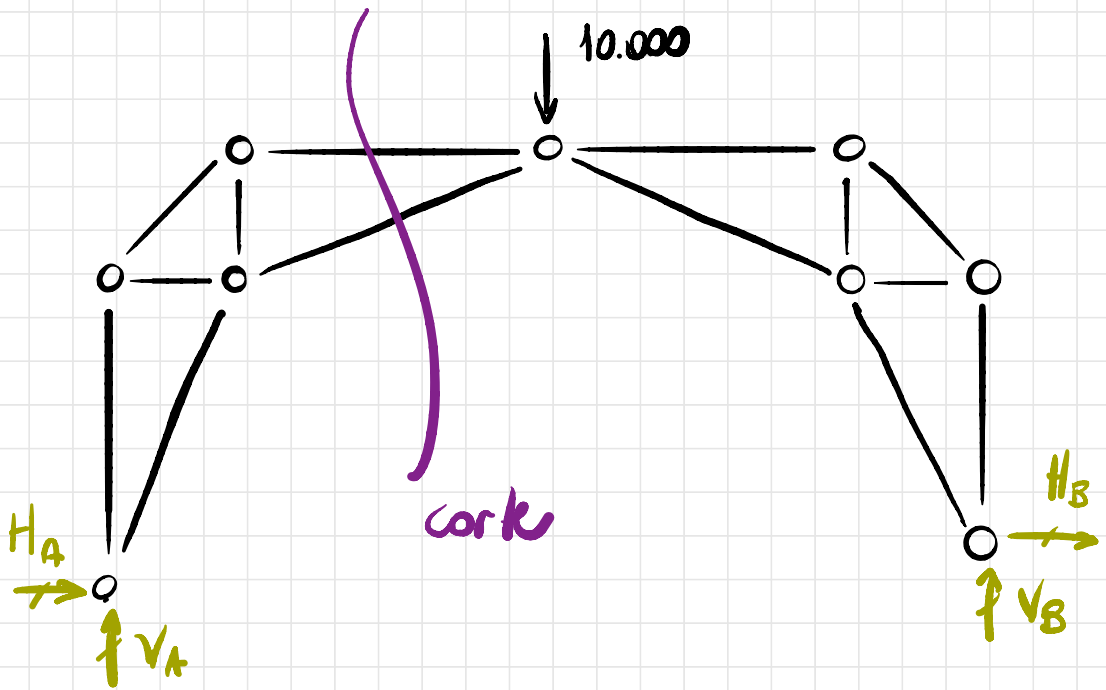


Determinar as normais nas treliças numeradas.

- treliça isostática
- 4 reações vinculores
- 14 barras
- 9 nós \Rightarrow 18 equações l.v. de equilíbrio.



$$\sum F_H = 0: H_A + H_B = 0$$

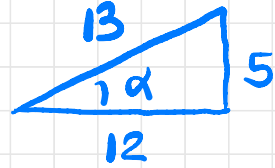
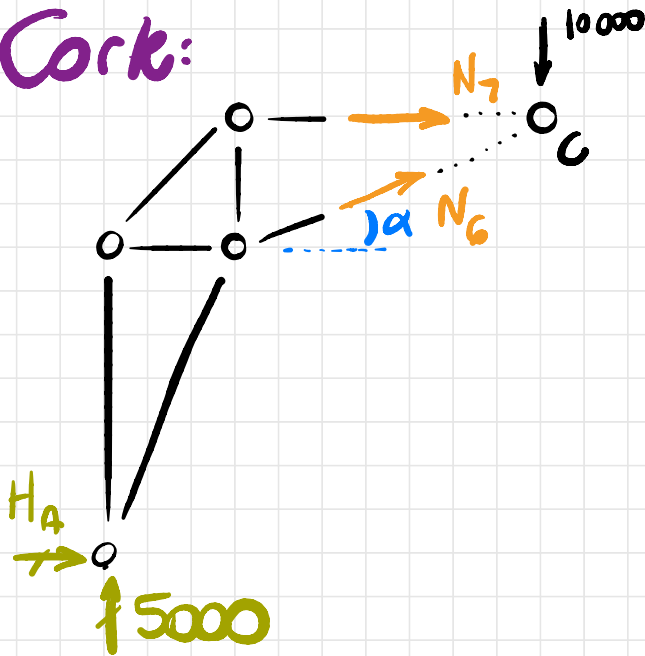
$$\sum F_V = 0: V_A + V_B = 10.000$$

