Eletromagnetismo

Eletrostática

Eletricidade: Ação à distância

Contradição ao senso comum (ação por contato)

Grécia: âmbar (*elektron*) − Tales de Mileto (circa 600 AC)

Charles du Fay (1698-1739):

eletricidade vítrea X eletricidade resinosa,

proposta de dois fluidos

Conceito posterior: um único fluido

→Willian Watson (1746), Benjamin Franklin (1747)

Positivo X Negativo

Triboeletricidade

	Materiais	⊕
↓	Pele de coelho	↑
↓	Vidro	↑
↓	Mica	↑
↓	Lã	↑
↓	Pele de gato	↑
↓	Seda	↑
↓	Algodão	↑
↓	Madeira	↑
↓	Âmbar	↑
↓	Resinas	↑
↓	Metais em geral (cobre, níquel, prata)	↑
↓	Enxofre	↑
↓	Celulóide	↑
⊖		

Tabela 3.1: Série triboelétrica para alguns materiais.

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Triboeletricidade

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Quantifying and understanding the triboelectric series of inorganic non-metallic materials

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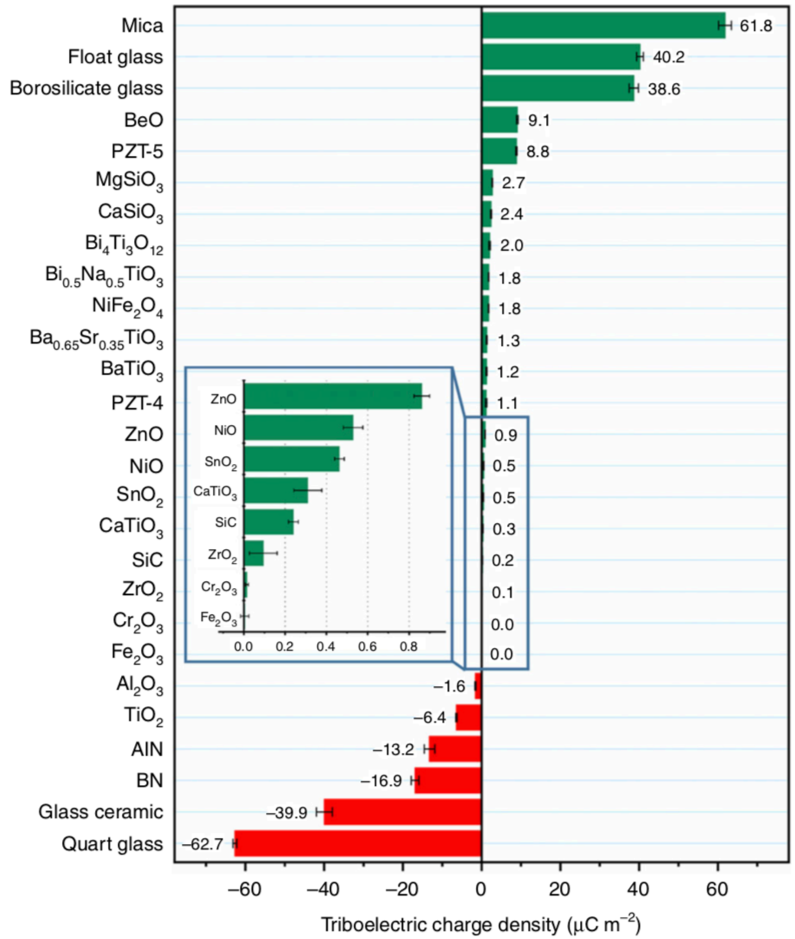


Fig. 3 Quantified triboelectric series of some common inorganic non-metalic materials. The error bar indicates the range within a standard deviation. Source data are provided as a Source Data file.

Table 1 Triboelectric series of materials and their TECD.

Materials	Average TECD ($\mu C\,m^{-2}$)	STDEV	α
Mica	61.80	1.63	0.547
Float glass	40.20	0.85	0.356
Borosilicate glass	38.63	1.18	0.342
BeO	9.06	0.21	0.080
PZT-5	8.82	0.16	0.078
$MgSiO_3$	2.72	0.07	0.024
$CaSiO_3$	2.38	0.15	0.021
$Bi_4Ti_3O_{12}$	2.02	0.21	0.018
$Bi_{0.5}Na_{0.5}TiO_3$	1.76	0.05	0.016
$NiFe_2O_4$	1.75	0.07	0.0155
$Ba_{0.65}Sr_{0.35}TiO_3$	1.28	0.11	0.011
$BaTiO_3$	1.27	0.08	0.0112
PZT-4	1.24	0.12	0.011
ZnO	0.86	0.04	0.008
NiO	0.53	0.05	0.005
SnO_2	0.46	0.02	0.004
SiC	0.31	0.07	0.003
$CaTiO_3$	0.24	0.02	0.002
ZrO_2	0.09	0.07	0.001
Cr_2O_3	0.02	0.01	0.00013
Fe_2O_3	0.00	0.02	0.000
Al_2O_3	−1.58	0.14	−0.014
TiO_2	−6.41	0.18	−0.057
AIN	−13.24	1.35	−0.117
BN	−16.90	0.97	−0.149
Clear very high-temperature glass ceramic	−39.95	2.04	−0.353
Ultra-high-temperature quartz glass	−62.66	0.47	−0.554

