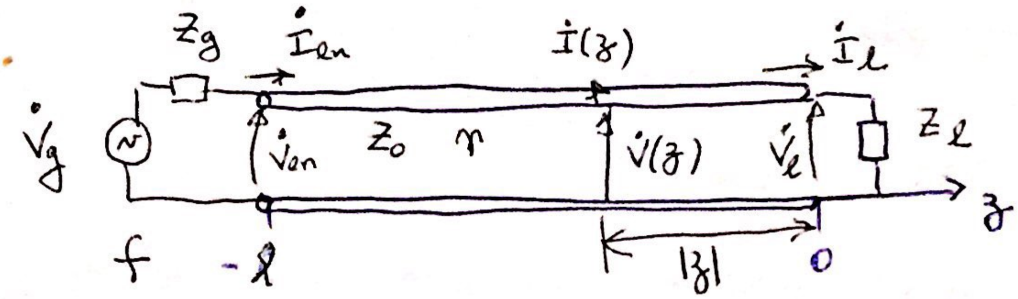
Resumo Linhas em RPS



$\gamma = \alpha + j\beta$ constante de propagação $[m^{-1}]$

$$\dot{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \qquad \beta = \frac{\omega}{v_f} = \frac{2\pi}{\lambda}$$

$$\dot{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \qquad \lambda = \frac{v_f}{f}$$ comprimento de onda

Coeficiente de reflexão da carga $\rho_\ell = \dfrac{Z_\ell - Z_0}{Z_\ell + Z_0} = \dfrac{V_0^-}{V_0^+}$

$$\dot{V}_\ell = \dot{V}(z=0) = V_0^+ + V_0^-$$

$$\dot{I}_\ell = \dot{I}(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} \qquad \frac{\dot{V}_\ell}{\dot{I}_\ell} = Z_\ell = Z_0 \frac{1+\rho_\ell}{1-\rho_\ell}$$

$$\dot{V}_{en} = \dot{V}(z=-\ell) = V_0^+ e^{\gamma \ell} + V_0^- e^{-\gamma \ell} \qquad \frac{\dot{V}_{en}}{\dot{I}_{en}} = Z_{en} = Z_0 \frac{1+\rho_{en}}{1-\rho_{en}}$$

$$\dot{I}_{en} = \dot{I}(z=-\ell) = \frac{V_0^+ e^{\gamma \ell}}{Z_0} - \frac{V_0^- e^{-\gamma \ell}}{Z_0}$$

$$\rho_{en} = \rho_\ell e^{-2\gamma \ell} = \rho_\ell e^{-2\alpha \ell} e^{-2j\beta \ell}$$

Num ponto z qualquer $-\ell \leq z \leq 0$

$$\rho(z) = \rho_\ell e^{2\gamma z} = \rho_\ell e^{-2\gamma |z|}$$

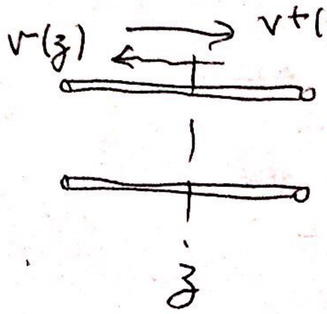
$|z|$ é a distância entre a carga e a posição de interesse.

Observa-se que $\rho(z)$ é um número complexo:

$$\rho(z) = |\rho(z)|\, e^{j\angle\rho(z)}$$

$|\rho(z)|$ módulo do coeficiente de reflexão em z

$\angle\rho(z)$ fase do coeficiente de reflexão em z



$$\dot{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\dot{I}(z) = \frac{V_0^+ e^{-\gamma z}}{Z_0} - \frac{V_0^- e^{\gamma z}}{Z_0}$$

$V^+(z) = V_0^+ e^{+\gamma z} = V_0^+ e^{+\gamma|z|}$ onda incidente

$V^-(z) = V_0^- e^{\gamma z} = V_0^- e^{-\gamma|z|}$ onda refletida

$$\rho(z) = \frac{V^-(z)}{V^+(z)} = \frac{Z(z) - Z_0}{Z(z) + Z_0}$$

onde $Z(z) = \dfrac{\dot{V}(z)}{\dot{I}(z)} = Z_0 \dfrac{1+\rho(z)}{1-\rho(z)}$ impedância no ponto z

