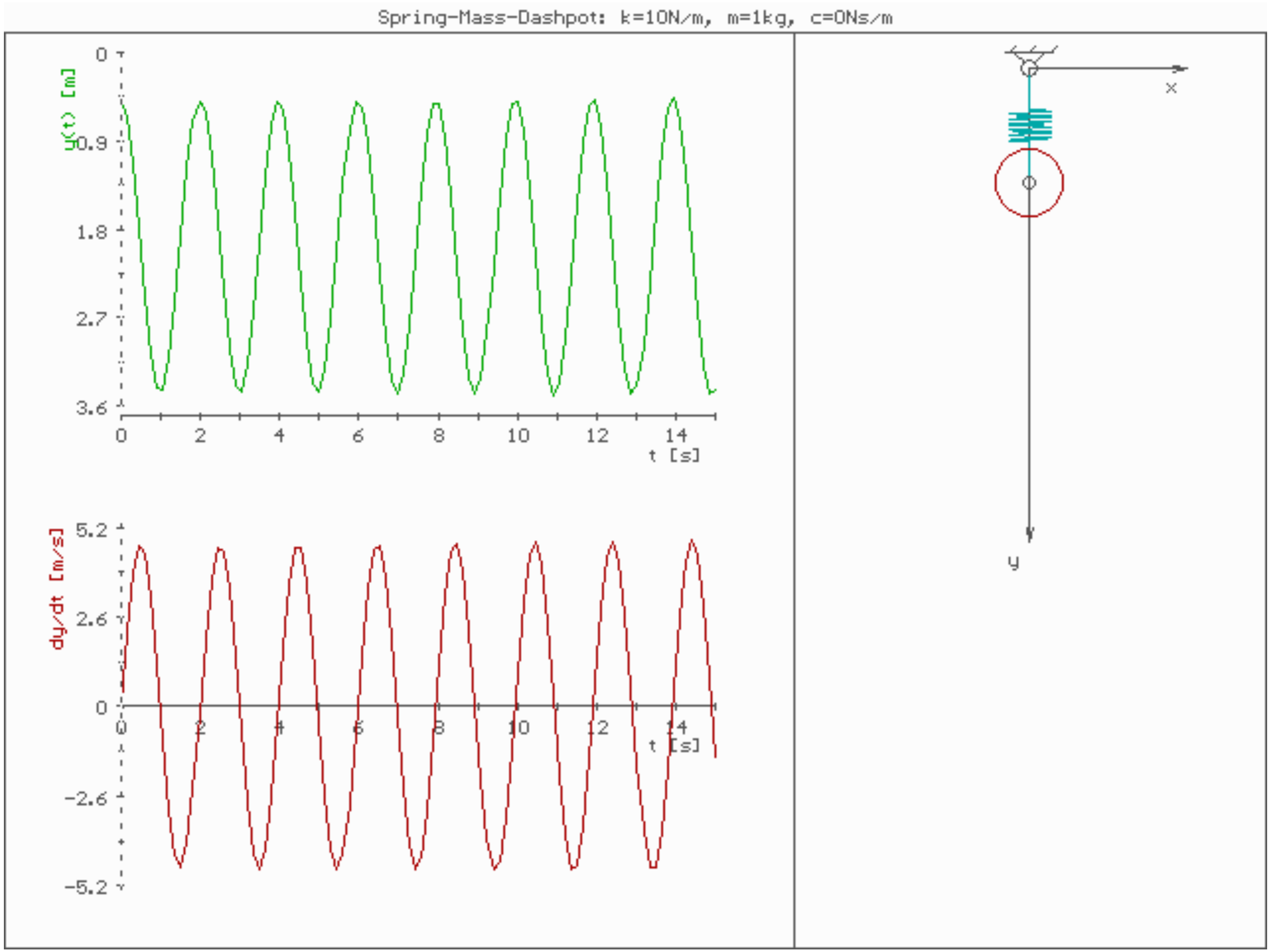
Thus the displacement, velocity, and acceleration can be expressed as[4]

$$
\begin{aligned}
\text{displacement} &= \mathrm{Re}[Ae^{i\omega t}] && = A\cos\omega t \\
\text{velocity} &= \mathrm{Re}[i\omega Ae^{i\omega t}] && = -\omega A\sin\omega t \\
& && = \omega A\cos(\omega t + 90^\circ) \\
\text{acceleration} &= \mathrm{Re}[-\omega^2 Ae^{i\omega t}] && = -\omega^2 A\cos\omega t \\
& && = \omega^2 A\cos(\omega t + 180^\circ)
\end{aligned}
$$

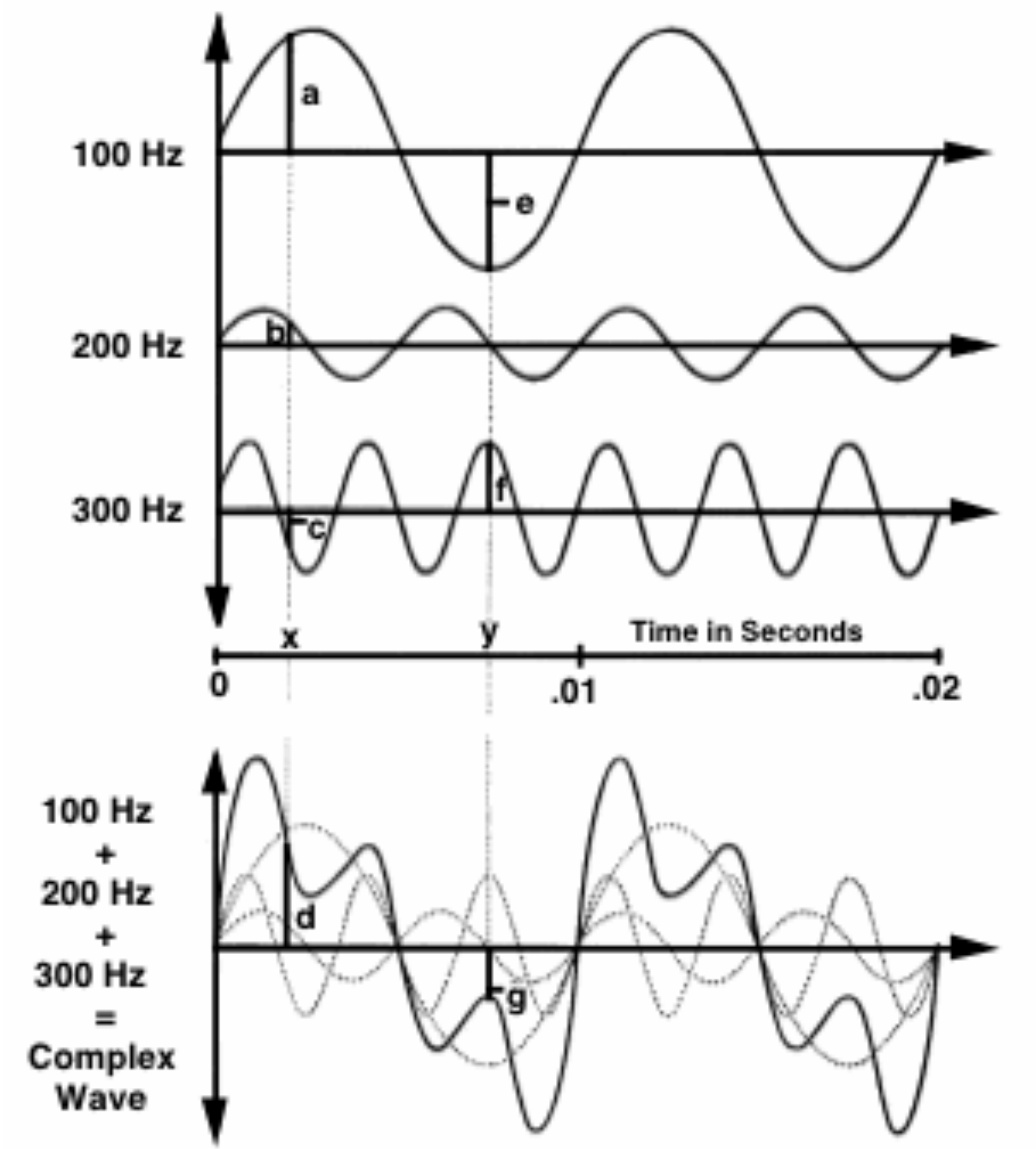


$$\tau = \frac{2\pi}{\omega} \qquad f = \frac{1}{\tau} = \frac{\omega}{2\pi} \qquad \begin{aligned} x_1 &= A_1 \sin \omega t \\ x_2 &= A_2 \sin(\omega t + \phi) \end{aligned}$$

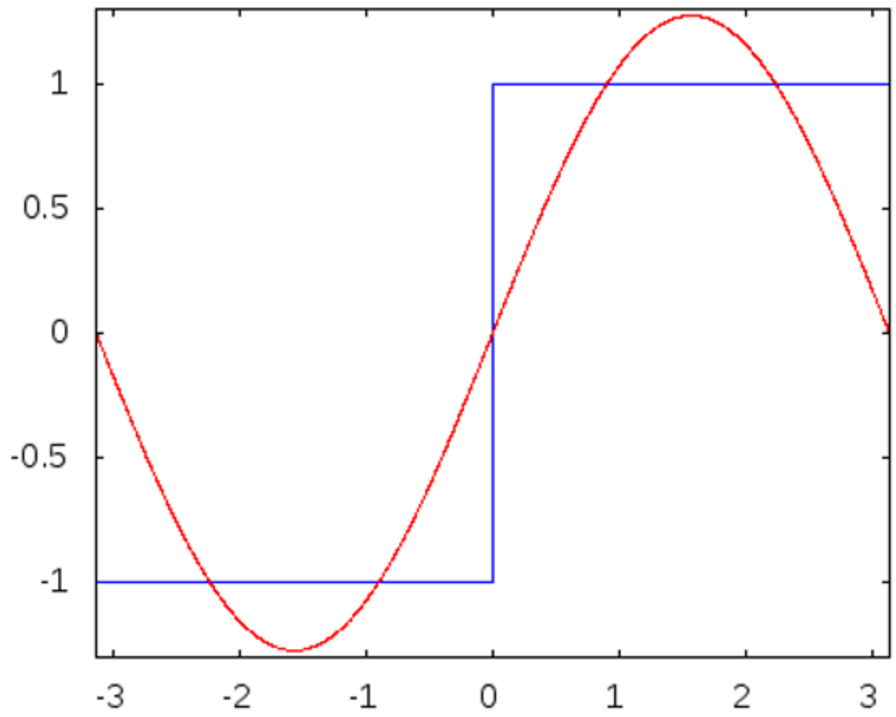



Spring-Mass-Dashpot: k=10N/m, m=1kg, c=0Ns/m
y(t) [m]
0
0.9
1.8
2.7
3.6
0
2
4
6
8
10
12
14
t [s]
dy/dt [m/s]
5.2
2.6
0
-2.6
-5.2
t [s]
x
y