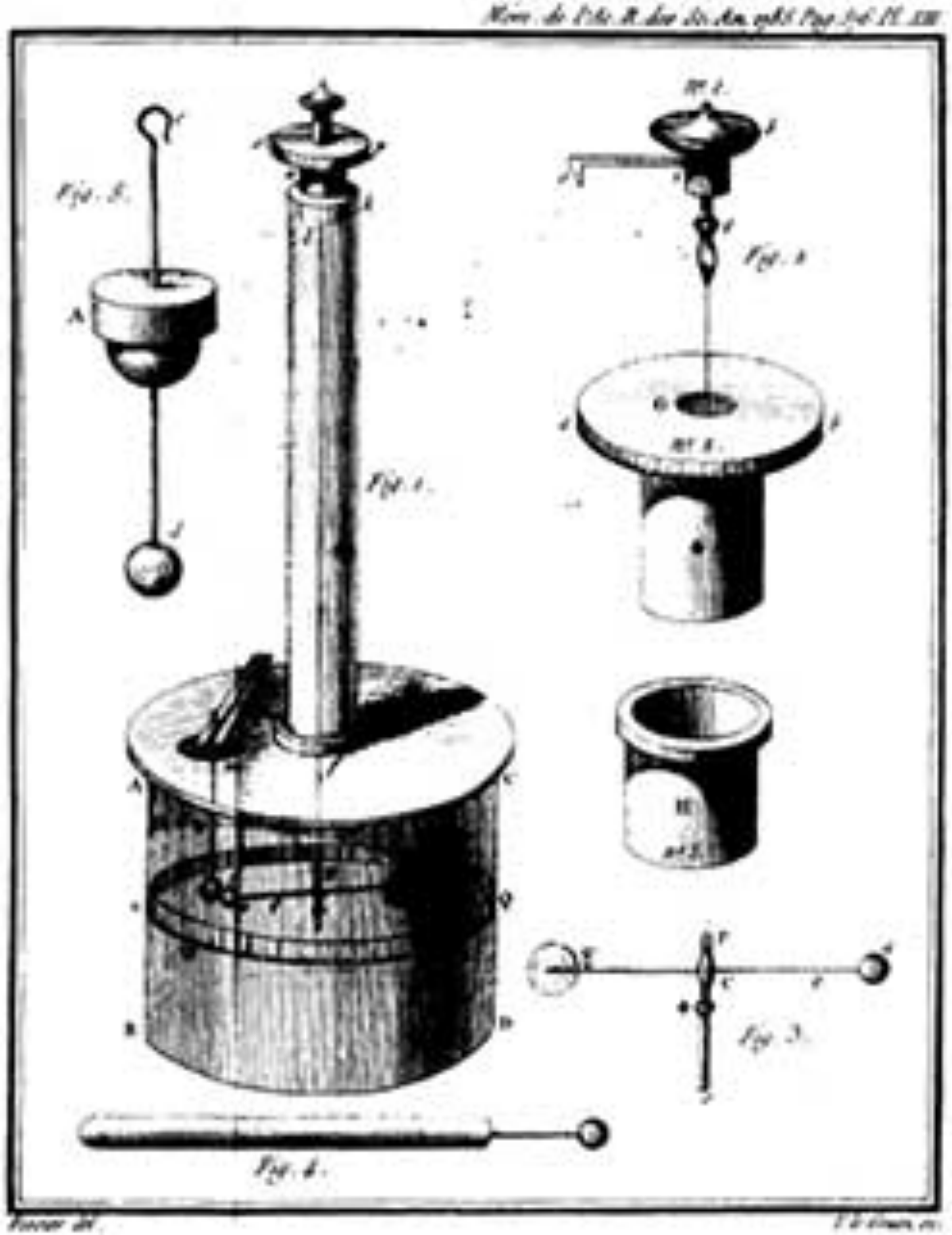
STDEV, standard deviation.
Note: The α refers to the measured triboelectric charge density of tested materials over the absolute value of the measured triboelectric charge density of the reference material (PTFE).

