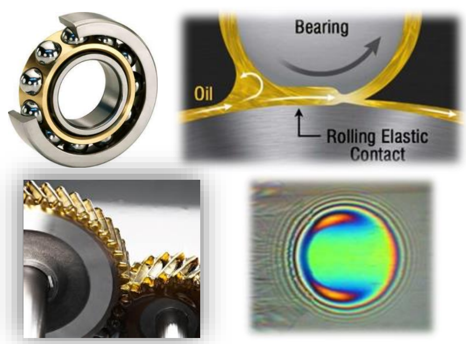
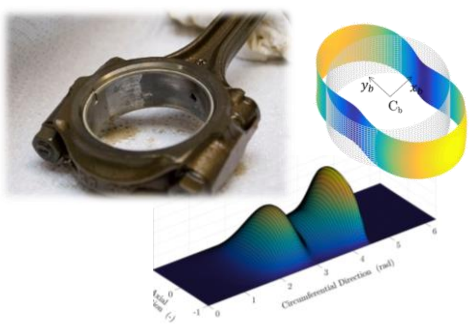
Fundamentals of Lubrication





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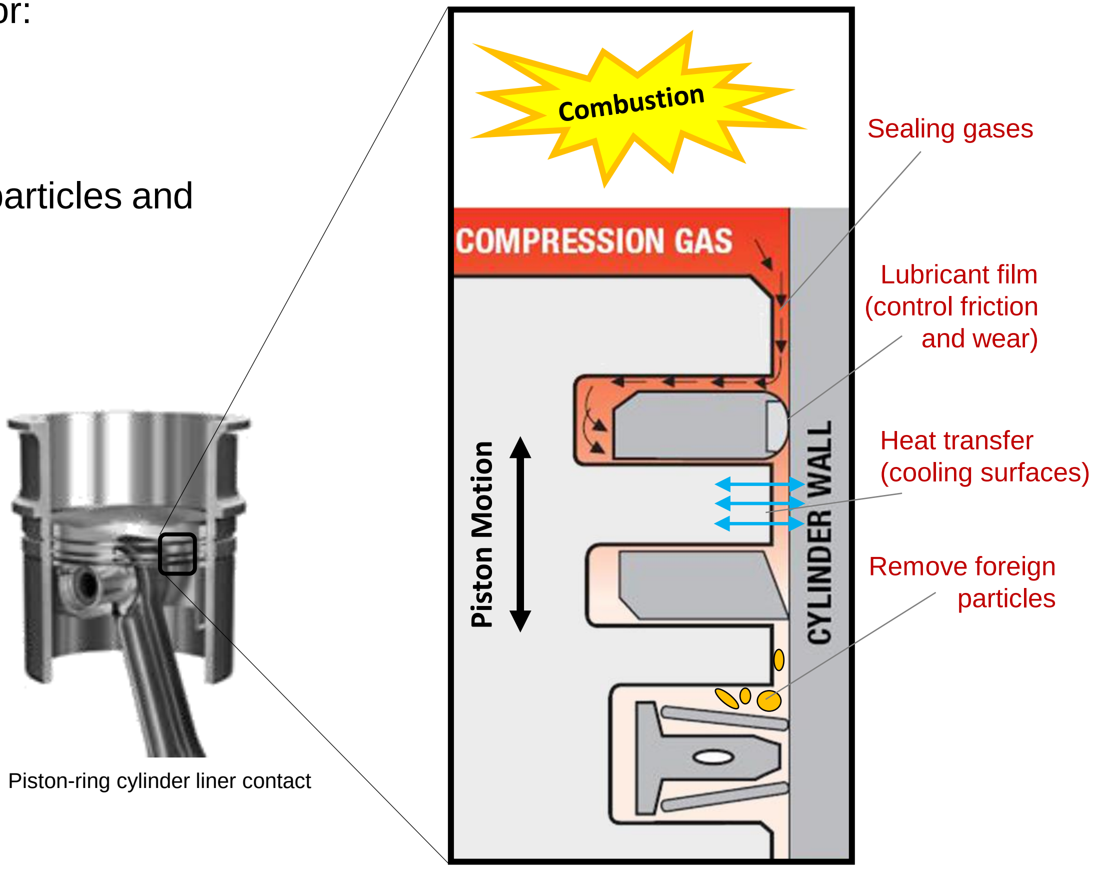
Outline

Outline

1. Roles & Functions of Lubrication

Fluid added at the contact interface for:

- Control friction and wear
- Carry away wear and foreign particles and contaminants
- Cool the contacting parts
- Sealing
- Easy mixing with chemicals (additives)



Piston-ring cylinder liner contact

Outline

2. Lubricant

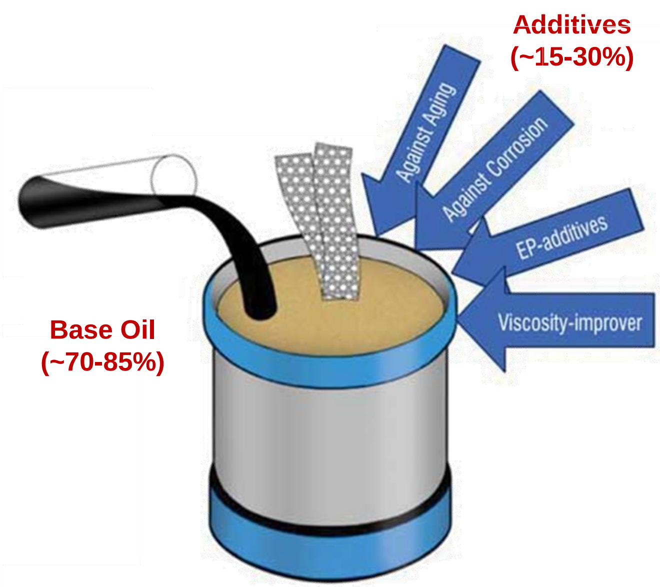
2.1 Composition

- ***Lubricant formulation:*** combines selected base oils and chemical additives for a specific application.
- ***Lubricant performance:*** maintains required friction and wear rates as well as thermochemical stability despite continuous degradation.
- Typical lubricating oils are composed of ~70-85% base oil and ~15-30% additives.

Moderate temperatures — **Mineral**
High temperatures — **Synthetic**

Refining crude oil

- Thermophysical properties and thermal stability depend significantly on the properties of the base oil.

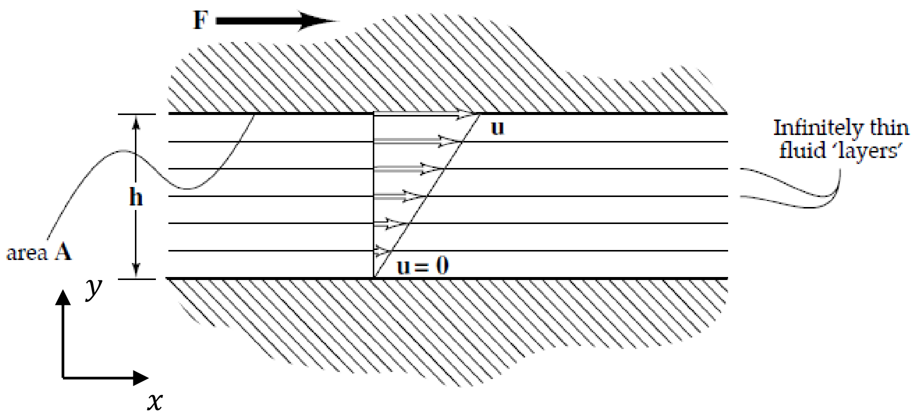


Composition of a typical automotive engine oil

2. Lubricant

2.2 Physical Properties :: Viscosity

- The viscosity is associated with the **resistance of a fluid to flow** due to the intermolecular interactions and internal friction between the molecules.



$$\frac{F}{A}\,\alpha\,\left(\frac{u}{h}\right) \;\Rightarrow\; \tau = \eta\,\frac{\partial u}{\partial y}$$

Linear relationship between shear stress and shear rate (Newtonian fluid)

Dynamic viscosity, η **[Pa.s]**

Kinematic viscosity, $\vartheta = \eta/\rho$ **[m²/s]**

- More viscous oils would not necessarily perform better; more viscous oils require more power to be sheared; thus, the power losses are higher and more heat is generated.

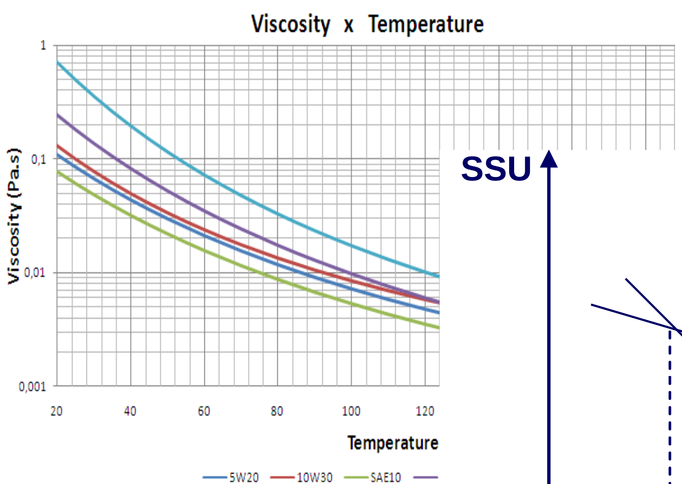
- Viscosity varies typically with temperature, pressure and shear rate.

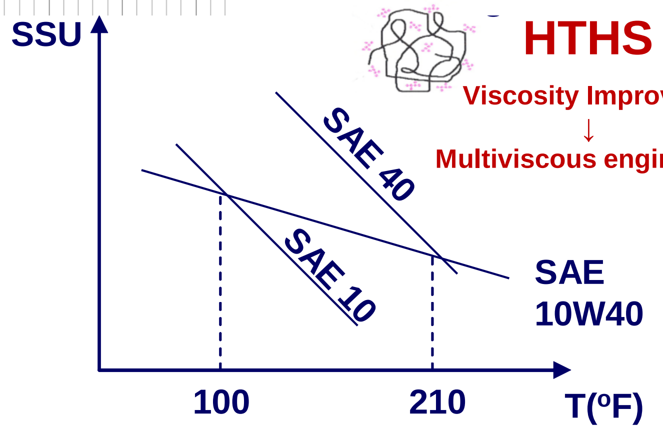
2. Lubricant

2.2 Physical Properties :: Viscosity

Viscosity-Temperature Relationship

- The viscosity of mineral and synthetic oils is extremely sensitive to temperature.
- With increasing temperature, the viscosity of oils falls rapidly (exponential behaviour, e.g. ASTM, Cameron, Vogel equations).



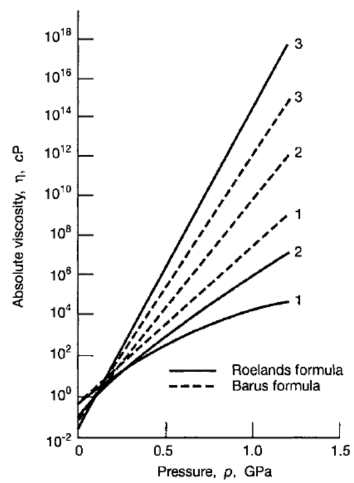


HTHS

Viscosity Improvers
↓
Multiviscous engine oils

Viscosity-Pressure Relationship (Piezoviscous effect)

- Lubricant viscosity generally increases with pressure (rise of molecular packing and intermolecular interactions).
- Particularly significant for heavily loaded concentrated contacts (e.g., line and point counterformal contacts).



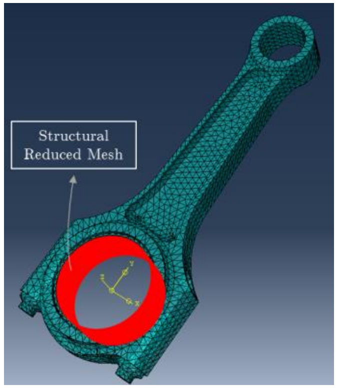
Barus and Roelands equations

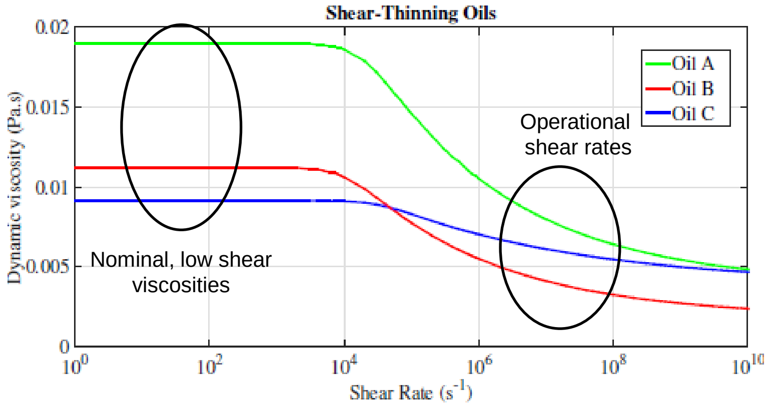
2. Lubricant

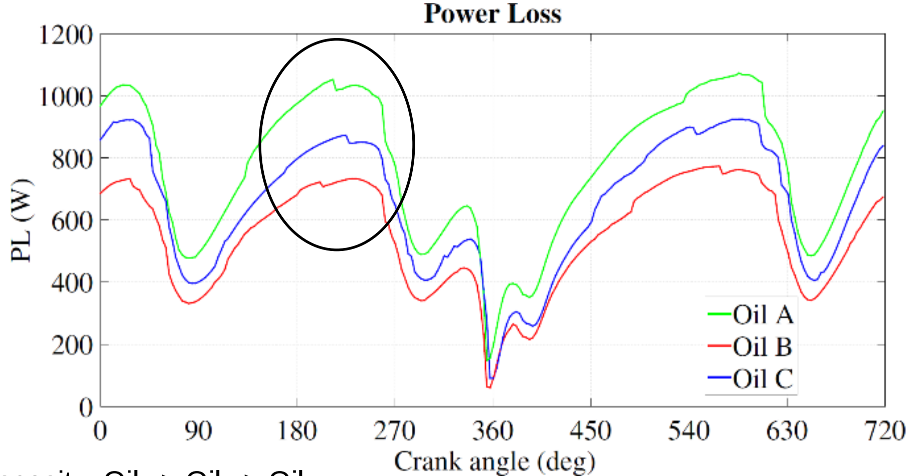
2.2 Physical Properties :: Viscosity

Viscosity-Shear Rate Relationship

- Non-Newtonian fluid behaviour: viscosity variation with shear rate.
- In engine oils, the shear-thinning effect is associated with VII additives (multiviscous oils).
- Viscosity decreases at relatively high shear rate conditions (e.g. Eyring and Carreau family equations).