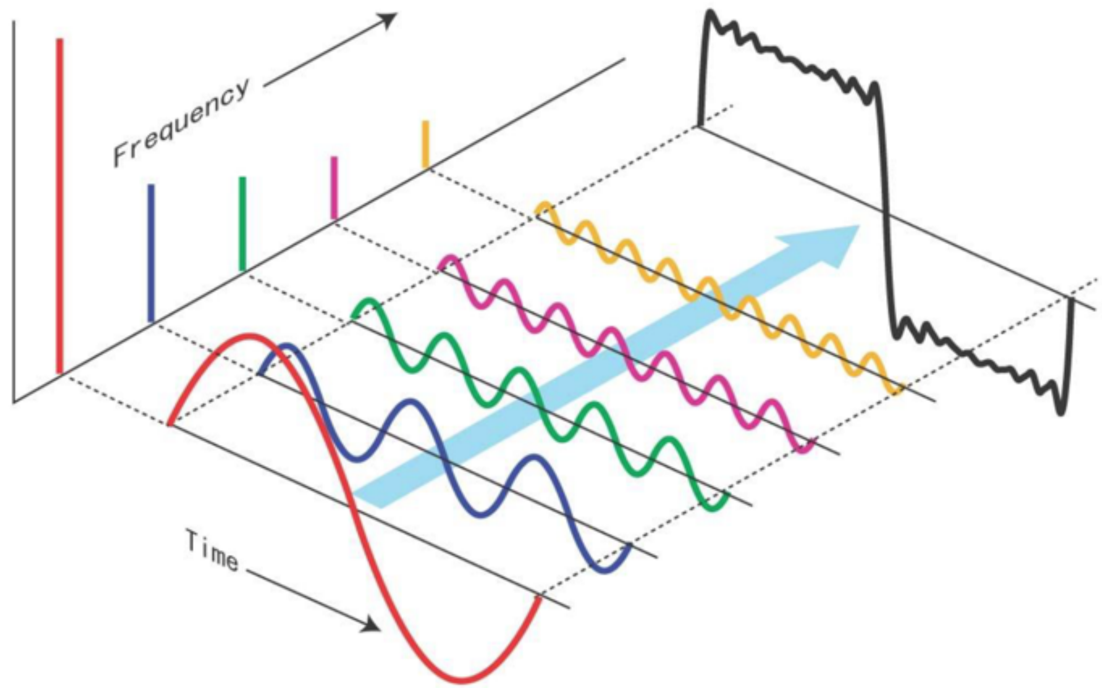
Composição



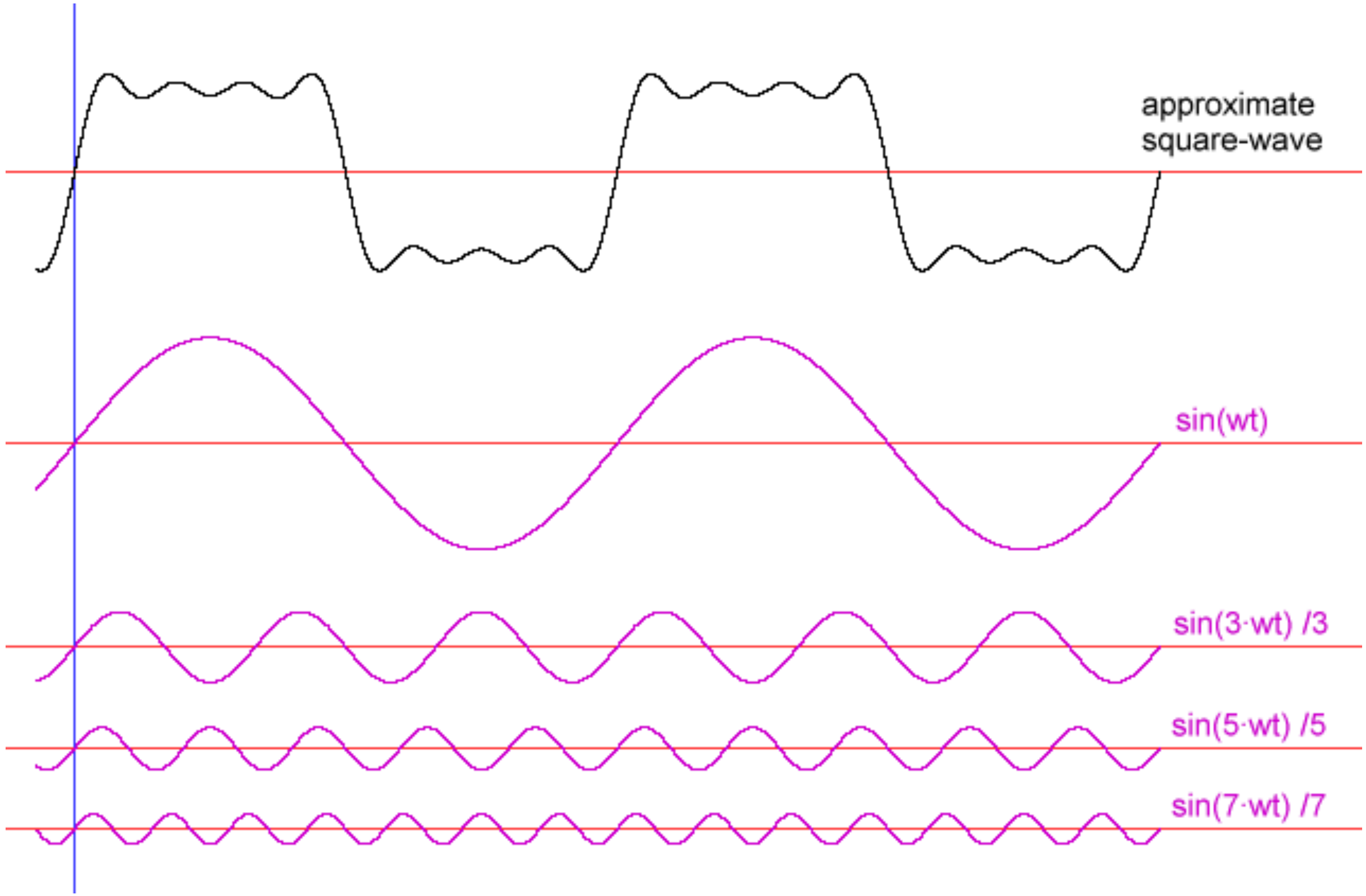
100 Hz
200 Hz
300 Hz
a
b
c
d
e
f
g
x
y
Time in Seconds
0
.01
.02
100 Hz + 200 Hz + 300 Hz = Complex Wave