Medindo a carga elétrica

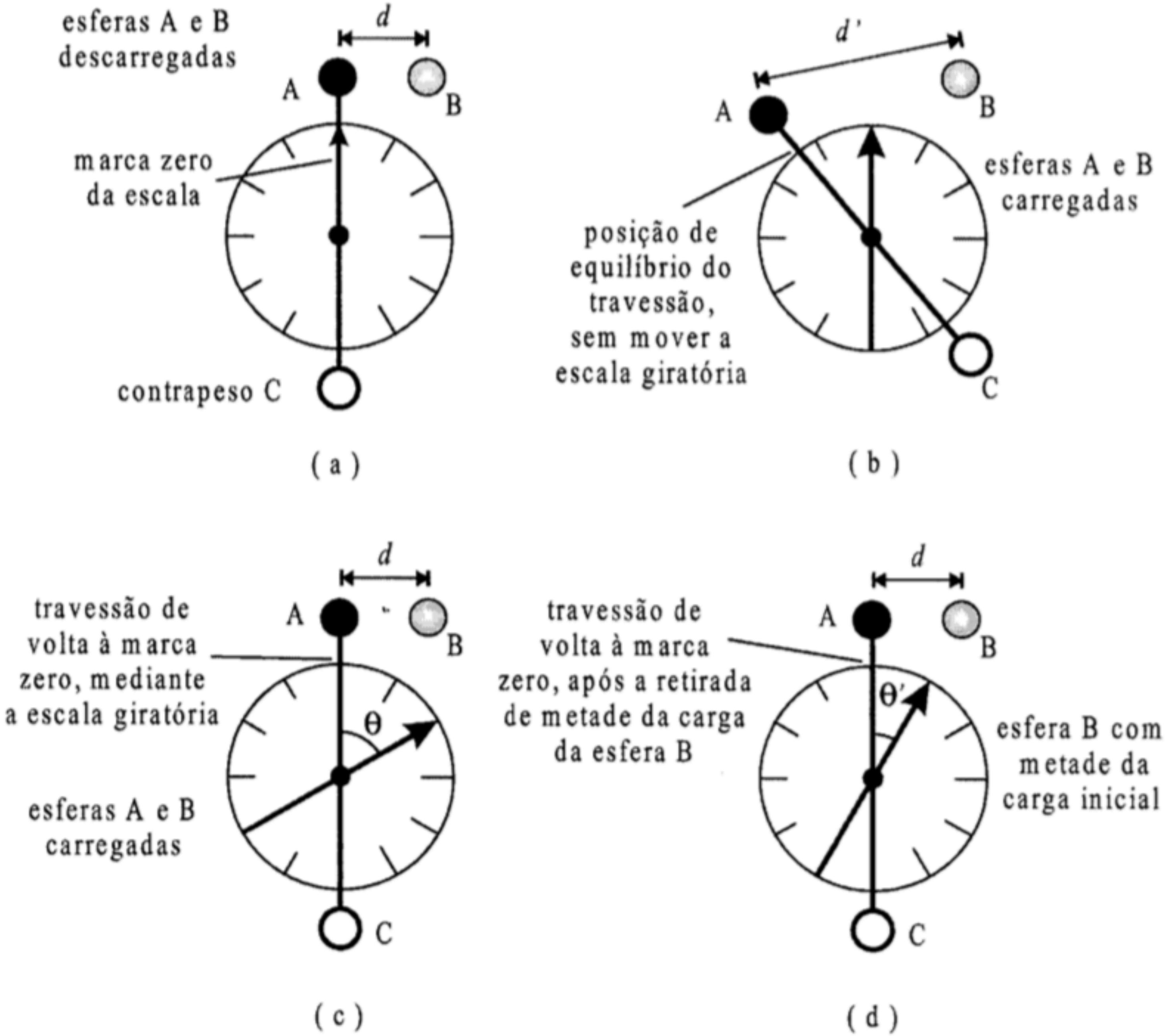
Equação de Força elétrica

introduzida por Bernoulli (1760)

Medida por Coulomb (Balança de torção)



https://www.aps.org/publications/apsnews/201606/physicshistory.cfm



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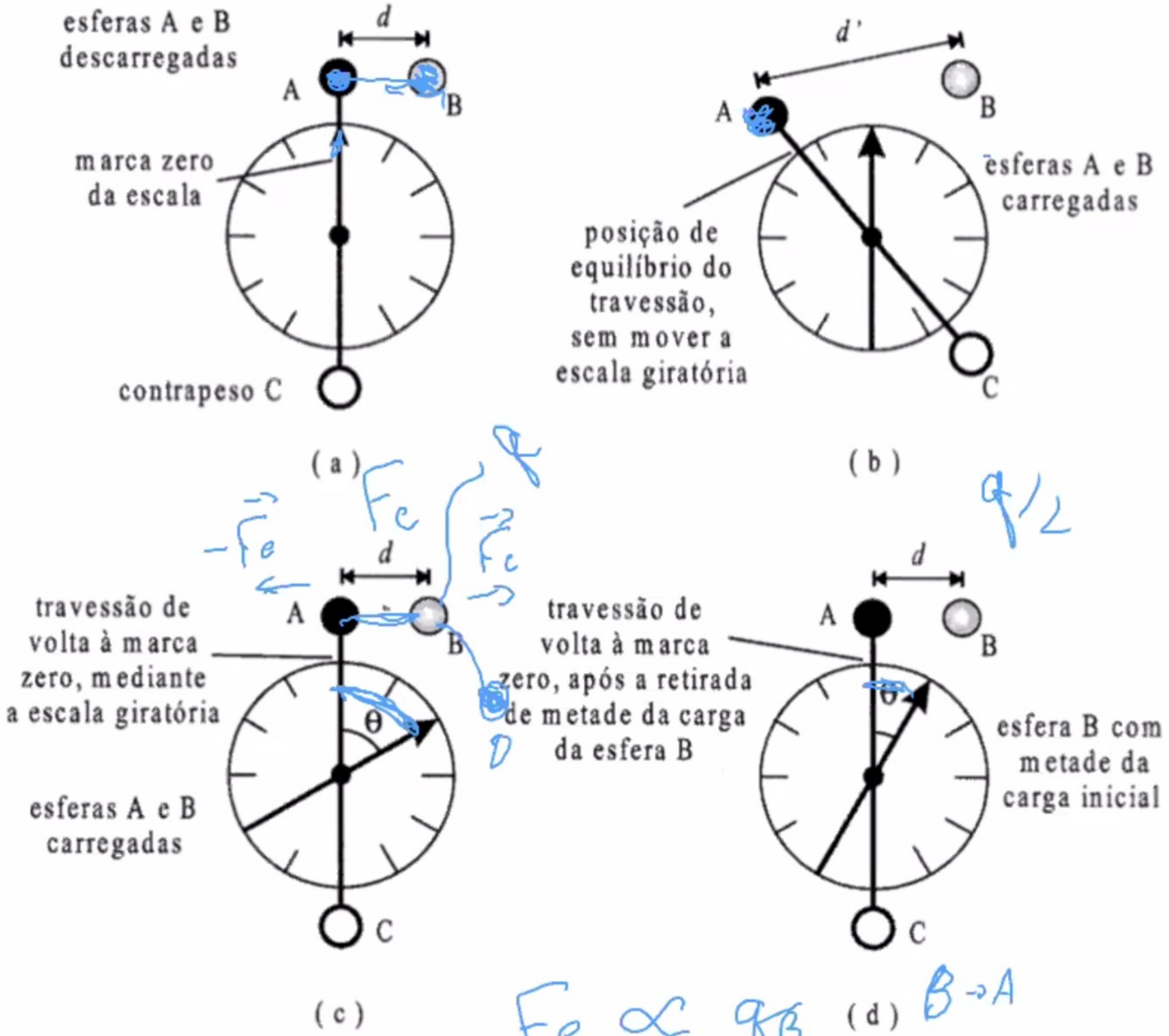

