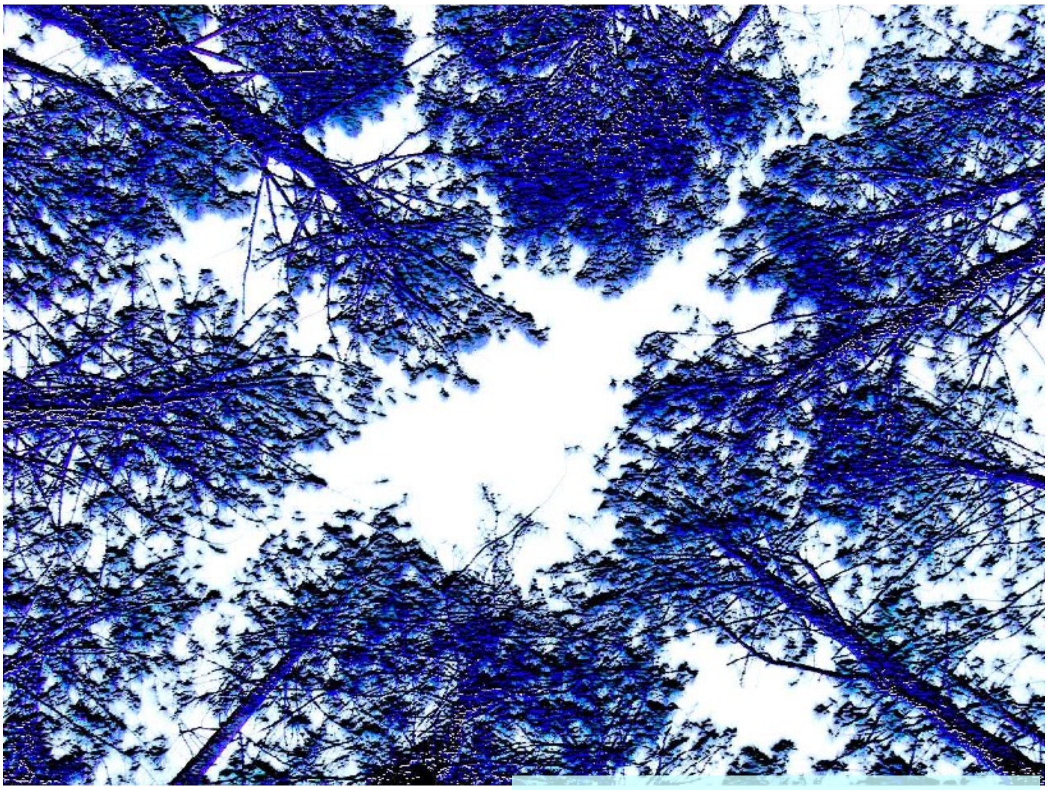

- Daddy, why are those trees so tall?
- Because they pump water, baby.

2nd VIRTUAL WATER PROBLEM
V. 3.3 Water Resources: Quantitative Aspects
SHS 5890

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Living Ram-Pump = Big-Leaf + Water-Pumps + CO_2 Fix

1. Background

A Virtual Water Problem (VWP) is a virtual narrative example of using hydraulic and hydrologic tools, **only** for academic purposes. A VWP lets students and scientists to play water scenarios by created storylines, with river basin approach, but without endorsements or commitments to institutions mentioned. The following VWP is the **second** of a series of VWPs. Its narratives encompass: global water policies, a case-study, river-basin driven problem, driving forces, international task force, water scenarios, executive summaries, adaptive intervention protocols and equations, tables, etc. References and links are presented at the end of the VWP narrative.

2. Virtual Water Problem Narratives

2.1 Global water policies

It´s the year 2023 and global protocols of SGD, DRR and COP21 are full running. Nations are more aware about disaster risk perception and invest into policies to reduce vulnerability of people, especially those refugees and migrant from/to climate-impacted regions. In Brazil, national economic incentives at the free-zone of Manaus, Amazonas, are being discussed to a new term: until year 2050. Carbon-trade and carbon assimilation mechanisms are currently part of Clean Development Mechanisms (CDM). Carbon assimilation is strongly influenced by the rate of real evapotranspiration and the ways water is pumped at forests regions. The mechanism of ram pumps (see in following sections) In order to mitigate regional impacts derived from changes occurring on the whole hydrological cycle, the United Nations (UN) will propose a new international program whose principal policy will state a central question: "*Virtual Water Problem under Global Crisis: Download or Footprint* ?". This cross-cutting, science-frontier program would encourage meteorologists, agronomists, plant physiologists/ biologists, soil physics, engineers, and physical chemists to make valuable contributions to a special hot-spot: "*how hydraulic concepts could regulate carbon sequestration through the transpiration process*?", poor-addressed during the 20th century. This hot-spot was wisely pointed by a whole vision of the mechanisms of transpiration, named as the Hendriks-Hansen Transpiration Disciplinary Vision (see **Box 1**).

Coping with global change risks, several Non-Government Organizations (NGOs) called UN on a Plea for Transpiration Process Revisited at Classroom (see **Box 2**). The objective of this Plea is to review, as Hendricks & Hansen had made four decades ago, the entire transpiration process from soil to atmosphere. The purpose of this review is twofold: (1), to attain a basic, unified understanding of principles affecting transpiration, and (2) that common people, especially pupils and students, could appreciate a qualitative evaluation of few fundamental concepts to the fields concerned with transpiration. Passing some decades, those pupil generations, who studied the essentials assets of hydrology and hydraulics at classrooms, became adults. That previous learning process permitted them to develop ideas of what are main societal needs to cope with water shortage and extremes. Many of those "old scholars" maintained that thinking in their business, thereby requesting politicians to put more efforts in real, practical solutions facing water.

Box 1. Hendricks-Hansen Transpiration Interdisciplinary Vision

In 1962, David W. Hendricks and Vaughn E. Hansen presented a useful contribution to approach the mechanisms of transpiration into a broadly, friendly manner. Their vision had been presented with a special attention to the professionals studying the problem, as follows.

- The Meteorologist, more concerned with water vapor as it affects the weather, and the weather as if affects vaporization of water.
- The Agronomist and the Soil Physicists, usually defining the relative ease with which water is available to the root system in different types of soils and at different moisture contents for the respective soil.
- The Plant Physiologist, by providing considerable information on the physical features of the plant conduction media and the interrelationship between transpiration and the condition of the plant. Usually (s)he describes the forces that lift the water within the plant, but in an intuitive manner inadequate for precise description
- The Physicist, having used the available energy approach to evaluate the basic mechanism of the evaporation process involving vaporization of the water and transport of the vapor.
- The Physical Chemist, using the Free Energy Concept, to evaluate all the energies involved for the entire transpiration process, with a precise language, not well understood by those of other fields.
- The Engineer, attacking in a more practical approach in order to estimate total area-wide evaporation quantities for existing and proposed projects, sometimes using empirical formulas to estimate consumptive use and easily available weather and crop information

In this way, the Hendricks-Hansen vision on transpiration went further, as they quoted in 1962:
"*…it is felt that the integrated hydraulic approach will consolidate (...) on the various phases of the transpiration problem. The following hypothesis appears to unify and to further explain the transpiration phenomenon in terms of a hydraulic engineering approach.*"

Box 2. A Common Plea for Transpiration Process Revisited at Classrooms

At the beginning of 21st century, several international continental trade blocks, especially from Europe, Asia, and North America, facing to the exploding water demand expected for the year 2025 and the uncertain scenarios of people migration, have suggested countries' governments to deeply encourage the study of the hydrological cycle and hydraulic concepts at secondary school classrooms. One emphasis will be the transpiration process because its the intrinsic nature of the process is, or must be, associated to how hydraulic concepts should help on the rationale of managing carbon sequestration and, at global scale, to counter-attack the incipient global warming. Other focus is how to represent those processes, integrating the "hydraulic pump" with the "big leaf" and with the "water (and carbon) footprint"

2.2 A case-study at scale of domiciliary urban lot

Pair decades passed and their effects emerged. For instance, during the period 2017-2019 some authorities of several countries submitted to UN and World Bank a lot of reports and bottom-up manifestos to how the occupation of human settlements should be in the short future. Newly mentioned, the "agropoles" could be urbanized through hydrologically-regarded lots and according to UN's global scenarios, especially from the Millennium Ecosystem Assessment, whose results were formerly hosted at www.MAweb.org.

All these projects should be standardized in terms of regional (spatial) impact, inter-generational (time) sustainability and located in different latitudes (international relevance). Details of water balance in agropoles were presented in the 1st Virtual Water Problem. In the following sections, the foci will be at the scale of domiciliary urban lot existing at the agropole (see **Box 3**).

Box 3. Lyrics for urban 'hydro-lots'

As the 20th-Century Hendriks-Hansen's Vision, the urban 'hydro-lots' of the 21st century are to be projected in respect to the elements of the soil, the plant and the atmosphere, as a whole. Very popular among communities, the 'hydro-lots' are daily part of people folk and feelings. Usually the 'hydro-lots' receive poems and lyrics, from common people and non-technician users who had made musical rhythms, in order to friendly keep in mind the role the 'hydro-lots' play in the hydrologic cycle. Lyrics, with verses separated by diagonal bar (' / '), have the power of being simpler for teaching pupils of the future and for guiding the performance of water regulation and carbon sequestration by futuristic urban hydro-lots.

- "***The soil***": "*…The first part of the soil / To lose moisture / Is that part in direct contact / With the root system / A difference in soil capillarity potential / Let's say ΔHs / Will be established / Between the soil adjacent / To the root system / And the outlying soil / Of a higher moisture content / And capillary potential is a capillary force / Holding the water, holding the water / Between soil particles / The first part of the soil / To lose moisture* (chorus)"

- "***The Plant***": These lyrics people used to sing are composite poems known as "***The Root***", "***The Xylem***", "***The Leaf***" and "***The Stomata***" ("*Around the* ***Root*** */ The maximum possible rate / Of moisture absorption / By the Plant / Is proportional to the amount / Of Root surface area / In direct contact with moisture //…// The* ***Xylem*** *is a water-conducting media / With low hydraulic resistance / Between the Root and the Leaf / It can be compared / As a series of tubes" //…// "The* ***Leaf*** *provides the water exit / In vapor form, in vapor form / Its vapor flow is regulated / By stomates that governs / Of course, the water movement / Through the Plant, all the Plant ! / Its major energy mechanisms / For moving and lifting / The liquid water though Plant / Are found in the Leaf"//…// "The* ***Stomates*** *are valves / On the end of the system/ When they are open / Water vapor moves readily out / From the leaf to the atmosphere / And carbon dioxide moves into the Leaf / It is photosynthesis / When Stomates are closed / Gaseous exchange is quite paused / Then they conserve water / To survive much longer/ When a moisture deficit develops / Within the Leaf / The Stomatal valve becomes important / As a major throttling device / To regulate the transpiration rate* (chorus)").

- "***The Atmosphere***" "*The vapor moves / From dense to less dense regions / Of water vapor / Technicians say 'Fick's Law' / With the atmosphere / As conducting media / And vapor is part of the air / A moist air, the moist air* (chorus)".

The International Decade 2003-2012 of 'PUB' Program—*Prediction in Ungauged Basins* formerly hosted at www.iahs.info site, was renewed for the period 2012-2021 and suggested UNESCO to give incentive research grants to any "**agropole**" that would encompass more hydrologically-regarded urban lots. The term 'hydrologically-regarded' herein means that the urban lots are to be planned in order to collaborate with global life cycles, especially the master cycles from water and carbon, and their relationships.

Each urban lot would be sized, projected, constructed and maintained, in a way, to mimic as far as possible the three key components: soil, plant and atmosphere. In other way, because these three elements have crucial points to processes like transpiration, they are capable of being regulated or managed by owners and residents. The future bio-technologies will work on how hydro-lots would seem more with plants in order to regulate water losses and carbon sequestration. Colloquially speaking, those futuristic urban lots would be known as "hydro-lots" and their key elements are more explained in **Box 3**.

From Box 3, it is clear that soil acts as a moisture reservoir for water utilized by plant; the rate at which soil moisture is available to plant depends on how much of the root system is in direct contact with moisture; this rate is dependent of (1) the total root surface area at any instant, and (2) the rate at which moisture can be conducted through the soil from moist regions away from the roots to drier regions adjacent to the root that have been depleted of moisture.