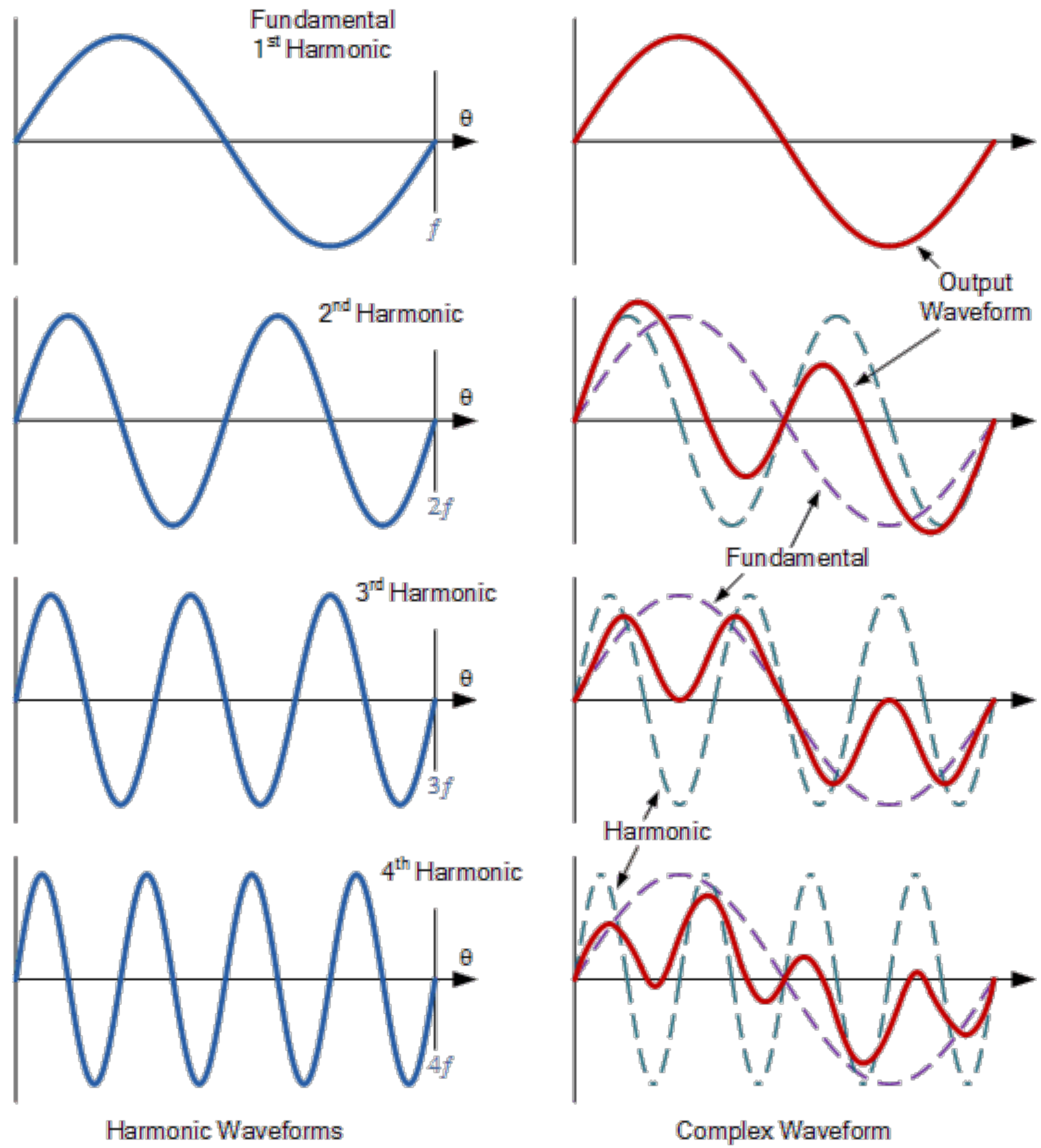


1
0.5
0
-0.5
-1
-3
-2
-1
0
1
2
3

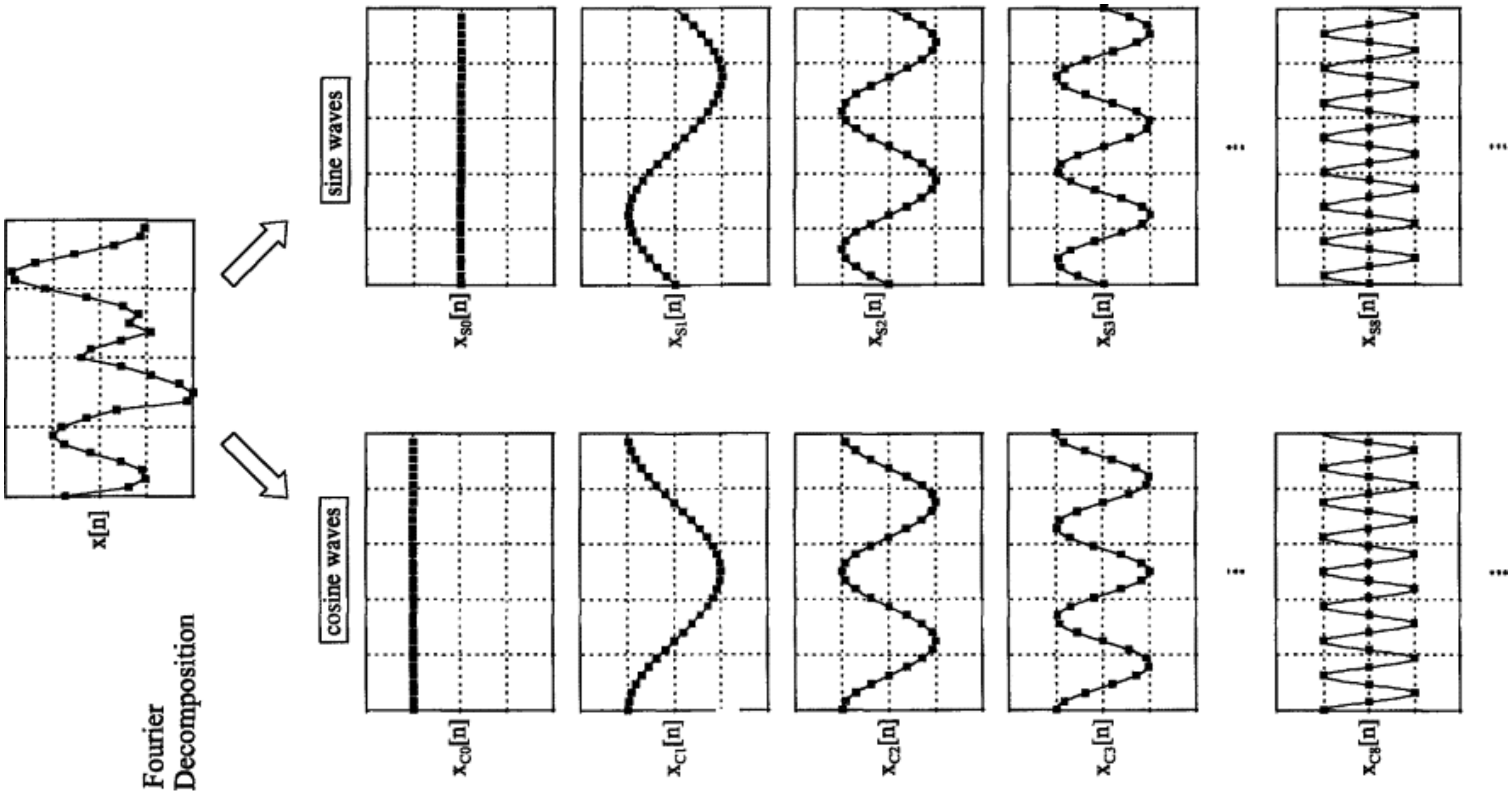


Frequency
Time


approximate
square-wave
sin(wt)
sin(3·wt) /3
sin(5·wt) /5
sin(7·wt) /7


Fundamental
1st Harmonic
θ
f
2nd Harmonic
θ
2f
3rd Harmonic
θ
3f
4th Harmonic
θ
4f
Output
Waveform
Fundamental
Harmonic
Harmonic Waveforms
Complex Waveform

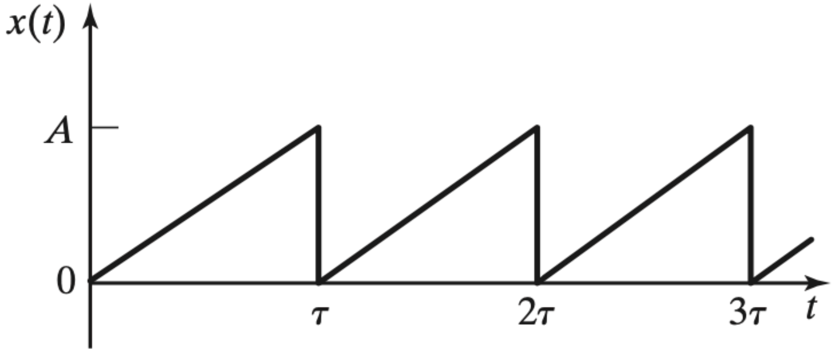
Decomposição

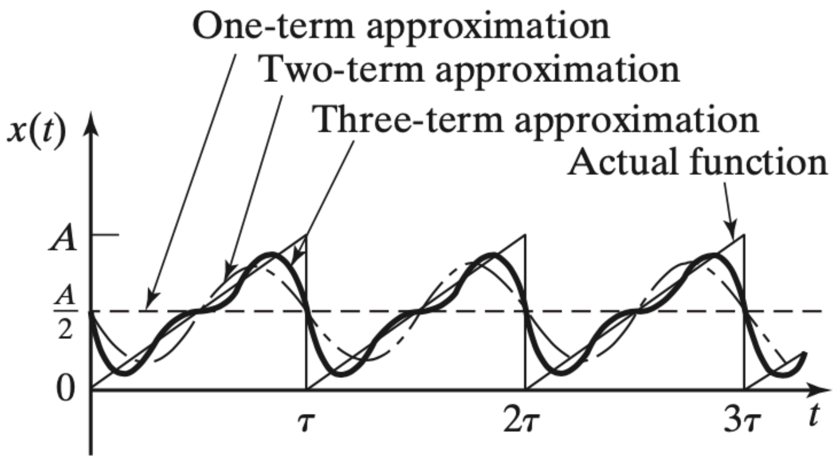


Análise harmônica: decomposição

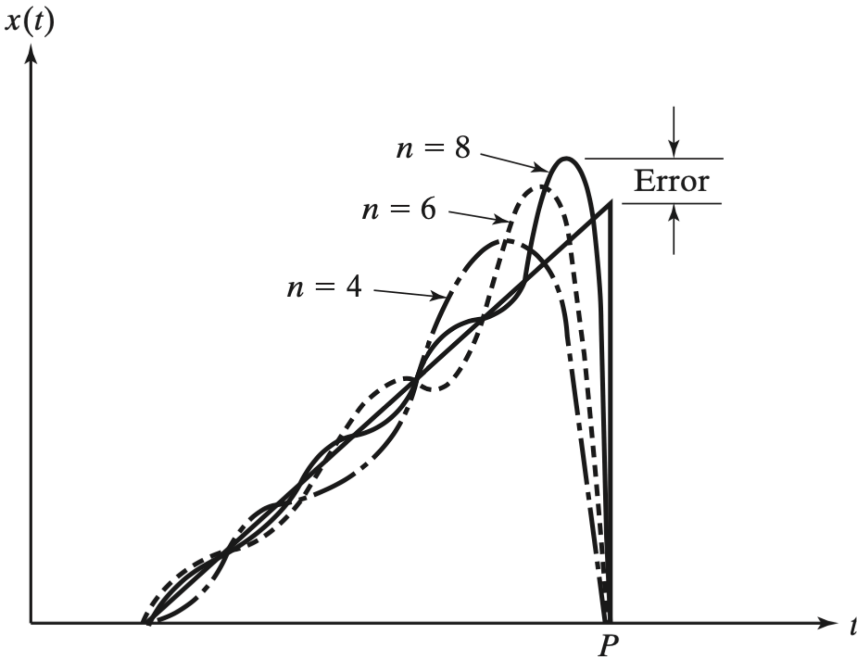
$$
\begin{aligned}
x(t) &= \frac{a_0}{2} + a_1 \cos \omega t + a_2 \cos 2\,\omega t + \cdots \\
&\quad + b_1 \sin \omega t + b_2 \sin 2\,\omega t + \cdots \\
&= \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t)
\end{aligned}
$$

Análise harmônica





Análise harmônica

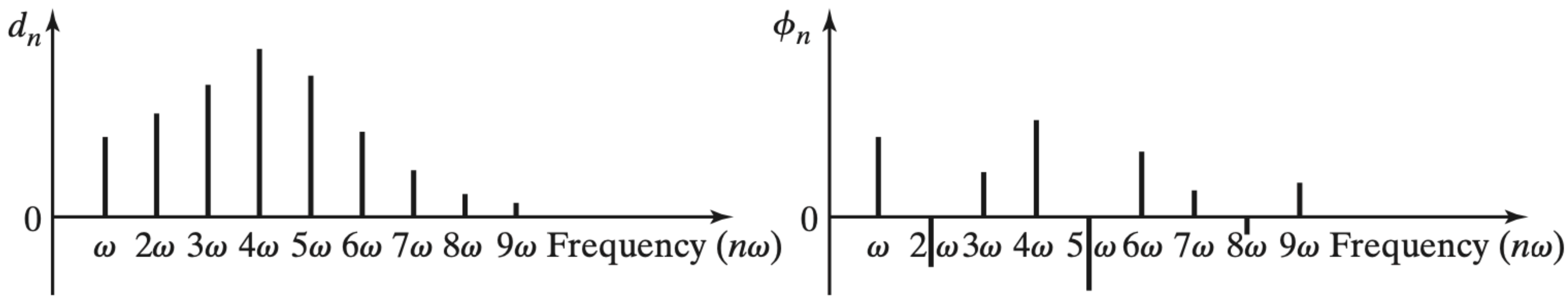


$$x(t) = d_0 + d_1 \cos(\omega t - \phi_1) + d_2 \cos(2\omega t - \phi_2) + \cdots$$