$$\odot \sum M_A = 0: -10.000 \cdot 17 + V_B \cdot 34 = 0$$

$$V_B = 5000 \text{ kgf}$$

$$V_A = 5000 \text{ kgf}$$

Cork:



$$\sin \alpha = 5/13$$

$$\cos \alpha = 12/13$$

$$\sum F_H = 0: H_A + N_7 + N_6 \cos \alpha = 0$$

$$\sum F_V = 0: 5000 + N_6 \sin \alpha = 0$$

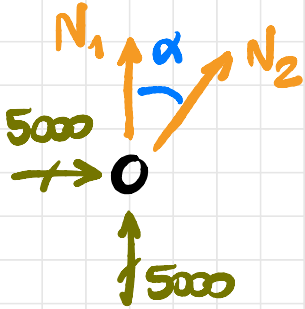
$$N_6 = -\frac{5000}{\sin \alpha} \Rightarrow N_6 = -13000 \text{ kgf}$$

$$\curvearrowright \sum M_C = 0: H_A \cdot 17 - 5000 \cdot 17 = 0$$

$$H_A = 5000 \text{ kgf}$$

$$N_7 = -H_A - N_6 \cos \alpha = -5000 + 12000 \Rightarrow N_7 = 7000 \text{ kgf}$$

No A:



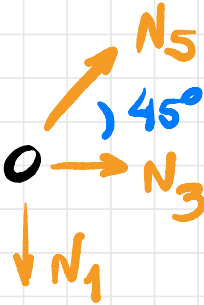
$$\sum F_H = 0: 5000 + N_2 \sin \alpha = 0$$

$$N_2 = -13000 \text{ kgf}$$

$$\sum F_V = 0: 5000 + N_1 + N_2 \cos \alpha = 0$$

$$N_1 = -5000 - N_2 \cos \alpha \Rightarrow N_1 = 7000 \text{ kgf}$$

No F:



$$\sum F_H = 0: N_5 \cos 45^\circ + N_3 = 0$$

$$\sum F_V = 0: -N_1 + N_5 \sin 45^\circ = 0$$

$$N_5 = 7000\sqrt{2} \text{ kgf}$$

$$N_3 = -N_5 \cos 45^\circ \Rightarrow N_3 = -7000 \text{ kgf}$$

Nó D:



$$\sum F_H = 0: -N_5 \sin 45^\circ + N_7 = 0$$

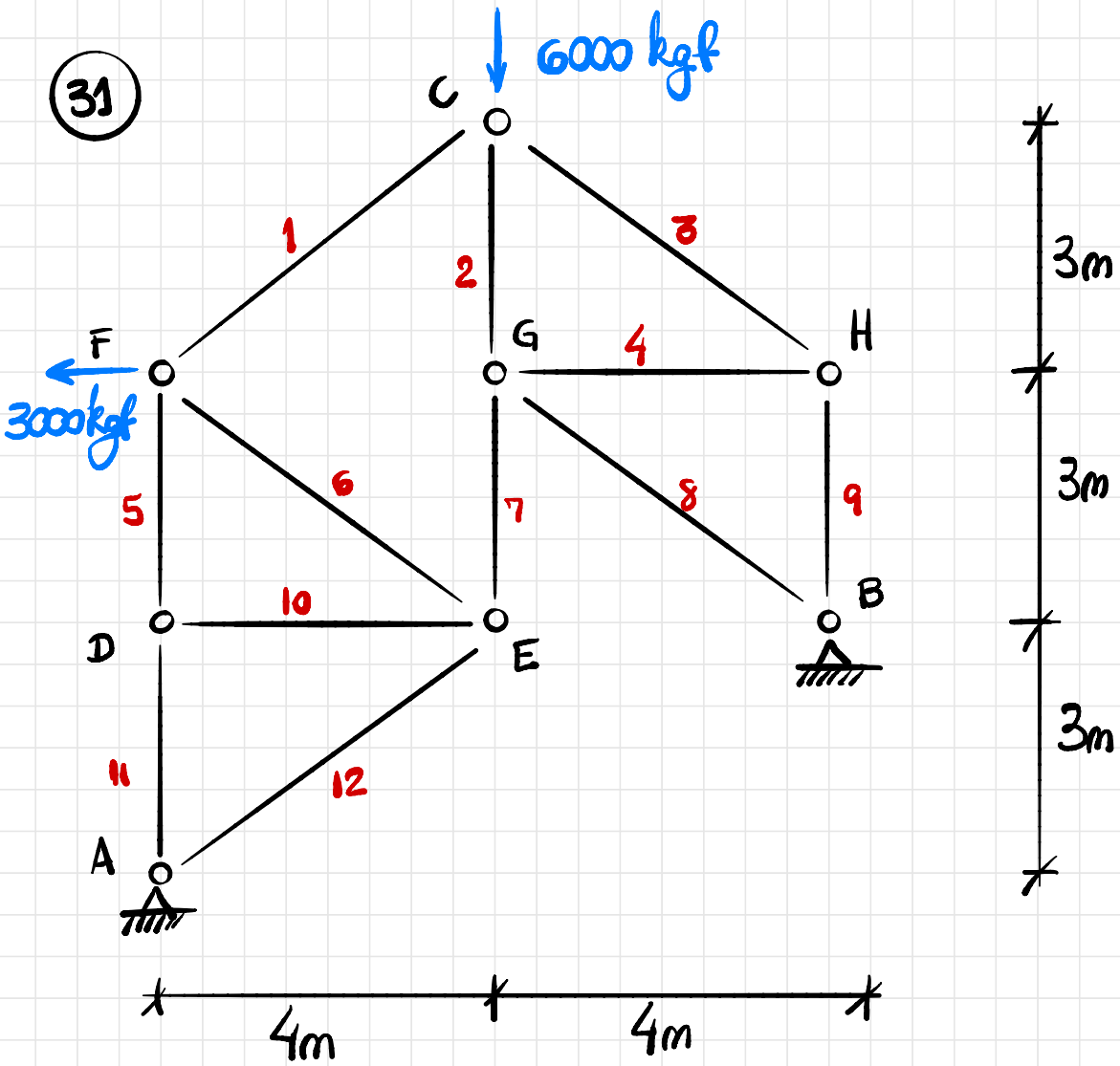
$$-7000\sqrt{2} \cdot \frac{1}{\sqrt{2}} + 7000 = 0$$