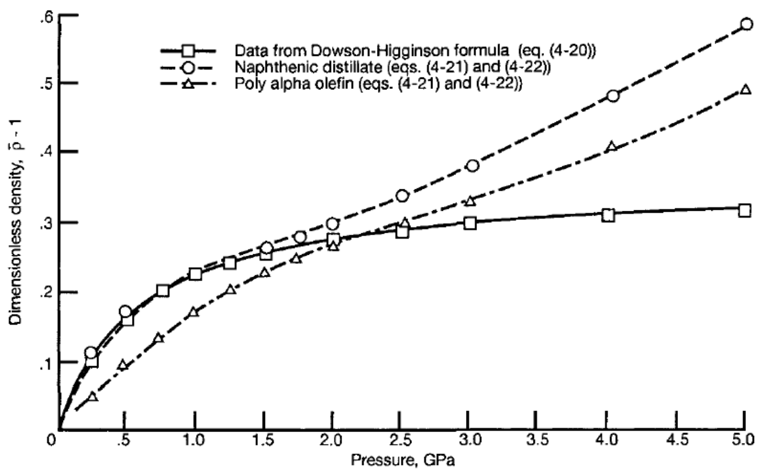


Viscosity: $Oil_A > Oil_C > Oil_B$
Power loss: $PL_A > PL_C > PL_B$

2. Lubricant

2.3 Physical Properties :: Density

- As the lubricant is compressed, the distance between the molecules becomes smaller; thus, its density is increased.
- For liquid lubricants at extremely high pressures (EHL), the oil can no longer be considered an incompressible medium; compressibility is also important for gaseous lubricants.
- The density-pressure relationship of lubricating oils is roughly linear at low pressures, but the rate of increase falls at high pressures.



- Dowson-Higginson equation:

$$\rho = \rho_0 \left(1 + \frac{0.6\, p}{1 + 1.7\, p} \right)$$

2. Lubricant

2.4 Physical Properties :: Thermal Properties

Important for assessing thermal effects in lubrication, e.g., the oil's cooling properties, heat transfer at the interface and local surface temperatures, etc.

- **Specific Heat:** c_p
- **Thermal Conductivity:** k
- **Thermal Diffusivity:** $\alpha = \frac{k}{\rho c_p}$
  - These properties usually vary linearly with temperature;
  - Increases with the increasing polarity or hydrogen bonding of the molecules.

2. Lubricant

2.5 Lubricant Additives

- Chemicals mixed with the base oils to change the properties of a lubricant and its overall performance.

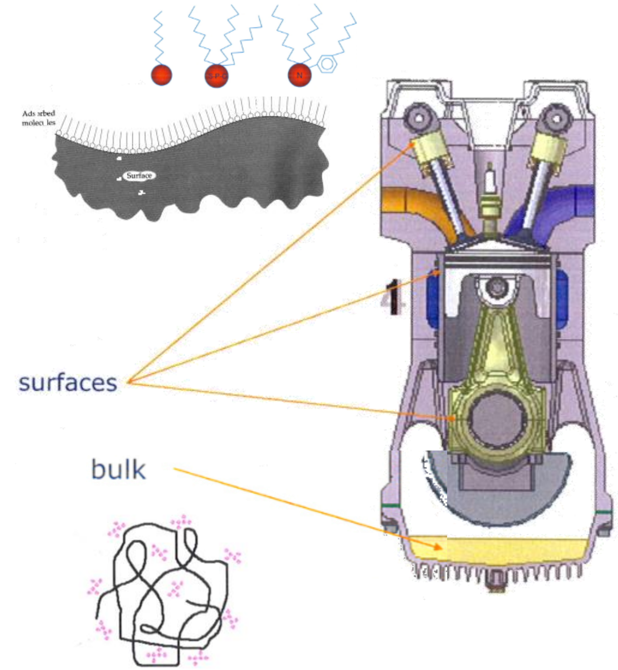
- Additives dictate specific characteristics of the lubricant, such as corrosion, oxidation, foaming, wear, friction and other physicochemical and tribological properties.

Surface Active

- **Friction Modifiers (FM)**
  - Molecular layer with low friction properties (reduced boundary friction). E.g., MoDTC → MoS_2.
  - Synergistic action of lubricant, topography, materials and ambient
- **Anti-Wear (AW)**
  - Reduce wear through a protective molecular film, e.g., ZDDP.
- **Extreme Pressure (EP)**
  - React with the iron on the surface, forming a sacrificing layer
  - Increase adhesion resistance and control severe wear

Bulk Active

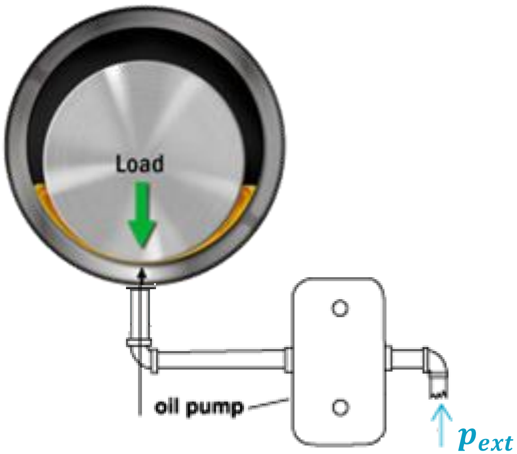
- **Viscosity Index Improvers (VII)**
  - Increases high-temperature viscosity (multiviscous lubricants);
  - Affects the lubricant shear-thinning properties (non-Newtonian behaviour



Outline

3. Types of Lubrication

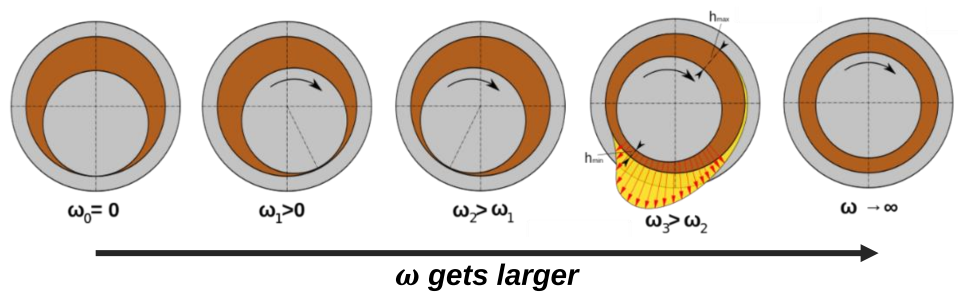
- Hydrostatic Lubrication



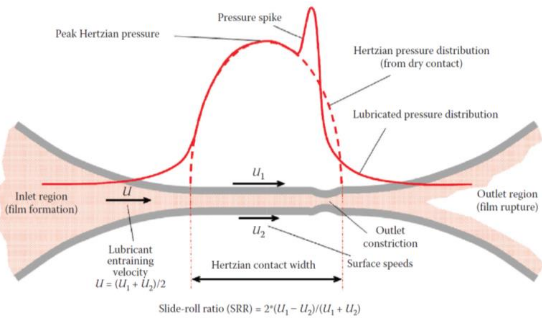
"External pressured bearings"
(often used in 'start-ups')

- (Elasto)-Hydrodynamic Lubrication
  (***"Self-Acting" Lubrication***)

Conformal Contacts (HL)

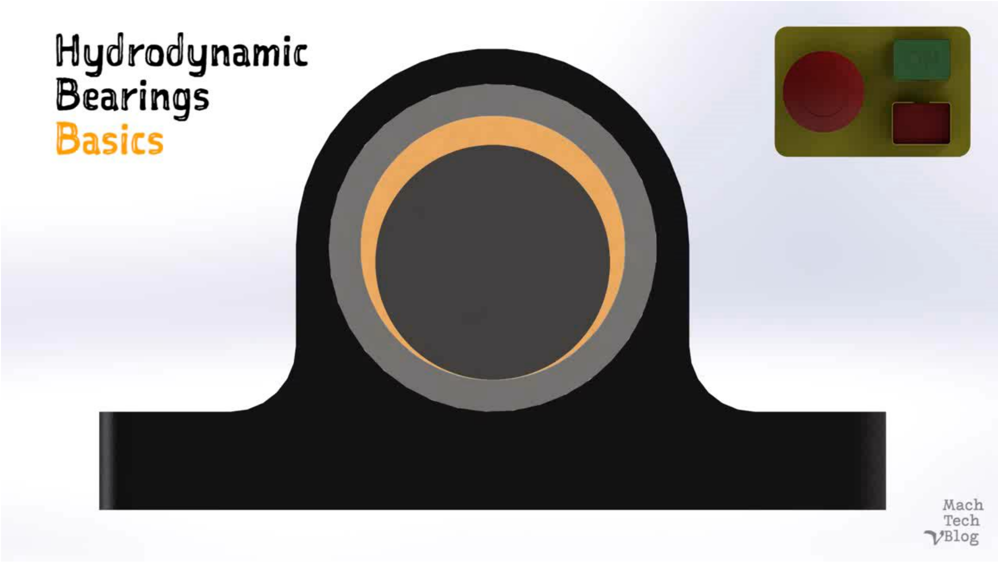


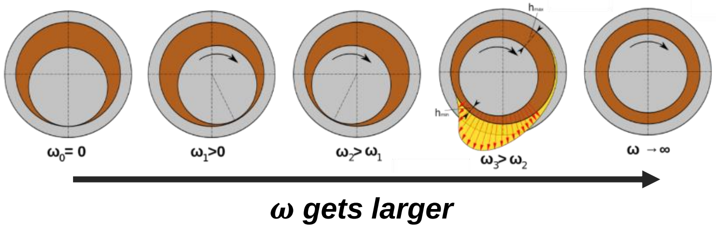
Non-Conformal Contacts (EHL)



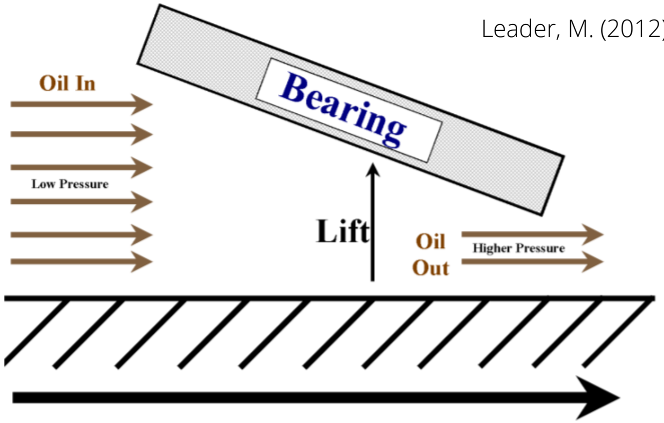
3. Types of Lubrication

- (Elasto)-Hydrodynamic Lubrication
  (“*Self-Acting” Bearings*)





Leader, M. (2012)