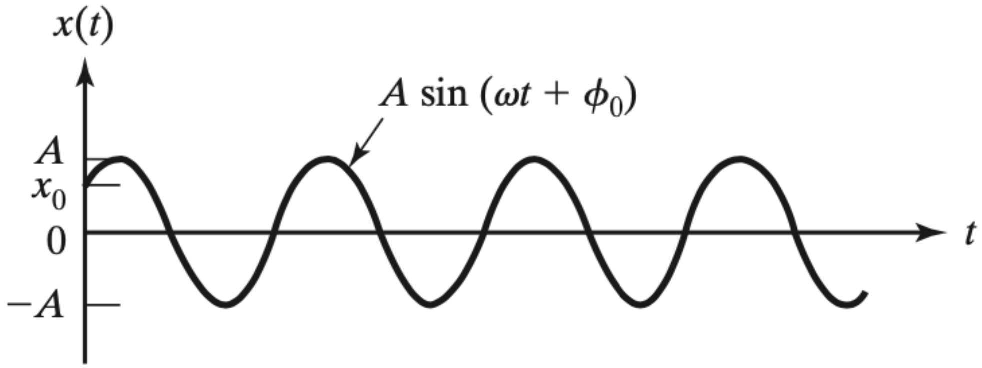
$$d_0 = a_0/2$$
$$d_n = (a_n^2 + b_n^2)^{1/2}$$

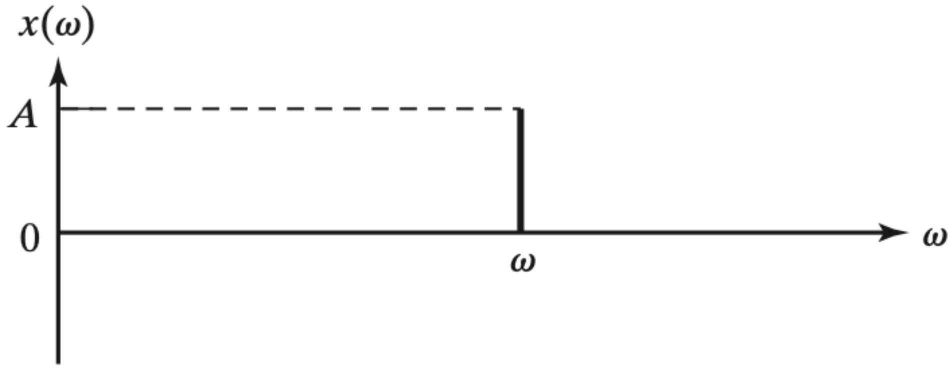
$$\phi_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

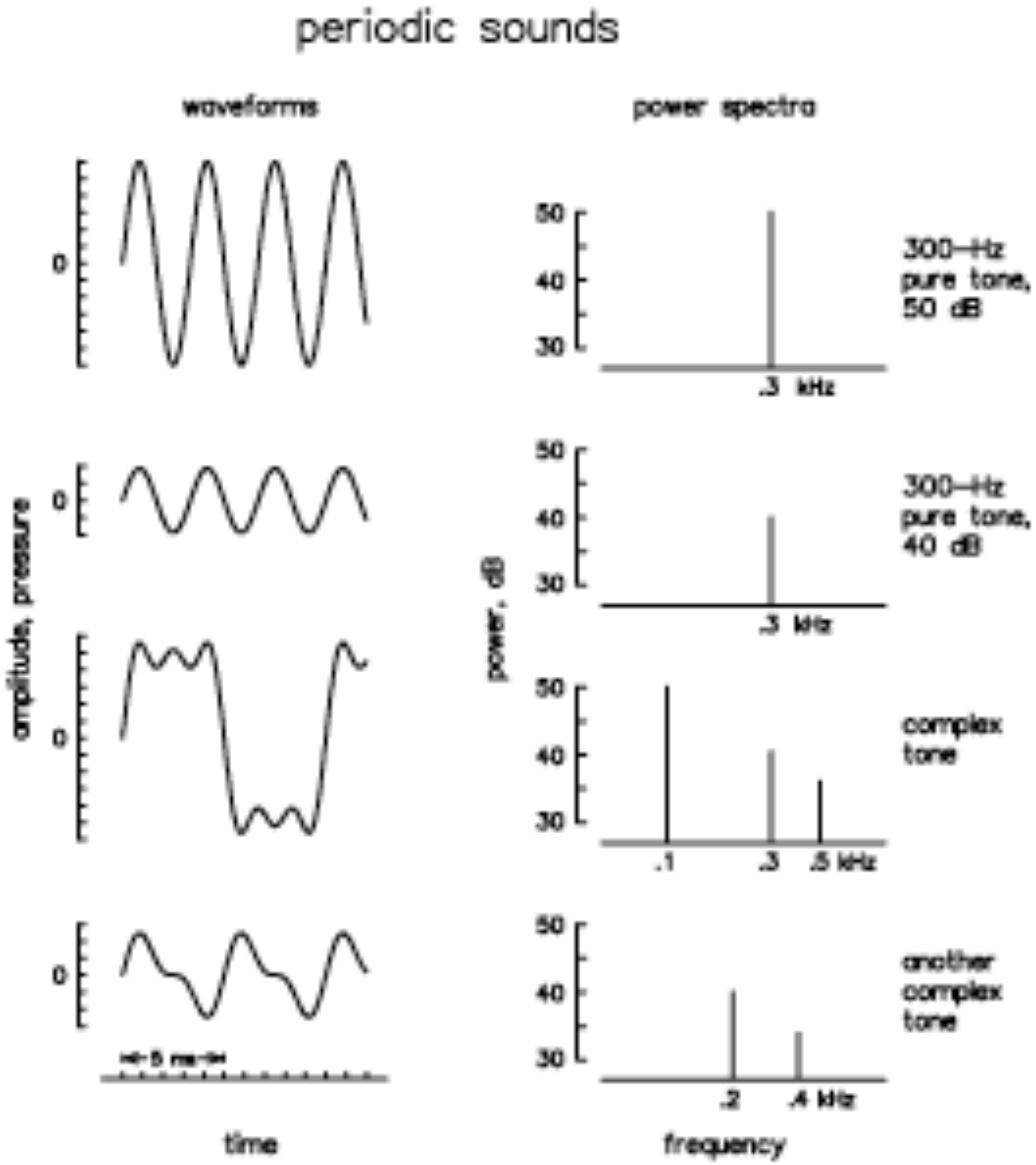
Análise harmônica: uma função qualquer



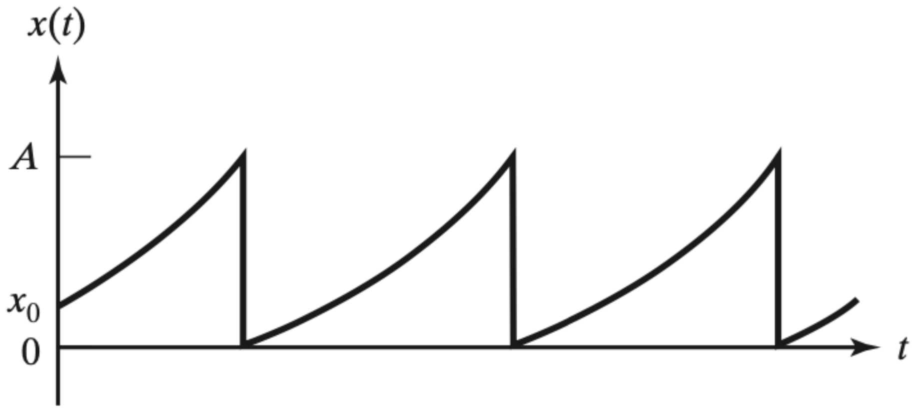
Análise harmônica: exemplo 1



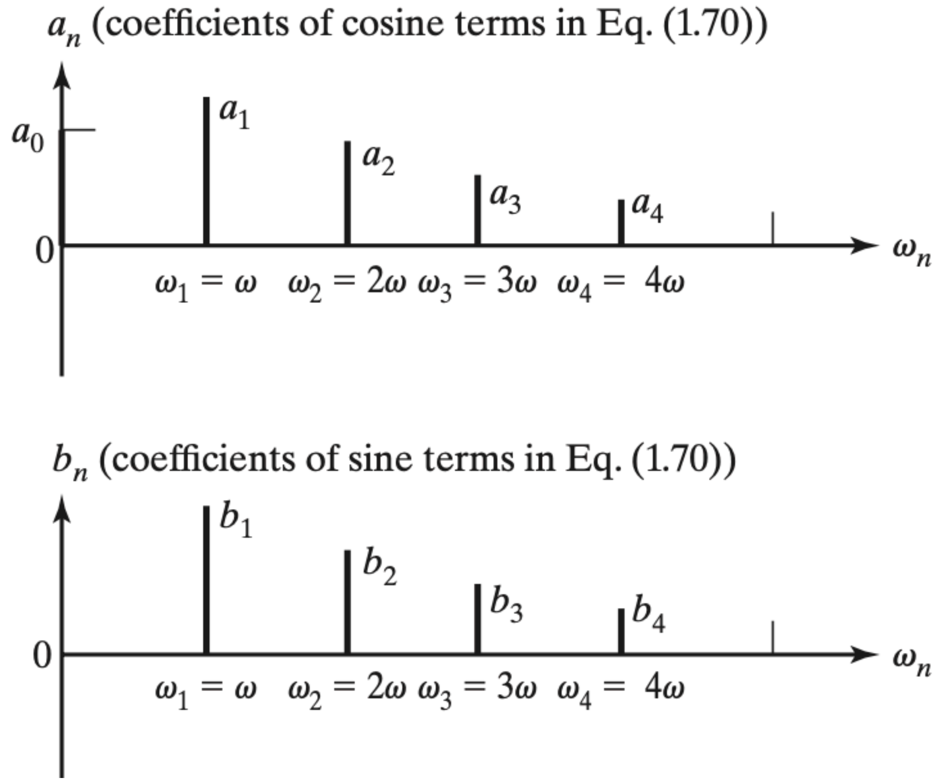



periodic sounds
waveforms
power spectra
amplitude, pressure
0
0
0
0
power, dB
50
40
30
.3 kHz
300-Hz
pure tone,
50 dB
50
40
30
.3 kHz
300-Hz
pure tone,
40 dB
50
40
30
.1
.3
.5 kHz
complex
tone
50
40
30
.2
.4 kHz
another
complex
tone
5 ms
time
frequency

