2020-1, "STATPHYS", AULA 08

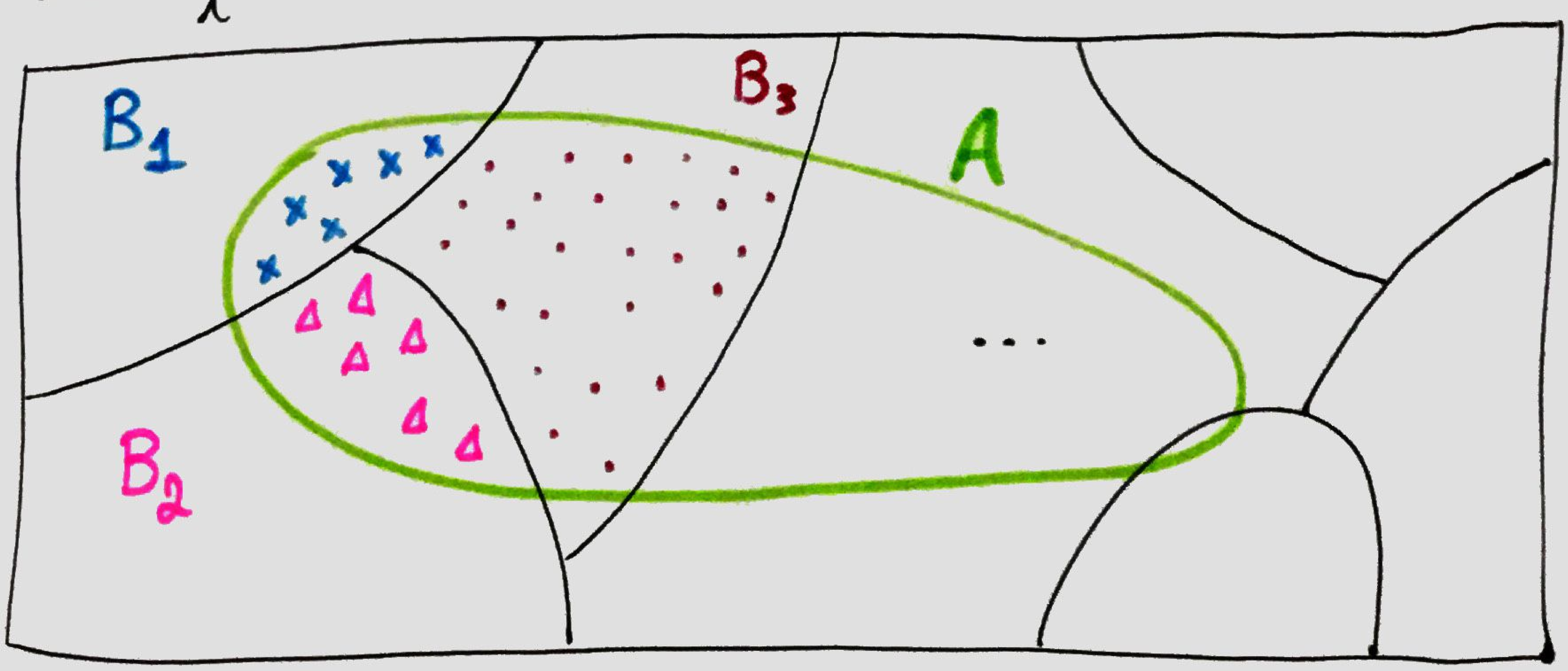
OBJETIVOS: DISCUTIR A LEI DA PROBABILIDADE TOTAL E APLICÁ-LA AO RANDOM WALK 1D.