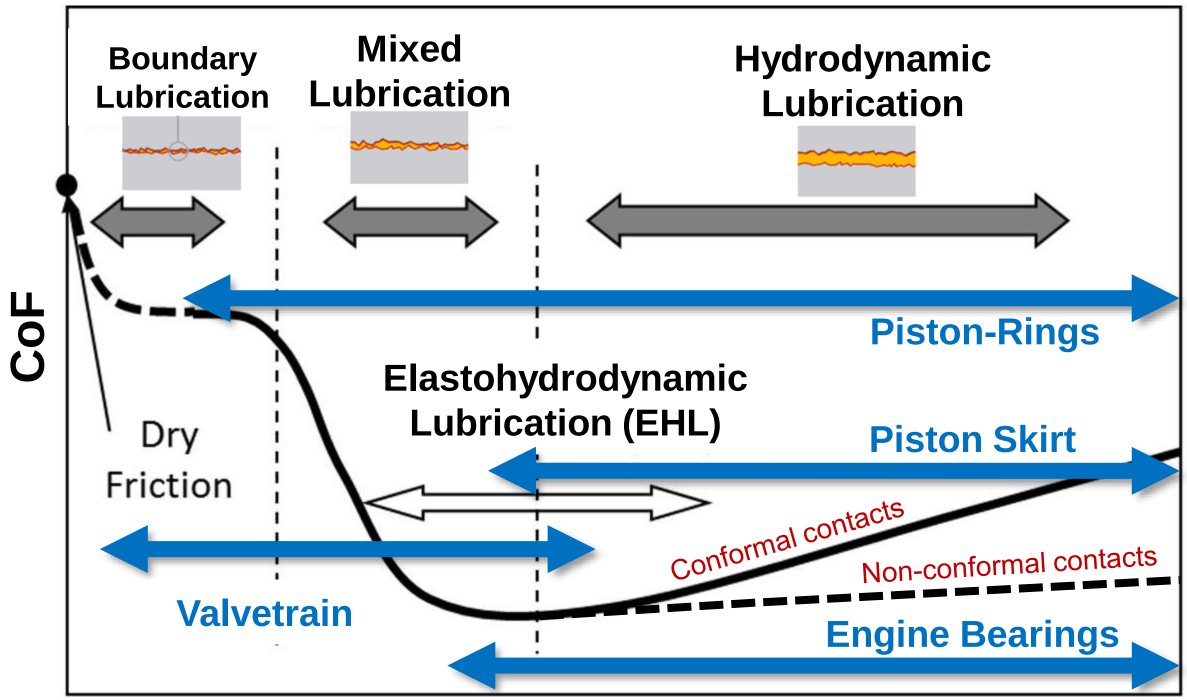
Outline

4. Lubrication Regimes

❑ Stribeck curve

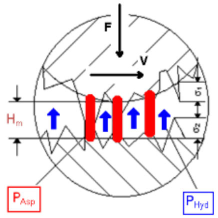
Engine Friction Losses in Passenger Cars

- Piston-rings: 35-40%
- Valve trains: 20-25%
- Bearings: 10-15%
- Cylinders: 10-15%
- Other: 5-20%



Hersey Number: $H = \frac{\eta U}{W}$ **Film Ratio:** $\Lambda = \frac{h_{min}}{Sq^*}$

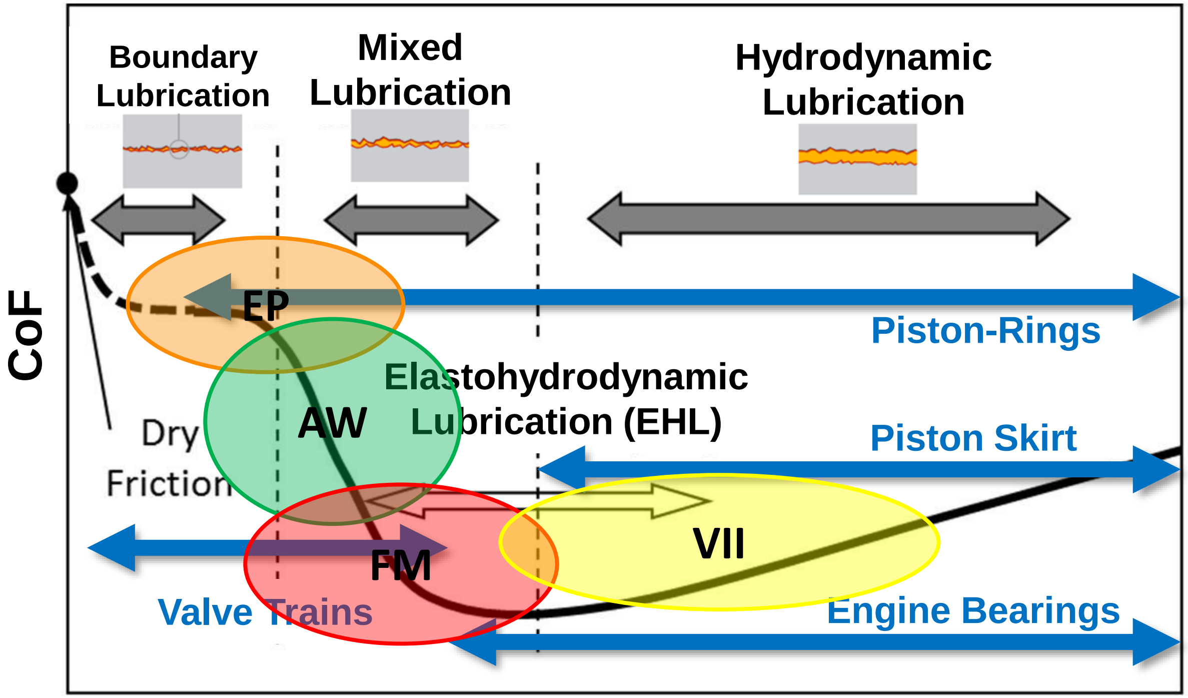
Inputs:
- Geometry
- Speed
- Load
- Materials
- Lubricant
- Temperature
- Ambient



Outputs:
- Friction
- Wear
- Heat
- Noise
- Failure tendency

4. Lubrication Regimes

- Stribeck curve (additives)

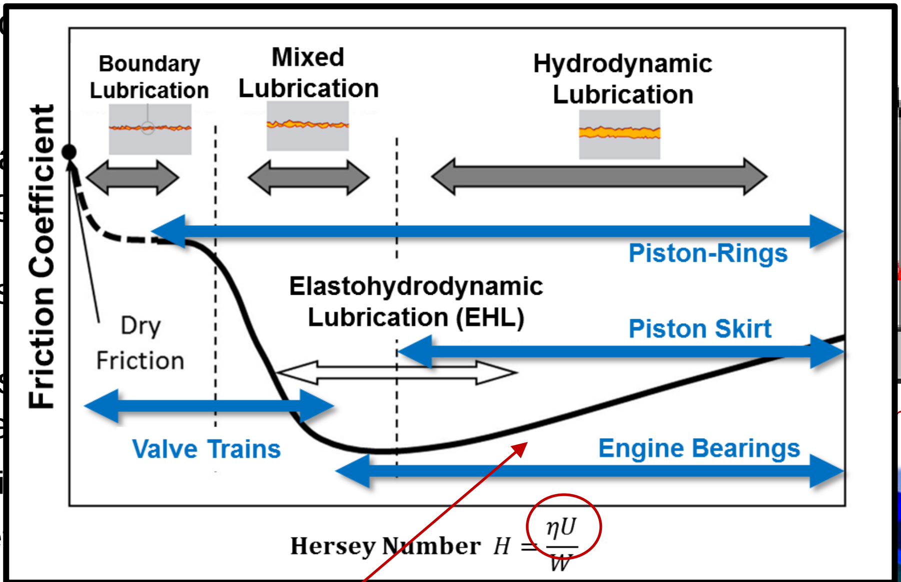


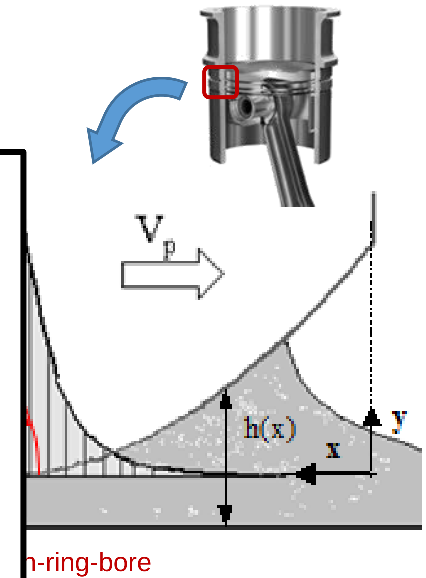
Hersey Number: $H = \frac{\eta U}{W}$ **Film Ratio:** $\Lambda = \frac{h_{min}}{Sq^*}$

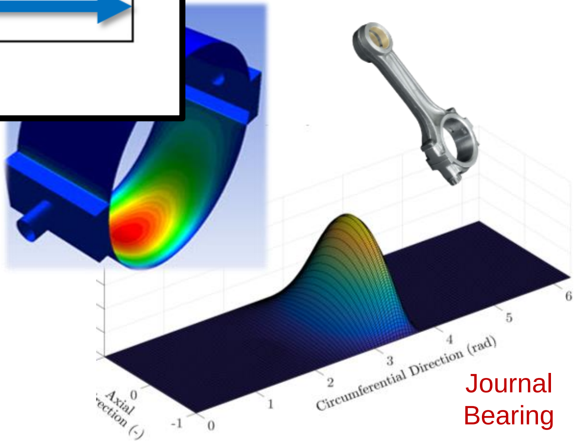
4. Lubrication Regimes

4.1 Hydrodynamic Lubrication (HL)

- Complete separati
  (~ 5–100 µm)
- Typical of conforma
  and journal bearing
- Moderate fluid pres
- Hydrodynamic pres
  significantly affecte
  - Lubricant viscosi
  - Surfaces geome
  - Minimum oil film thickness (MOFT)
  - Surface velocities







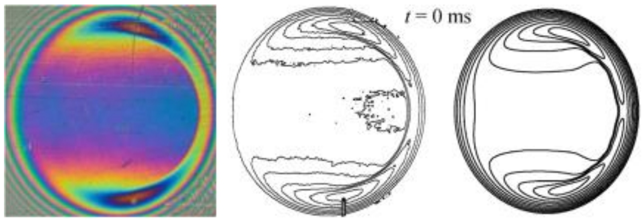
- Mathematical modelling: Reynolds equation

$$W \;\alpha\; \frac{\eta U_r}{h^2} \qquad F_t \;\alpha\; \frac{\eta U_s}{h}$$

4. Lubrication Regimes

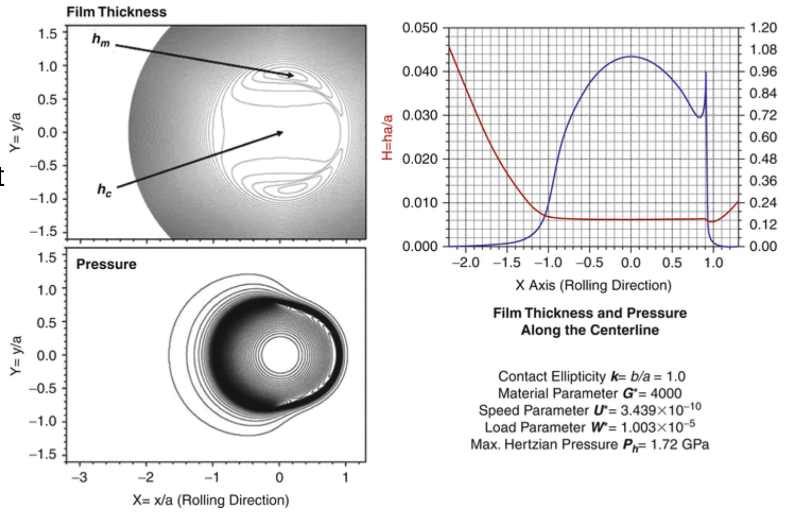
4.2 Elastohydrodynamic Lubrication (EHL)

- Thin-film thickness **without** asperity contact (~ 0.5–5 μm)
- Significant surfaces deformation induced by the fluid pressure (fluid-structure interaction problem)
- **Hard-EHL**
  - Typical of non-conformal lubricated contacts
  - Surfaces with higher elastic modulus (*e.g.* metals)
  - High fluid pressures (~0.5-5.0 GPa)
  - Elastic deformations due to surfaces compression
  - Film thickness is governed by the rolling speed and fluid properties at the inlet (*e.g.* Hamrock-Dowson and Masjedi-Khonsari formulas)
  - Friction governed by the sliding speed and fluid properties (rheology, thermal and piezoviscous effects) at the contact
  - Shear-thinning, viscosity-pressure and density-pressure effects significantly affect the lubricant rheology
  - Modelling: Reynolds equation, half-space contact mechanics, heat transfer and rheological models
  - Examples: gear tooth flanks, rolling element bearings, cams-tappet, etc



Film thickness measurement
(optical interferometry technique)

Simulation

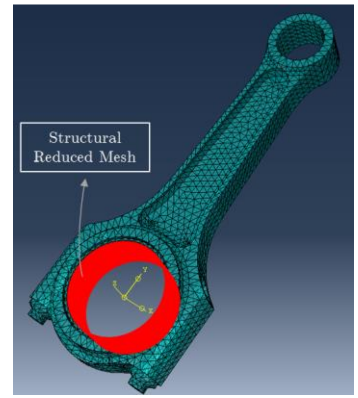


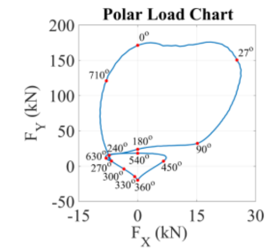
Simulation (point contact)

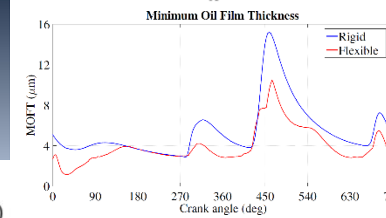
4. Lubrication Regimes

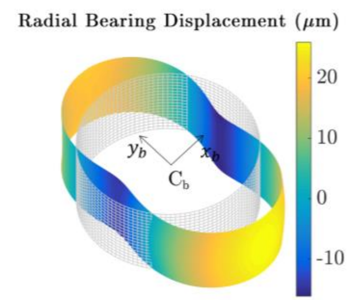
4.2 Elastohydrodynamic Lubrication (EHL)

