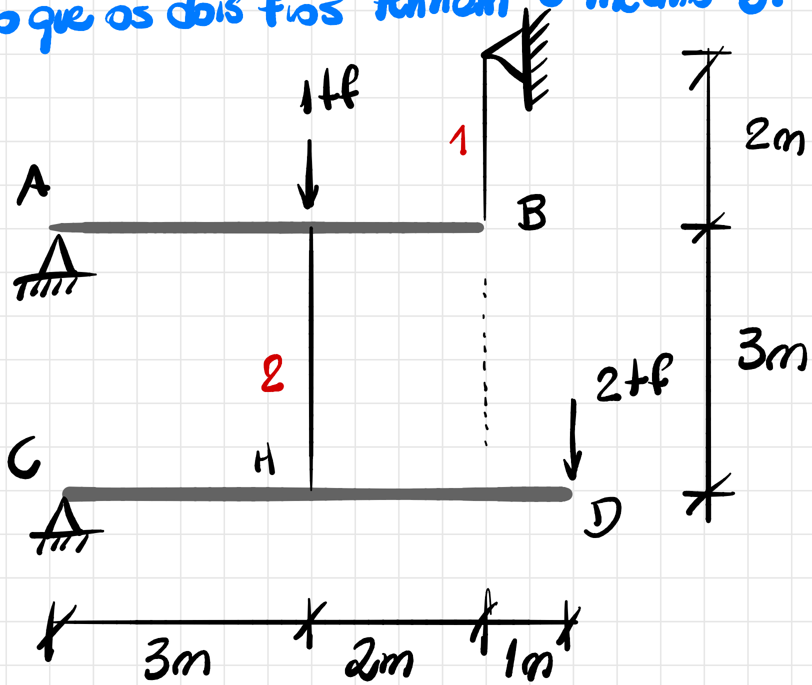
As barras AB e BC são rígidas. Dimensionar os fios 1 e 2, sabendo-se que:

a) $\sigma_e = 4000\ kgf/cm^2$ $(s = 1{,}6)$: $\bar{\sigma} = 2500\ kgf/cm^2$

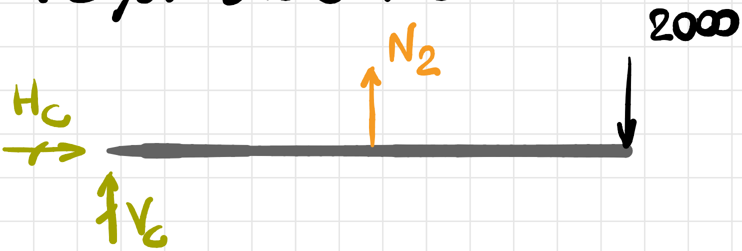
b) $\theta_{A,máx} = 0{,}002\ rad$

c) $v_{D,máx} = 3{,}5\ cm$

O módulo de elasticidade do material dos fios é $E = 4 \times 10^5\ kgf/cm^2$. O dimensionamento deve ser feito de modo que os dois fios tenham o mesmo S.



- Equilíbrio barra CD:

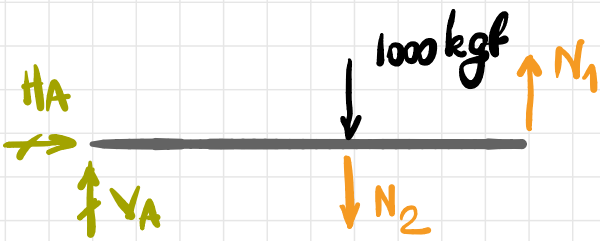


$$\sum F_H = 0: \quad \boxed{H_c = 0}$$

$$\sum F_V = 0: \quad V_c + N_2 = 2000 \qquad \boxed{V_c = -2000\ \text{kgf}}$$

$$\circlearrowleft(+) \sum M_A = 0: \quad N_2 . 3 - 2000 . 6 = 0 \Rightarrow \boxed{N_2 = 4000\ \text{kgf}}$$