The major components of the plant transpiration system are the root system, the xylem, and the leaf system of which the stomata are a part, and will be considered only from a functional aspect and not necessarily in anatomical detail.

3. Driving forces: energy changes at the 'hydro-lots'

Obviously, energy changes encompass the liquid and the vapor phase as explained below.

3.1 Liquid phase

The liquid phase is dependent on hydraulics of flow and the sources of energy, as Figure 1.

3.1.1 Hydraulics of Flow

The transpiration system can be hydraulically analyzed as any other closed system that transports fluid, as a pipeline for instance. Figure 1 shows the energy grade line concept applied to liquid and vapor phases of the system. Because of low velocities, velocity head is neglected due to laminar flow.

Energy is required to: (1) extract water, (2) transport water through the soil, (3) transport it through the plant, (4) vaporize water and (5) transport water into the atmosphere. All these energy phases have losses. In Figure 1, the change in the point-to-point Bernoulli energy level gives the head losses and energy sources. Flow direction is towards negative energy gradient.

Accounting for the head losses through the liquid phase of the transpiration process, the total frictional head loss is:

(Equation 1),

in which h is the dissipated head with the subscripts "s", "r", "x" and "l" referring to the soil, root, xylem and the leaf components, respectively. These subscripts will designate the same flow components when used hereafter with other hydraulic terms. Assuming that the flow is laminar, Darcy's flow ($q_{Darcy} = k \cdot h/L$) and continuity equation ($Q = q_{Darcy} \cdot a$) may be applied to each component of flow in the liquid phase. For any given hydraulic component, q_{Darcy} is the velocity, k is the hydraulic conductivity, L is the length towards the flow directs, a is the cross-sectional area to the flow, and Q is the total flow rate. If the expression $h = Q/a \cdot L/k$ is substituted into each component, it yields the hydraulic factors that affect transpiration in the liquid phase.

$$h_f = Q \cdot \left[\frac{L_s}{a_s k_s} + \frac{L_r}{a_r k_r} + \frac{L_x}{a_x k_x} + \frac{L_l}{a_l k_l} \right]$$

(Equation 2)

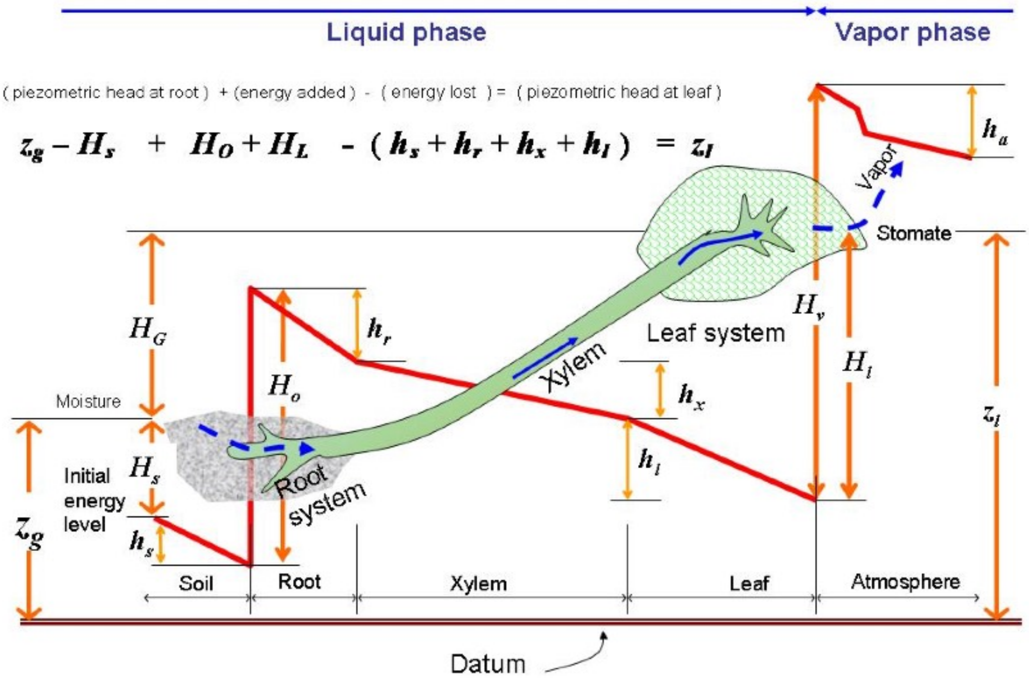


Figure 1 – Hypothetical energy grade line in the transpiration process for condition $h_f = H_f$. (from Hendricks and Hansen, 1962). See explanation on the text, through Figures 2 and 3.

The terms within brackets in equation 2 are resistance terms, and could be replaced as R_s for the soil's resistance and R_p for the plant's resistance, as follows:

$$R_s = \frac{L_s}{a_s k_s}$$

(Equation 3)

$$R_p = \frac{L_r}{a_r k_r} + \frac{L_x}{a_x k_x} + \frac{L_l}{a_l k_l}$$

(Equation 4)

in which R_s and R_p are the equivalent hydraulic resistances of the soil and plant respectively. R_s is dependent on the soil moisture content and the type of the soil, and R_p depends on the type of the plant and the stage of growth, that includes the extent of the root system development. From Equation 2, a more simplified equation is obtained:

$$Q = \frac{h_f}{R_s + R_p}$$

(Equation 5),

which states that the flow rate that can be delivered in the liquid phase is dependent on the ratio of the total head available and the hydraulic resistances of the soil (depending on water content) and the plant (depending on the type and growth phase). From equation 5, it can be seen that the lower the value for R_p and the more head, h_f , that can be developed, the more drought resistance is the plant. Keep in mind that the vertical distance between the plant-conduit and the energy grade line (EGL) represents pressure head because velocity head is negligible.

It can be seen as Q increased during the day, causing h_f to increase, the EGL drops further below the plant, thereby increasing the negative pressure in the water stream. This explains, in hydraulic terms, the diurnal fluctuation in stem diameter noted by plan physiologists, that is, an elastic phenomenon caused by changes in pressure within the stem.

3.1.2 Sources of energy

Sources of energy whether from electrostatic effects or temperature differentials may exist. But the most important fact is that sufficient energy must exist to transport the water and the several types of energies do exist and are additive algebraically. There are positive and negative energies. Figure 1, Figure 2 and Figure 3 help to understand the statements in following sections.

3.1.2.1 Positive energies

These include capillary potential in the leaf, H_L (capital 'H'), and osmotic pressure, H_o.

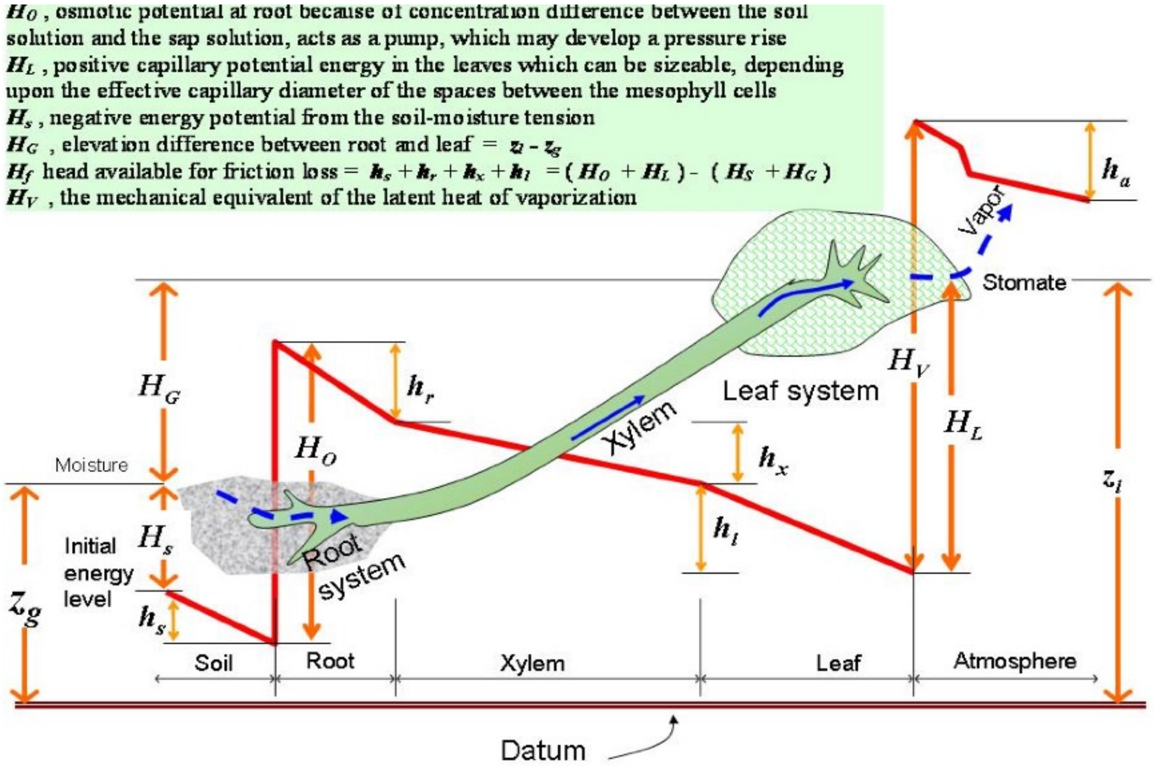


Figure 2- Energy considerations from hypothetical mechanisms of evaporation (from Hendricks and Hansen, 1962).

For the mathematical description of the capillary energy at leaf, it is convenient to consider leaf interstices as having an “effective” capillary diameter, that can be sizeable during the day, as:

$$H_L = \frac{4 \cdot \sigma \cdot h}{\gamma_w \cdot d} \cos\beta \qquad \text{(Equation 6)}$$

in which σ is the surface tension of water (“sap solution”), γ_w is the specific weight of water (“sap”), β is the angle of contact between the fluid and the walls of “tube” with diameter d, and h is the height of rising fluid inside the walls.

Osmotic pressure, H_o, is dependent on the difference in concentration between the leaf sap solution and the soil solution. Osmotic pressure can also develop into a significant potential energy. This potential could act negatively for a highly saline solution and a weak sap solution. Osmotic pressure has the hydraulic effect of a pump, and can be calculated as:

$$H_O = n \cdot R \cdot T \qquad \text{(Equation 7)}$$

where n is the molar concentration of solution (moles per liter) for soil or sap, R is the gas constant, and T is the absolute temperature.

3.1.2.2 Negative energies

Negative potentials include soil moisture capillary potential, Hs, and elevation (gravity) energy, H_G.

Box 4. Hydraulic analogy at 'hydro-lot' scale from plant physiology

The soil may represent a reservoir having a lower head that may go down to H_L, but not necessarily unless the flow rate demands it. The water in the leaf reservoir is higher than in the soil reservoir by an elevation difference H_G. An osmotic pump between the two reservoirs aids the flow from soil to leaf. The osmotic pump can develop a head H_O. A summary of all positive and negative energies yields the net potential energy in the liquid state, H_f, which is available, if needed, as frictional head loss by the moving water

$$H_f = H_O + H_L - (H_S + H_G)$$

In terms of forces, a similar approach can be used, as:

$$F_f = F_O + F_L - (F_S + F_G)$$

in which F_f is the resultant force available for moving the water. Because the system is in a dynamic state, a frictional shearing force f_f opposing the motion should also be depicted. This shearing force corresponds to frictional head loss, h_f, in the energy approach[1]. It is important to remember that H_o and H_L may exceed even the largest values of H_G , and this physically explain the ability of the highest trees to transpire water.

Because there is a limit on H_f, there is also an upper limit on the flow capacity of the transpiration system, as it must always be true that $h_f \leq H_f$. Figure 1 is drawn for the condition in which $h_f = H_f$. If it should occur that the rate of vaporization is such that this upper limit flow rate of the liquid phase of the transpiration system is exceeded, then the plant tissues will supply the deficit. The plant will eventually wilt if the vaporization rate continues to exceed the hydraulic capacity of the transpiration system. However, the stomates provide a throttling effect when there is a moisture deficit. Also, positive potentials, H_O and H_L, may exceed even the largest values of H_G, explaining the ability of the highest trees to transpire water.