- Thin-film thickness **without** asperity contact (~ 0.5–5 μm)
- Significant surfaces deformation induced by the fluid pressure (fluid-structure interaction problem)
- **Soft-EHL**
  - Typical of conformal lubricated contacts and/or non-conformal contacts with lower elastic modulus (*e.g.* rubber)
  - Moderate fluid pressures (< 0.5 GPa)
  - For conformal contacts with high elastic modulus, surface deformations are induced by the flexibility of the bearing structure (*e.g.* conrod bearings)
  - Viscosity-pressure and density-pressure effects do not affect the lubricant rheology significantly
  - Friction governed by the sliding velocities, film thickness and lubricant viscosity
  - Modelling: Reynolds equation, reduce FEM model of the structure
  - Examples: elastomers, biomechanics devices, journal bearings, etc

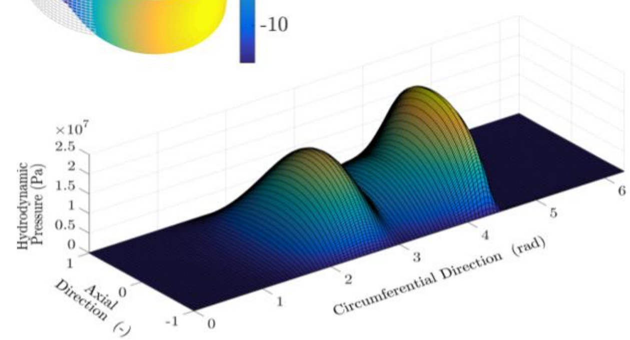








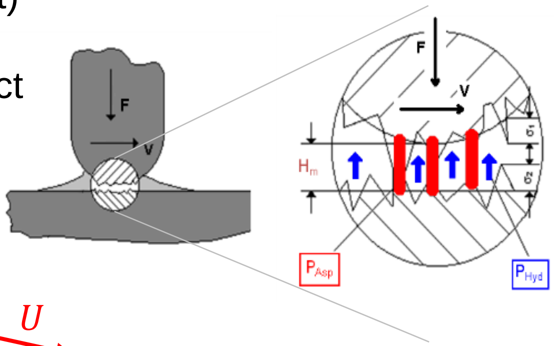
Simulation
(conrod big-end bearing)
LUBST code

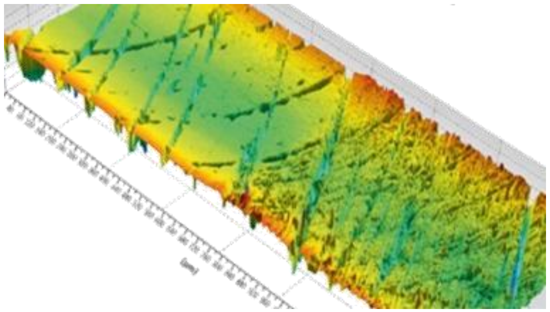


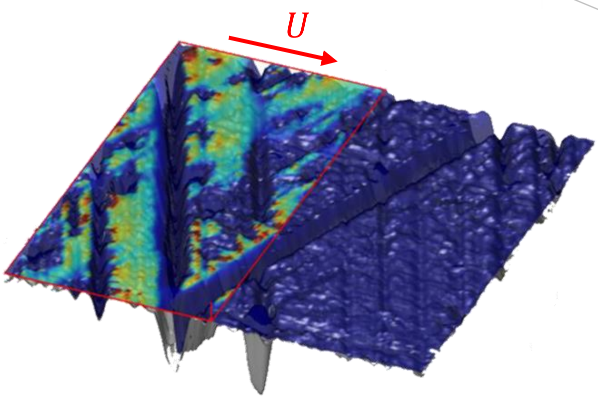
4. Lubrication Regimes

4.3 Mixed Lubrication (ML)

- (Very) thin film thickness **with** asperity interactions (rough contact)
- External loads balanced by the hydrodynamic and asperity contact pressures
- Surface roughness (pattern, heights distribution, etc.) plays a crucial role in the tribological performance







- Friction losses governed by the fluid viscosity and mechanochemical properties of the tribofilm formed at the interface (asperity level)

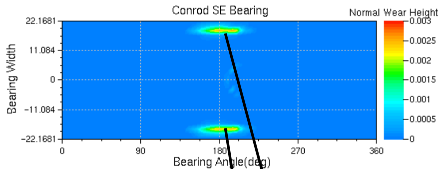
4. Lubrication Regimes

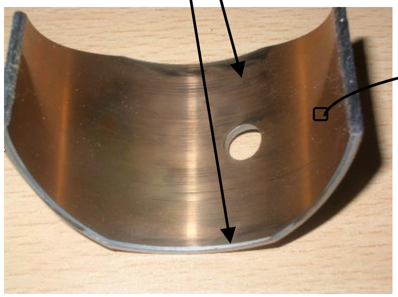
4.3 Mixed Lubrication (ML)

- Stochastic modelling
  - Modelling and simulation in the **macroscopic** component scale.
  - **Microscopic** (roughness) effects considered through statistical models (*e.g.* flow and contact factors).
  - Fluid problem: ***Patir & Cheng average flow model or homogenisation methods***.
  - Contact problem: ***Greenwood-Tripp model****.*
  - Flow and contact factors calculated from deterministic simulations.

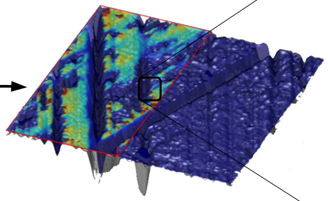
- Deterministic modelling
  - Modelling and simulation in the ***microscopic*** roughness scale.
  - Contact geometry is defined considering the actual (measured) surface roughness heights.
  - Fluid problem: ***Reynolds equation with inter-asperity cavitation***
  - Contact problem: ***Deterministic contact models***

Global scale

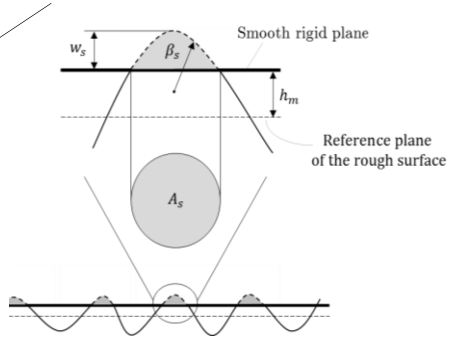




Comparison of simulation results with actual wear marks.



Local scale

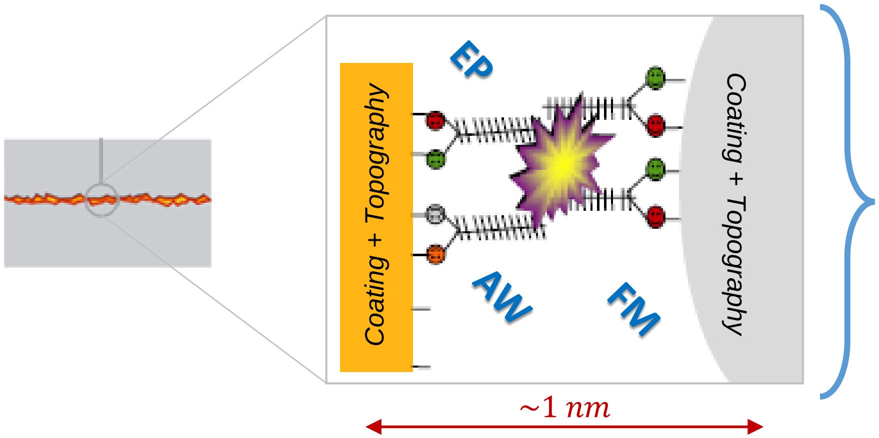


Individual asperity

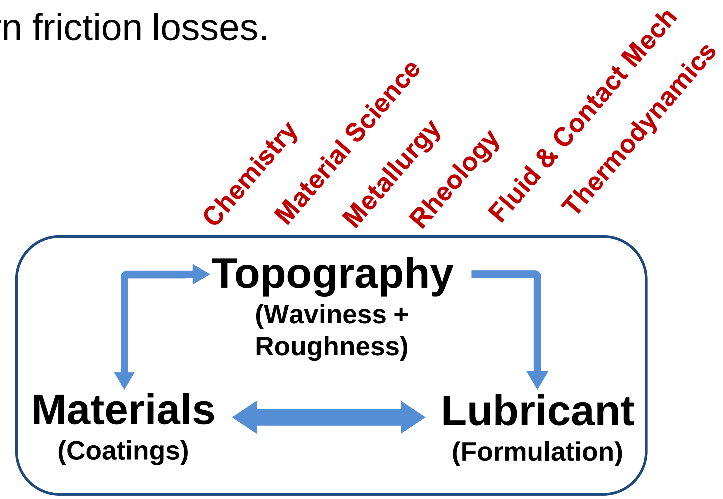
4. Lubrication Regimes

4.4 Boundary Lubrication (BL)

- Practically no lubricant film is developed on the contact interface.
- Contacting surfaces separated solely by a **molecular film** (tribofilm) attached to the surfaces ("surface acting" additives play an essential role in this regime).
- The mechanochemical properties of the tribofilm govern friction losses.



Nanoscale molecular interactions
(van der Walls / covalent bonds)

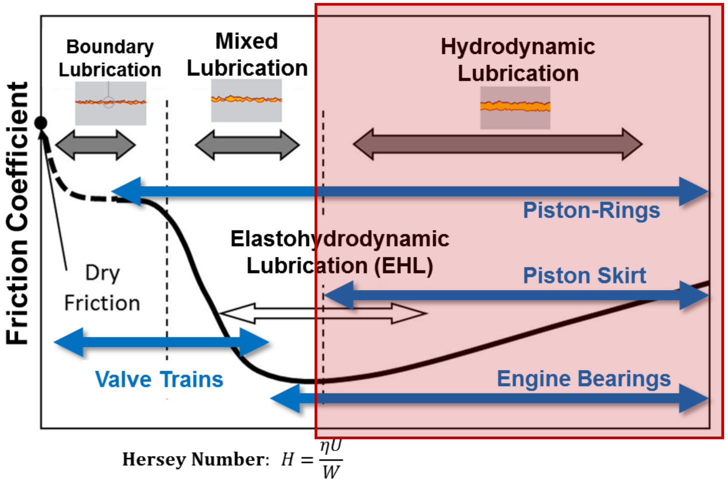


Synergetic interactions and multidisciplinary research

Outline

1. Roles & Functions of Lubrication
2. Lubricants
   - 2.1 Composition
   - 2.2 Viscosity
   - 2.3 Density
   - 2.3 Thermal Properties
   - 3.4 Lubricant Additives
3. Types of Lubrication
4. Lubrication Regimes (Stribeck curve)
   - 4.1 Hydrodynamic Lubrication (HL)
   - 4.2 Elastohydrodynamic Lubrication (EHL)
   - 4.3 Mixed Lubrication (ML)
   - 4.4 Boundary Lubrication (BL)

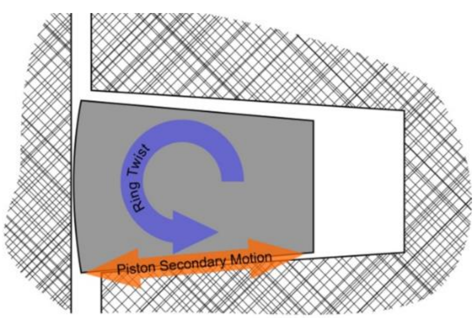
5. **Theory of Fluid Film Lubrication**
   - 5.1 Roles & Principles
   - 5.2 Assumptions of the Lubrication Theory
   - 5.3 Derivation of the Generalized Reynolds Equation
   - 5.4 Physical Interpretation of Reynolds Equation
   - 5.5 Fluid Film Cavitation
   - 5.6 Operational Parameters



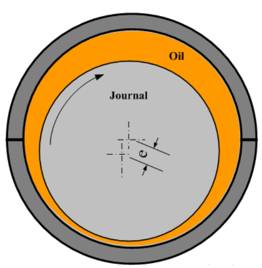
5. Theory of Fluid Film Lubrication

5.1 Roles & Principles

- Fluid pressure generated within a thin lubricant layer which separates the contacting surfaces;
- Lubricant layer with low shear strength (oil viscosity);
- Reduction of frictional losses and superficial damage;
- Conditions for the occurrence of hydrodynamic lubrication:
  - Significant tangential and/or normal surface velocities;
  - Contact geometry should NOT be flat and parallel (wedge effect), except for ‘pure’ squeeze conditions.



Piston-ring barrel shaped profile



Plane journal bearing clearance

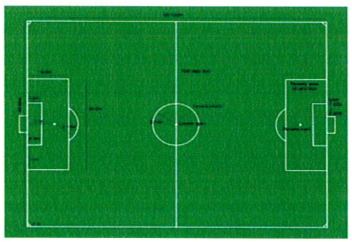
Curiosity

“Why does the lubricant not escape?”