$$\sum F_V = 0: -N_5 \cos 45^\circ - N_4 = 0$$

$$N_4 = -N_5 \cos 45^\circ \Rightarrow N_4 = -7000 \text{ kgf}$$

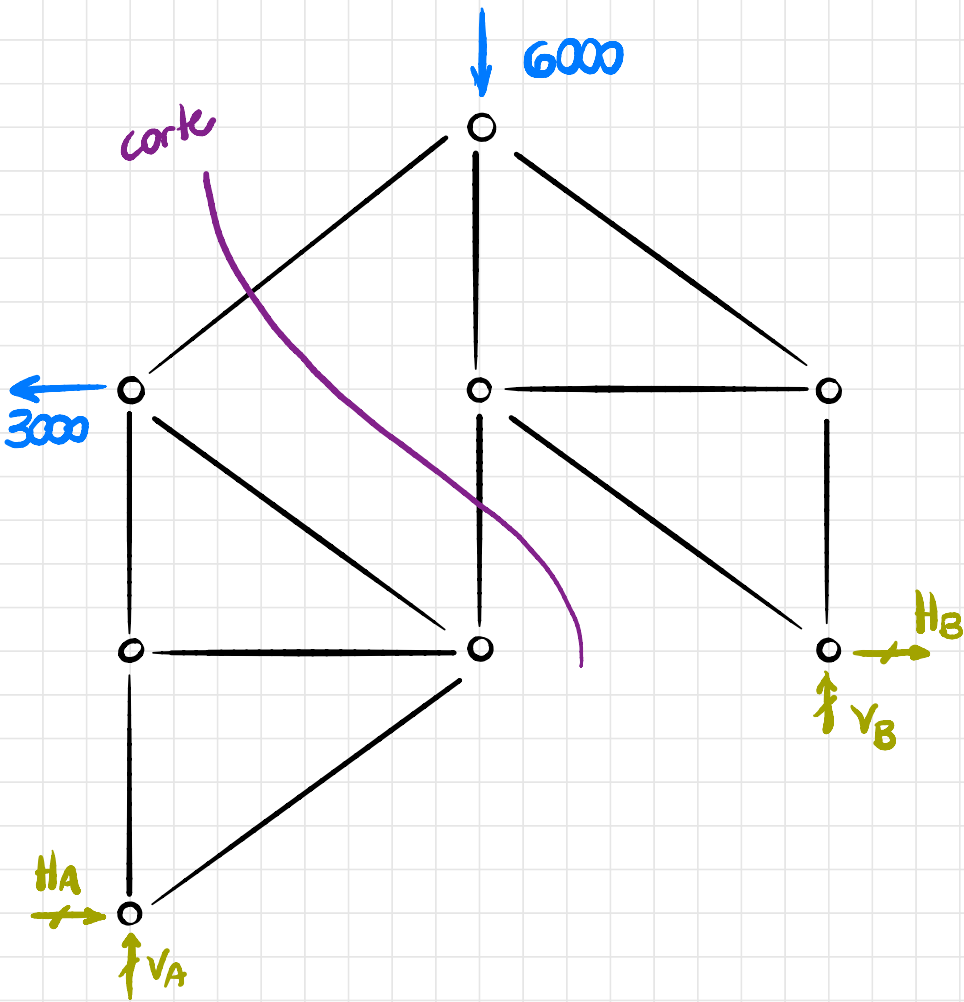
barra	N [kgf]
1	7000
2	-13000
3	-7000
4	-7000
5	$7000\sqrt{2}$
6	-13000
7	7000

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Treliza isostática (8 nós, 4 reações, 12 barras)

Reações de apoio:



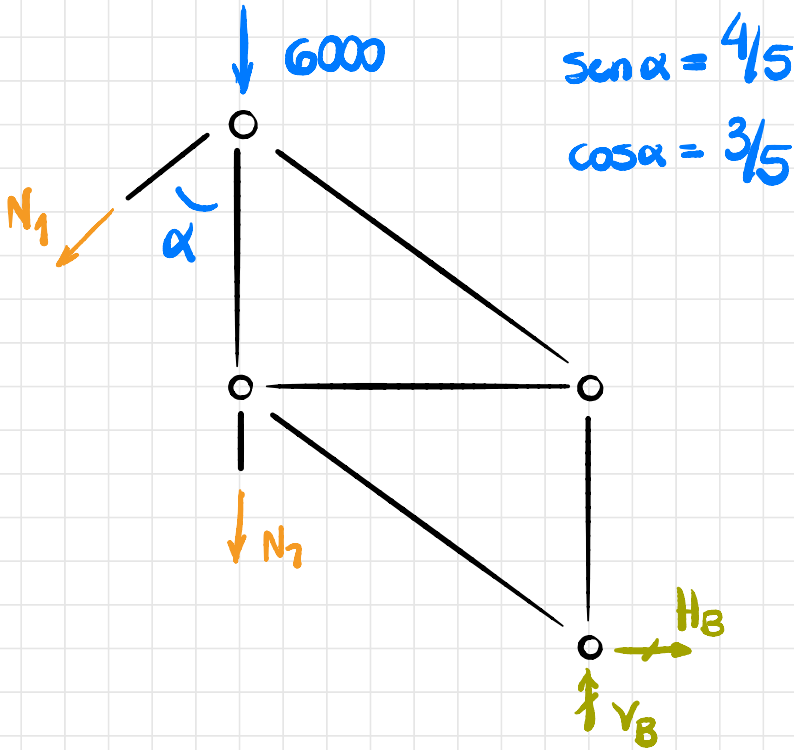
$$\sum F_H = 0: H_A + H_B = 3000$$

$$\sum F_V = 0: V_A + V_B = 6000$$

$$\textcircled{+} \sum M_A = 0: 3000 \cdot 6 - 6000 \cdot 4 - H_B \cdot 3 + V_B \cdot 8 = 0$$