3.2 Vapor phase

When the rate at which water can be delivered to the leaf is <u>not</u> a limiting factor, the vapor phase controls the transpiration rate. Excepting as regulated by leaf structures and stomata, transpiration is essentially evaporation from the leaf because the water evaporates from the moist spongy mesophyll cells into the sub-stomata cavity and passes through the stomata into the atmosphere.

[1] As in the energy approach, in which it must always be true that head loss, h_f, does not exceed available head, H_f, in the force approach, the shearing force, f_f, developed by the moving liquid cannot exceed the resultant available force, F_f , or $f_f \leq F_f$. The force approach and the energy approach are essentially identical because force times length equals energy, or F is proportional to dH/dL

Two conditions are necessary for a net evaporation to occur and either may govern evaporation rate. First, an energy source must exist to raise the energy level of the water molecules to the vapor state; therefore, the rate of evaporation is limited by the rate at which this energy is available. Second, a transport mechanism must exist to remove the water vapor from the neighborhood of its source to the external atmosphere.

Vapor phase is described in terms of available power and vapor transport, explained below.

The weight rate of vaporization *G* is first dependent on the rate at which energy *P* is available for evaporation. *P* has the dimensions of a power term. If considered analogous to a pump, the latent heat of vaporization, H_v (585 calories per gram of water), may be considered a fixed operating head and the terms may be combined to yield the power equation:

$$G = \frac{P}{H_v}$$ (Equation 8),

noting that $G = Q \cdot \gamma_w$, in which *Q* is the volume flow rate and γ_w is the specific weight of water. The main sources of this power are radiant energy from sun and advective heat. If incoming radiant energy is utilized, the *P* term can be expressed as an unknown and evaluated.

Two conclusions may be drawn from Equation 8. First, the rate of vaporization *G* is limited by the power *P* available to vaporize the water. Second, if the rate of vaporization *G* is throttled by some other mechanism, i.e. the hydraulic system of the liquid phase or the vapor transport process, then the power consumed by evaporation may not necessarily be the maximum power available. In this case, the excess power will be consumed by other processes such as heating the air.

3.2.1 Vapor transport

A net evaporation cannot occur in a saturated atmosphere unless the water-vapor molecules in that saturated atmosphere are removed. The weight rate of vapor transport, *G*, from the saturated local atmosphere, adjacent to the liquid source, is a key phase of the transpiration process. The actual vaporization from liquid to a vapor results in an increase in internal energy of the fluid by *Hv*. The H_v rising in the 'energy grade line' (EGL) of Figure 3 implies such an increase in internal energy. The rate of vapor transport is described by Fick's Law of diffusion, similar o Darcy's Law, as:

$$\frac{G}{A} = (k_a \cdot R \cdot T) \cdot \frac{d\gamma}{dL_a}$$ (Equation 9),

in which the weight rate of vapor transport per unit area, *A*, is dependent on k_a, a diffusion constant, *R*, the gas constant, *T*, the absolute temperature, and $d\gamma/dL_a$, the vapor concentration gradient. For practical purposes, here the notation γ is not the specific weight, and is equivalent to water vapor as absolute humidity ($\gamma = q_v$).

Because the diffusion constant is dependent on the specific weight of the water vapor, $k_a = f(\gamma)$, it will vary throughout the transport path L_a, which is the distance between the saturated leaf surface and a point in the atmosphere. If Equation 9 is to be expressed in terms of relative humidity ($r_h = \gamma/\gamma_s$), the differential term is depicted as:

$$d\gamma \approx \Delta\gamma \approx (\gamma_s - \gamma) = \left(1 - \frac{\gamma}{\gamma_s}\right)\gamma_s = (1 - r_h)\gamma_s$$

and the Fick's Law re-expressed as:

$$\frac{G}{A} = (\overline{k_a} \cdot R \cdot T) \cdot \frac{(1 - r_s)\gamma_s}{dL_a} \qquad \text{(Equation 10).}$$

The term $\overline{k_a}$ would be an integrated average k_a over the distance L_a. The EGL for this vapor transport in terms of useful energy is seen in **Figure 3**. The energy drop between two points in the atmosphere, h_a, and the vapor transport is a function of the logarithm of the ratio of the vapor pressures at the two respective points. The discontinuity shown in the EGL of **Figure 3** for the vapor phase is to account for any energy loss through the stomata.

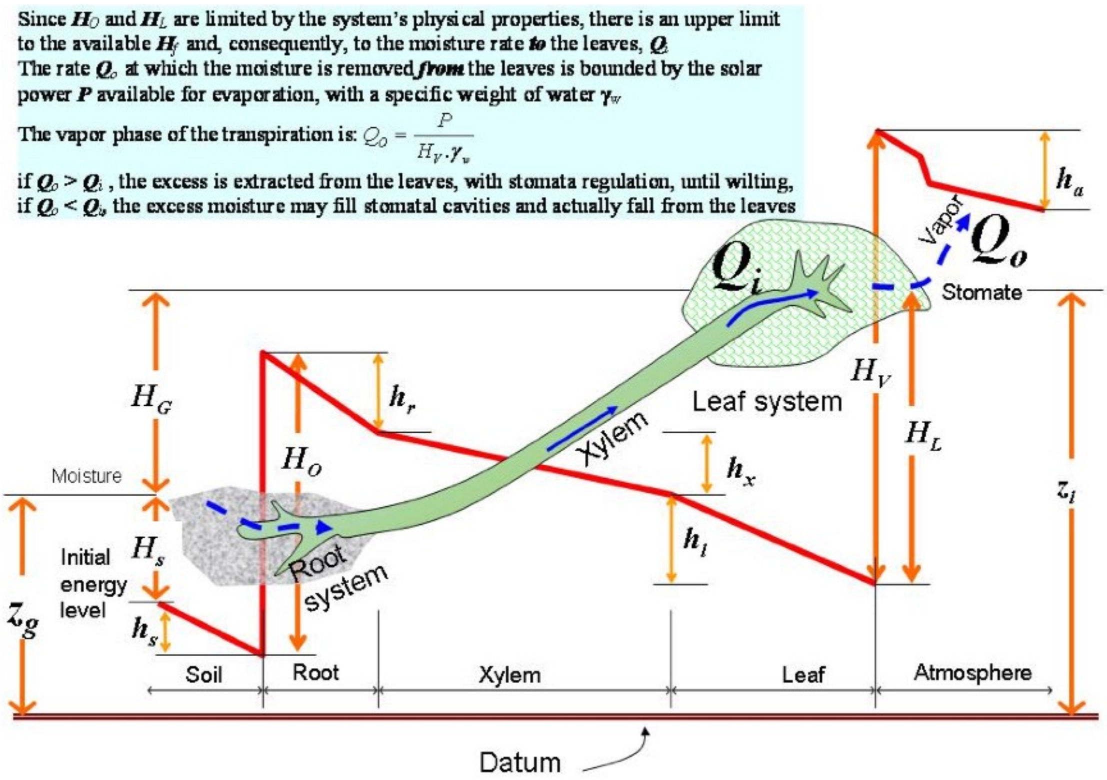


Figure 3- The water balance at plant scale (Eagleson, 1970), after Hendriks & Hansen 1962).

3. Practical aspects for the 'hydro-lot'

The equations of mechanisms of evaporation are addressed to understand the entire transpiration process and the manner in which the physical factors in the process are mutually interdependent in governing the transpiration rate (see **Box 5**). In order to comprehend better the significance of these equations in terms of 'hydro-lots', quantitative utilization would involve Equation 8 and Equation 10 as families of curves. This would allow for the variation of $\overline{k_a}$ and T in Equation 10, and H_v in Equation 8. Single curves are shown in **Figure 4**. Practical utilization of Figure 4 would most likely involve expressing the variables in terms of other easily measurable parameters. For instance, the term R_s could be expressed graphically in terms of soil moisture content and type of soil and R_p could be expressed graphically in terms of the type of plant and the stage of root system development. Other terms could be expressed in a similar manner. According to Hendriks-Hansen conclusions, the arrangement of curves of Figure 4 in an easily, practical form would involve additional effort and perhaps some modification of equations.

Box 5. Practical implications of transpiration process at 'hydro-lot' scale

The equation whose respective curve gives the lowest value of G/A determines the throttling phase of the transpiration process. Figure 4 reveals which of the meteorological conditions will govern rate of vaporization. For example, under conditions of high relative humidity and high available power, the relative-humidity curve (Eq. 10) will govern the vaporization process. Moreover, a dry wind that causes an increase in the relative-humidity gradient will increase the rate of vaporization. If the rate of vaporization exceeds the rate of liquid delivery to the leaf (Fig.3), then the deficit must come from storage within the plant tissues and the stomata closure will throttle the transpiration rate. The maximum rate of liquid deliverable from the soil is given by the expression $H_f \div (R_s+R_p)$, that occurs when $h_f = H_f$. This hydraulic capacity can be increased by decreasing R_s, i.e. through irrigation the soil, or by decreasing R_p, i.e. with more extensive root system, or by increasing H_f, i.e. with irrigation of the soil, coarser soil particles or purer soil solution.

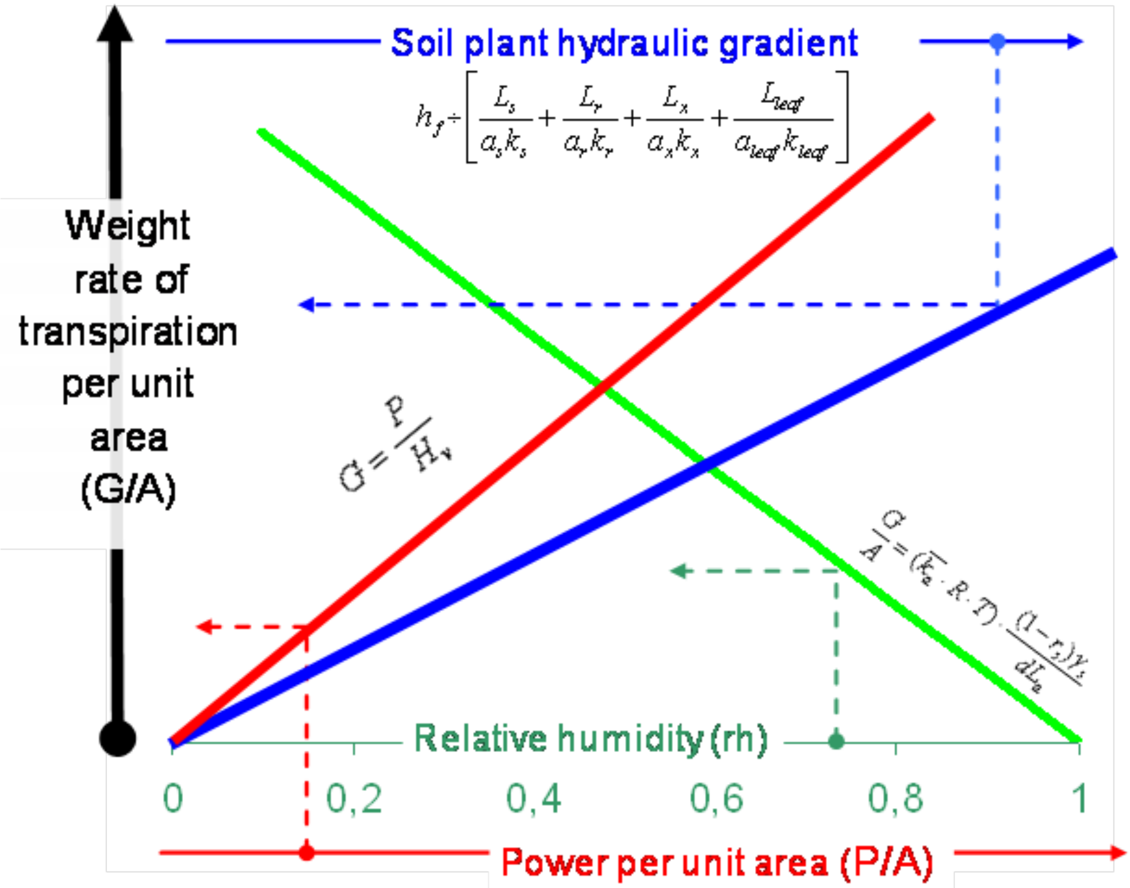


Figure 4- Summary relationships of weight rate of transpiration per unit area (ordinate) regarding: proportional to soil plant hydraulic gradient (upper abscissa; equation 5), proportional to power per unit area (lower abscissa), and inversely proportional to relative humidity (intermediate abscissa, equation 10).