Grass heights

$h \sim 1 - 10$ cm

$10^{-4} \leq \frac{h}{L} \leq 10^{-3}$



$L \sim 100$ m

Typical lubricated contacts:

$L \sim 1 - 100\ mm$

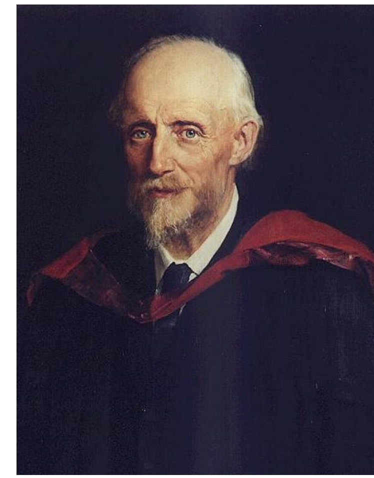
$h \sim 0.1 - 100\ \mu m$

$10^{-4} \leq \frac{h}{L} \leq 10^{-3}$

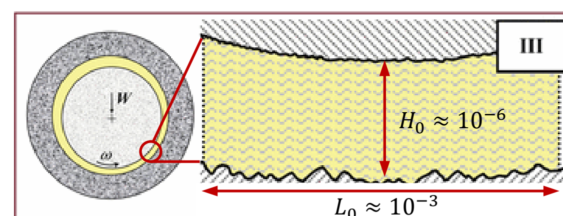
5. Theory of Fluid Film Lubrication

5.2 Assumptions of the Lubrication Theory

- Lubricant assumed to be a continuum medium;
- Lubricant assumed to be Newtonian*;
  (except under extreme lubrication conditions, in which viscosity-pressure, density-pressure and shear-thinning effects are important, *e.g.* EHL contacts)
- Lubricant flow assumed to be laminar*;
  (except for gas bearings and bearings with high clearances and/or speeds)
- External body forces are neglected*;
  (except for magnetorheological fluids)
- Dimensions across the film thickness $\mathcal{O}(10^{-6})$ are much smaller than the other dimensions along the contact $\mathcal{O}(10^{-3})$;
  (pressure gradient across film thickness is negligible)
- No-slip condition at the fluid-solid interface*;
  (except for contacts under high shear stress and shear rate conditions; hydrophobic and/or porous surfaces)
- Contact surfaces assumed to be perfectly smooth*
  (Reynolds roughness)



Osborne Reynolds (1842-1912)
British scientist
“Father” of the lubrication theory



*May be relaxed in more advanced analysis.

5. Theory of Fluid Film Lubrication

5.3 Generalized Reynolds Equation

- Governing Equations of Fluid Dynamics for Newtonian Fluids

  - Conservation of Mass

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v_j)}{\partial x_j} = 0$$

  - Conservation of Linear Momentum

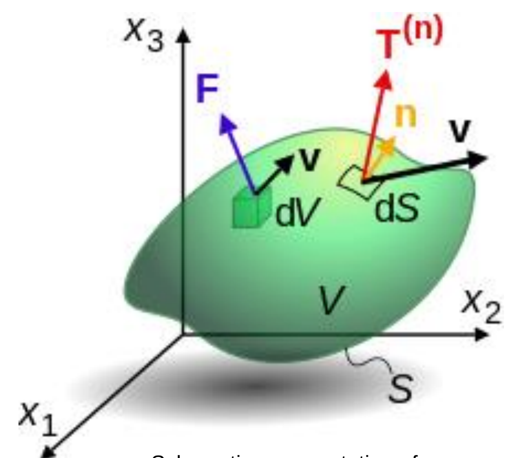
$$\rho\left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right] + \frac{\partial}{\partial x_i}\left(\lambda \frac{\partial v_j}{\partial x_j}\right) + \rho b_i$$

  - Conservation of Energy

$$\rho c_p\left(\frac{\partial T}{\partial t} + v_j \frac{\partial T}{\partial x_j}\right) = \alpha T\left(\frac{\partial p_H}{\partial t} + v_j \frac{\partial p_H}{\partial x_j}\right) + \frac{\partial}{\partial x_j}\left(k \frac{\partial T}{\partial x_j}\right) + \mu \Phi + \rho S_E$$

$$\Phi = \frac{\partial v_i}{\partial x_j}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \left(\lambda \frac{\partial v_j}{\partial x_j}\right)^2$$

(Viscous) dissipation function



Schematic representation of a continuum medium

5. Theory of Fluid Film Lubrication

5.3 Generalized Reynolds Equation

- Dimensionless Parameters

  - Independent Variables

$$\bar{x} = \frac{x}{L_0} \qquad \bar{y} = \frac{y}{H_0} \qquad \bar{z} = \frac{z}{L_0} \qquad \bar{t} = \frac{tV_0}{L_0}$$

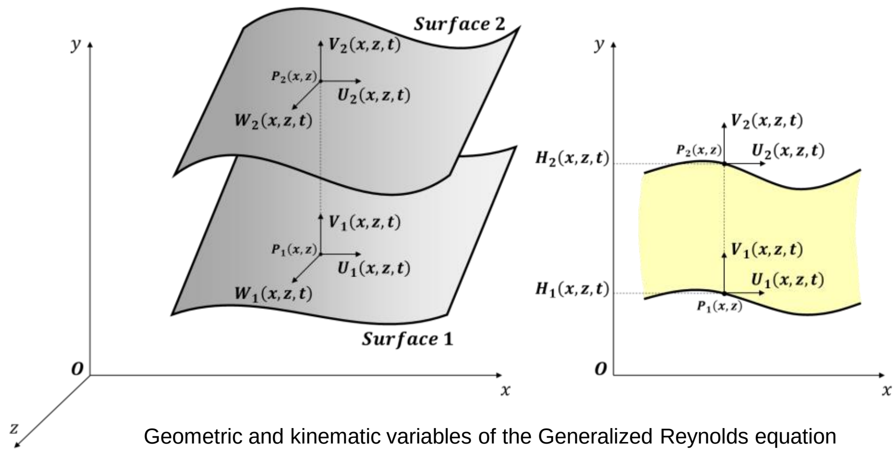
  - Fluid Properties

$$\bar{\mu} = \frac{\mu}{\mu_0} \qquad \bar{\rho} = \frac{\rho}{\rho_0} \qquad \bar{\lambda} = \frac{\lambda}{\lambda_0}$$

$$\bar{c_p} = \frac{c_p}{c_{p_0}} \qquad \bar{k} = \frac{k}{k_0} \qquad \bar{\alpha} = \frac{\alpha}{\alpha_0}$$

  - Dependent Variables

$$\bar{u} = \frac{u}{V_0} \qquad \bar{v} = \frac{vL_0}{V_0 H_0} \qquad \bar{w} = \frac{w}{W_0} \qquad \bar{p} = \frac{pH_0^2}{\mu_0 V_0 L_0} \qquad \bar{T} = \frac{T}{T_0}$$



Geometric and kinematic variables of the Generalized Reynolds equation

- Subscript 0: reference quantities
- L_0: characteristic transverse length of the contact interface (Oxz plane)
- H_0: characteristic normal length across the film thickness (Oy direction)

5. Theory of Fluid Film Lubrication

5.3 Generalized Reynolds Equation

- Dimensionless Equations

  - Conservation of Linear Momentum

Scale factor

$$\varepsilon = \frac{H_0}{L_0}$$

Reynolds Number

$$\mathrm{Re} = \frac{\rho_0 V_0 H_0}{\mu_0}$$

$$\frac{\partial \bar{p}}{\partial \bar{x}} = -\varepsilon \mathrm{Re}\bar{\rho}\left(\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}_j \frac{\partial \bar{u}}{\partial \bar{x}_j}\right) + \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\frac{\partial \bar{u}}{\partial \bar{y}}\right) + \varepsilon^2\left\{\left(\frac{\lambda_0}{\mu_0}\right)\frac{\partial}{\partial \bar{x}}\left(\bar{\lambda}\frac{\partial \bar{v}_j}{\partial \bar{x}_j}\right) + \frac{\partial}{\partial \bar{x}}\left(2\bar{\mu}\frac{\partial \bar{u}}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\frac{\partial \bar{v}}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{z}}\left[\bar{\mu}\left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{x}}\right)\right]\right\}$$

$$\frac{\partial \bar{p}}{\partial \bar{y}} = \varepsilon^2\left\{-\varepsilon \mathrm{Re}\bar{\rho}\left(\frac{\partial \bar{v}}{\partial \bar{t}} + \bar{v}_j \frac{\partial \bar{v}}{\partial \bar{x}_j}\right) + \left(\frac{\lambda_0}{\mu_0}\right)\frac{\partial}{\partial \bar{y}}\left(\bar{\lambda}\frac{\partial \bar{v}_j}{\partial \bar{x}_j}\right) + \frac{\partial}{\partial \bar{x}}\left(\bar{\mu}\frac{\partial \bar{u}}{\partial \bar{y}}\right) + \frac{\partial}{\partial \bar{y}}\left(2\bar{\mu}\frac{\partial \bar{v}}{\partial \bar{y}}\right) + \frac{\partial}{\partial \bar{z}}\left(\bar{\mu}\frac{\partial \bar{w}}{\partial \bar{y}}\right) + \varepsilon^2\left[\frac{\partial}{\partial \bar{x}}\left(\bar{\mu}\frac{\partial \bar{v}}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{z}}\left(\bar{\mu}\frac{\partial \bar{v}}{\partial \bar{z}}\right)\right]\right\}$$

$$\frac{\partial \bar{p}}{\partial \bar{z}} = -\varepsilon \mathrm{Re}\bar{\rho}\left(\frac{\partial \bar{w}}{\partial \bar{t}} + \bar{v}_j \frac{\partial \bar{w}}{\partial \bar{x}_j}\right) + \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\frac{\partial \bar{w}}{\partial \bar{y}}\right) + \varepsilon^2\left\{\left(\frac{\lambda_0}{\mu_0}\right)\frac{\partial}{\partial \bar{z}}\left(\bar{\lambda}\frac{\partial \bar{v}_j}{\partial \bar{x}_j}\right) + \frac{\partial}{\partial \bar{x}}\left[\bar{\mu}\left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{x}}\right)\right] + \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\frac{\partial \bar{v}}{\partial \bar{z}}\right) + \frac{\partial}{\partial \bar{z}}\left(2\bar{\mu}\frac{\partial \bar{w}}{\partial \bar{z}}\right)\right\}$$

  - Conservation of Energy

$$\bar{\rho}\bar{c}_p \mathrm{Pe}\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}_j \frac{\partial \bar{T}}{\partial \bar{x}_j}\right) = \alpha_0 T_0 \mathrm{N_d}\bar{\alpha}\bar{T}\left(\frac{\partial \bar{p}}{\partial \bar{t}} + \bar{v}_j \frac{\partial \bar{p}}{\partial \bar{x}_j}\right) + \frac{\partial}{\partial \bar{y}}\left(\bar{k}\frac{\partial \bar{T}}{\partial \bar{y}}\right) + \bar{\mu}\mathrm{N_d}\left[\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \left(\frac{\partial \bar{w}}{\partial \bar{y}}\right)^2\right] + \varepsilon^2\left\{\frac{\partial}{\partial \bar{x}}\left(\bar{k}\frac{\partial \bar{T}}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{z}}\left(\bar{k}\frac{\partial \bar{T}}{\partial \bar{z}}\right) + \left(\frac{\lambda_0}{\mu_0}\right)N_d\bar{\lambda}\left(\frac{\partial \bar{v}_j}{\partial \bar{x}_j}\right)^2 +\right.$$

$$\left.\bar{\mu}\mathrm{N_d}\left[2\left(\frac{\partial \bar{u}}{\partial \bar{x}}\right)^2 + 2\left(\frac{\partial \bar{v}}{\partial \bar{y}}\right)^2 + 2\left(\frac{\partial \bar{w}}{\partial \bar{z}}\right)^2 + \left(\frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\partial \bar{w}}{\partial \bar{x}}\right)^2 + 2\left(\frac{\partial \bar{u}}{\partial \bar{y}}\frac{\partial \bar{v}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{z}}\frac{\partial \bar{w}}{\partial \bar{y}}\right) + \varepsilon^2\left(\left(\frac{\partial \bar{v}}{\partial \bar{x}}\right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{z}}\right)^2\right)\right]\right\}$$

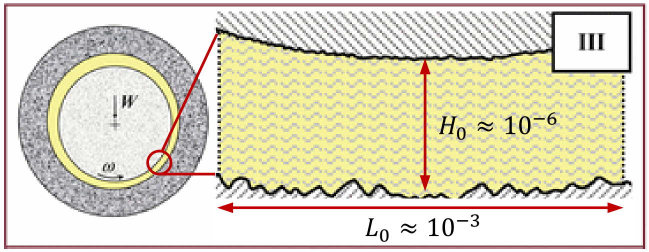
5. Theory of Fluid Film Lubrication

5.3 Generalized Reynolds Equation

- Order of magnitude analysis

  - From the lubrication theory: $\mathcal{O}(L_0) \approx 10^{-3}$ **(mm)**, $\mathcal{O}(H_0) \approx 10^{-6}$ **(μm)** $\Rightarrow$ $\mathcal{O}(\varepsilon) \approx 10^{-3}$

  - Thus, terms of $\mathcal{O}(\varepsilon)^n \ll 1$ may be neglected from the analysis.



  - In most lubrication systems, $\mathcal{O}(\varepsilon \mathrm{Re}) \ll 1$, thus this term can also be neglected from the analysis (neglecting inertia flow effects, i.e. laminar flow)

  - Such an assumption may not be valid when fluid viscosity is small and film thickness is large. In these cases, the Stokes equation can be used.