$$8V_B - 3H_B = 6000$$

Apenas com o equilíbrio, não é possível encontrar as reações do apoio. Fazendo o corte:



$$\sum F_H = 0: -N_1 \sin \alpha + H_B = 0$$

$$\sum F_V = 0: -6000 - N_1 \cos \alpha - N_7 + V_B = 0$$

$$\text{A) } \sum M_C = 0: V_B \cdot 4 + 6H_B = 0$$

$$H_B = -\frac{2}{3} V_B$$

$$8V_B - 3(-2/3 V_B) = 6000$$

$$10V_B = 6000 \rightarrow V_B = 600 \text{ kgf}$$

$$H_B = -2/3 \cdot 600 \rightarrow H_B = -400 \text{ kgf}$$

$$H_A = 3000 - (-400) \rightarrow H_A = 3400 \text{ kgf}$$

$$V_A = 6000 - 600 \rightarrow V_A = 5400 \text{ kgf}$$

$$N_1 = H_B / \sin \alpha = -400 \cdot \frac{5}{4}$$

$$N_1 = -500 \text{ kgf}$$

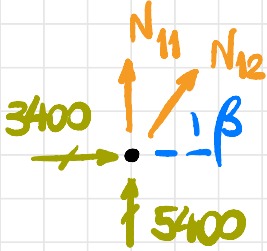
$$N_2 = -6000 - N_1 \cos \alpha + V_B$$

$$N_2 = -6000 - (-500) \cdot 3/5 + 600$$

$$N_2 = -5100 \text{ kgf}$$

Nó A:

$$\sin\beta = 3/5 ; \cos\beta = 4/5$$



$$\sum F_H = 0: 3400 + N_{12} \cos\beta = 0$$

$$N_{12} = -4250 \text{ kgf}$$

$$\sum F_V = 0: 5400 + N_{11} + N_{12} \sin\alpha = 0$$

$$N_{11} = -2850 \text{ kgf}$$

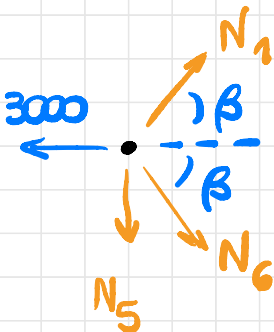
Nó D:



$$\sum F_H = 0: N_{10} = 0$$

$$\sum F_V = 0: N_5 = N_{11} \rightarrow N_5 = -2850 \text{ kgf}$$

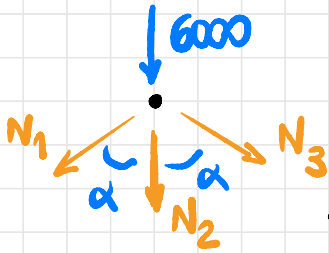
Nó F:



$$\sum F_H = 0: N_1 \cos\beta + N_6 \cos\beta = 3000$$

$$N_6 = 4250 \text{ kgf}$$

Nó C:



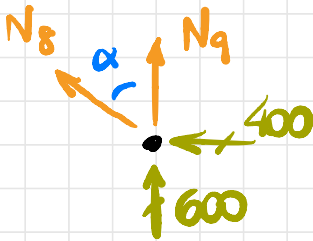
$$\sum F_H = 0: N_1 \sin \alpha - N_3 \sin \alpha = 0$$

$$N_3 = -500 \text{ kgf}$$

$$\sum F_V = 0: -6000 - N_2 - N_1 \cos \alpha - N_3 \cos \alpha = 0$$

$$N_2 = -5400 \text{ kgf}$$

Nó B:



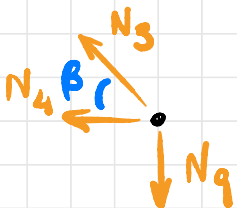
$$\sum F_H = 0: -N_8 \sin \alpha - 400 = 0$$

$$N_8 = -500 \text{ kgf}$$

$$\sum F_V = 0: 600 + N_8 \cos \alpha + N_9 = 0$$

$$N_9 = -300 \text{ kgf}$$

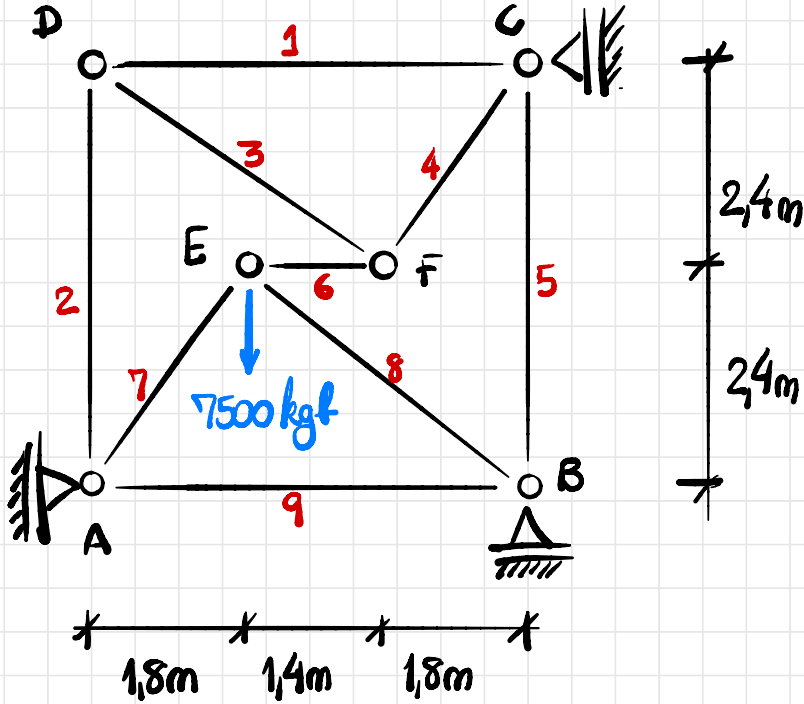
Nó H:



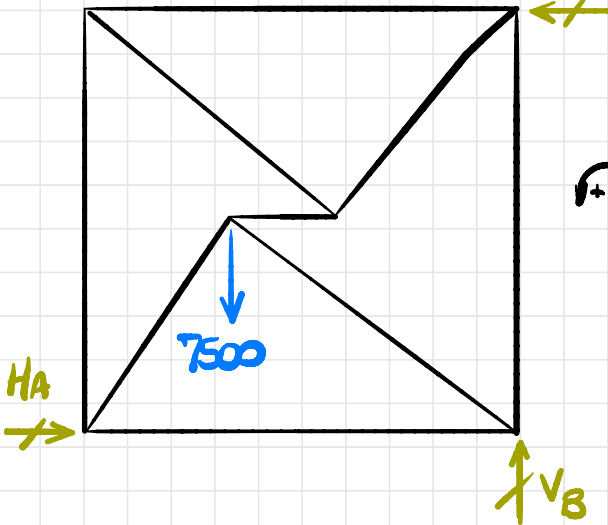
$$\sum F_H = 0: -N_4 - N_3 \cos \beta = 0$$