4. Integration of "hydraulic pump", "big leaf" and "water-carbon-footprint"

One problem pointed is the great interconnection related to:

- hydraulic similarity of transpiration process, with EGL scheme, as “hydraulic pump”,
- transpiration mechanism, through a “big leaf” approach,
- stocks of water transpired and carbon stored, as a “water-and-carbon footprint”.

All these restrictions are related to the basic data and results from 1st Virtual Water Problem, which again reutilized throughout this 2nd Virtual Water Problem, as follows.

4.1 Hydraulic similarity of transpiration process (hydraulic pump)

To apply the pressure drop in the biological pipes inside a “big-leaf” approach, the hydraulic behavior could be approached, for laminar flow, in terms of:

- geometric properties: pipe diameter (*d*) and roughness (z_o),
- fluid properties: fluid density (ρ) and dynamic viscosity (μ),
- flow properties: bulk mean velocity (*V*) and the change of pressure along the “pipe” ($\Delta p/L$),

as laminar flow:

$$\Delta p = \frac{32 \cdot \mu \cdot L \cdot V}{\gamma \cdot d^2}$$

(Equation 11).

When $\Delta p \equiv h = Q/a \cdot L/k$, this equation produces through the Hagen-Poiseuille law:

$$Q = \frac{\pi \cdot d^2}{4} \cdot \frac{\Delta p \cdot \gamma \cdot d^2}{32 \cdot \mu \cdot L} \equiv \frac{\pi \cdot d^2}{4} \cdot \left(\frac{1}{8} \cdot \left(\frac{d}{2} \right)^2 \cdot \frac{\rho \cdot g}{\mu} \right) \cdot \frac{\Delta p}{L} = a \cdot \frac{h \cdot k}{L}$$

(Equation 12),

where k_i is the intrinsic permeability [m²], a geometry a feature. That says the hydraulic conductivity is a function of geometrical, k_i, and fluid properties, $\gamma \cdot \mu^{-1}$. Under a steady state condition, the EGL is under equilibrium. In this situation,

$$Q = \frac{\pi \cdot d^2}{4} \cdot \left(\frac{1}{8} \cdot \left(\frac{d}{2} \right)^2 \cdot \frac{\rho \cdot g}{\mu} \right) \cdot \frac{\Delta p}{L} = \overbrace{h_f}^{total\ losses} \cdot \left[\frac{L_s}{a_s k_s} + \frac{L_r}{a_r k_r} + \frac{L_x}{a_x k_x} + \frac{L_l}{a_l k_l} \right]^{-1} = \frac{h_f}{R_s + R_p}$$

(Eq.13).

4.2 Transpiration mechanism, through a “big leaf” approach

A particular case is the water dynamical equilibrium in which the flow is assumed constant in wherever part of the system (Reichardt, 1979; 23). For short periods, when transpiration is constant, the system is considered under equilibrium, says:

steady transpiration flux = $Q = Q_o = Q_i = \dfrac{h_s}{\dfrac{L_s}{a_s k_s}} = \dfrac{h_r}{\dfrac{L_r}{a_r k_r}} = \dfrac{h_x}{\dfrac{L_x}{a_x k_x}} = \dfrac{h_l}{\dfrac{L_l}{a_l k_l}} \cong \dfrac{h_{plant}}{\dfrac{L_{plant}}{a_{plant} k_{plant}}}$ (Eq.14).

Transpiration rates are difficult to measure and hence are usually dealt with in conjunction with evaporation as ***potentials*** for evapotranspiration. For **potential evapotranspiration**, the Penman-Monteith equation adjusted assumes saturation vapor pressure at the leaf surface linearly according to the total diffusion resistance, as:

$$\lambda E = \frac{\Delta \cdot (R_n - G) + \dfrac{\rho \cdot C_p}{r_a} \cdot (e_s - e)}{\Delta + \gamma_{psicro} \dfrac{(r_{air} + r_{stomata})}{r_{air}}}$$ (Equation 15),

in which r_{air} is the atmospheric diffusion resistance, γ_{psicro} is the psicrometric constant, C_p is the specific heat, e_s and e are, respectively, the saturation vapor pressure and actual vapor pressure, and Δ is the tangent of the curve of saturation vapor pressure with temperature, and l_v is the latent heat of vaporization, as follows:

$$r_{air}[\text{s} \cdot \text{m}^{-1}] = \frac{K_m}{K_h} \cdot \frac{C_1^2}{\overline{u}_2} \cdot \ln\left(\frac{z_2}{z_o}\right)^2$$ (Equation 16),

$$\gamma[\text{Pa} \cdot^{\text{o}} \text{C}^{-1}] = \frac{C_p K_h p}{0{,}622.l_v.K_w}$$ (Equation 17),

$$\Delta[\text{Pa} \cdot^{\text{o}} \text{C}^{-1}] = \frac{de_s}{dT} = \frac{d\left(611 \cdot \exp\left(\dfrac{17{,}27.T}{237{,}3 + T}\right)\right)}{dT} = \frac{4098 \cdot e_s}{(237{,}3 + T)^2}$$ (Equation 18).

with C_1 the Karman constant (≈0.4), K_m/K_h ≈1 (the ratio of diffusivities of heat conduction and air convection), C_p is the specific heat at constant pressure (≈ 1005 J kg^{-1} $^{\text{o}}$K^{-1}), p is the atmospheric pressure, R_a is the gas constant for wet air (= 287 J kg^{-1} $^{\text{o}}$K^{-1}), ρ_{w} is the water density, which varies with temperature, z_o is the evaporating surface roughness in the boundary limit of aerodynamic profile of wind, l_v is the latent heat of vaporization, u_2 is the average wind speed, referred to 2 m height.

A second resistance ($r_{stomata}$) represents the surface or stomatal diffusion resistance from the "big leaf". It is composed of a parallel combination of all the separate resistances to moisture flux through the leaves and the soil surface. The series combination (r_{air} + $r_{stomatal}$) represents the total resistance to moisture flux in evapotranspiration from dry surfaces. The resistance r_{air}

can be evaluated using the Kármán value of 0.4, assuming $K_m \approx K_h$, and selecting the roughness height from experimental tables. The stomatal resistance will vary seasonally according to availability of moisture.

Table 3– Surface roughness and specific roughness of species

Surface	Wind speed u_2 at z=2m [m/s]	Roughness (z_o) [cm]
←-----------------------------Surface roughness--------------------------------→		
Open water	2.1	0.001
Wet soil	1.8	0.02
Mown Grass (4.5 cm tall)	2	2.4
Mown Grass (4.5 cm tal)	6 to 8	1.7
Long Grass "*capim*" (60-70cm)	1.5	9.0
Long Grass "*capim*" (60-70cm)	6.2	3.7
Sugar cane (100 cm)	…	4.0
Sugar cane (200 cm)	…	5.0
Sugar cane (400 cm)	…	9.0
Brush (135 cm)	…	14.0
Orange orchard (350 cm)	…	50.0
Pine forest (500 cm)	…	65.0
Pine forest (2,700 cm)	…	300.0
Deciduous forest (1,700 cm)	…	270.0

Table 3 (cont.)- Specific roughness of species

Species	Resistance to water vapour (sec cm^{-1})		
	r_{st}	r_c	r_a
Betula verrucosa	0.9	83	0.8
Quercus robur	6.7	380	0.7
Acer platanoides	4.7	85	0.7
Circaea lutetiana	16.1	90	0.6
Lamium galeobdolon	10.6	37	0.7
Helianthus annuus	0.4	-	

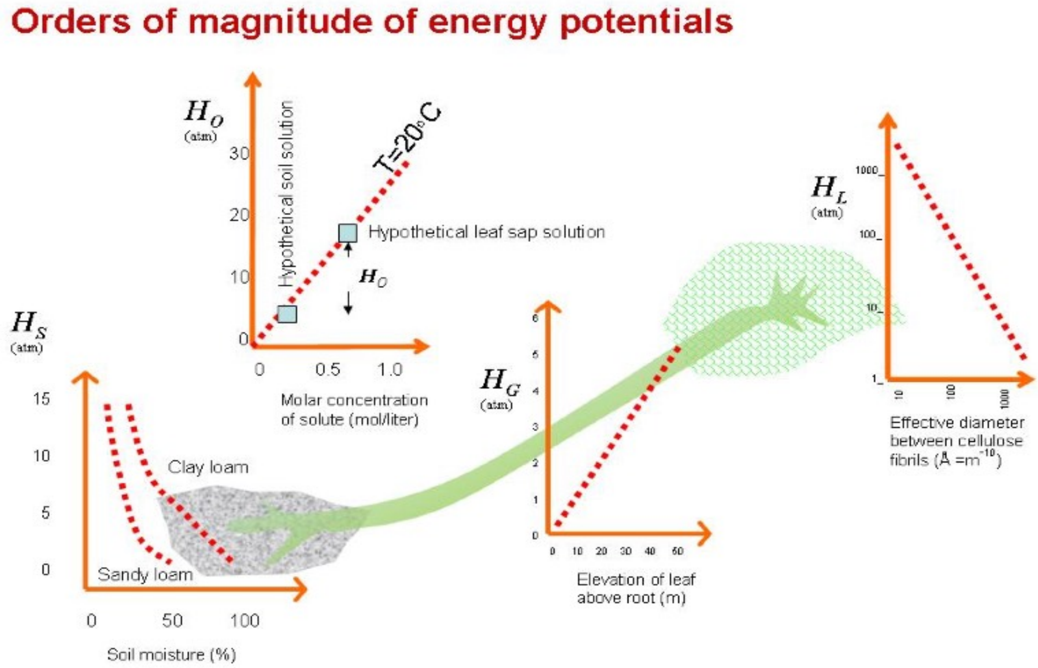
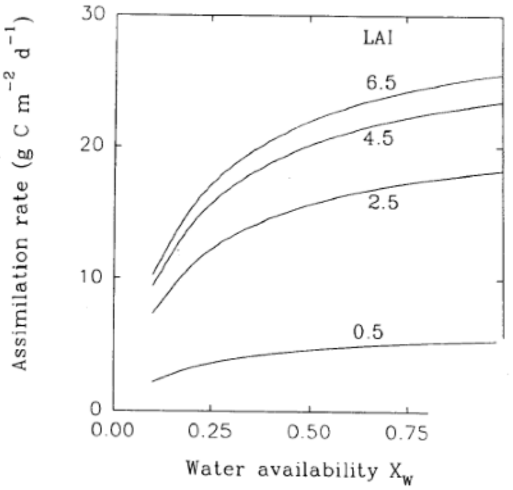


Figure 5- Example of soil, osmotic, gravity and leaf-driven potentials acting through the "big leaf".

4.3 The water-and-carbon footprint

The photosynthesis process, with water transpiration, is also related to the carbon fixation in the vegetal tissue of the plant, says into the "big leaf" tissue. This carbon fixation is related to the Leaf Area Index, *LAI*, which is the ratio between the sum of all small evaporating area from stomata surface in a plant and the vertical projected area. Usually the referred unit area is 1 m^2 and *LAI* represents the increment of vegetal evaporating surface, i.e. LAI > 1 when the plant grows up and its evaporating surface increases. **Figure 6** and **Figure 7** outline results from Slavich et al (1998) with carbon assimilation, water transpiration and the water use efficiency, in terms of grams of carbon assimilated per kilogram of water transpirated, as functions of LAI and the water availability, expressed as

$Xw = (S - Smin) / (Smax - Smin)$, (Equation 19).