5. Theory of Fluid Film Lubrication

5.3 Generalized Reynolds Equation

- Order of magnitude analysis

  - By neglecting terms of order $\mathcal{O}(\varepsilon)^n$ and $\mathcal{O}(\varepsilon Re)$, the dimensionless equations are reduced to:

$$\frac{\partial \bar{p}}{\partial \bar{x}} = \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\frac{\partial \bar{u}}{\partial \bar{y}}\right) \qquad \frac{\partial \bar{p}}{\partial \bar{y}} = 0 \qquad \frac{\partial \bar{p}}{\partial \bar{z}} = \frac{\partial}{\partial \bar{y}}\left(\bar{\mu}\frac{\partial \bar{w}}{\partial \bar{y}}\right)$$

$$\bar{\rho}\bar{c_p}Pe\left(\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v}_j\frac{\partial \bar{T}}{\partial \bar{x_j}}\right) = \alpha_0 T_0 N_d \bar{\alpha}\bar{T}\left(\frac{\partial \bar{p}}{\partial \bar{t}} + \bar{v}_j\frac{\partial \bar{p}}{\partial \bar{x_j}}\right) + \frac{\partial}{\partial \bar{y}}\left(\bar{k}\frac{\partial \bar{T}}{\partial \bar{y}}\right) + \bar{\mu}N_d\left[\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 + \left(\frac{\partial \bar{w}}{\partial \bar{y}}\right)^2\right]$$

  - Rewriting these equations in the dimensional form, the original set of governing equations is considerably simplified as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \quad \frac{\partial p}{\partial y} = 0, \quad \frac{\partial p}{\partial z} = \frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) \qquad (1)$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + v_j\frac{\partial T}{\partial x_j}\right) = \alpha T\left(\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + w\frac{\partial p}{\partial z}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right]$$

5. Theory of Fluid Film Lubrication

5.3 Generalized Reynolds Equation

- Calculation of the flow velocity components (u, v, w)

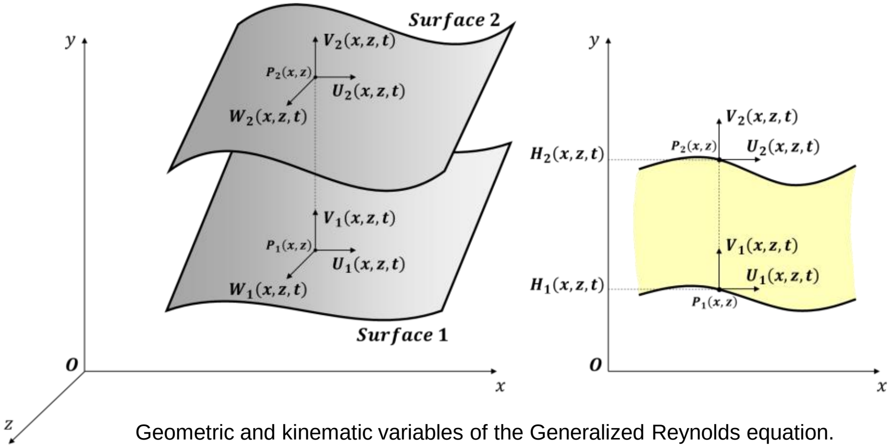
  - Velocity boundary conditions (no-slip condition):

$$\begin{cases} u = U_1 \quad v = V_1 \quad w = W_1 \quad \text{at} \quad y = H_1(x,z,t) \\ u = U_2 \quad v = V_2 \quad w = W_3 \quad \text{at} \quad y = H_2(x,z,t) \end{cases}$$

  - Assuming that the fluid viscosity and density **do not** change across the film thickness:

$$\mu = \mu(x,z,t) \qquad \rho = \rho(x,z,t)$$

This assumption is relaxed in the **generalised Reynolds equation**.



Geometric and kinematic variables of the Generalized Reynolds equation.

  - By integrating the linear momentum equations in the y-direction and applying the non-slip wall velocity boundary conditions, one obtains:

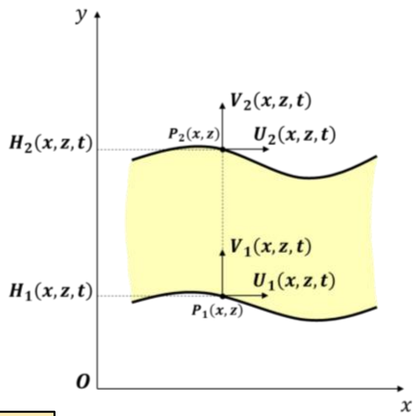
$$\begin{cases} u(x,y,z,t) = \dfrac{1}{2\mu}\dfrac{\partial p}{\partial x}\left[y^2 - y(h + 2H_1) + H_1(h + H_1)\right] + \left(\dfrac{U_2 - U_1}{h}\right)(y - H_1) + U_1 \\ w(x,y,z,t) = \dfrac{1}{2\mu}\dfrac{\partial p}{\partial z}\left[y^2 - y(h + 2H_1) + H_1(h + H_1)\right] + \left(\dfrac{W_2 - W_1}{h}\right)(y - H_1) + W_1 \end{cases} \quad (2)$$

5. Theory of Fluid Film Lubrication

5.3 Generalized Reynolds Equation

- Integration of the conservation of mass equation across the film thickness:

$$\int_{H_1}^{H_2} \frac{\partial \rho}{\partial t} dy + \int_{H_1}^{H_2} \frac{\partial (\rho u)}{\partial x} dy + \int_{H_1}^{H_2} \frac{\partial (\rho v)}{\partial y} dy + \int_{H_1}^{H_2} \frac{\partial (\rho w)}{\partial z} dy = 0$$



Leibniz rule for integration by parts
+
Velocity boundary conditions

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_z}{\partial z} + \rho\left(-U_2 \frac{\partial H_2}{\partial x} - W_2 \frac{\partial H_2}{\partial z} + U_1 \frac{\partial H_1}{\partial x} + W_1 \frac{\partial H_1}{\partial z}\right) + \rho(V_2 - V_1) + (H_2 - H_1)\frac{\partial \rho}{\partial t} = 0 \quad (3)$$

- Average flow across the lubricant film:

$$\begin{cases} q_x = \int_{H_1}^{H_2} \rho u dy = -\frac{\rho (H_2 - H_1)^3}{12\mu} \frac{\partial p}{\partial x} + \rho (H_2 - H_1)\left(\frac{U_1 + U_2}{2}\right) \\ q_z = \int_{H_1}^{H_2} \rho w dy = -\frac{\rho (H_2 - H_1)^3}{12\mu} \frac{\partial p}{\partial z} + \rho (H_2 - H_1)\left(\frac{W_1 + W_2}{2}\right) \end{cases} \quad (4)$$

The velocity components of Eq. (2) were substituted into the integrals that define q_x and q_z

5. Theory of Fluid Film Lubrication

5.3 Generalized Reynolds Equation

- (Isothermal) Generalized Reynolds equation
  - The isothermal Generalized Reynolds Equation is obtained by substituting Eq. (4) in Eq. (3):

$$\underbrace{\frac{\partial}{\partial x}\left[\frac{\rho(H_2-H_1)^3}{12\mu}\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[\frac{\rho(H_2-H_1)^3}{12\mu}\frac{\partial p}{\partial z}\right]}_{\text{Pressure Flow (Poiseuille)}}=\underbrace{\frac{\partial}{\partial x}\left[\frac{\rho(U_2+U_1)}{2}(H_2-H_1)\right]+\frac{\partial}{\partial z}\left[\frac{\rho(W_2+W_1)}{2}(H_2-H_1)\right]}_{\text{Wedge Flow (Couette)}}+$$
$$\underbrace{\rho\left[\left(U_1\frac{\partial H_1}{\partial x}-U_2\frac{\partial H_2}{\partial x}\right)+\left(W_1\frac{\partial H_1}{\partial z}-W_2\frac{\partial H_2}{\partial z}\right)\right]}_{\text{Translation Squeeze}}+\underbrace{\rho(V_2-V_1)}_{\text{Normal Squeeze}}+\underbrace{(H_2-H_1)\frac{\partial \rho}{\partial t}}_{\text{Local Expansion}} \quad (5)$$

  - Equation in the conservation vector form:

$$\nabla\cdot(\boldsymbol{\Gamma}^{\mathbf{p}}\nabla p_H)=\nabla\cdot(\boldsymbol{\Gamma}^{\mathbf{c}}\vec{\boldsymbol{v}})+[S_{TS}+S_{NS}]+S_T\frac{\partial \rho}{\partial t}$$

Suitable for numerical solutions
(Tensor, vector and source terms defined accordingly)

  - Shear rate and shear stress components:

$$\begin{cases}\tau_{xy}=\mu\dfrac{\partial u}{\partial y}=\dfrac{1}{2}\dfrac{\partial p}{\partial x}[2y-(h+2H_1)]+\mu\left(\dfrac{U_2-U_1}{h}\right)\\[2ex]\tau_{zy}=\mu\dfrac{\partial w}{\partial y}=\dfrac{1}{2}\dfrac{\partial p}{\partial z}[2y-(h+2H_1)]+\mu\left(\dfrac{W_2-W_1}{h}\right)\end{cases}$$

The velocity fields of Eq. (2) were substituted on the shear rate components

5. Theory of Fluid Film Lubrication

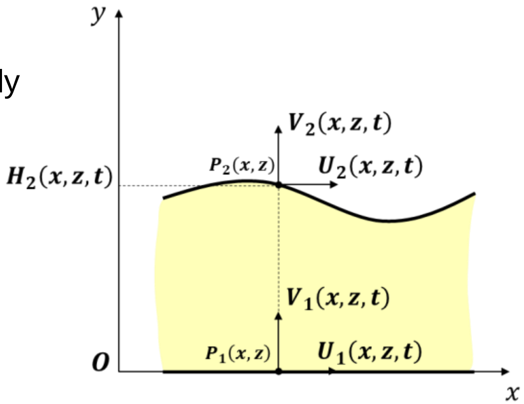
5.3 Generalized Reynolds Equation

- (Isothermal) Generalized Reynolds equation

  - In many applications, the origin of the coordinate system can be conveniently attached to one of the contacting surfaces, *e.g.* bottom surface (surface 1). Thus:

$$\begin{cases} H_1 = 0 \\ H_2 = h \end{cases}$$

← **film gap**



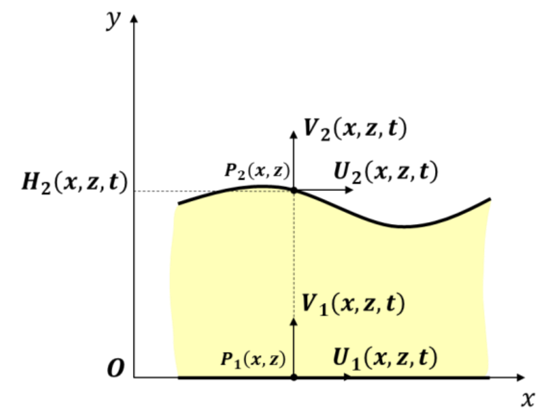
  - Thus, Eq. (5) assumes a simpler form that can be written as ('Simplified' Reynolds equation) :

$$\underbrace{\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial z}\right)}_{\text{Pressure-Flow (Poiseuille)}}=\underbrace{\frac{\partial}{\partial x}\left[\frac{\rho h(U_2+U_1)}{2}\right]+\frac{\partial}{\partial z}\left[\frac{\rho h(W_2+W_1)}{2}\right]}_{\text{Wedge-Flow (Couette)}}-\underbrace{\rho\left(U_2\frac{\partial h}{\partial x}+W_2\frac{\partial h}{\partial z}\right)}_{\text{Translation-Squeeze}}+\underbrace{\rho(V_2-V_1)}_{\text{Normal Squeeze}}+\underbrace{h\frac{\partial\rho}{\partial t}}_{\text{Local Expansion}} \quad (6)$$

5. Theory of Fluid Film Lubrication

5.4 Interpretation of the Reynolds Equation

- Fluid pressure generation is affected by:
  - Interfacial geometry (surface geometries and rigid body displacements)
  - Lubricant properties (viscosity and density)
  - Surface velocities (tangential and normal components)
  - Boundary pressures and fluid cavitation
  - Fluid and surface temperatures, surface deformations (EHL), surface roughness



$$\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial z}\right)=$$
$$\frac{\partial}{\partial x}\left[\frac{\rho h(U_2+U_1)}{2}\right]+\frac{\partial}{\partial z}\left[\frac{\rho h(W_2+W_1)}{2}\right]-$$
$$\rho\left(U_2\frac{\partial h}{\partial x}+W_2\frac{\partial h}{\partial z}\right)+\rho(V_2-V_1)+h\frac{\partial \rho}{\partial t}$$

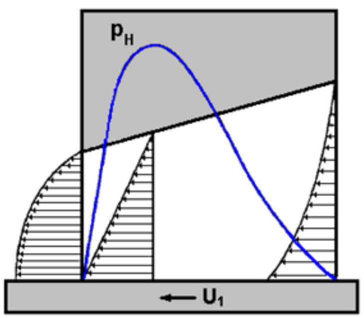
- Physical interpretation of the Reynolds equation
  - The **Reynolds equation corresponds to an alternative form of the mass conservation equation** written in terms of contact geometry, kinematics, and lubricant properties.
  - The lubricant flow through the contact interface comprises interchangeable flow components that promote fluid pressure build-up (hydrodynamic pressure distribution) under appropriate conditions.
  - The lubricant flow components are associated with the following physical mechanisms:

5. Theory of Fluid Film Lubrication

5.4 Interpretation of the Reynolds Equation

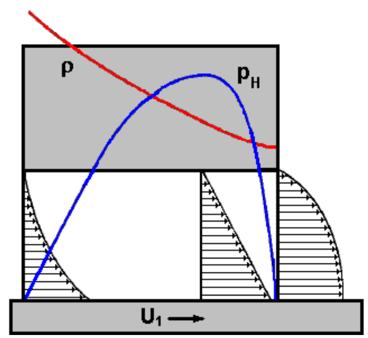
- Wedge-Flow terms

$$\underbrace{\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial z}\right)}_{\text{Pressure-Flow (Poiseuille)}} = \underbrace{\frac{\partial}{\partial x}\left[\frac{\rho h(U_2+U_1)}{2}\right]+\frac{\partial}{\partial z}\left[\frac{\rho h(W_2+W_1)}{2}\right]}_{\text{Wedge-Flow (Couette)}} - \underbrace{\rho\left(U_2\frac{\partial h}{\partial x}+W_2\frac{\partial h}{\partial z}\right)}_{\text{Translation-Squeeze}} + \underbrace{\rho(V_2-V_1)}_{\text{Normal Squeeze}} + \underbrace{h\frac{\partial \rho}{\partial t}}_{\text{Local Expansion}}$$



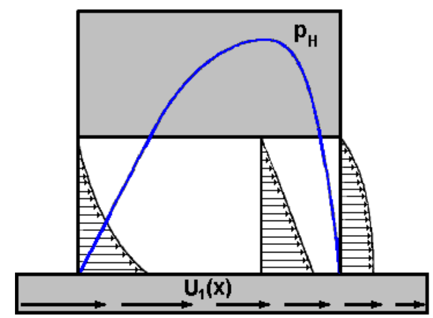
Physical-Wedge

- Best known mechanism for pressure generation
- As the film thickness varies along the bearing, there is a different Couette flow rate at each section



Density-Wedge

- Rate at which lubricant density changes in the sliding direction
- This effect could be introduced by raising the temperature of the lubricant as it passes through the bearing



Stretch-Wedge

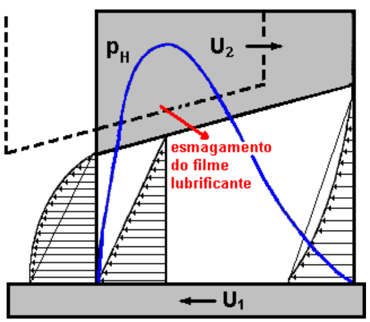
- Rate at which velocity changes in the sliding direction
- For positive pressure to be developed, the surface velocities have to decrease in the sliding direction.

5. Theory of Fluid Film Lubrication

5.4 Interpretation of the Reynolds Equation

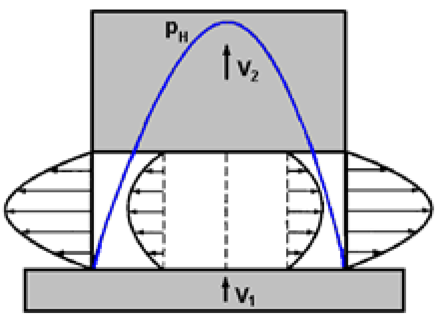
- Squeeze and Expansion terms

$$\underbrace{\frac{\partial}{\partial x}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\rho h^3}{12\mu}\frac{\partial p}{\partial z}\right)}_{\text{Pressure-Flow (Poiseuille)}}=\underbrace{\frac{\partial}{\partial x}\left[\frac{\rho h(U_2+U_1)}{2}\right]+\frac{\partial}{\partial z}\left[\frac{\rho h(W_2+W_1)}{2}\right]}_{\text{Wedge-Flow (Couette)}}\underbrace{-\rho\left(U_2\frac{\partial h}{\partial x}+W_2\frac{\partial h}{\partial z}\right)}_{\text{Translation-Squeeze}}+\underbrace{\rho(V_2-V_1)}_{\text{Normal Squeeze}}+\underbrace{h\frac{\partial \rho}{\partial t}}_{\text{Local Expansion}}$$



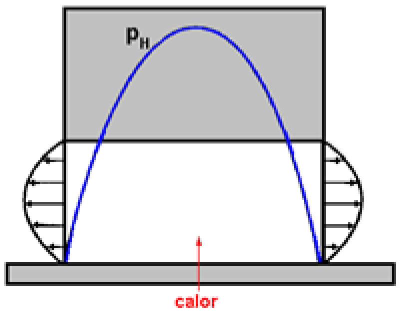
Translation-Squeeze

- Results from the translation of inclined surfaces
- The local film thickness is squeezed by the sliding of the inclined surface



Normal-Squeeze

- Positive pressure generated when the film thickness is diminished
- Normal squeeze action provides a valuable effect when bearing surfaces tend to be pressed together



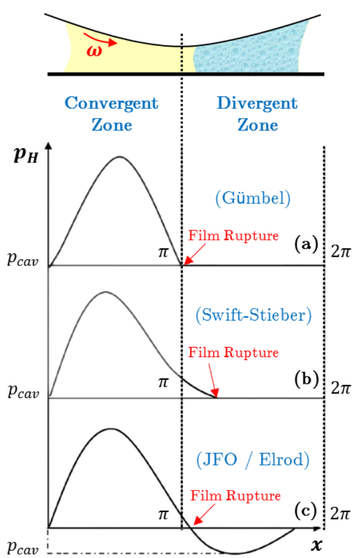
Local Expansion

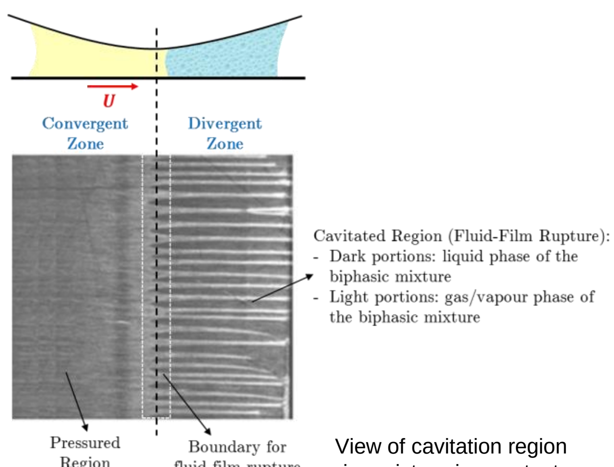
- Local time rate of density often achieved considering thermal expansion

5. Theory of Fluid Film Lubrication

5.5 Fluid Film Cavitation

- Fluid capacity to sustain **tensile** stress is unlikely in realistic lubrication environments.
- As the **fluid pressure falls below the saturation or vapour pressure of the lubricant**, the lubricant film is broken, and a mixture of gases/vapour and liquid is formed in some regions of the interface (cavitation zones).
- This phenomenon plays an essential role in the performance of lubrication systems.





View of cavitation region in a piston-ring contact

Cavitation region (within the cavitation region pressure is approx. constant)

Cavitation region in a steadily loaded journal bearing

5. Theory of Fluid Film Lubrication

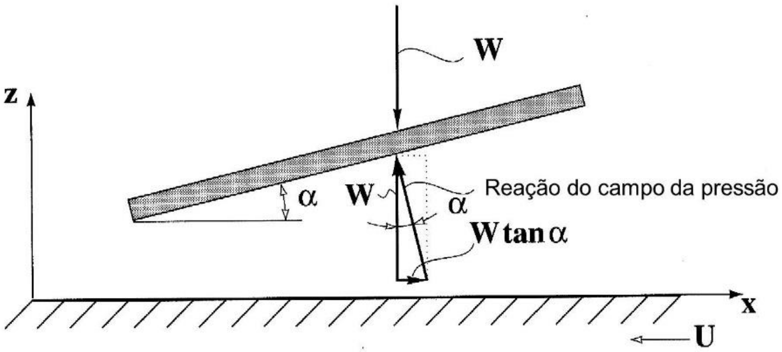
5.6 Operational Parameters

- After the solution of the Reynolds equation for given gap geometry, lubricant properties and surface velocities, the following operational parameters are commonly calculated:

  - Hydrodynamic pressure distribution: $p(x,z,t)$
  - Load-carrying capacity: $\vec{\boldsymbol{W}}(t)$

$$W(t) = \iint_A p(x,z,t)\,dxdz$$

Example for sliding bearings

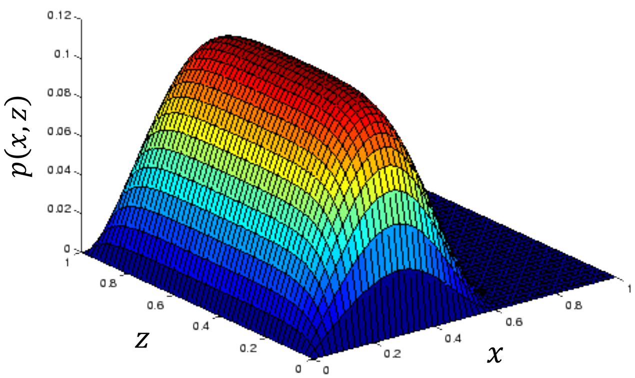


  - Friction force $\vec{\boldsymbol{F}}(t)$ and power loss $P(t)$

$$F(t) = \iint_A \tau_{xy}(x,z,t)\,dxdz$$

$$P(t) = F(t)U(t)$$

Example for sliding bearings



5. Theory of Fluid Film Lubrication

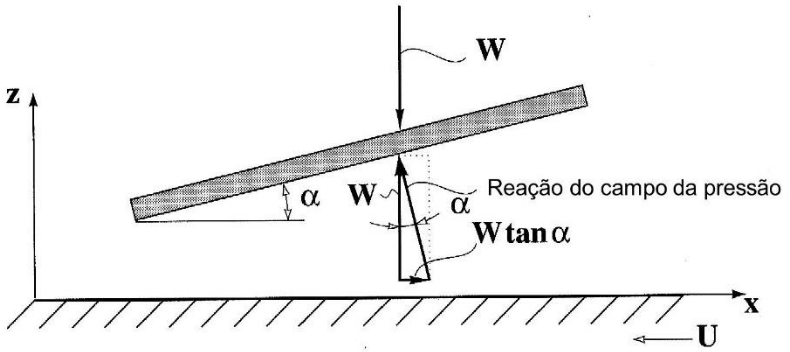
5.6 Operational Parameters

- After the solution of the Reynolds equation for given gap geometry, lubricant properties and surface velocities, the following operational parameters are commonly calculated:

  - Coefficient of friction: $COF(t)$

$$COF(t) = \frac{F(t)}{W(t)} = \frac{\iint_A \tau_{xy}(x,z,t)\,dxdz}{\iint_A p(x,z,t)\,dxdz}$$

Example for sliding bearings

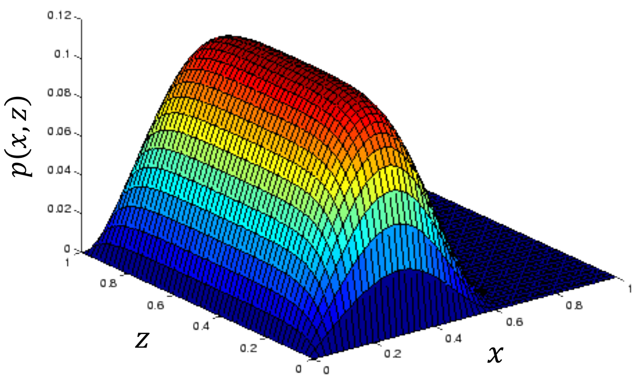


  - Minimum oil film thickness: $MOFT(t) = min[h(x,z,t)]$

  - Peak pressure: $PP(t) = max[p(x,z,t)]$

  - Lubricant flow (leakage): $Q(t)$

$$Q_{x=0} = \int_0^{Lz} q_x(0,z,t)dz \qquad Q_{z=0} = \int_0^{Lx} q_z(x,0,t)dx$$



Outline

1. **Roles & Functions of Lubrication**

2. **Lubricants**
   - 2.1 Composition
   - 2.2 Viscosity
   - 2.3 Density
   - 2.3 Thermal Properties
   - 3.4 Lubricant Additives

3. **Types of Lubrication**

4. **Lubrication Regimes (Stribeck curve)**
   - 4.1 Hydrodynamic Lubrication (HL)
   - 4.2 Elastohydrodynamic Lubrication (EHL)
   - 4.3 Mixed Lubrication (ML)
   - 4.4 Boundary Lubrication (BL)

5. **Theory of Fluid Film Lubrication**
   - 5.1 Roles and Principles
   - 5.2 Assumptions of the Lubrication Theory
   - 5.3 Derivation of the Generalized Reynolds Equation
   - 5.4 Physical Interpretation of Reynolds Equation
   - 5.5 Fluid Film Cavitation
   - 5.6 Operational Parameters