$F_e \propto q_B$ $B \to A$

$F_e \propto q_A$ $A \to B$

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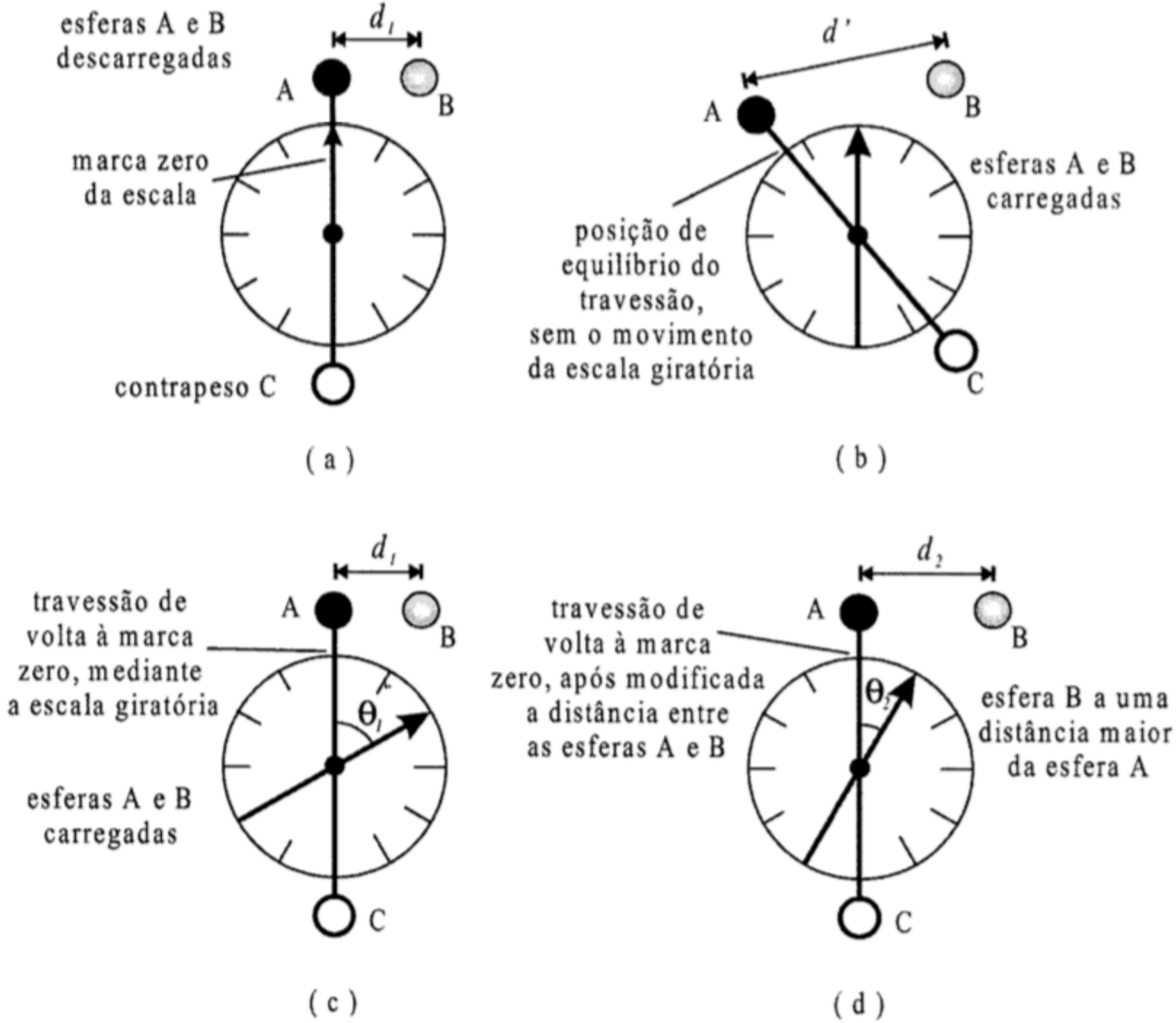

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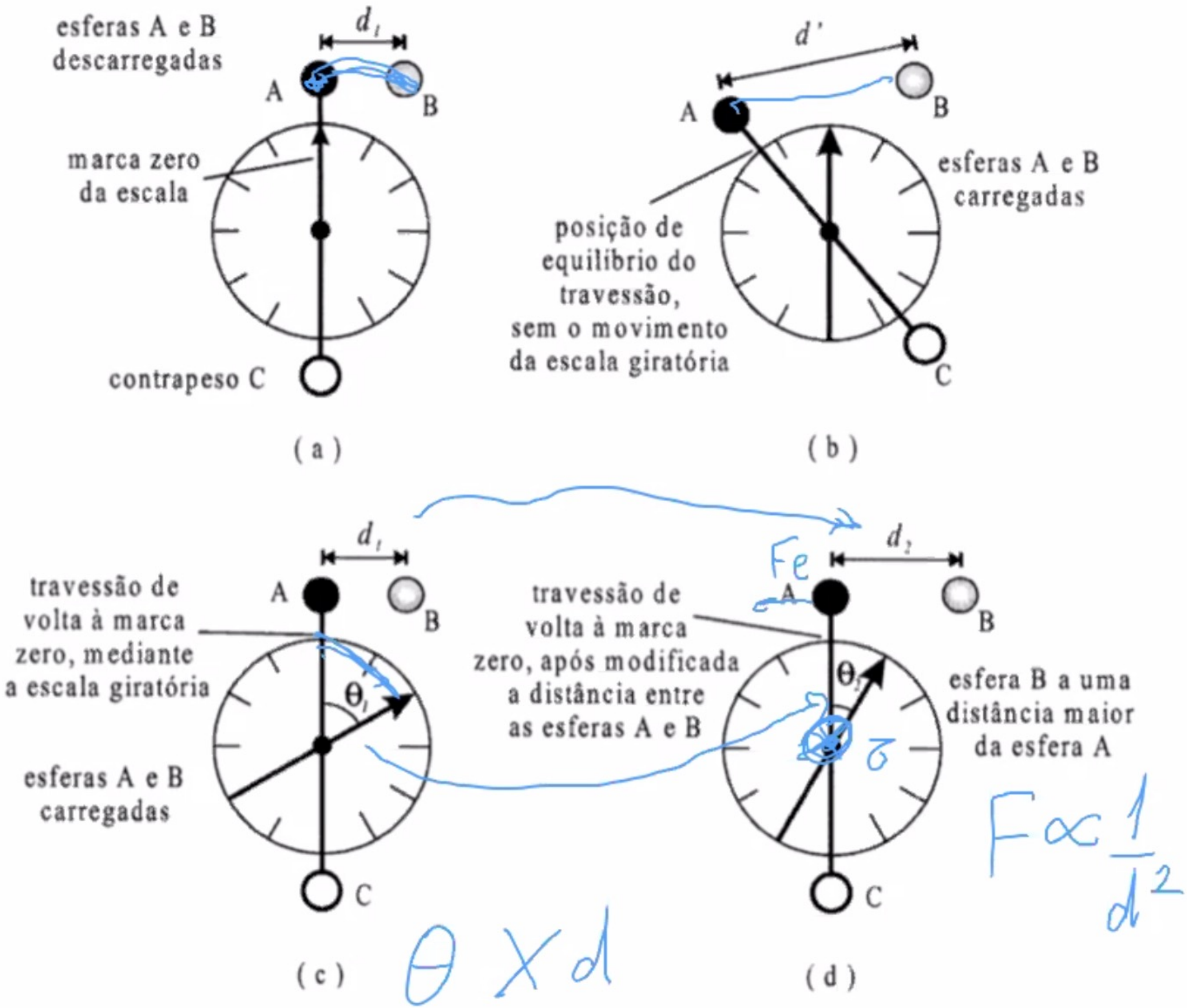