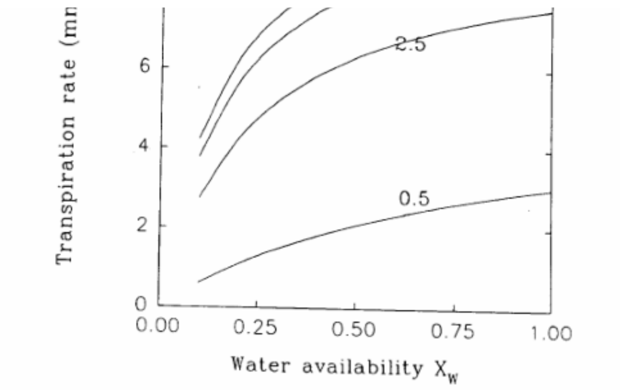


Figure 6- Carbon assimilation (left) and transpiration flux (right) for varying soil water availability (*Xw*) and leaf area index (*LAI*). Source: Slavich et al (1998).

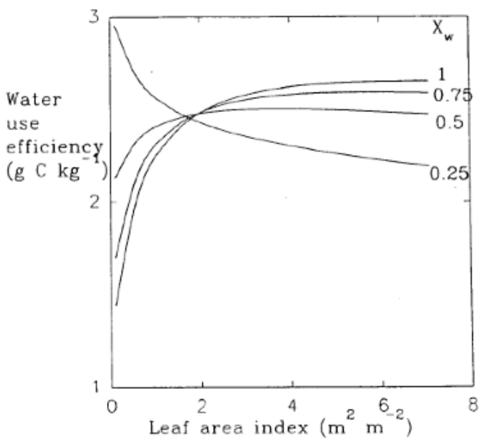


Figure 7- Water use efficiency (g C assimilated per kg of water transpirated) for varying soil water availability (*Xw*) and leaf area index (*LAI*). Source: Slavich et al (1998).

5. Assessment of "hydraulic pump", "big leaf" and "water-carbon footprint"

Given previous lumped, monthly water balance, with data of temperatures, vapor pressures, precipitation, maximum and minimum soil water storage, and actual evapotranspiration rates it is possible to relate three parts: the hydraulic pump (section 4.1), the big leaf approach (section 4.2) and the "water-carbon-footprint" (section 4.3), as follows. The Figure 8 depicts how the hydraulic pump machine, as a "ram pump", works under general framework of pulsatile flow. It is assumed throughout this VWP there is an analogy between the ram pump phases and the way the big leaf pumps water. Dimensional analysis are outlined in **Appendix 1.**

The vessels of the big leaf (Fig. 2, 3 and 4) are under flow regimes derived upon Moody`s Diagram of Figure 9. Table 1 and Table 2 depict dimensionless numbers useful for flow calculations

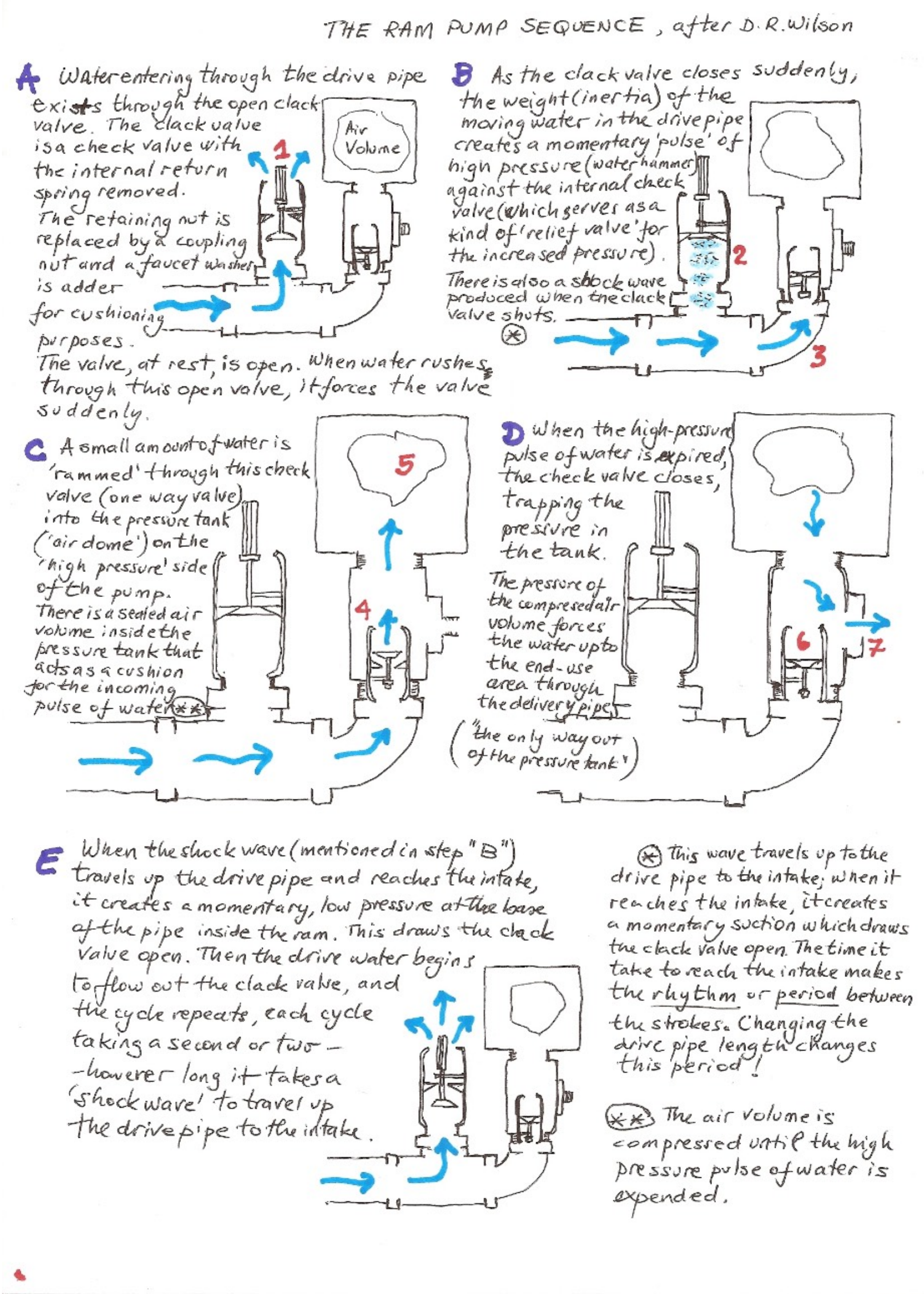


Figure 8- How the ram pump works

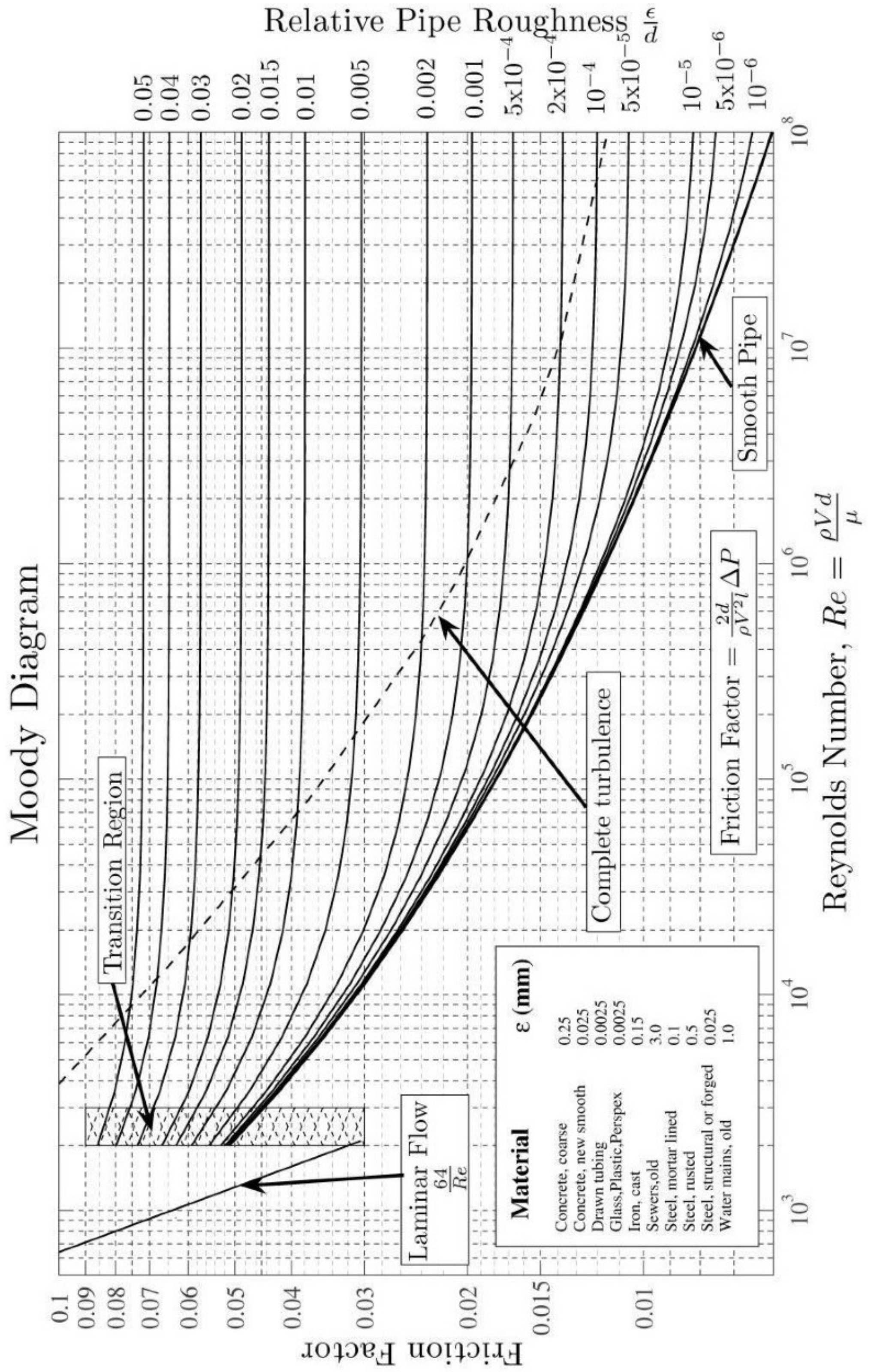


Figure 9- Moody's Diagram

Table 1. Dimensionless Numbers. SHS5890. 2009. Prof. E. M. Mendiondo (emm@sc.usp.br)