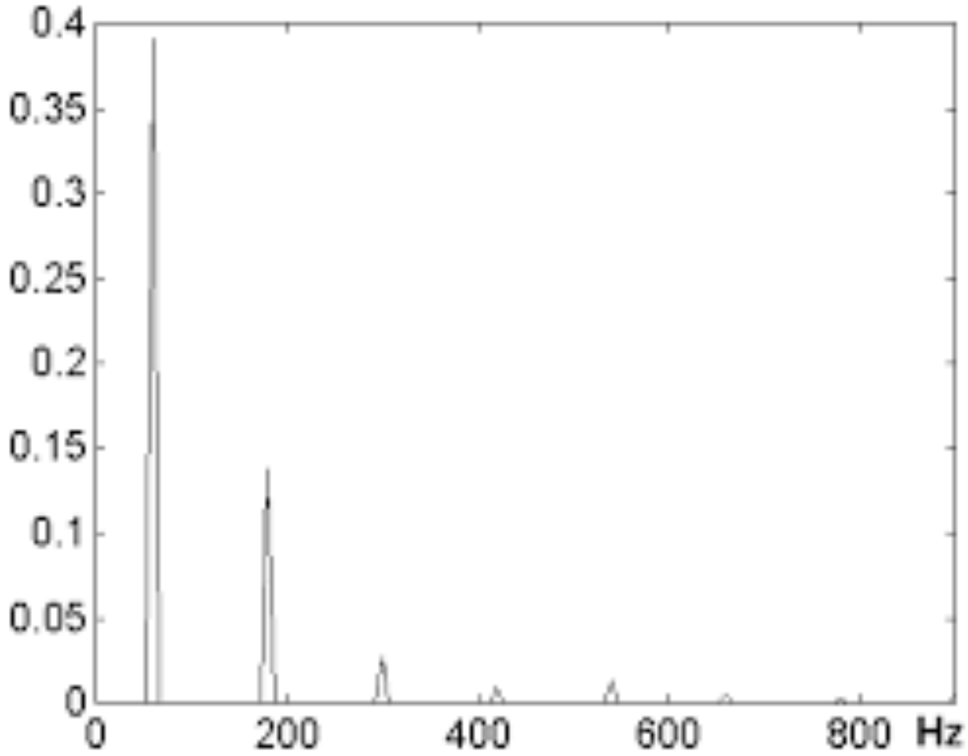
Análise harmônica: exemplo 2



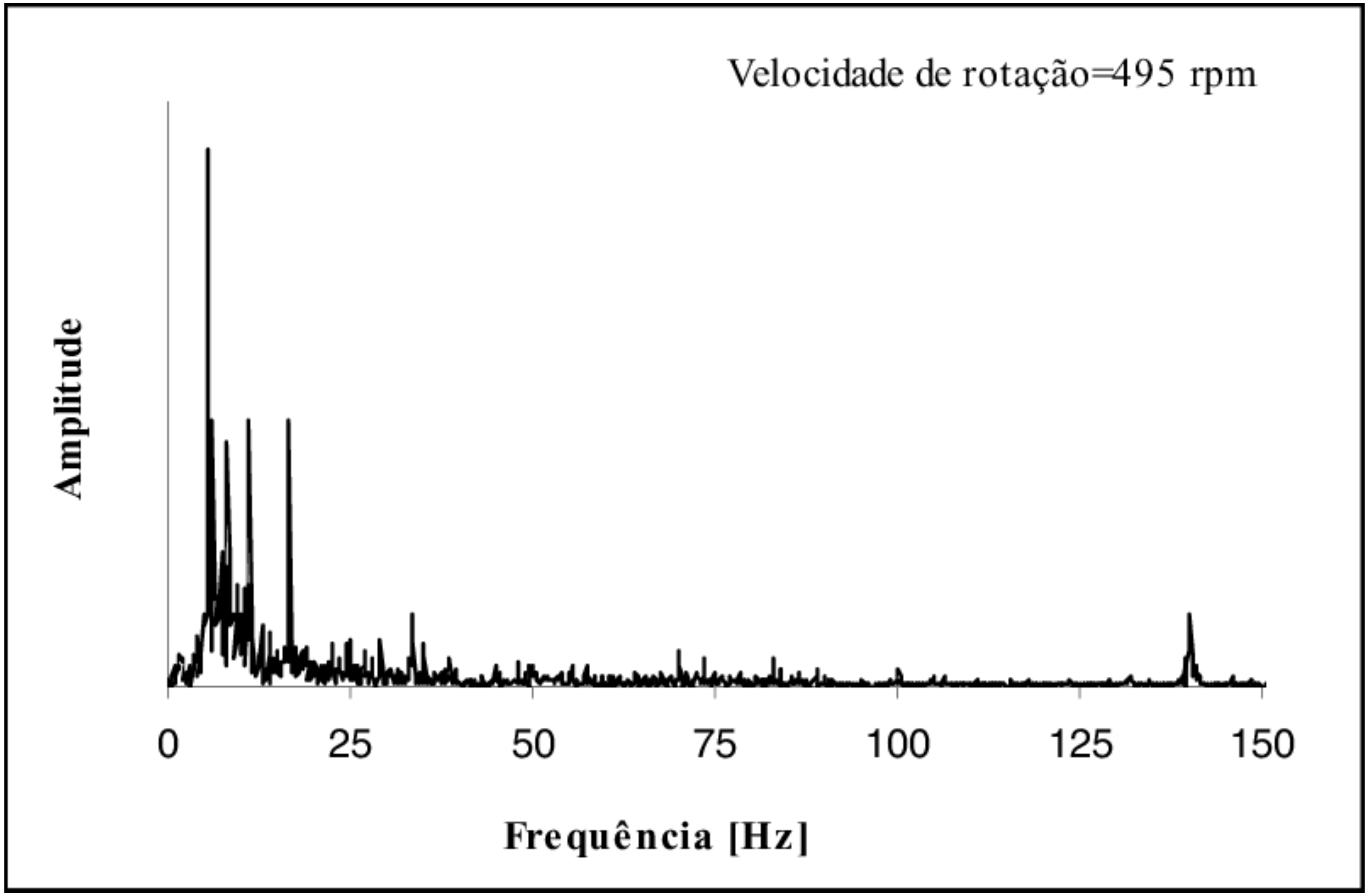
(c)