$\theta \times d$

$\theta \propto \tau \propto F$

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Força Elétrica

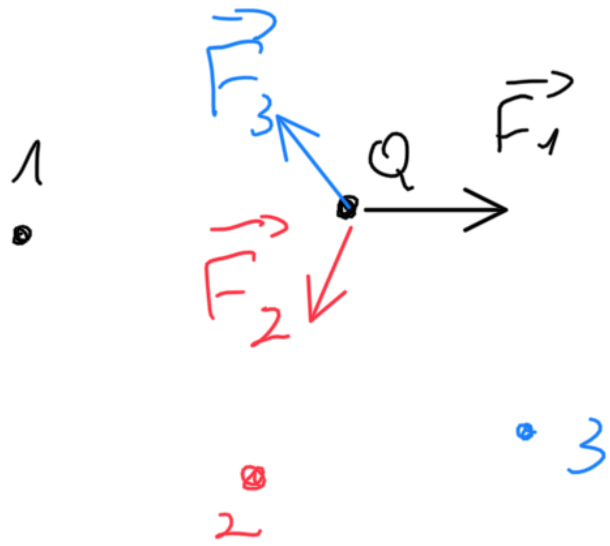
$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} q_1 \cdot q_2 \cdot \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3} = \vec{F}(\vec{r}_2)$$

$$\varepsilon_0 \cong 8{,}854 \cdot 10^{-12} \frac{C^2}{Nm^2}$$

$$k = \frac{1}{4\pi\varepsilon_0} \cong 9{,}0 \cdot 10^9 \frac{Nm^2}{C^2}$$

Superposição:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots =$$



$$k Q q_1 \frac{(\vec{r} - \vec{r}_1)}{|\vec{r} - \vec{r}_1|^3} +$$

$$k Q q_2 \frac{(\vec{r} - \vec{r}_2)}{|\vec{r} - \vec{r}_2|^3} +$$

$$k Q q_3 \frac{(\vec{r} - \vec{r}_3)}{|\vec{r} - \vec{r}_3|^3} + \dots$$

$$\vec{F} = \frac{Q}{4 \pi \varepsilon_0} \sum_{i=1}^{n} q_i \frac{(\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^3}$$

$$\vec{F} = \frac{Q}{4\pi\varepsilon_0} \sum_{i=1}^{n} q_i \frac{(\vec{r}-\vec{r}_i)}{|\vec{r}-\vec{r}_i|^3}$$

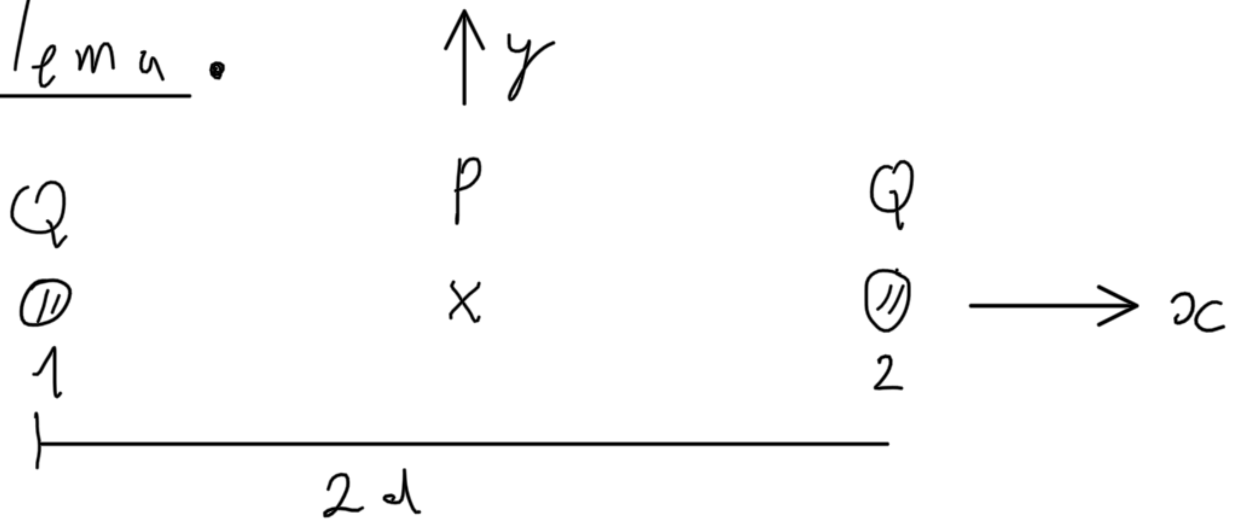
Limite contínuo → distribuição de cargas $\rho(\vec{r}\,')$

$$\vec{F}(\vec{r}) = \frac{Q}{4\pi\varepsilon_0} \int \rho(\vec{r}\,') \cdot \frac{(\vec{r}-\vec{r}\,')}{|\vec{r}-\vec{r}\,'|^3} \cdot dV$$