- Equilíbrio barra AB:



$$\sum F_H = 0: \quad \boxed{H_A = 0}$$

$$\sum F_V = 0: \quad V_A - N_2 - 1000 + N_1 = 0$$

$$\circlearrowleft(+) \sum M_A = 0: \quad -1000 . 3 - N_2 . 3 + N_1 . 5 = 0$$

$$\begin{cases} V_A - 4000 - 1000 + N_1 = 0 \\ -1000 \cdot 3 - 4000 \cdot 3 + N_1 \cdot 5 = 0 \end{cases}$$

$\boxed{N_1 = 3000 \text{ kgf}} \Rightarrow \boxed{V_A = 2000 \text{ kgf}}$

Logo $N_1 = 3000$ kgf ; $N_2 = 4000$ kgf.

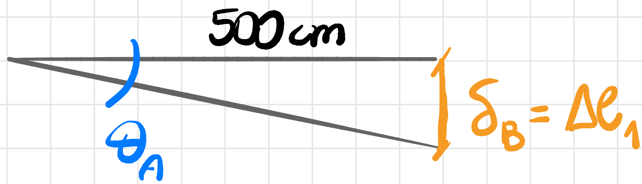
Dimensionamento:

a) $\sigma \leq \bar{\sigma}$

$$\frac{N_1}{A_1} \leq \bar{\sigma} \Rightarrow A_1 \geq \frac{N_1}{\bar{\sigma}} \Rightarrow \boxed{A_1 \geq 1{,}2 \text{ cm}^2}$$

$$\frac{N_2}{A_2} \leq \bar{\sigma} \Rightarrow A_2 \geq \frac{N_2}{\bar{\sigma}} \Rightarrow \boxed{A_2 \geq 1{,}6 \text{ cm}^2}$$

b) $\theta_{A,máx} = 0{,}002 \text{ rad}$



$$\theta_A \cong tg\,\theta_A = \frac{\delta_B}{500} = \frac{\Delta\ell_1}{500} \leqslant \theta_{A,máx}$$

$$\frac{N_1\ell_1}{500EA_1} \leqslant \theta_{A,máx} \Rightarrow A_1 \geqslant \frac{N_1\ell_1}{500E\theta_{A,máx}}$$

$$A_1 \geqslant \frac{3000 \cdot 200}{500 \cdot 4 \cdot 10^5 \cdot 0{,}002} \Rightarrow \boxed{A_1 \geqslant 1{,}5 \text{cm}^2}$$

Para os fios terem o mesmo coeficiente de segurança:

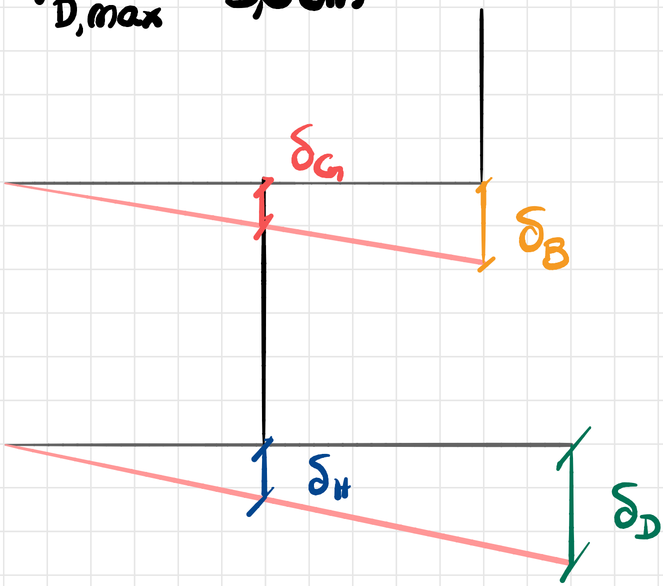
$$\frac{N_1}{A_1} = \bar{\sigma} = \frac{N_2}{A_2} \quad \therefore \quad A_2 = \frac{N_2}{N_1} A_1 = \frac{4}{3} A_1$$