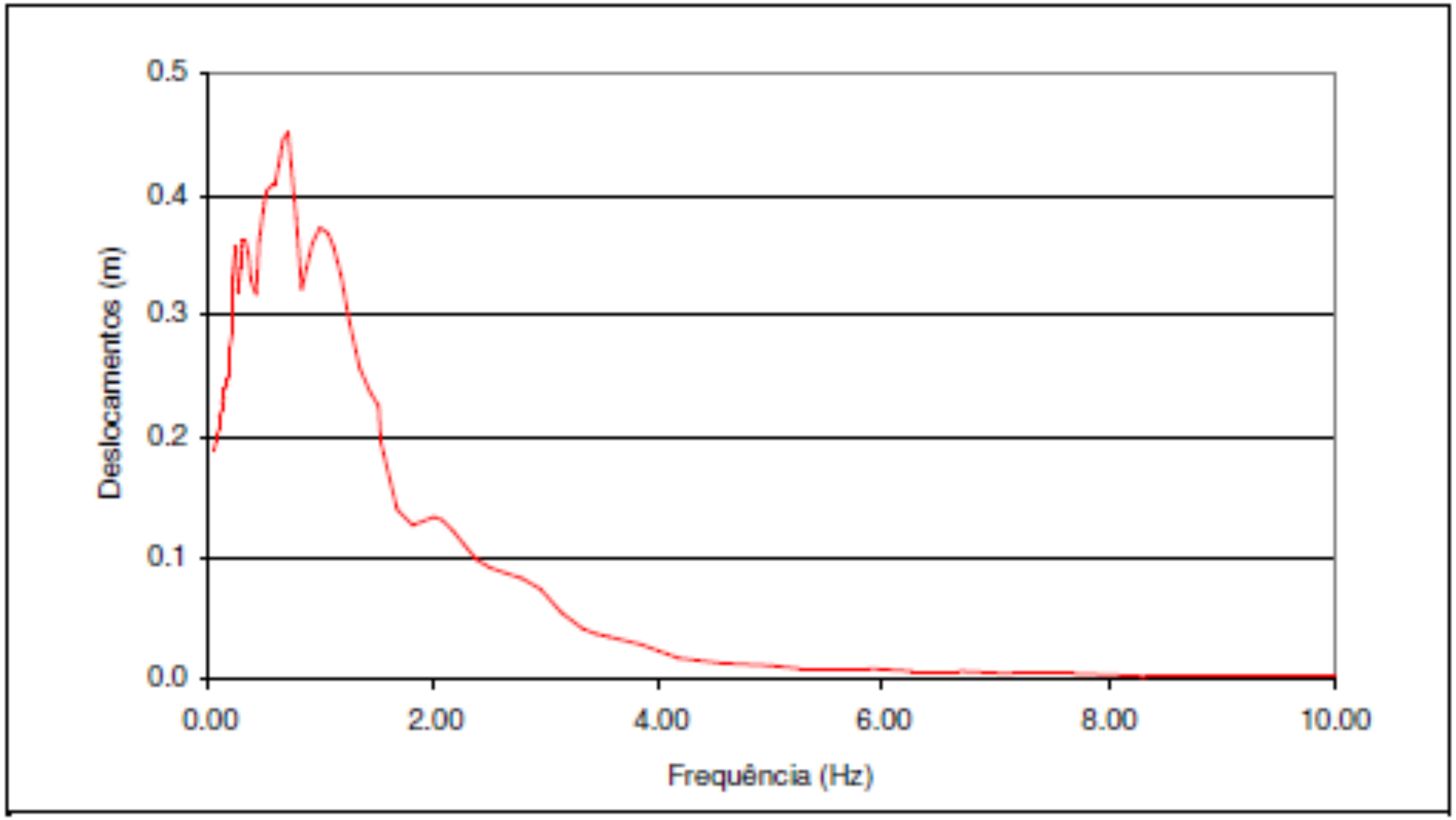
Espectro de frequência: exemplos



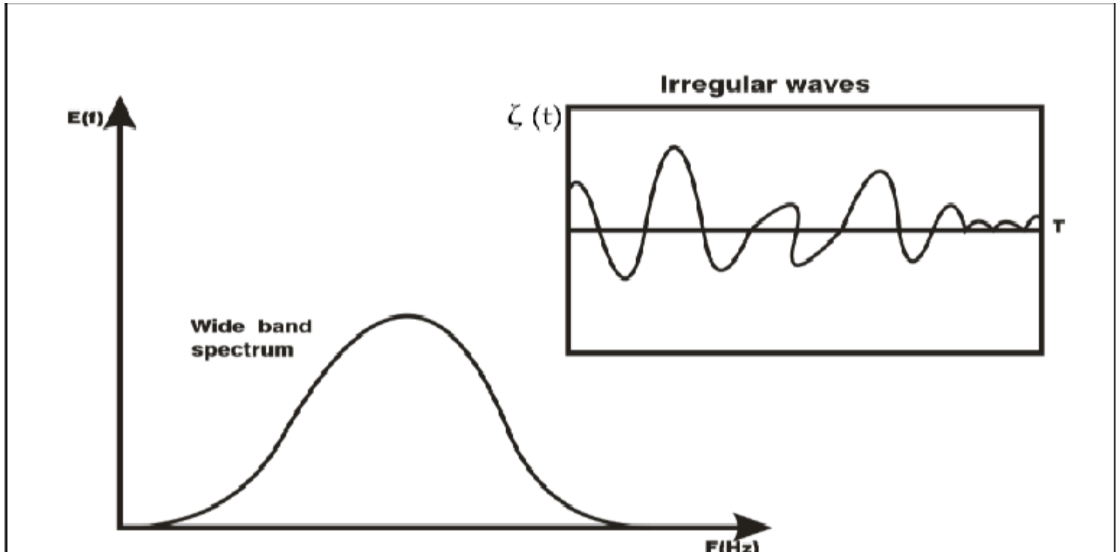
Espectro de frequência: exemplos

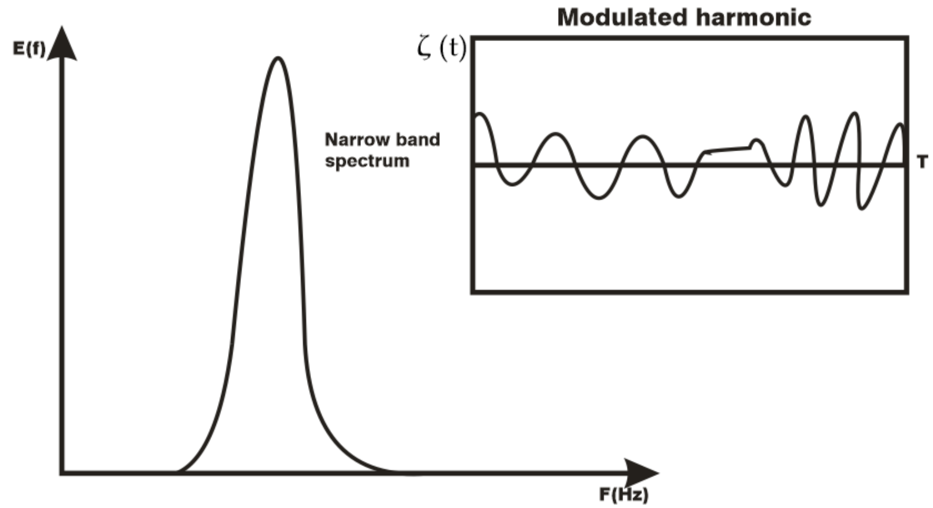


Espectro de frequência: exemplos

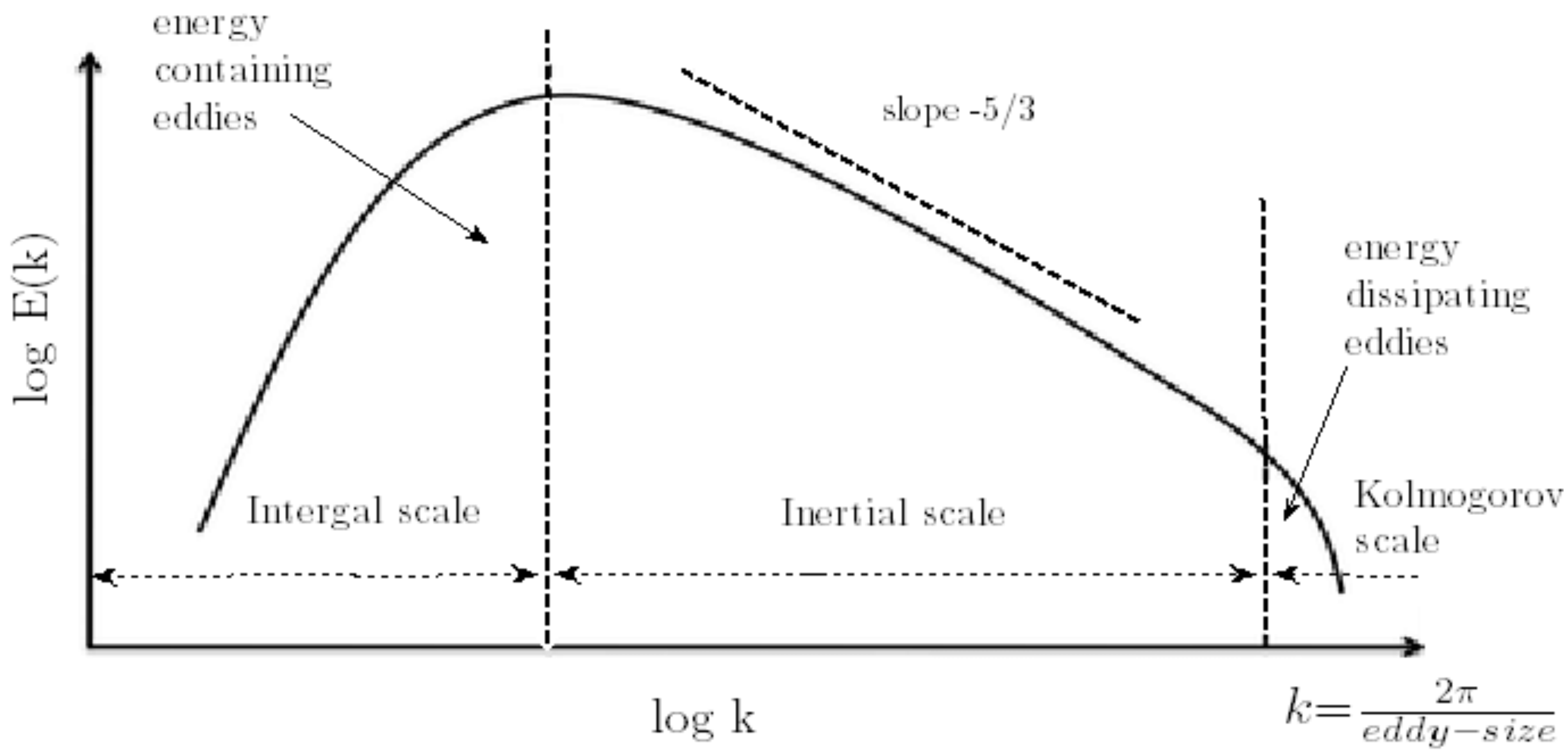


Espectro de frequência: exemplos





Espectro de frequência: exemplos