$\rho(\vec{r}\,')\,dV = dq$ → carga infinitesimal em $\vec{r}\,'$

$\rho(\vec{r}\,')$ → densidade de carga em $\vec{r}\,'$

Problema:



Carga $-Q$ @ P $\quad \vec{F}_1 = -\vec{F}_2 \rightarrow$ equilíbrio!

Estabilidade: deslocamento em torno do equilíbrio

$$\vec{r} = x\hat{i} + y\hat{j}$$

Estabilidade: deslocamento em torno do equilíbrio

$$\vec{r} = x\hat{i} + y\hat{j}$$

$$\vec{F_1} = kQ \cdot (-Q) \frac{(x+d)\hat{i} + y\hat{j}}{\left[(x+d)^2 + y^2\right]^{3/2}}$$

$$\vec{F_2} = -kQ^2 \frac{(x-d)\hat{i} + y\hat{j}}{\left[(x-d)^2 + y^2\right]^{3/2}}$$

Ao longo do eixo $x \rightarrow y = 0$, $|x| < d$

$$\vec{F_1} = F_{1x}\hat{i}$$

$$F = F_{1x} + F_{2x} = -kQ^2 \left[\frac{x+d}{\left[(x+d)^2\right]^{3/2}} + \frac{x-d}{\left[(x-d)^2\right]^{3/2}} \right]$$

$$F = F_{1x} + F_{2x} = -k\,Q^2 \left[\frac{x+d}{[(x+d)^2]^{3/2}} + \frac{x-d}{[(x-d)^2]^{3/2}} \right]$$

$$= -k\,Q^2 \left[\frac{x+d}{|x+d|^3} + \frac{x-d}{|x-d|^3} \right]$$

$$x+d > 0 \quad , \quad x-d < 0 \Rightarrow \frac{x-d}{|x-d|^3} = -\frac{d-x}{(d-x)^3}$$

$$F = -k\,Q^2 \left[\frac{1}{(d+x)^2} - \frac{1}{(d-x)^2} \right] = -k\frac{Q^2}{d^2} \left[\frac{1}{\left(1+\frac{x}{d}\right)^2} - \frac{1}{\left(1-\frac{x}{d}\right)^2} \right]$$

$$F = -kQ^2\left[\frac{1}{(d+x)^2} - \frac{1}{(d-x)^2}\right] = -k\frac{Q^2}{d^2}\left[\frac{1}{\left(1+\frac{x}{d}\right)^2} - \frac{1}{\left(1-\frac{x}{d}\right)^2}\right]$$

expandindo $|\alpha| << 1$

$$f(\alpha) = f(0) + \frac{\partial}{\partial\alpha} f(\alpha)\Big|_{\alpha=0} \cdot \alpha + \frac{1}{2!}\frac{\partial^2}{\partial\alpha^2} f(\alpha)\Big|_{\alpha=0} \alpha^2 + O(3)$$

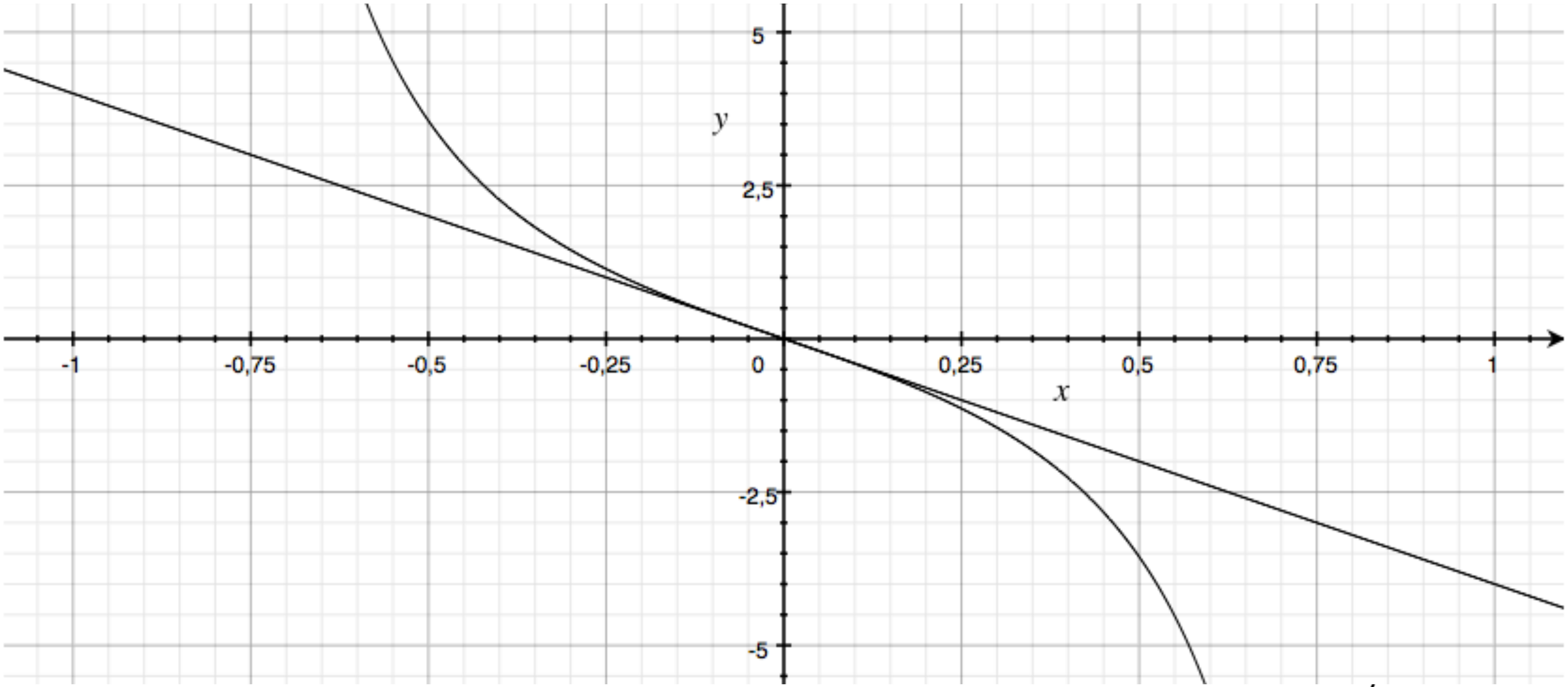
$$f(\alpha) = \frac{1}{(1+\alpha)^2} \qquad f(0) = 1$$