$$\therefore \boxed{A_2 \geqslant 2 \text{cm}^2}$$

c) $V_{D,máx} = 3{,}5\,cm$

1,8

2,4



δ_G está relacionado a δ_B: $\dfrac{\delta_B}{500} = \dfrac{\delta_G}{300}$

δ_D está relacionado a δ_H: $\dfrac{\delta_D}{600} = \dfrac{\delta_H}{300}$

O fio 2 'encurta' δ_G e 'estica' δ_H:

$$\Delta l_2 = \delta_H - \delta_G$$

$$\Delta \ell_2 = \frac{\delta_D}{2} - \frac{3}{5}\delta_B$$

$$\therefore \delta_D = 2\left(\Delta \ell_2 + \frac{3}{5}\delta_B\right) \leq V_{D,máx}$$

$$2\left[\frac{N_2 \ell_2}{EA_2} + \frac{3}{5} \cdot \frac{N_1 \ell_1}{EA_1}\right] \leq V_{D,máx}$$

$$\frac{4000 \cdot 300}{4 \cdot 10^5 A_2} + \frac{3}{5} \cdot \frac{3000 \cdot 200}{4 \cdot 10^5 \cdot A_1} \leq 1{,}75$$

$$\frac{3}{A_2} + \frac{9}{10A_1} \leq 1{,}75$$

Para o mesmo coeficiente de segurança:

$$\frac{3}{\frac{4}{3}A_1} + \frac{9}{10A_1} \leq 1{,}75 \qquad \frac{9}{4A_1} + \frac{9}{10A_1} \leq 1{,}75$$

$$\frac{90+36}{40A_1} \leq 1{,}75$$

$$70A_1 \geq 126$$

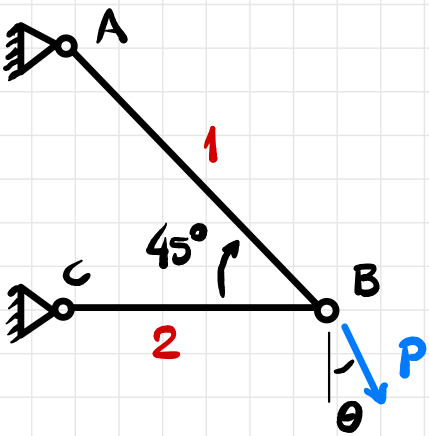
$$\therefore \boxed{A_1 \geq 1{,}8\,cm^2}$$

$$e \quad \boxed{A_2 \geq 2{,}4\,cm^2}$$

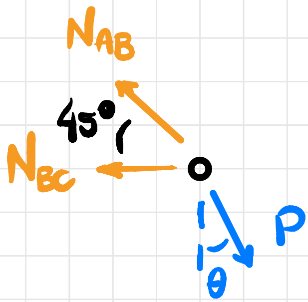
O conjunto de áreas que atendem a todos os critérios é:

$$A_1 = 1{,}8\,cm^2 \quad e \quad A_2 = 2{,}4\,cm^2.$$

A estrutura ABC suporta no nó B uma força P que atua segundo um ângulo θ com a vertical. As áreas das seções transversais dos elementos AB e AC são A_1 e A_2, respectivamente. Achar o valor do ângulo θ de modo que o deslocamento do nó B seja na direção da força P.



Fazendo o equilíbrio do nó B:



$$\sum F_H = 0: \; -N_{BC} - N_{AB} \cdot \cos 45° + P \cdot \text{sen}\,\theta = 0$$

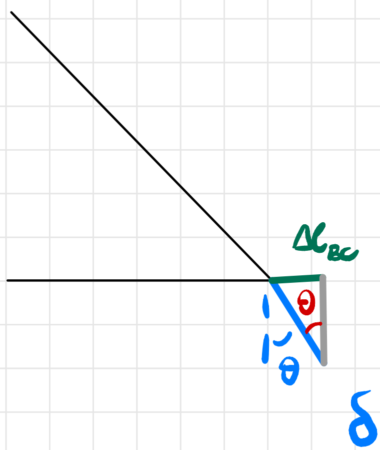
$$\sum F_V = 0: \; -P\cos\theta + N_{AB} \cdot \text{sen}\,45° = 0$$