Name	Process summary	Numerator	Denominator	Equation and comments
Archimedes	motion of fluids due to density differences	gravitational force	viscous force	***Ar*** = (g rho_s L^3) (rho_s - rho_f) / (mu^2) ; where g: gravitational acceleration, L: characteristic length, mu: viscosity, rho_f: fluid density, rho_s: solid density; used in momentum transfer, buoyancy, and motion due to density difference
Biot	heat transfer, unsteady state	"in"-thermal resistance	surface film resistance	***Bi*** = (h_T (x_2-x_1)) / k ; where h_T: heat transfer coefficient, (x_2-x_1): mid-plane distance, k: thermal conductivity
Blake	momentum transfer	inertial force	viscous force	***Bl*** = (V rho) / (mu (1-epsilon) s) OR Bl = G / (mu (1-epsilon) s) ; epsilon: void fractrion, G: mass velocity, mu: viscosity, rho: density, s: particle area/particle volume, V: velocity
Bond	capillary action driven by buoyancy	gravitational force	surface tension force	***Bo*** = (g (rho-rho_f) d^2) / (g_c sigma) ; where d: droplet/bubble diameter, g: gravitational acceleration, g_c: dimensional constant, rho: droplet/bubble density, rho_f: surrounding fluid density, sigma: surface tension
Capillary	fluid flow influenced by surface tension	viscous force	surface tension force	***Ca*** = (mu V) / (g_c sigma) ; g_c: dimensional constant, mu: viscosity, sigma: surface tension, V: velocity ; it is equivalent to (We / Re), used in momentum transfer, atomization and 2-phase flow in beds of solids
Cauchy	momentum transfer, compressible flow	inertial force	compressibility force	***C*** = (rho V^2) / (g_c E_b) ; E_b: bulk modulus of fluid, g_c: dimensional constant, rho: density, V: velocity
Cavitation	cavitation	excess of local static head	vapor pressure head	***Sigma*** = (P-P_{vap}) / (0.5 rho v^2) ; P: pressure, P_{vap}: vapor pressure of fluid, rho: density, v: velocity; a lower cavitation number (especially Cavitation Numbers < 1) implies a high degree of cavitation
Courant	Courant-Friedrichs-Lewy condition (CFL)	convergence's solving of partial differential equations (usually hyperbolic PDEs) numerically		***C*** = $u.\Delta t/\Delta x$, *u* : velocity (L/T), Δt: time step (T), Δx: length interval (L); *C* depends on the particular equation to be solved and not on Δt and Δx; the timestep must be less than a certain time in many explicit time-marching computer simulations, otherwise the simulation will produce wildly incorrect results
Drag Coefficient	flow calculations	gravitational force	inertial force	C_D = (g (rho-rho_f) L) / (rho V^2) ; g: gravitational acceleration, L: characteristic dimension of object, rho: density of object, rho_f: density of surrounding fluid, V: velocity
Elasticity	viscoelastic flow	elastic force	inertial force	***El*** = (theta mu) / (rho r^2) ; r: pipe/conduit radius, mu: viscosity, rho: density, theta: relaxation time
Euler	compressible flow	pressure force	inertial force	***EU*** = p / (ρ v^2) ; p = pressure (or Δp = pressure difference), ρ = density, v = velocity ; two variants of Euler No. (both dimensionless) are: Pressure coefficent = Δp / (½ ρ v^2) and Cavitation No.
Froude	wave and surface behavior	Inertial force	gravitational force	***Fr*** = V^2 / (g L) ;V: Velocity, g: gravitational acceleration, L: characteristic length ; used in momentum transfer in general and open channel flow and wave and surface behavior
Galileo	heat transfer, viscous flow	regravity force	viscous force	***Ga*** = (g D^3 rho^2) / mu^2 ; g: gravitational acceleration, D: diameter, mu: viscosity, rho: density ; used in momentum and heat transfer in general and viscous flow and thermal expansion calculations
Grashof	heat transfer, convection	buoyancy force	viscous force	***Gr*** = (L^3 rho^2 g beta (T_2-T_1)) / mu^2 ; beta: coefficient of expansion, (T_2-T_1): temperature difference, g: gravitational acceleration, L: characteristic length, mu: viscosity, rho: density
Knudsen	momentum transfer, gas flow calculations	length of mean free path	characteristic dimension	***Kn*** = lambda / L ; lambda: length of mean free path (m), L: characteristic dimension (m) ; used in momentum and mass transfer in general and very low pressure gas flow calculations
Lewis	heat-and-mass transfer	equivalent to Sc/Pr		***Le*** = k / (D_V rho C_p) OR Le = alpha / D_V ; alpha: thermal diffusivity, C_P: heat capacity, D_V: diffusivity, k: thermal conductivity, rho: density

(cont.)

Further Reading:
Courant, K. Friedrichs, H. Lewy, (1967) On the partial difference equations of mathematical physics, *IBM Journal*, March, pp. 215-234, German Translated [www.stanford.edu/class/cme324/classics/courant-friedrichs-lewy.pdf]
Gioia, G, Bombardelli, F A (2002) Scaling and similarity in rough channel flows, *Phys. Rew. Letters* 88(1): 014501-1-4 [http://cee.engr.ucdavis.edu/faculty/bombardelli/PRL14501.pdf]
Purcell, E M (1976) Life at low Reynolds Number, *American Journal of Physics* 45, pp. 3-11 [jilawww.colorado.edu/perkinsgroup/Purcell_life_at_low_reynolds_number.pdf]
Womersley, J.R. (1955) Method for the calculation of velocity, rate flow, and viscous drag in arteries when the pressure gradient is known, *J. Physiol.*, Vol 127, pp. 553–563.[jp.physoc.org/cgi/reprint/127/3/553.pdf]

Table 1. Dimensionless Numbers. SHS5890. 2009. Prof. E. M. Mendiondo (emm@sc.usp.br)

Name	Process summary	Numerator	Denominator	Equation and comments
Lift Coefficient	flow calculations	3D or 2D problems		A flying body's (3D) lift coefficient C_L is the dimensionless value defined as: C_L=L/(1/2) ρ V^2 S; a related value is the section, or 2D, lift coefficient, c_l defined by the similar formula (Γ represents lift per unit wingspan): c_l= Γ /(1/2) ρ V^2 c
Mach	general-and-near ultrasonic flow	velocity	velocity of sound in fluid	***Ma*** = V / V_{sound} ;V: velocity, V_{sound}: velocity of sound in fluid ; it is used in momentum transfer in general and near/ultra sonic flow and throttling calculations
Nusselt	heat transfer	total heat transfer	conductive heat transfer	***Nu*** = (alpha d)/ λ ; alpha: heat transfer coefficient (W/(m^2K)), d: characteristic length (m), λ : thermal conductivity (W/(m K))
Peclet	heat transfer, convection	bulk heat transfer	conductive heat transfer	***Pe*** = (D V rho C_p) / k ; D: characteristic length, V: velocity, rho: density, C_p: heat capacity, k: thermal conductivity
Power	momentum transfer, power consumption in agitators / pumps	drag force	inertial force	N_P = (g_c P) / N^3 rho D^5) ; D: characteristic length, g_c: dimensional constant, N: rate of rotation, P: power, rho: density
Prandtl	heat transfer, forced convection	momentum diffusivity	thermal diffusivity transfer	***Pr*** = (C_p mu) / k ; C_P: heat capacity, k: thermal conductivity, mu: viscosity
Rayleigh	heat transfer, free convection	equivalent to Gr · Pr		***Ra*** = (L^3 rho^2 g beta (T_2-T_1) C_P) / (mu k) OR Ra = (L^3 rho g beta (T_2-T_1)) / (mu alpha) ; alpha: thermal diffusivity, beta: coefficient of expansion, C_P: heat capacity, (T_2-T_1): temperature difference, g: gravitational acceleration, k: thermal conductivity, L: characteristic length, mu: viscosity, rho: density.
Reynolds	dynamic similarity	inertial force	viscous force	***Re*** = (rho v D) / mu ; rho: density, v: velocity, D: characteristic length or diameter, mu: viscosity
Schmidt	mass transfer	kinetic viscosity	molecular diffusivity	***Sc*** = mu / (rho D_V) ; D_V: diffusivity, mu: viscosity, rho: density
Sherwood	mass-transfer operation	convective transport	diffusive mass transport	***Sh*** = *K. L /D; L* :characteristic length (m); *D* : mass diffusivity ($m^2.s^{-1}$) ; *K* is the mass transfer coefficient ($m.s^{-1}$)
Stanton	heat transfer, forced convection	heat transferred	thermal capacity of fluid	***St*** = h / (C_P rho V) OR St = h / (C_P G) ; C_P: heat capacity, G: mass velocity, h: heat transfer coefficient, rho: density, V: velocity.It is equivalent to Nu/(Re Pr).
Stokes	behavior of particles suspended in fluid flow	stopping distance of a particle	characteristic dimension of the obstacle	***Stk*** = τ U_o / d_c ; τ : the relaxation time of the particle, U_o : the fluid velocity of the flow well away from the obstacle; d_c : the characteristic dimension of the obstacle ; for *Stk* >> 1, particles will continue in a straight line as the fluid turns around the obstacle therefore impacting on the obstacle; for *Stk* <<1, particles will follow the fluid streamlines closely
Weber	bubble formation	inertial force	surface tension force	***We*** = (D V^2 rho) / (g_c sigma) OR We = (D G^2) / (g_c rho sigma) ; g_c: dimensional constant, G: mass velocity, D: characteristic length, rho: density, sigma: surface tension, v: velocity
Womersley	biofluid mechanics	pulsatile flow frequency	viscous effects	***Alfa2*** = *R* $(\omega/v)^{1/2}$ = R $(\omega\rho/\mu)^{1/2}$, where *R* is an appropriate length scale (for example the radius of a pipe), ω is the angular frequency of the oscillations, and v, ρ, μ are the kinematic viscosity, density, and dynamic viscosity of the fluid, respectively; it can be written in terms of Reynolds number (*Re*) and Strouhal number (*Sr*): Alfa2 = $(2\pi \text{ Re Sr})^{1/2}$

Further Reading:
Courant, K. Friedrichs, H. Lewy, (1967) On the partial difference equations of mathematical physics, *IBM Journal*, March, pp. 215-234, German Translated [www.stanford.edu/class/cme324/classics/courant-friedrichs-lewy.pdf]
Gioia, G, Bombardelli, F A (2002) Scaling and similarity in rough channel flows, *Phys. Rew. Letters* 88(1): 014501-1-4 [http://cee.engr.ucdavis.edu/faculty/bombardelli/PRL14501.pdf]
Purcell, E M (1976) Life at low Reynolds Number, *American Journal of Physics* 45, pp. 3-11 [jilawww.colorado.edu/perkinsgroup/Purcell_life_at_low_reynolds_number.pdf]
Womersley, J.R. (1955) Method for the calculation of velocity, rate flow, and viscous drag in arteries when the pressure gradient is known, *J. Physiol.*, Vol 127, pp. 553–563.[jp.physoc.org/cgi/reprint/127/3/553.pdf]

1. Monthly potential evapotranspiration ($Etp_{Thornthwaite}$)

Using Thornthwaite´s potential evapotranspiration,

ETp [mm]= d . 16 . (10. T / $\boldsymbol{I}$)^a ; T [^{o}C] (Equation 20).,

with coefficient "d" varying according to monthly temperature and remains constant, and coefficient "a" = $67.5 \cdot 10^{-8} I^3 - 7.71 \cdot 10^{-5} I^2 + 0.01791 \cdot I + 0{,}492$, with coefficient

$$I = \sum_{i=1}^{12} \left(\frac{T}{5}\right)^{1{,}514}$$ (Equation 21).

5.2 Assuming that $Etp_{Thornthwaite} \approx Etp_{Penman\text{-}Monteith}$

Using Equation 15, with values of e_s, e, Δ, γ_{psicro}, $G \approx 0$, Cp, ρ_w, and assuming z_o and u_2 according to a given land-use of **Table 3**, it is expected to calculate, indirectly the net radiation, R_n. If estimated Rn has negative values, a new iteration, with new r_{air} and $r_{stomatal}$ should be made.

5.3 Using R_n for transpiration flux

Net radiation R_n is further used to estimate the output flux of transpiration from the "big leaf", Q_o, according to Equation 8, with $P = R_n$. It is worth noting that transpiration should be equal or less than the actual evapotranspiration rate, says $Q_{o(month)} \leq ETR_{(month)}$. If not, a new iteration, returning to section 5.2 should be made.

5.4 Steady state transpiration flux

The steady state condition permits to initiate the assessment of EGL at the liquid and vapor phase. For the vapor phase, it is used the Equation 9 and 10.

For the liquid phase, equations 11 to 14 are used in order to delineate the EGL at the "big leaf" plant. When necessary, the orders of magnitude of potentials at Figure 5 could be used.

For the water availability related to soil potential, it is possible to assume a retention curve of:

$$(\theta_{month} \cdot z) = (\theta_{\min} \cdot z) + \frac{\left[(\theta_{\max} \cdot z) - (\theta_{\min} \cdot z)\right]}{\left[1 + (\alpha \cdot (H_s)^n\right]^{1-1/n}}$$ (Equation 22)

with α and n as empirical parameters, as functions of soil structure, pedo-genesis and land-use, z is the average depth at which the water balance is made (at the river basin scale). Thus we could outline that the water storage, resulting of the water balance, is $S_{month} = \theta_{month} \cdot z$, and the maximum and minimum water value are, respectively $S_{month} = \theta_{max} \cdot z$, and $S_{month} = \theta_{min} \cdot z$.

The water availability, X_w, is expressed as:

$$X_{w,month} = \frac{\left[(\theta_{month} \cdot z) - (\theta_{\min} \cdot z)\right]}{\left[(\theta_{\max} \cdot z) - (\theta_{\min} \cdot z)\right]}$$ (Equation 23).