$$\frac{\partial}{\partial\alpha} f(\alpha) = -2 \cdot \frac{1}{(1+\alpha)^3} \qquad \frac{\partial}{\partial\alpha} f(\alpha)\Big|_{\alpha=0} = -2$$

$$F(x) = -k\frac{Q^2}{d^2}\left[-2\frac{x}{d} + \left(-2\frac{x}{d}\right)\right] = \underline{4kx\frac{Q^2}{d^3}}$$

repulsão

$$F = -kQ^2\left[\frac{1}{(d+x)^2} - \frac{1}{(d-x)^2}\right] = -k\frac{Q^2}{d^2}\left[\frac{1}{\left(1+\frac{x}{d}\right)^2} - \frac{1}{\left(1-\frac{x}{d}\right)^2}\right]$$



$$F(x) = -k\frac{Q^2}{d^2}\left[-2\frac{x}{d} + \left(-2\frac{x}{d}\right)\right] = \underline{4kx\frac{Q^2}{d^3}}$$

repulsão

Ao longo do eixo y, $x = 0$

$$\vec{F_1} = k Q^2 \frac{(x+d)\,\hat{i} + y\,\hat{j}}{\left[(x+d)^2 + y^2\right]^{3/2}} \quad \Big/ \quad \vec{F_2} = -k Q^2 \frac{(x-d)\,\hat{i} + y\,\hat{j}}{\left[(x-d)^2 + y^2\right]^{3/2}}$$

$$\vec{F_1} = \vec{F_{1y}}\,\hat{j}$$

$$F_y = F_{1y} + F_{2y} = -2kQ^2 \frac{y}{(d^2 + y^2)^{3/2}}$$

$$|y| << d$$

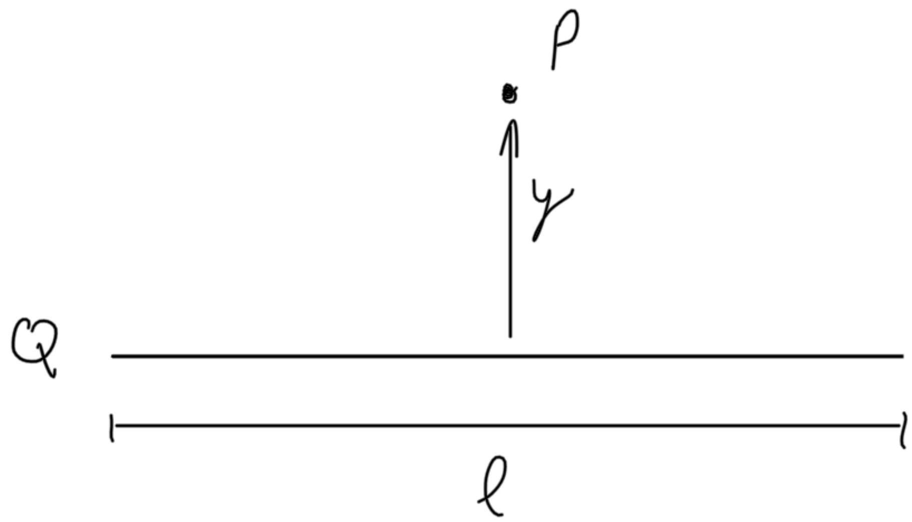
$$F_y = -2kQ^2 \frac{y}{d^3} \rightarrow \text{restauro}$$

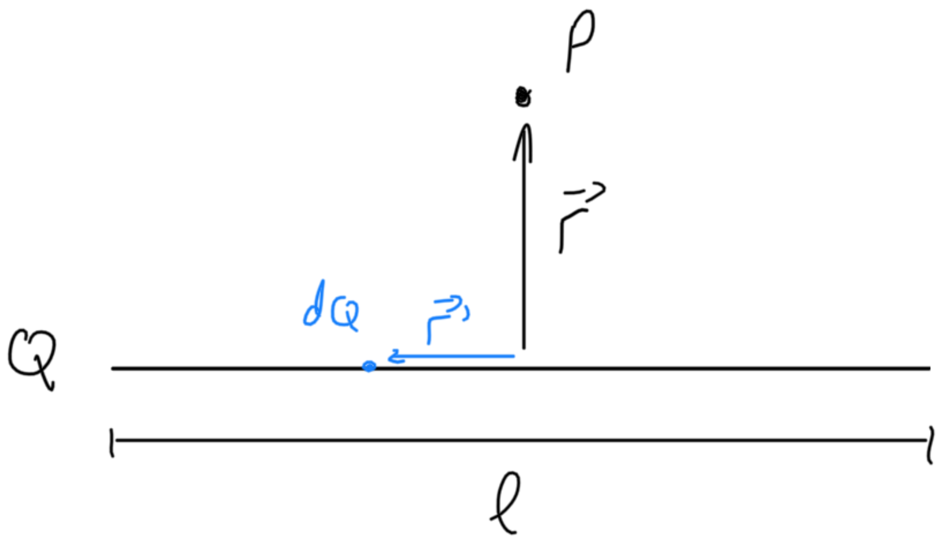
$$\vec{F} = \vec{F}_1 + \vec{F}_2 = k \frac{Q^2}{d^3} \cdot (2x\,\hat{i} - y\,\hat{j})$$

Como podemos jogar com as cargas para ter equilíbrio?