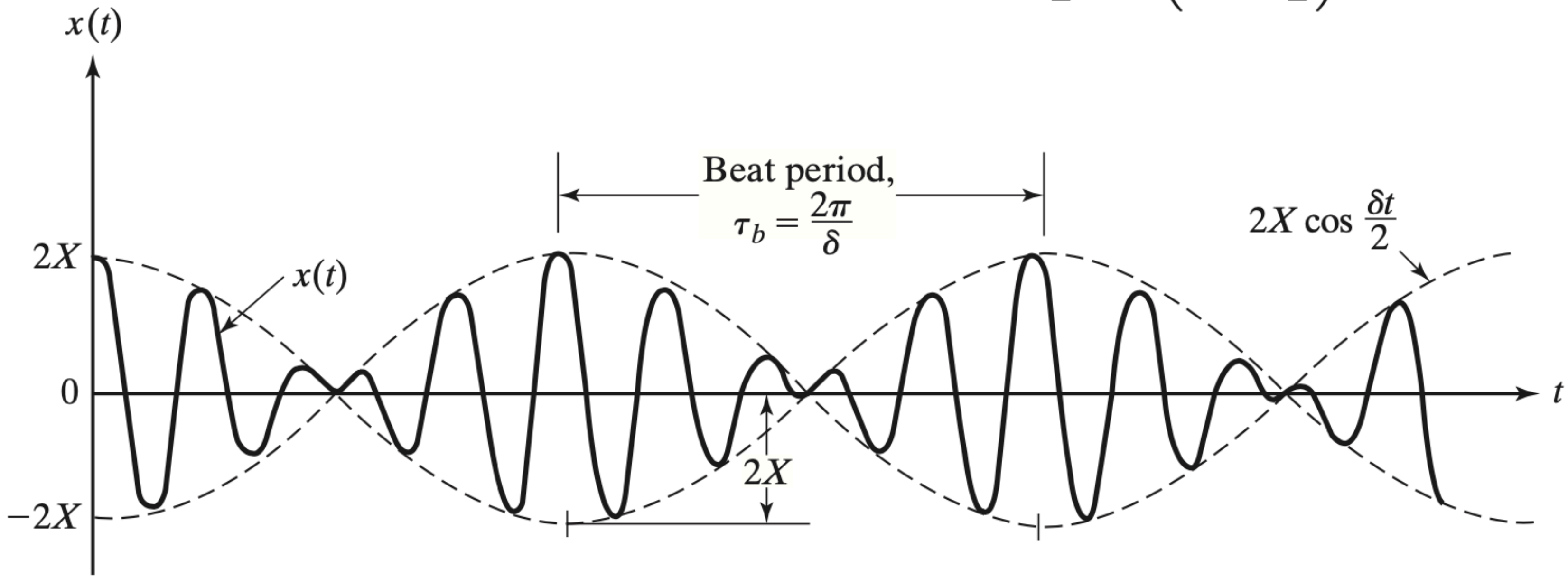


Batimento

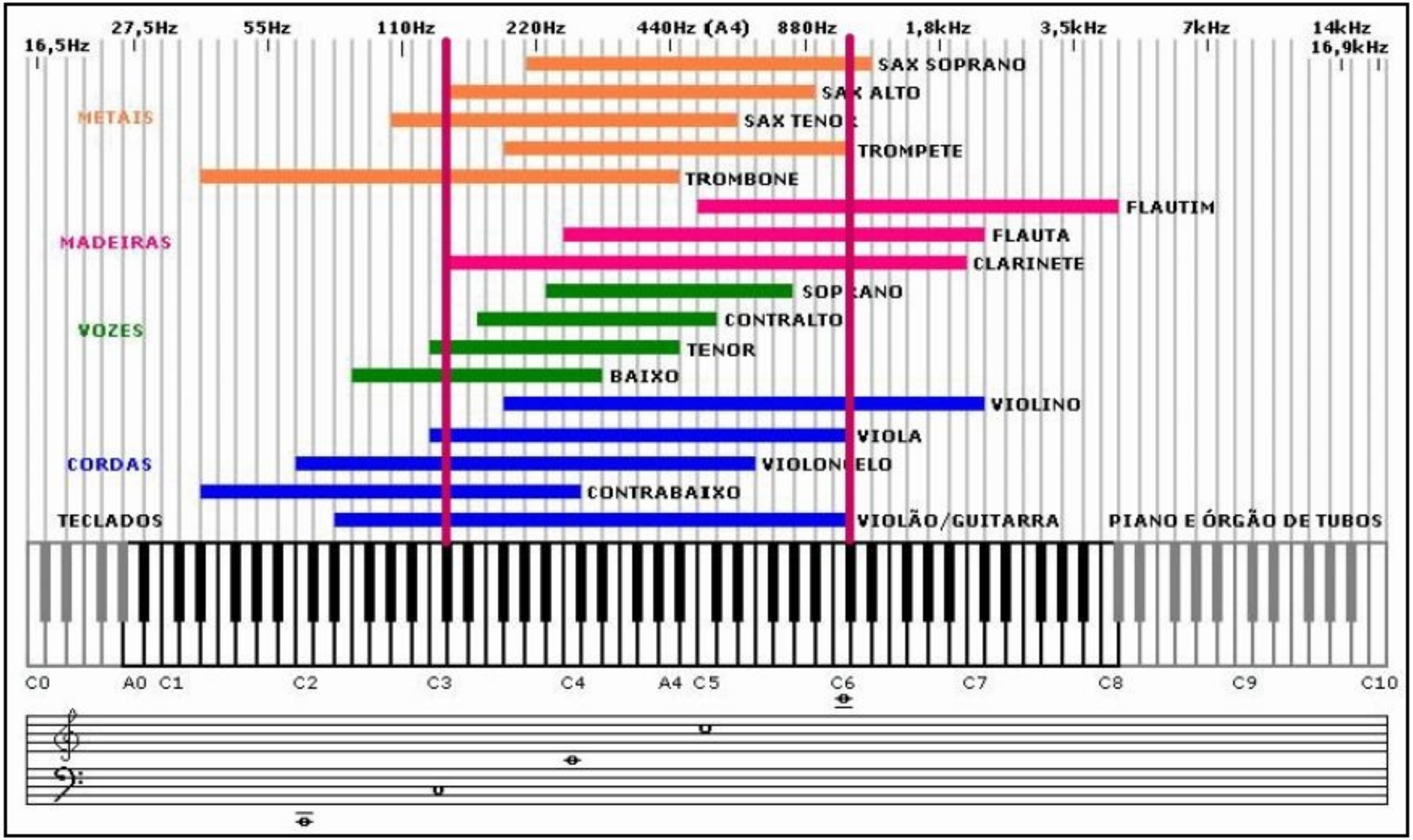
$$x_1(t) = X\cos\omega t$$
$$x_2(t) = X\cos(\omega + \delta)t$$

$$x(t) = x_1(t) + x_2(t) = X[\cos\omega t + \cos(\omega + \delta)t]$$

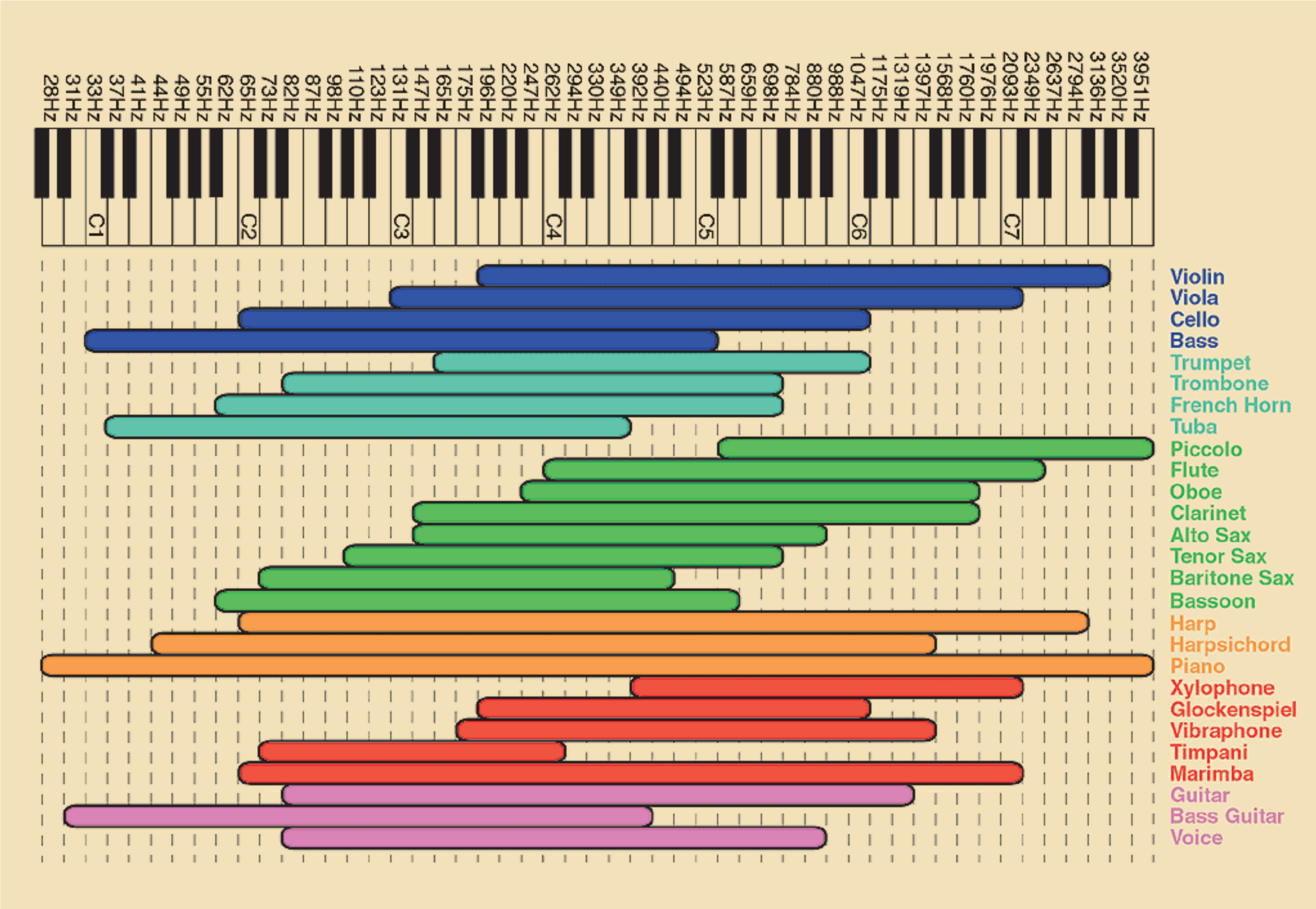
$$x(t) = 2X\cos\frac{\delta t}{2}\cos\left(\omega + \frac{\delta}{2}\right)t$$