ONDE ESTAMOS: 1.2 ELEMENTOS DE PROBABILIDADE

INÍCIO DA AULA

* LEI DA PROBABILIDADE TOTAL (LPT)

$$\Omega = \bigcup_i B_i$$



$$\boxed{\mathbb{P}(A) = \sum_i \mathbb{P}(A \mid B_i) \cdot \mathbb{P}(B_i)}$$

$i \neq j$

$B_i \cap B_j = \phi$

□ DEMONSTRAÇÃO:

$$\mathbb{P}(A) = \mathbb{P}(A \cap \Omega) = \mathbb{P}\left[A \cap \left(\bigcup_i B_i\right)\right] =$$

$$= \mathbb{P}\left[\bigcup_i (A \cap B_i)\right] = \sum_i \mathbb{P}(A \cap B_i)$$

$$= \sum_i \mathbb{P}(A \mid B_i) . \mathbb{P}(B_i) \quad \square$$

→ TÉCNICA PARA "INCORPORAR INFORMAÇÃO E UTILIZÁ-LA DE FORMA EFICIENTE".

→ BASE P/ CONSTRUÇÃO DE PROCESSOS ESTOCÁSTICOS

* RANDOM WALK 1D

S_n: POSIÇÃO DO ANDARILHO APÓS n PASSOS

$$\boxed{p_{k,n} \equiv \mathbb{P}(S_n = k)}$$

$$\left.\begin{aligned} A &= \{\omega \in \Omega \,/\, S_n(\omega) = k\} \\ B_i &= \{\omega \in \Omega \,/\, S_{n-1}(\omega) = i\} \end{aligned}\right]$$

ω: CADA POSSÍVEL PASSEIO DE n PASSOS

$$\mathbb{P}(S_n = k) = \sum_i \mathbb{P}(S_n = k \mid S_{n-1} = i) . \mathbb{P}(S_{n-1} = i) =$$

$$= \mathbb{P}(S_n = k \mid S_{n-1} = k-1) . \mathbb{P}(S_{n-1} = k-1) + \mathbb{P}(S_n = k \mid S_{n-1} = k+1) . \mathbb{P}(S_{n-1} = k+1)$$

$$\therefore \boxed{p_{k,n} = p \cdot p_{k-1,n-1} + q \cdot p_{k+1,n-1}}$$

→ EDP DISCRETA!

→ SOLUÇÃO? FAMÍLIA DE GERADORAS (OU CARACTERÍSTICAS)

$$\boxed{g_n(z) = \sum_{k=-n}^{n} p_{k,n} \cdot z^k}$$

$$\sum_k p_{k,n} \cdot z^k = pz \sum_k p_{k-1,n-1} \cdot z^{k-1} + \frac{q}{z} \sum_k p_{k+1,n-1} \cdot z^{k+1} \Rightarrow$$

$$\Rightarrow g_n(z) = p \cdot z \, g_{n-1}(z) + \frac{q}{z} g_{n-1}(z) \Rightarrow$$

$$\Rightarrow \boxed{g_n(z) = \left(pz + \frac{q}{z}\right) \cdot g_{n-1}(z)} \Rightarrow$$

$$\Rightarrow g_n(z) = \left(pz + \frac{q}{z}\right)^n \cdot \boxed{g_0(z)}$$

$= 1$, POIS $p_{k,0} = \delta_{k,0}$

$$\therefore \boxed{g_n(z) = \left(pz + \frac{q}{z}\right)^n} \quad \text{OU} \quad \boxed{\phi_{S_n}(k) = \left(pe^{+ik} + qe^{-ik}\right)^n}$$

• SOLUÇÃO DISCRETA

$$g_n(z) = \left(\frac{pz^2+q}{z}\right)^n = \sum_{i=0}^{n} \binom{n}{i} p^i q^{n-i} \cdot z^{2i-n} \Rightarrow$$

$m \equiv 2i-n$

$\binom{n}{l} \equiv 0$

l NÃO INTEIRO

$$\Rightarrow g_n(z) = \sum_{m=-n}^{+n} z^m \cdot p^{\frac{n+m}{2}} \cdot q^{\frac{n-m}{2}} \binom{n}{\frac{n+m}{2}}$$

$$\boxed{p_{m,n} = \binom{n}{\frac{n+m}{2}} p^{\frac{n+m}{2}} \cdot q^{\frac{n-m}{2}}}$$