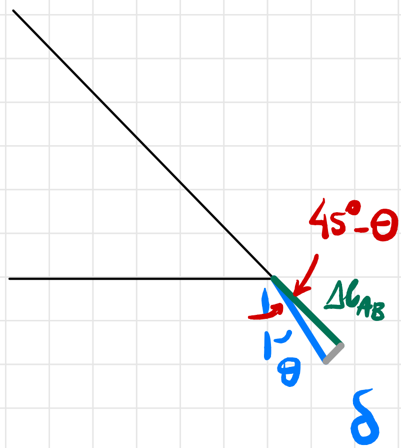
$$N_{AB} = \frac{P\cos\theta}{\text{sen}\,45°} \longrightarrow \boxed{N_{AB} = \sqrt{2}\,P\cos\theta}$$

$$N_{BC} = P\,\text{sen}\,\theta - N_{AB} \cdot \cos 45° = P\,\text{sen}\,\theta - \cancel{\sqrt{2}}P\cos\theta \cdot \frac{1}{\cancel{\sqrt{2}}}$$

$$\therefore \; \boxed{N_{BC} = P(\text{sen}\,\theta - \cos\theta)}$$

Traçando o diagrama de Williot:





$$\operatorname{sen}\theta = \frac{\Delta \ell_{BC}}{\delta} \qquad \cos(45° - \theta) = \frac{\Delta \ell_{AB}}{\delta}$$

$$\delta = \frac{\Delta \ell_{BC}}{\operatorname{sen}\theta} \qquad \delta = \frac{\Delta \ell_{AB}}{\cos(45° - \theta)}$$

$$\cos(45° - \theta) = \cos 45° \cos\theta + \operatorname{sen} 45° \operatorname{sen}\theta$$

$$\cos(45° - \theta) = \frac{\cos\theta + \operatorname{sen}\theta}{\sqrt{2}}$$

Logo:

$$\frac{\Delta l_{BC}}{\operatorname{sen}\theta} = \frac{\sqrt{2}\,\Delta l_{AB}}{\operatorname{sen}\theta + \cos\theta}$$

$$\frac{N_{BC} \cdot l_{BC}}{\cancel{E}A_2 \operatorname{sen}\theta} = \frac{\sqrt{2}\,N_{AB} \cdot l_{AB}}{\cancel{E}A_1(\operatorname{sen}\theta + \cos\theta)}$$

se $l_{BC} = l$, então $l_{AB} = l\sqrt{2}$:

$$\frac{N_{BC} \cdot \cancel{l}}{A_2 \operatorname{sen}\theta} = \frac{\sqrt{2}\,N_{AB}\,\cancel{l}\sqrt{2}}{A_1(\operatorname{sen}\theta + \cos\theta)}$$

Substituindo as forças normais:

$$\frac{\cancel{P}(\operatorname{sen}\theta - \cos\theta)}{A_2 \operatorname{sen}\theta} = \frac{2\sqrt{2}\,\cancel{P}\cos\theta}{A_1(\operatorname{sen}\theta + \cos\theta)}$$

$$A_1(\text{sen}\,\theta - \cos\theta)(\text{sen}\,\theta + \cos\theta) = 2\sqrt{2}\,A_2\,\text{sen}\,\theta\cos\theta$$

$$A_1(\text{sen}^2\theta - \cos^2\theta) = \sqrt{2}\,A_2(2\,\text{sen}\,\theta\cos\theta)$$

$$-A_1\cos 2\theta = \sqrt{2}\,A_2\,\text{sen}\,2\theta$$

$$\frac{\cos 2\theta}{\text{sen}\,2\theta} = \frac{-\sqrt{2}\,A_2}{A_1}$$

E assim:

$$\boxed{\text{cotg}\,2\theta = \frac{-\sqrt{2}\,A_2}{A_1}}$$