Música

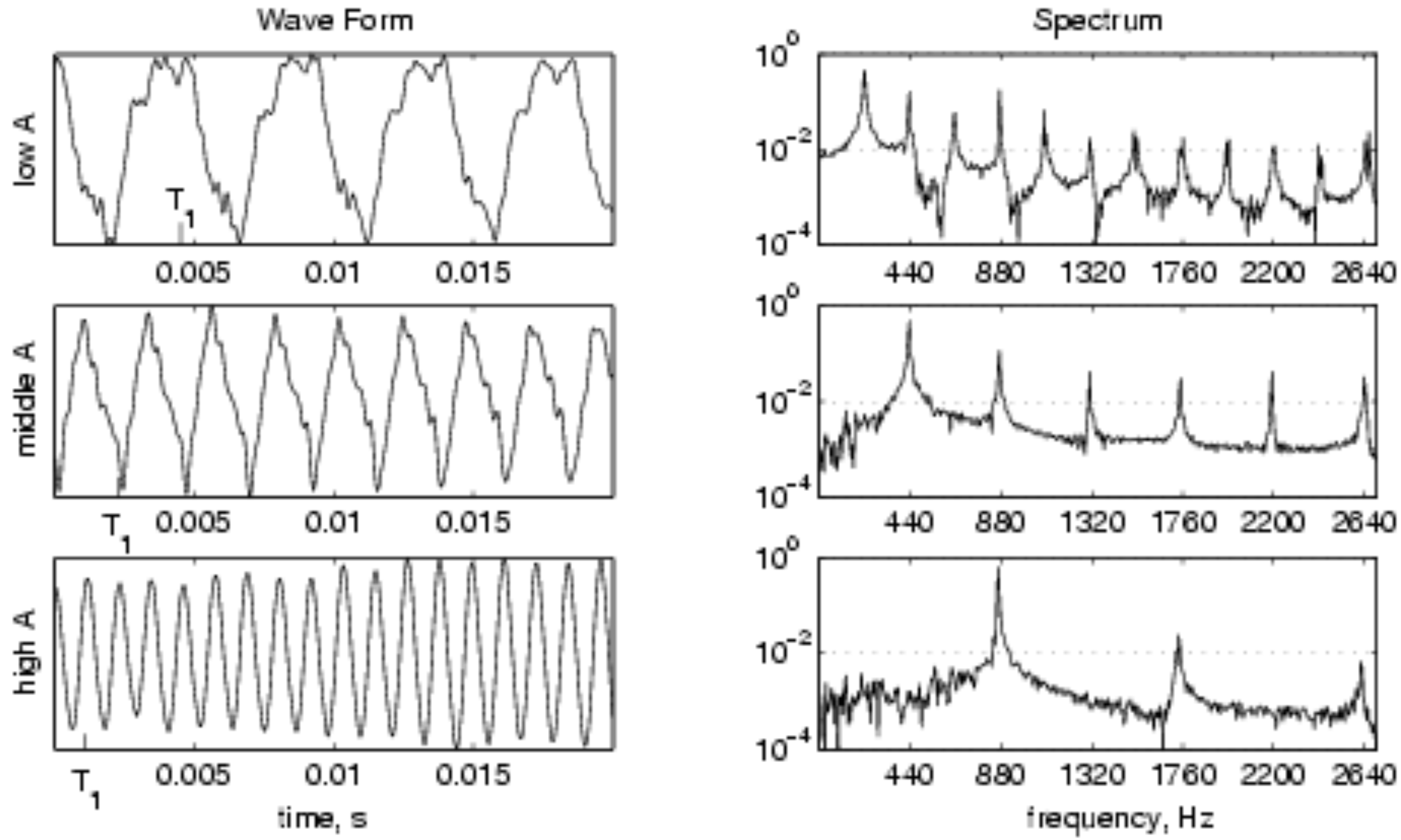


Mascaramento das faixas de freqüência dos instrumentos musicais e vozes

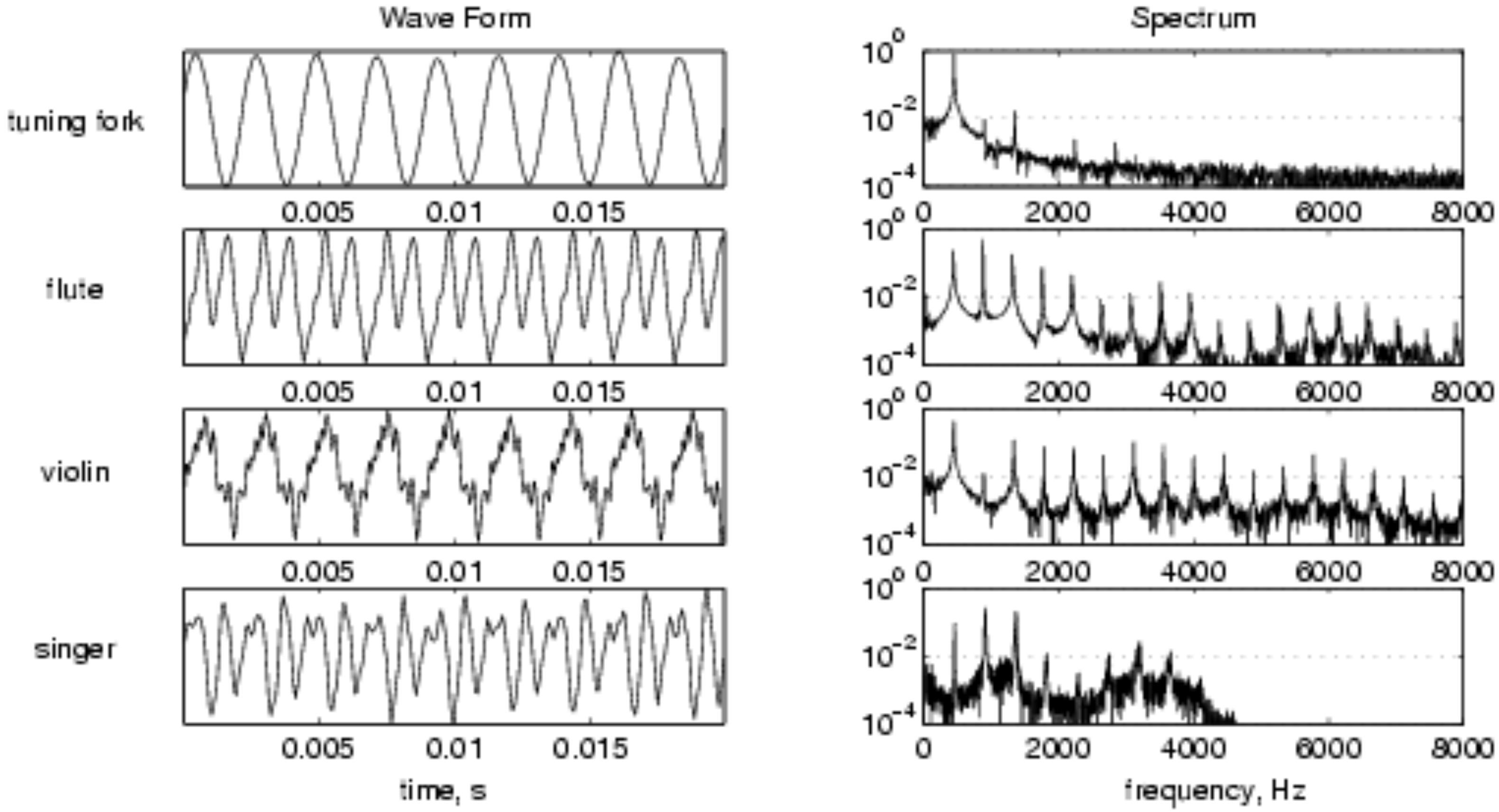


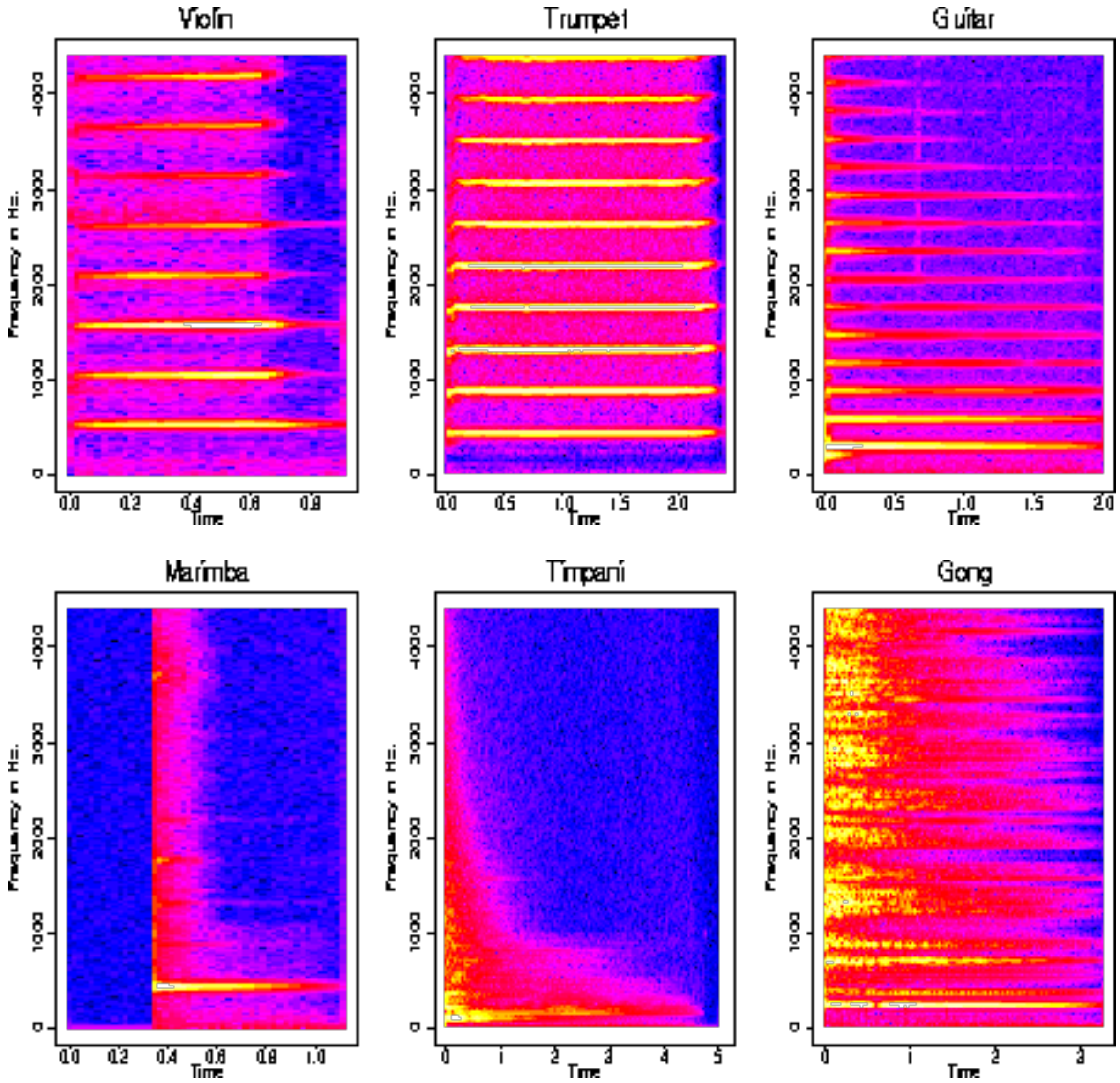
28Hz
31Hz
33Hz
37Hz
41Hz
44Hz
49Hz
55Hz
62Hz
65Hz
73Hz
82Hz
87Hz
98Hz
110Hz
123Hz
131Hz
147Hz
165Hz
175Hz
196Hz
220Hz
247Hz
262Hz
294Hz
330Hz
349Hz
392Hz
440Hz
494Hz
523Hz
587Hz
659Hz
698Hz
784Hz
880Hz
988Hz
1047Hz
1175Hz
1319Hz
1397Hz
1568Hz
1760Hz
1976Hz
2093Hz
2349Hz
2637Hz
2794Hz
3136Hz
3520Hz
3951Hz
C1
C2
C3
C4
C5
C6
C7
Violin
Viola
Cello
Bass
Trumpet
Trombone
French Horn
Tuba
Piccolo
Flute
Oboe
Clarinet
Alto Sax
Tenor Sax
Baritone Sax
Bassoon
Harp
Harpsichord
Piano
Xylophone
Glockenspiel
Vibraphone
Timpani
Marimba
Guitar
Bass Guitar
Voice

Análise harmônica: musical



Análise harmônica: musical





Violin
Trumpet
Guitar
Marimba
Timpani
Gong
Frequency in Hz.
Time