• SOLUÇÃO CONTÍNUA

$$\rho_{S_n}(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} dk\, e^{-ikx} \phi_{S_n}(k) =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} dk\, e^{-ikx}\left[\sum_{j=0}^{n}\binom{n}{j}(pe^{ik})^j \cdot (qe^{-ik})^{n-j}\right] =$$

$$= \sum_{j=0}^{n}\binom{n}{j} p^j \cdot q^{n-j} \underbrace{\left\{\frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-ik[x-(2j-n)]} dk\right\}}_{= \delta[x-\underbrace{(2j-n)}_{\equiv m}]} =$$

$$= \sum_{m=-n}^{+n} \binom{n}{\frac{n+m}{2}} p^{\frac{n+m}{2}} \cdot q^{\frac{n-m}{2}} \cdot \delta(x-m)$$

* LIMITE DO CONTÍNUO, RW 1D

$$P_{x,t} = p \cdot P_{x-\Delta x,\, t-\Delta t} + q \cdot P_{x+\Delta x,\, t-\Delta t}$$

$$\Downarrow$$

$$\cancel{\Delta x} \cdot \rho(x,t) = \left[p \cdot \rho(x-\Delta x, t-\Delta t) + q \cdot \rho(x+\Delta x, t-\Delta t)\right] \cdot \cancel{\Delta x}$$

$$\Downarrow$$

$$\rho(\cancel{x},t) =$$

$$= p \cdot \Big\{ \rho(\cancel{x},t) - \Delta x\, \rho_x(x,t) - \Delta t\, \rho_t(x,t) + \frac{(\Delta x)^2}{2} \rho_{xx}(x,t) + $$
$$+ (-\Delta x)(-\Delta t) \rho_{x,t}(x,t) + \frac{(\Delta t)^2}{2} \rho_{tt}(x,t) \Big\} +$$

$$+ q \Big\{ \rho(\cancel{x},t) + \Delta x\, \rho_x(x,t) - \Delta t\, \rho_t(x,t) + \frac{(\Delta x)^2}{2} \rho_{xx}(x,t) +$$
$$+ (+\Delta x)(-\Delta t) \rho_{x,t}(x,t) + \frac{(\Delta t)^2}{2} \rho_{tt}(x,t) \Big\} +$$
$$+ \text{ERRO}$$

PAUSA! $1^{\underline{A}}$ ORDEM: $\Delta t \cdot \rho_t = -(p-q) \Delta x\, \rho_x$

SE $(p-q) \dfrac{\Delta x}{\Delta t} \longrightarrow v$, $\rho(x,t) \propto \delta(x - vt)$

<u>SEM</u> DIFUSÃO.

E SE $\dfrac{(\Delta x)^2}{\Delta t} \longrightarrow D$? AFINAL, $\mathbb{V}(S_n) \propto n \ldots$

$\hookrightarrow t$

PAUSA DE NOVO!

$$0 = -(p-q)\Delta x \rho_x - \Delta t \rho_t + \frac{(\Delta x)^2}{2}\rho_{xx} +$$

$$+ (p-q)\Delta x \Delta t \rho_{xt} + \frac{(\Delta t)^2}{2}\rho_{tt} +$$

$$+ \text{ERRO}$$

$$\frac{(\Delta x)^2}{\Delta t} \longrightarrow D$$

$$0 = -\underbrace{(p-q)}_{=\lambda.\Delta x}\frac{\Delta x}{\Delta t}\rho_x - \rho_t + \frac{D}{2}\rho_{xx} + \text{ERRO}'$$

$$\therefore \boxed{\rho_t = -\alpha\rho_x + \frac{D}{2}\rho_{xx}} \qquad \alpha = \lambda.D$$

EQ. DE DIFUSÃO OU "DO CALOR"