Some equations are useful as follows:

$$E_r(m/s) = \frac{1}{l_v \rho_w}[R_N - H_s - G]$$, R_N(W/m^2)

$$l_v(\text{J/kg}) = 2500000 - 2360.T(^oC)$$,

$$\gamma(Pa/^oC) = \frac{C_p K_h p}{0{,}622.l_v.K_w}$$

$$E_a(m/s) = B(e_{as} - e_a)$$

$$B(m/(Pa.s)) = \frac{0{,}622.k^2.\rho_a.u_2}{p.\rho_w.[\ln(\frac{z_2}{z_o})]^2}$$, T (^{o}C)

$$HR(Pa/Pa) = \frac{e}{e_s}$$

$$e_s = 611.\exp\left(\frac{17{,}27.T}{237{,}3 + T}\right)$$

$$\Delta(Pa/^oC) = \frac{de_s}{dT} = \frac{4098 e_s}{(237{,}3 + T)^2}$$

$$q_v(\text{kg kg}^{-1}) = 0{,}622 \cdot \frac{e}{p}$$

Note1 : for interested readers in linking living pumps with head analysis, a short introduction to dimensional analysis is outlined in **Appendix 1**

5.5 Water Use Efficiency

Water use efficiency can be expressed in agronomic or physiological terms

$$\text{Agropole economic efficiency} = \frac{\text{economic yield}}{\text{water used}}$$

$$\text{Agronomic water use efficiency} = \frac{\text{dry matter produced}}{\text{water used}}$$

For irrigation, the water used also includes water outputs, water excess (drainage and runoff) plus evaporation and transpiration, in order to minimize losses.

$$\text{Physiological water use efficiency} = \frac{\text{dry matter produced}}{\text{water transpired}} = \frac{\text{net } CO_2 \text{ fixed}}{\text{transpiration}}$$

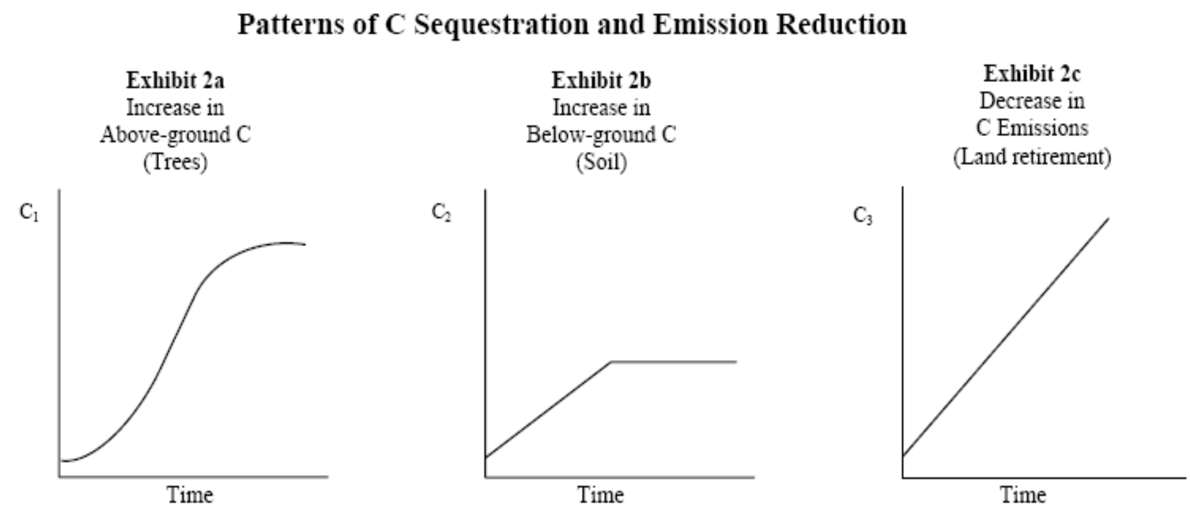
$$\text{Physiological water use efficiency} = \frac{\dfrac{\Delta CO_2}{r_{stomata} + r_{air}}}{\dfrac{e_l - e_a}{r_{stomata} + r_{air}}}$$

If the CO_2 concentration in the atmosphere increases, water-use efficiency increases. If the atmosphere is is more humid, it also increases WUE.

6. Carbon sequestration

By comparison to the California Registry price of about \$2/ton, recent carbon prices on the European market have been typically in the range of \$14-15 Euros per ton, or about \$16-18. Markets for carbon are still in the early stages of development and are likely to change substantially over the next several years. Thus, current market prices may not be a good indicator of even near-term, let alone mid- or long-term prices.

- Comparing current carbon prices to timber stumpage prices indicates that current markets value timber more as lumber than as sequestered carbon.
- Using European carbon market prices of \$18/ton, a thousand board feet (MBF) of timber would be worth about \$10 as sequestered carbon, as compared to current Douglas-fir timber prices of \$280-350/MBFor redwood prices of \$680-830/MBF



C_1 = metric tons of Carbon (tC) above ground in tree trunks, limbs, leaves, leaf litter, dead wood, etc.
C_2 = metric tons of Carbon (tC) below ground in forest soils
C_3 = metric tons of Carbon (tC) emission reductions from reducing energy and fertilizer use
C_T = Total Reduction in Atmospheric carbon
$= C_1 + C_2 + C_3$

Note: a metric ton of Carbon (C) is equivalent to 3.67 metric tons of atmospheric carbon dioxide (tCO_2)

Appendix 1 – Dimensional Analysis

Application to the pressure drop in a pipe:

We wish to develop an expression for $\Delta P / L$ in a pipe as a function of the relevant parameters. We wish to do it in a <u>general</u> way.

$$\Delta P / L = f_n(D, k, V, \rho, \mu)$$

In the above expression, we have identified the following relevant parameters:

D	Pipe diameter (m)
k	rms roughness (m)
V	Bulk mean velocity (m/sec)
ρ	Density (kg/m^3)
μ	Dynamic viscosity (N-sec/m^2)

We wish to describe these effects in terms of dimensionless parameters, π .

$$\pi_1 = f_n(\pi\ ,\pi_3,\pi_4,........\pi_n)$$

In this case, the pipe, there are 6 quantities (n=6). Also, there are 3 primary dimensions; length, time, and mass (m=3). We can create a "force" dimension from these three, thus, force is not new.

Buckingham Pi Theorem:
The number of independent π terms, j, of all the relevant π terms is $n - m$.

Note that the total number of possible terms is: $\frac{n!}{(m+1)!\,(n-(m+1))!}$, but many of these are dependent on others. For this example, m=3, n=6, so j=3 of the possible 15 terms.
Next, we find these terms in a way that will assure that the three we find are independent of one-another. Each π term will have m <u>repeating variables</u> (which are quantities in common) plus a new variable. For this system, repeating variables could be taken from the following list:

Geometric Parameters	D, k
Fluid Properties	ρ, μ
Flow Properties	$V, \Delta P / L$

I will choose one from each group; I'll take D, ρ, V as repeating variables. Next, create the π terms, first taking $\Delta P / L$ as the dependent variable, to construct π_1:

π_1	from	$\Delta P / L, D, \rho, V$
π_2	from	μ, D, ρ, V
π_3	from	k, D, ρ, V

Next, form the dimensionless groups:

$$[\pi_1] = [L]^w \left[\frac{M}{L^3}\right]^X \left[\frac{L}{T}\right]^y \left[\frac{ML}{T^2}\frac{1}{L^2}\frac{1}{L}\right]^z$$

where [ ] means “the units of.” Note $\left[\frac{ML}{T^2}\right]$ is the unit of “force.” We want π_1 to be unitless.

Therefore, the exponents for the various units must individually sum to zero.

L units w-3x+y-2z = 0

M units x+z = 0

T units -y-2z = 0

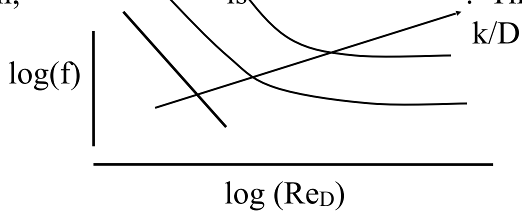
Solution of this set of simultaneous equations yields: z = 1, x = -1, y = -2, and w = 1.

so: $$\pi_1 = \frac{\left(\Delta P / L\right)}{\rho V^2 / D}$$ We may recognize this π_1 term to be f/2, where f is the Darcy-Weisbach friction factor.

Similarly, $$\pi_2 = \frac{\mu}{\rho V D}$$ We recognize this to be Re^{-1}.

and $$\pi_3 = k/D$$ the relative roughness.

Thus, our expression; $\pi_1 = f\left(\pi_2, \pi_3\right)$ is $f = f_n\left(\mathrm{Re}, k/D\right)$. This is the expression plotted in the Moody diagram:



For example, see Fig. 8.12, p. 364 of *Introduction to Fluid Mechanics*, 2nd ed. by Fox and McDonald or just look up “Moody chart” in any undergraduate fluid mechanics book..

Imagine testing a pump-

First, determine the controlled parameters that influence the pump performance:

N Rotational speed (rpm)

Q Flow rate allowed to pass through the pump (m^3/sec)

ρ Density of the fluid used in the test (kg/m^3)

μ Dynamic viscosity of the fluid used in the test (N-sec/m^2)

Second, identify the quantities which describe the pump geometry:

D Diameter of the impeller (m)

$\ell_1, \ell_2, \ell_3, \ell_4, ...\ell_n$ Various length scales which describe the geometry such as those which you would give to a manufacturer to construct the pump (m).

Third, list the performance parameters:

H Head developed by the pump = $\Delta p / \rho g$ (m)

η Efficiency of the pump (%)

Use dimensional analysis techniques to describe the dependent parameters in terms of the controlled parameters:

$$\psi = \frac{gH}{N^2 D^2} = f\left(\frac{Q}{ND^3}, \frac{\rho ND^2}{\mu}, \frac{?_1}{D}, \frac{?_2}{D}, \frac{?_3}{D}, \frac{?_4}{D}, \ldots \frac{?_n}{D}\right)$$

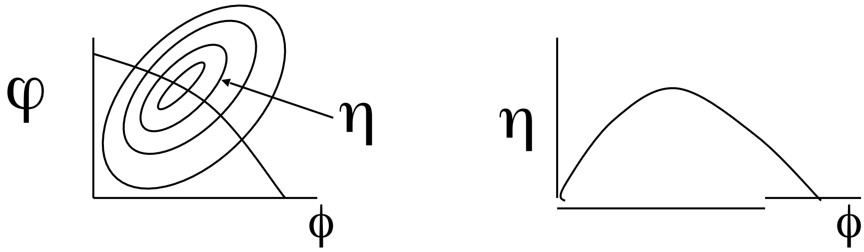
ψ is the head coefficient; $\phi = \frac{Q}{ND^3}$ is the flow coefficient.

$$\eta = f\left(\frac{Q}{ND^3}, \frac{\rho ND^2}{\mu}, \frac{?_1}{D}, \frac{?_2}{D}, \frac{?_3}{D}, \frac{?_4}{D}, \ldots \frac{?_n}{D}\right)$$

, and, though not independent,

$$C_P = \frac{\rho(gH)(Q)}{\eta \rho N^2 D^2 ND^3} = \frac{P}{\rho N^3 D^5} = f\left(\frac{Q}{ND^3}, \frac{\rho ND^2}{\mu}, \frac{?_1}{D}, \frac{?_2}{D}, \frac{?_3}{D}, \frac{?_4}{D}, \ldots \frac{?_n}{D}\right)$$

is the power coefficient. If the list is complete, all effects are captured and pumps of all sizes and shapes pumping any fluid behave similarly when parameters are matched.
If we restrict to pumps of the same geometry and fix the fluid type.



Additional parameters, generated from the above (as was done with Power) are added:

Specific Speed - a non-dimensional speed.

$$N_s = \frac{\phi^{1/2}}{\psi^{3/4}} = \frac{NQ^{1/2}}{(gH)^{3/4}}$$

High Specific speed usually means smaller, faster, higher-power-density pumps (given the same geometry).