$$N_4 = 400 \text{ kgf}$$

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Reações do apoio



$$\sum F_H = 0: H_A = H_C$$

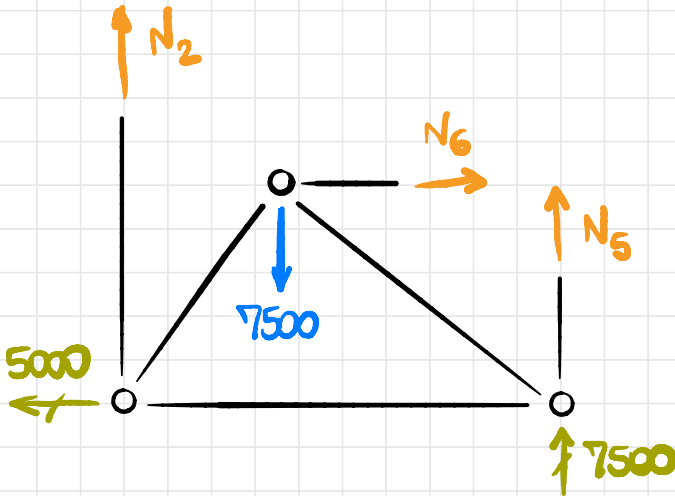
$$\sum F_V = 0: V_B = \underline{7500 \text{ kgf}}$$

$$\begin{aligned} \textcircled{+} \sum M_A = 0: & -7500 \cdot 1,8 \\ & + 5V_B + 4,8H_C = 0 \end{aligned}$$

$$H_C = \underline{-5000 \text{ kgf}}$$

$$H_A = \underline{-5000 \text{ kgf}}$$

Fazendo um corte:



$$\sum F_H = 0: -5000 + N_6 = 0 \rightarrow N_6 = 5000 \text{ kgf}$$

$$\sum F_V = 0: N_2 + N_5 = 0$$

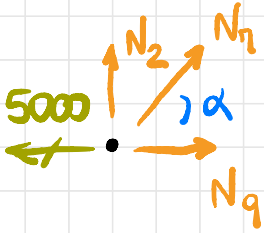
$$\text{A) } \sum M_A = 0: -7500 \cdot 1,8 - N_6 \cdot 2,4 + 7500 \cdot 5 + N_5 \cdot 5 = 0$$

$$5N_5 = -12000 \rightarrow N_5 = -2400 \text{ kgf}$$

$$N_2 = 2400 \text{ kgf}$$

No A:

$$\sin \alpha = 0,8 ; \cos \alpha = 0,6$$



$$\sum F_H = 0: -5000 + N_7 \cos \alpha + N_9 = 0$$

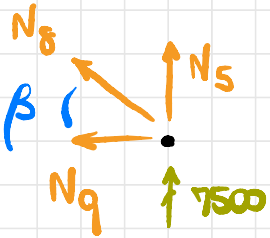
$$\sum F_V = 0: N_2 + N_7 \sin \alpha = 0$$

$$N_7 = -N_2 / \sin \alpha \rightarrow N_7 = -3000 \text{ kgf}$$

$$N_9 = 5000 - N_7 \cos \alpha \rightarrow N_9 = 6800 \text{ kgf}$$

No B:

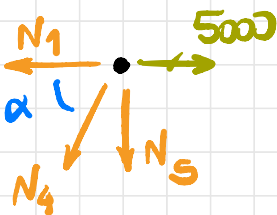
$$\sin \beta = 0,6 ; \cos \beta = 0,8$$



$$\sum F_H = 0: -N_9 - N_8 \cos \beta = 0$$

$$N_8 = -N_9 / \cos \beta \rightarrow N_8 = -8500 \text{ kgf}$$

No C:



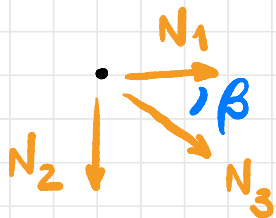
$$\sum F_H = 0: -N_1 - N_4 \cos \alpha + 5000 = 0$$

$$\sum F_V = 0: -N_5 - N_4 \sin \alpha = 0$$

$$N_4 = -N_5 / \sin \alpha \rightarrow N_4 = 3000 \text{ kgf}$$

$$N_1 = 5000 - N_4 \cos \alpha \rightarrow N_1 = 3200 \text{ kgf}$$

Nº D:



$$\sum F_H = 0: N_1 + N_3 \cos \beta = 0$$

$$N_3 = -N_1 / \cos \beta \rightarrow N_3 = -4000 \text{ kgf}$$

barra	N [kgf]
1	3200
2	2400
3	-4000
4	3000
5	-2400
6	5000
7	-3000
8	-8500
9	6800