Distribuição Contínua:

Carga q em P





$$dQ = \lambda \cdot d\ell$$

$$\lambda = Q/\ell$$

$$\vec{F} = \int \frac{q}{4\pi\varepsilon_0}\, \lambda(\vec{r})\, \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} \cdot d\ell$$

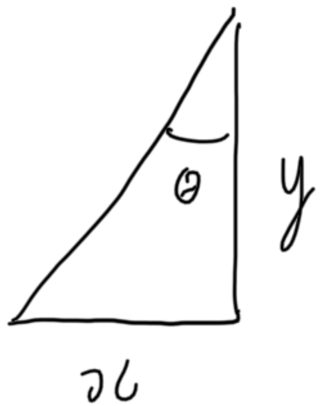
$$\vec{r} = y\,\hat{j} \qquad \vec{r}' = x\,\hat{i} \qquad |\vec{r}-\vec{r}'| = \sqrt{x^2+y^2}$$

$$\vec{F} = \vec{F}_x + \vec{F}_y = \frac{q\lambda}{4\pi\varepsilon_0} \int_{-l/2}^{l/2} -\frac{x}{(x^2+y^2)^{3/2}} \cdot dx \; \hat{i}$$

$$+ \frac{q\lambda}{4\pi\varepsilon_0} \; y \int_{-l/2}^{l/2} \frac{1}{(x^2+y^2)^{3/2}} \cdot dx \; \hat{j}$$

$$\vec{F}_x = 0 \; !$$

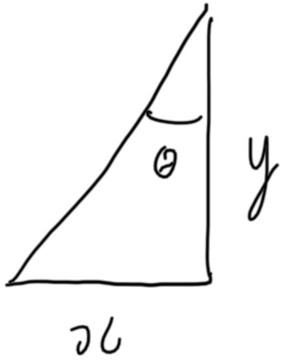
$$\vec{F} = \frac{q\lambda y}{4\pi\varepsilon_0} \hat{j} \int_{-l/2}^{l/2} \frac{1}{(y^2 tg^2\theta + y^2)^{3/2}} y \sec^2\theta \, d\theta$$



$$x = y \; tg\,\theta$$

$$dx = y \frac{\partial}{\partial\theta}(tg\,\theta) \cdot d\theta = y \sec^2\theta \, d\theta$$

$$\vec{F}_x = 0 \; ! \qquad \vec{F} = \frac{q\lambda y}{4\pi\varepsilon_0}\hat{j}\int_{-l/2}^{l/2} \frac{1}{(y^2 tg^2\theta + y^2)^{3/2}}\, y \sec^2\theta \, d\theta$$



$$x = y\, tg\,\theta$$

$$dx = y \frac{\partial}{\partial\theta}(tg\,\theta)\cdot d\theta = y \sec^2\theta\, d\theta$$

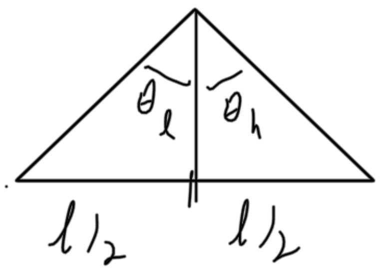
$$\vec{F} = \frac{q\lambda}{4\pi\varepsilon_0 y}\hat{j}\int_{\theta_l}^{\theta_h} \frac{\sec^2\theta}{(1+tg^2\theta)^{3/2}}\, d\theta$$

$$1 + tg^2\theta = 1 + \frac{sen^2\theta}{cos^2\theta} = \frac{1}{cos^2\theta} = \sec^2\theta$$

$$\Rightarrow \vec{F} = \frac{q\lambda}{4\pi\varepsilon_0 y}\hat{j}\int_{\theta_l}^{\theta_h} \frac{1}{\sec\theta}\, d\theta$$

$$\vec{F} = \frac{q\lambda}{4\pi\varepsilon_0 y}\,\hat{j}\int_{\theta_l}^{\theta_h}\cos\theta\, d\theta = \frac{q\lambda}{4\pi\varepsilon_0 y}\,\hat{j}\ \operatorname{sen}\theta\Big|_{\theta_l}^{\theta_h}$$

$$= \frac{q\lambda}{4\pi\varepsilon_0 y}\,\hat{j}\left(\operatorname{sen}\theta_h - \operatorname{sen}\theta_l\right) = \frac{q\lambda}{2\pi\varepsilon_0 y}\ \operatorname{sen}\theta_h\ \hat{j}$$



$$\operatorname{sen}\theta_h = \frac{l/2}{\sqrt{y^2 + l^2/4}}$$

$$= \frac{l}{\sqrt{4y^2 + l^2}}$$

$$\vec{F} = \frac{q\lambda l}{2\pi\varepsilon_0 y}\ \frac{1}{\sqrt{4y^2+l^2}}\ \hat{j} = \frac{q\cdot Q}{4\pi\varepsilon_0}\cdot\frac{1}{y\sqrt{y^2+(l/2)^2}}\cdot\hat{j}$$

se $l \to 0$ $\quad \vec{F} = \dfrac{qQ}{4\pi\varepsilon_0 y^2}\,\hat{j}$!