Reference: Pump Handbook, Second Ed. [...]assik, Krutzsch, Fraser and Messina, McGraw-Hill, 1986, pp. 2-198 and 2-199

CENTRIFUGAL PUMPS

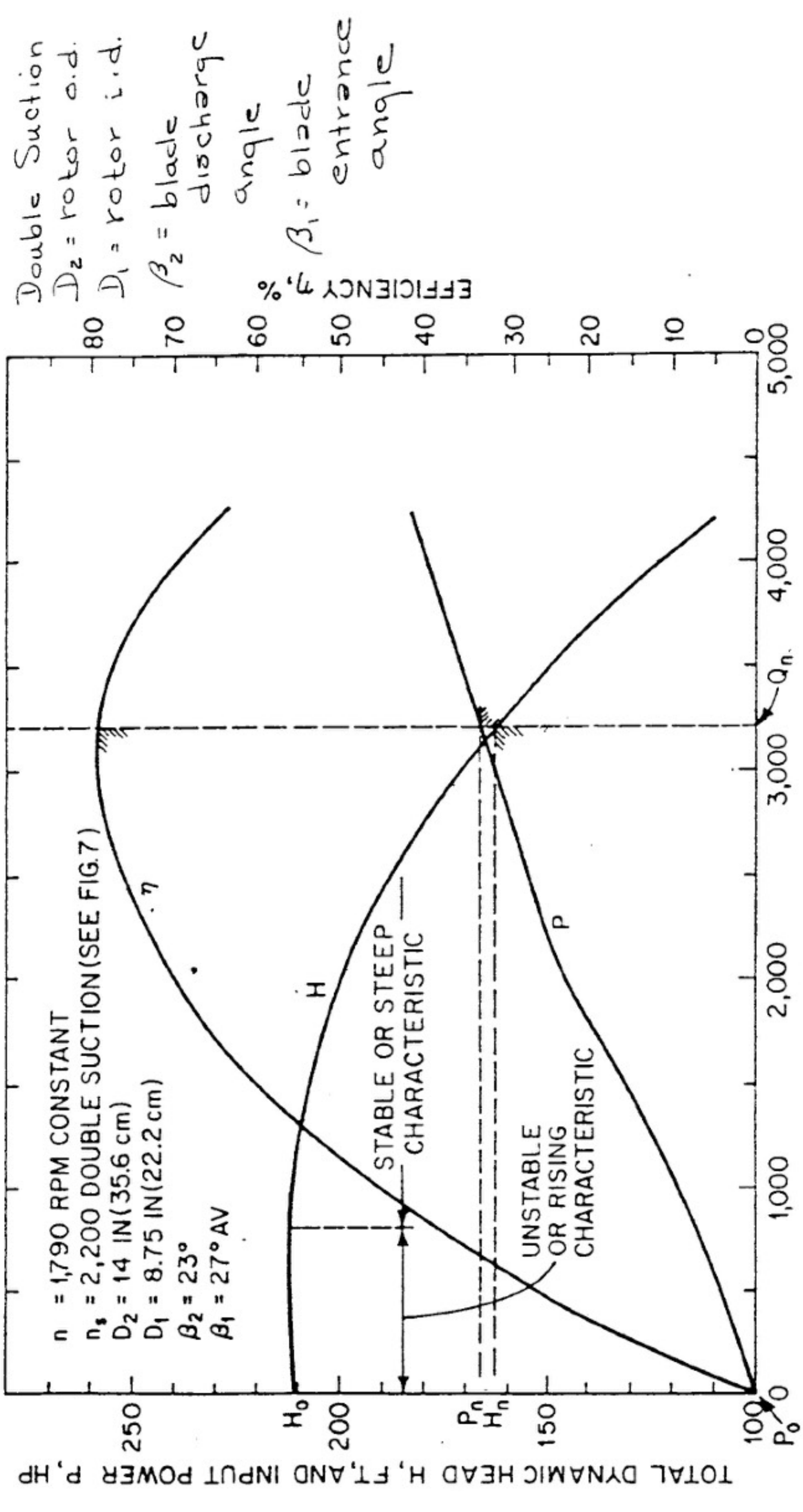


FIG. 4 Typical pump characteristics, backward-curved vanes (ft × 0.3048 = m; hp × 745.7 = W; gpm × 0.06309 = l/s).

Reference: Pump Handbook, Second Ed., Karassik, Krutzsch, Fraser and Messina, McGraw-Hill, 1986, pp. 2-198 and 2-199

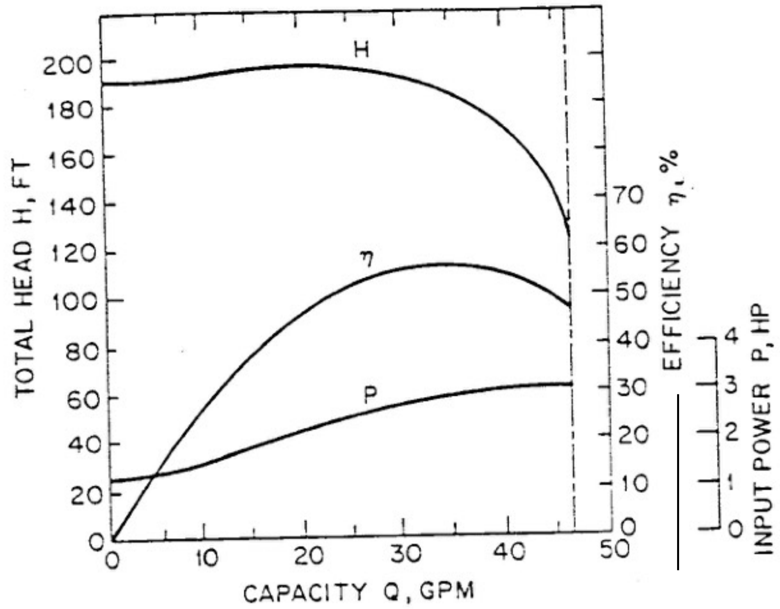


FIG. 5 Pump characteristics, radial vanes (ft × 0.3048 = m; hp × 745.7 = W; gpm × 0.06309 = l/s). (Ref. 3)

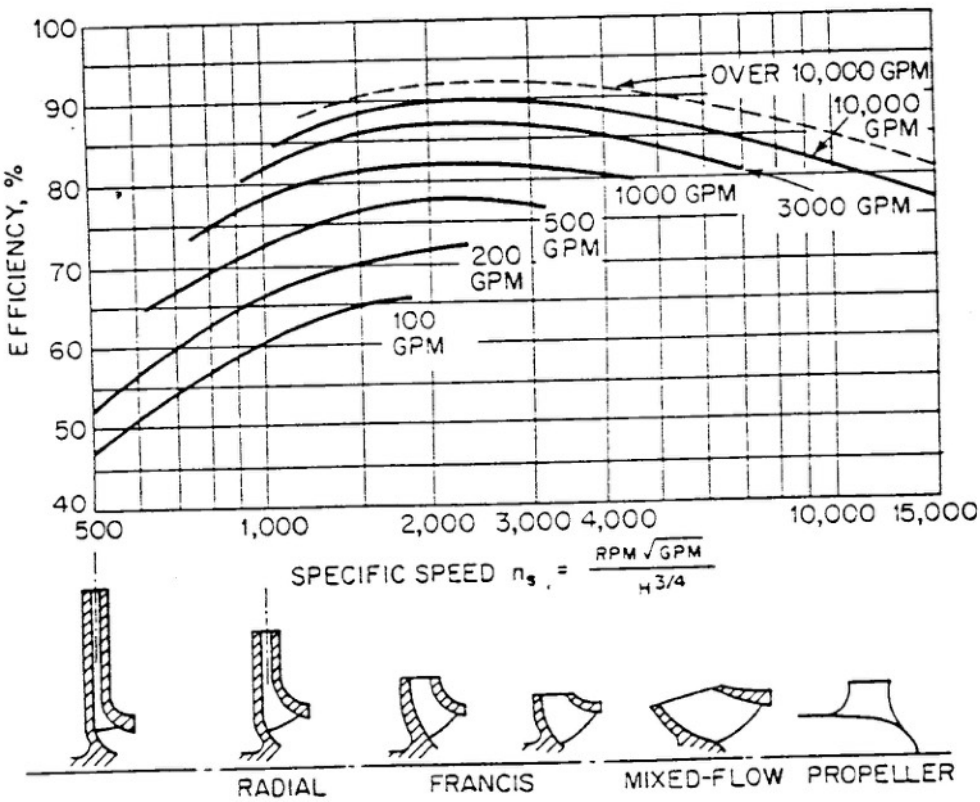


FIG. 6 Pump efficiency versus specific speed and size (gpm × 0.06309 = l/s). (Worthington Pump)

Example:

A water pump has the following rated-flow characteristics:

Q=1000 gpm, H=20 ft; η =0.80; at $N/2\pi$ =3600 rpm. Then:

$$P = \frac{\rho Q g H}{\eta} = \frac{8 Lbm}{gal}\frac{1000 gal}{\min}\frac{32.2 ft}{\sec^2}\frac{20 ft}{0.80}\frac{\min}{60 \sec} = 107{,}333\frac{Lbm\ ft^2}{\sec^3}\frac{Lbf\ \sec^2}{32.2\ ft\ Lbm}$$

$$P = 3{,}333\frac{ft\ Lbf}{\sec} = 3{,}333\frac{ft\ Lbf}{\sec}\frac{hp\ \sec}{550\ ft\ Lbf} = 6.06\ hp$$

What would we expect if the rotational speed were slowed to $5/6$ th rated speed:

$$Q = Q_R\left(5/6\right) = 833 gpm$$

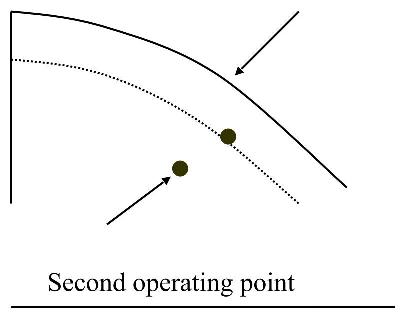
$$H = H_R\left(5/6\right)^2 = 13.9\ ft$$

$$P = P_R\left(5/6\right)^3 = 3.5\ hp$$

$$\phi = Q/ND^3$$

$$\psi = gH/N^2D^2$$

$$C_P = \frac{\phi\psi}{\rho\eta}$$



P requires knowing the $\eta(N)$ relationship. $P \approx P_R\left(5/6\right)^3 = 3.5\ hp$ if η is constant.

Example:

Consider an axial flow fan: D=1.83 m; $N/2\pi = 1400\ rpm$; $V_{axial} = 12.2\ m/\sec$

We wish to test the performance of a $3/4$ th scale fan (which is dynamically similar) with the same flow rate, Q: Thus,

$\frac{Q}{ND^3}$ is fixed between actual fan and scaled model

thus: $$\frac{Q_a}{N_a D_a^{\ 3}} = \frac{Q_m}{N_m D_m^{\ 3}} \rightarrow \frac{N_m}{N_a} = \left(\frac{D_a}{D_m}\right)^3 = \left(\frac{4}{3}\right)^3 = 2.37$$

$$N_m = 2.37(1400\ rpm) = 3318\ rpm$$

$\frac{gH}{N^2D^2}$ is fixed between actual fan and scale model.

thus: $$\frac{gH_a}{N_a^{\ 2}D_a^{\ 2}} = \frac{gH_m}{N_m^{\ 2}D_m^{\ 2}} \rightarrow \frac{gH_m}{gH_a} = \left(\frac{N_m}{N_a}\right)^2\left(\frac{D_m}{D_a}\right)^2 = \left[\left(\frac{4}{3}\right)^3\left(\frac{3}{4}\right)\right]^2 = \left(\frac{4}{3}\right)^4 = 3.16$$

The velocities will be similar between the scaled fan and the actual fan -- scaling parameters were chosen to do so.

Let's see:

$$\frac{V_{axial,m}}{V_{axial,a}} = \frac{\left(\frac{Q}{D^2}\right)_m}{\left(\frac{Q}{D^2}\right)_a} = \left(\frac{D_a}{D_m}\right)^2 = \left(\frac{4}{3}\right)^2 = 1.78$$

$$\frac{U_m}{U_a} = \frac{N_m}{N_a}\frac{D_m}{D_a} = \left(\frac{4}{3}\right)^3\left(\frac{3}{4}\right) = \left(\frac{4}{3}\right)^2 = 1.78$$; U is the rotor velocity.

So, the axial velocity scales on the rotor velocity. Now, let's look at specific speed:

$N_s = \frac{\phi^{1/2}}{\psi^{3/4}}$; ϕ and ψ do not change with scale so $N_{s,model} = N_{s,actual}$.

There is a related, but different set of parameters used for compressors and turbines (through which ρ changes because the flow is compressed) these will be discussed later.