IOC5815 - Dinâmica do Fluido Geofísico I

1 Lista 02

1. Considere a propagação de uma onda bidimensional, tal como mostrado na Figura 6.1 de Vallis. Suponha que podemos decompor os parâmetros da onda nas componentes x e y, assim, temos para comprimento de onda λ, λ_x e λ_y, para velocidade de fase c, c_x e c_y, e para número de onda K, k e l.

   Encontre uma relação única e mais simples (ou direta) entre λ, λ_x e λ_y. Da mesma forma, encontre uma relação entre c, c_x e c_y, e para o número de onda. Os vetores de λ, velocidade de fase e do número de onda podem ser considerados típicos? Explica a sua resposta. Nota: a resposta desta questão não são as equações 6.10 e 6.11 do Vallis. Você deve encontrar um relação mais direta entre as 3 componentes.

2. Consider the passage of surface gravity waves over the free surface of a flat bottomed ocean with depth H, initially at rest. Starting from the general Navier-Stokes equation as a function of pressure:

$$\rho_0 \frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p - \rho_0 g \boldsymbol{k}$$

   determine the expressions for u', v', w', and p' due to the perturbation. Consider all necessary boundary conditions.

3. (a) Derive equation 7.48 (Vallis) in a detailed way, that is, show and explain all algebraic manipulations. Just a reminder, those equations are derived for a two-layer shallow water model. Discuss what is the consequence for that assumption? How it would affect the result if the shallow water conditions were not applied?

   (b) Show that for the baroclinic mode the sea surface and the interface elevations are out of phase in a two-layer ocean model. In this case, the shallow model conditions can be used.

4. The conservation of energy in a surface gravity waves is given by equation 7.31 (V). Derive the expressions for the mean kinetic energy average over the wave period ($\overline{KE}$), the perturbation potential energy $\overline{PE}$ and the time averaged energy flux $\overline{F}$. What is the expression of the energy conservation and how do we interpret that result? This should not be just a copy of Vallis. You have to understand how each term was derived and show that explicitly.

5. This problem is taken from Cushman-Roisin and Beckers (2011), page 782. Given a dispersion relation

$$\omega = \frac{k}{(k^2+1)},$$

   analyze the signal composed of two waves:

$$a(x,t) = A_1 cos(k_1 x - \omega(k_1)t) + A_2 cos(k_2 x - \omega(k_2)t),$$

   where ω is calculated using the dispersion relation. As before, show the evolution for $A_1 = A_2 = 1$ in the following situations:

- $k_1 = k_2 = 0.5$,
- $k_1 = k_2 = 2$,
- $k_1 = 1.95$, $k_2 = 2.05$,
- $k_1 = 0.45$,$k_2 = 0.55$.

Can you explain what you have observed? (Hint: Plot the dispersion relation.)