PRO 5971 - Statistical Process Monitoring

Shewhart control chart: X-bar chart - dealing with the sampling interval

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Linda Lee Ho

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Department of Production Engineering
University of São Paulo

Outline

Parameters of the control chart

- Until now, the parameters to design the control charts are: the control limits and the sample size
- However there is other parameter equally relevant: the sampling interval (h) which depends on:
  1. time to produce one item
  2. cost of inspection
  3. time of inspection
  4. stability of the process
  5. losses in operating under an special cause
- In practice, the value of h is arbitrary chosen, however low values of h yield higher inspection cost and increase of false alarms.
- For example: Control limits: $\mu_0 \pm 3\sigma/\sqrt{n}$
  - $n = 4$ and $h = 1$ hour - a false alarm for every 370.4 hours
  - $n = 4$ and $h = 0.5$ hour - a false alarm for every 185.2 hours

Procedure for less arbitrary choice of the sampling interval h

- Efficiency measure: time between the shift and the signal
- It is desirable to very short

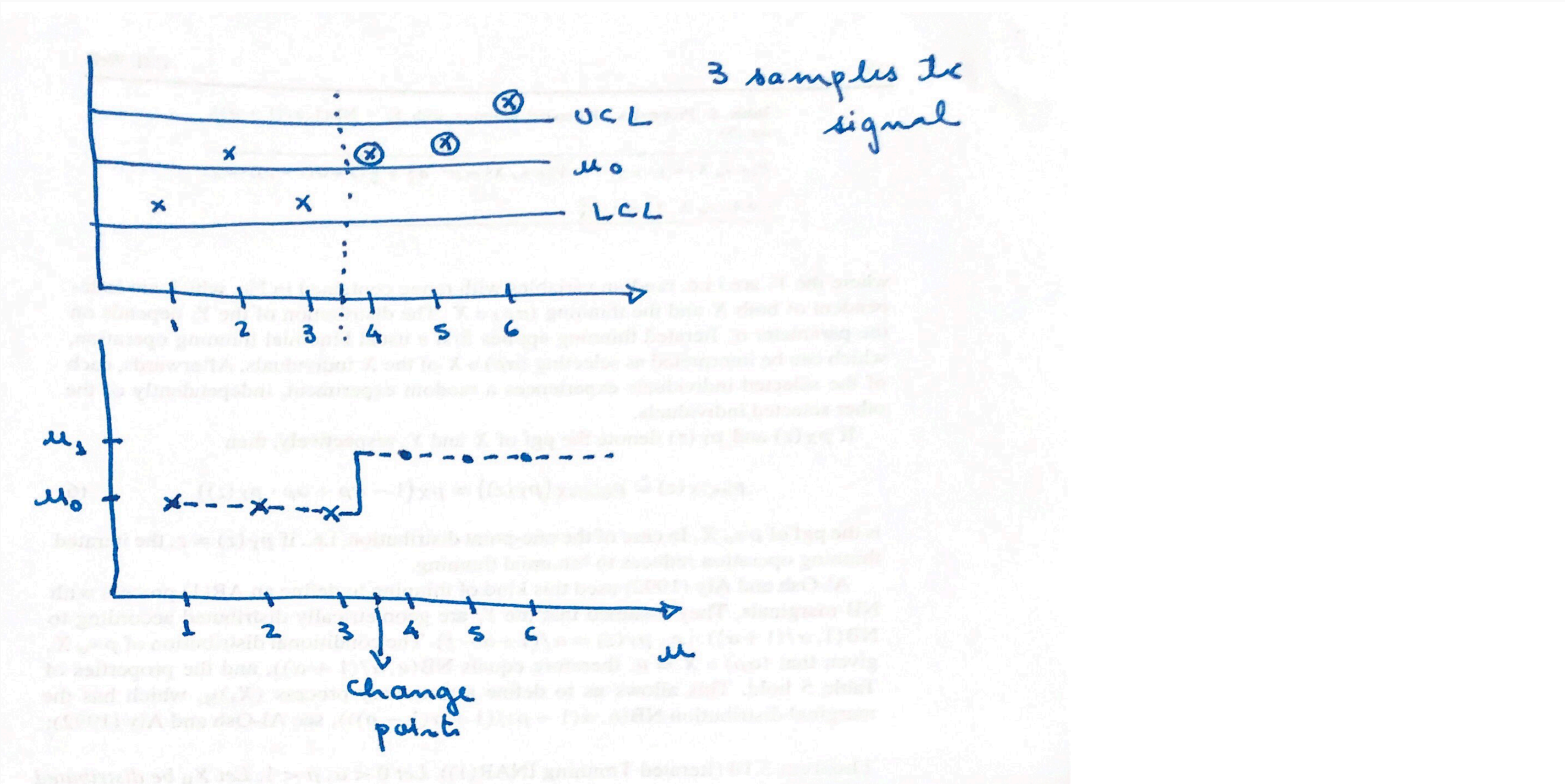

Figure 1: Time to signal

Average time to signal - ATS

- Assumption- the time for a shift is a random variable with a uniform distribution in the interval [0;h].
- ATS as function of h is expressed as $ATS = \frac{h}{power} - \frac{h}{2}$
- For in-control situation ATS is the average time for a false alarm (ATFA)$= \frac{h}{\alpha}$
- For $h = 1$ hour and under alternative hypothesis (out-of-control): ATS_1=ARL_1-0.5 and under null hypothesis (in-control): ATS_0=ARL_0
- $\frac{n}{h}$ is known as the sampling rate

Exercise

- Consider an $\bar{X}$ control chart and $ATFA = \dfrac{h}{\alpha} = 500$ hours
- Find ATS for the following plans: a)n=2 and h=0.25 hours; b)n=4 and h=0.5; c) n=8 and h=1.0 and d) n=16 and h=2 for the shift sizes $\mu_0 \rightarrow \mu_0 + \delta\sigma$, $\delta = 0.5, 0.75, 1, 1.25, 1.5, 2$
- Discuss which plan is more appropriate to each shift size.