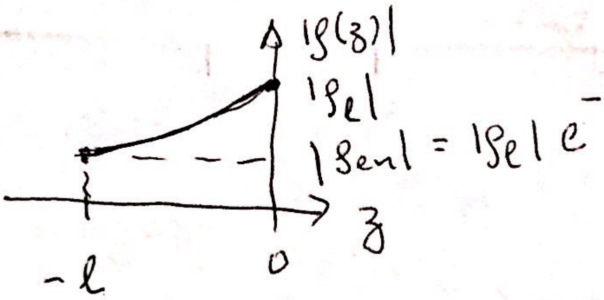
$Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$ impedância característica (Ω)

$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)}$ constante de propagação (m^{-1})

α (Np/m) constante de atenuação

β (rd/m) constante de fase

$$|\rho(z)| = |\rho_\ell| e^{2\alpha z} = |\rho_\ell| e^{-2\alpha|z|}$$



$$\angle\rho(z) = \angle\rho_\ell + 2\beta z = \angle\rho_\ell - 2\beta|z|$$

Casos particulares

Linha sem perdas

$$R = G = 0$$
$$\alpha = 0$$
$$Z_0 = \sqrt{\frac{L}{C}}$$
$$\beta = \omega\sqrt{LC}$$

Linha sem distorção

$$\frac{R}{L} = \frac{G}{C}$$
$$\alpha = \sqrt{RG}$$
$$Z_0 = \sqrt{\frac{L}{C}}$$
$$\beta = \omega\sqrt{LC}$$

Linha de baixas perdas

$$R \ll \omega L$$
$$G \ll \omega C$$
$$\alpha \cong \frac{1}{2}\left(\frac{R}{Z_0} + G Z_0\right)$$
$$\beta \cong \omega\sqrt{LC}$$
$$Z_0 \cong \sqrt{\frac{L}{C}}$$

Linha sem perdas $\quad \gamma = j\beta$

$|\rho(z)| = |\rho_\ell|$ qualquer ponto da linha

$$\dot{V}(z) = V_0^+ e^{-j\beta z}\left(1 + \rho_\ell e^{2j\beta z}\right)$$

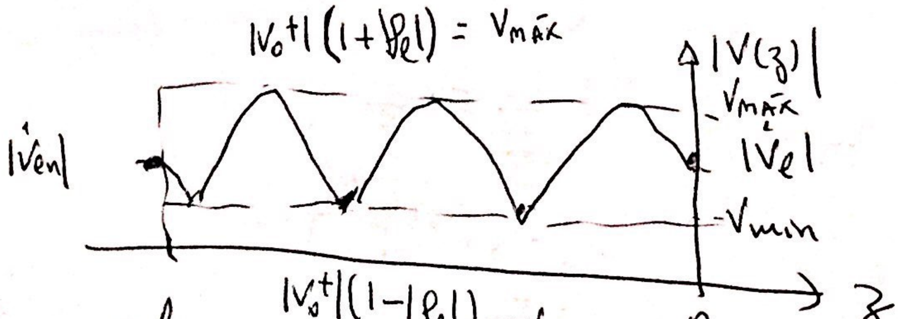
$$\dot{I}(z) = \frac{V_0^+ e^{-j\beta z}}{Z_0}\left(1 - \rho_\ell e^{2j\beta z}\right) \qquad -\ell \le z \le 0$$

ou

$$\dot{V}(z) = V_0^+ e^{j\beta|z|}\left(1 + \rho_\ell e^{-2j\beta|z|}\right)$$

$$\dot{I}(z) = \frac{V_0^+ e^{j\beta|z|}}{Z_0}\left(1 - \rho_\ell e^{-2j\beta|z|}\right)$$

Padrão de Onda Estacionária



$$\frac{V_{m\acute{a}x}}{V_{min}} = \frac{1 + |\rho_\ell|}{1 - |\rho_\ell|} = \text{COE ou TOE}$$

COE: Coeficiente de Onda estacionária

ou

TOE: Taxa de onda estacionária.