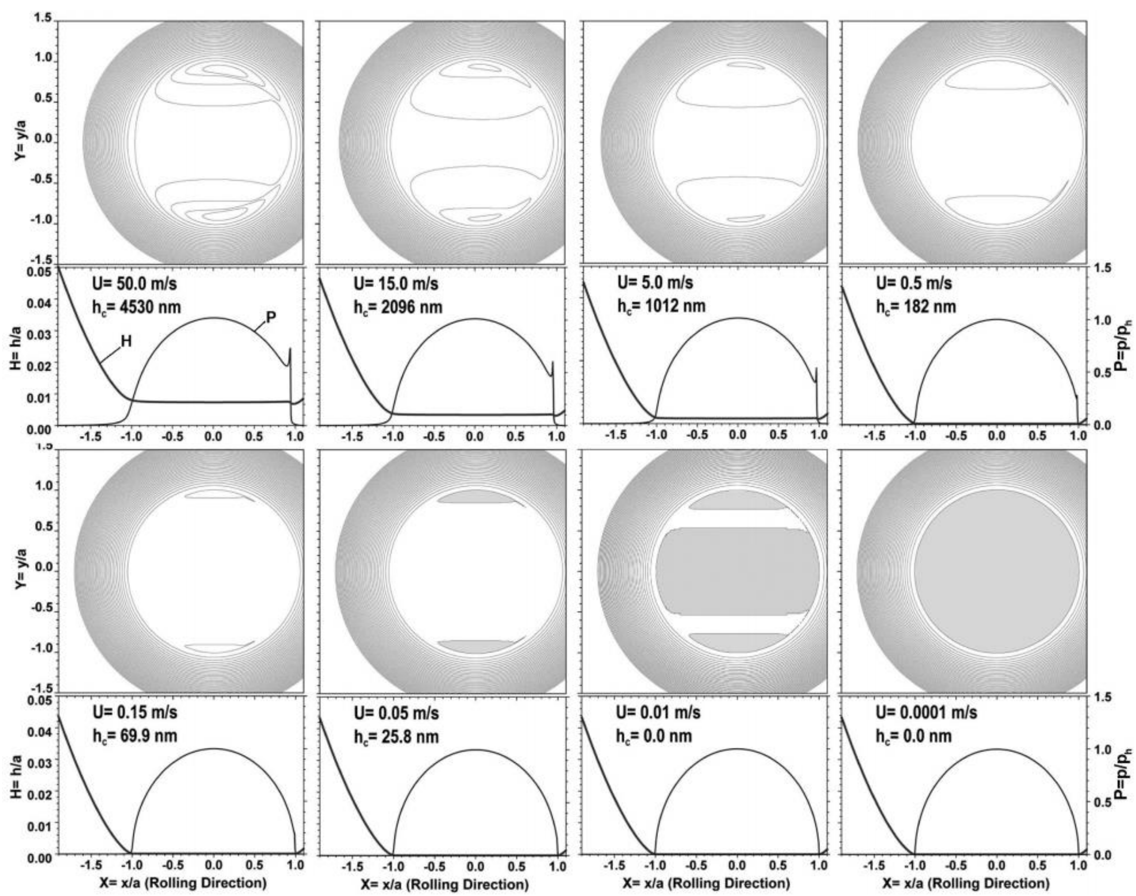
6. **Advanced Topics**
   - 6.1 Mixed Lubrication with Rough Surfaces
   - 6.2 Thermal Behaviours
   - 6.3 Plasto-Elastohydrodynamic Lubrication (PEHL)

6. Advanced Topics

6.1 Mixed Lubrication with Rough Surfaces

- Film thickness contours showing the entire **transition from full-film and mixed to boundary lubrication** in a circular contact EHL case with smooth surfaces. Source: [1].

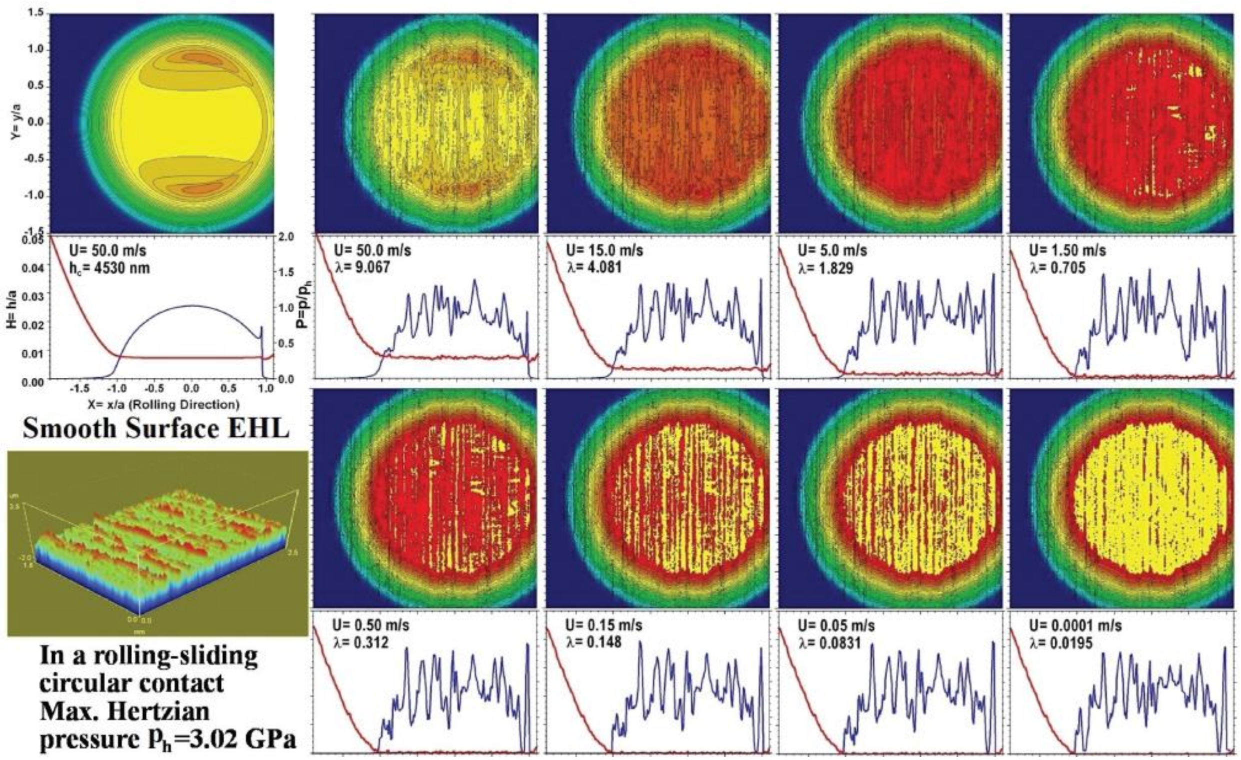
- Full coupling of Reynolds and elastic half-space equations for **smooth surfaces**.



6. Advanced Topics

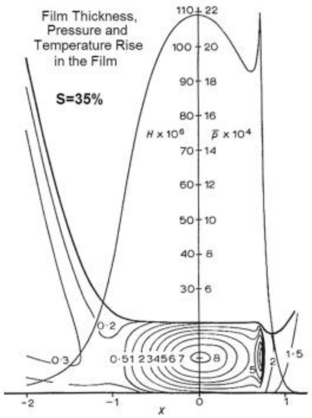
6.1 Mixed Lubrication with Rough Surfaces

- Film thickness contours showing the entire transition from full-film and mixed to boundary lubrication in a circular contact EHL case with two ground rough surfaces. Source: [1].
- Full coupling of Reynolds and elastic half-space equations for **rough surfaces**.

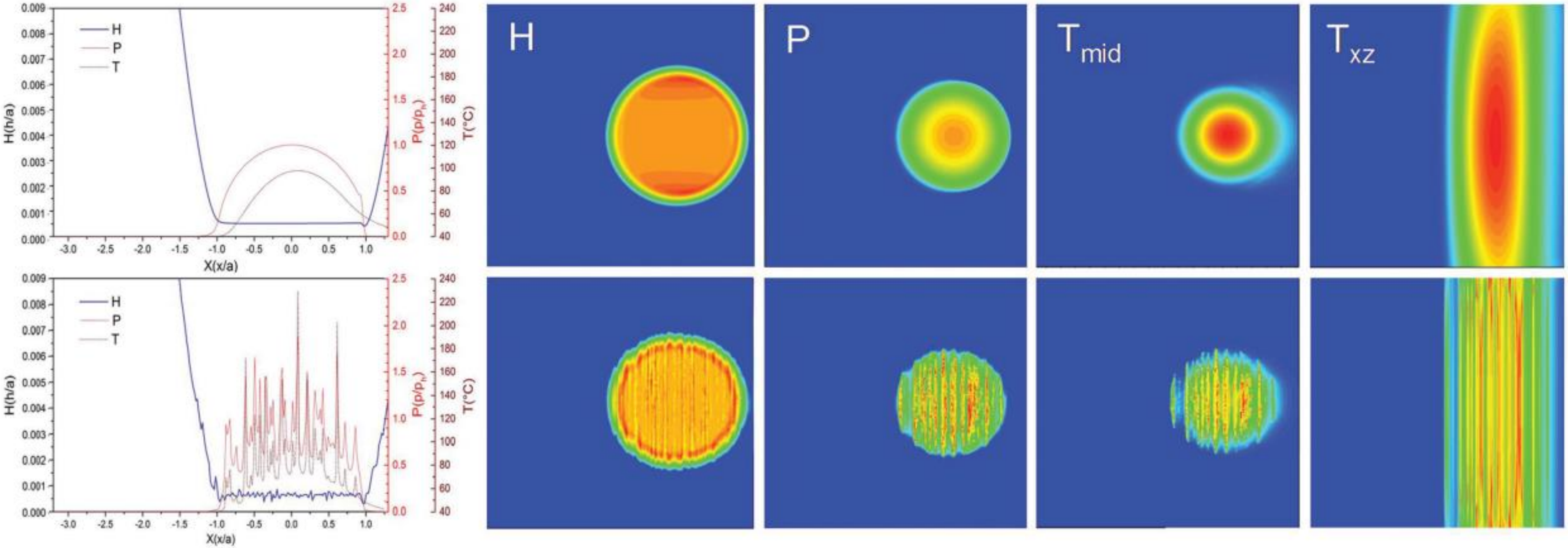


6. Advanced Topics

6.2 Thermal Behaviours



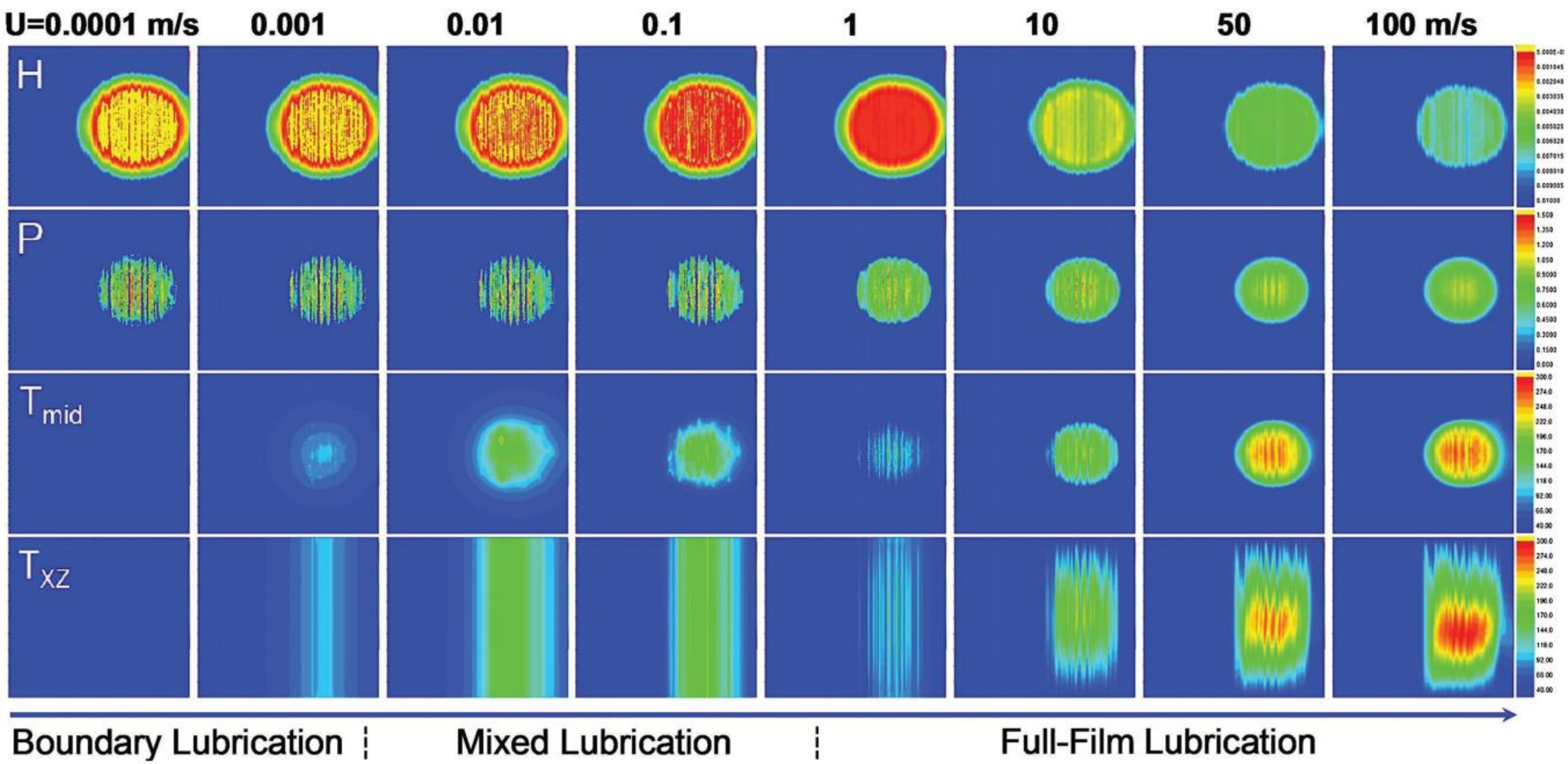
- Comparison between TEHL solutions without (top) and with (bottom) real machined roughness. From left to right: film thickness, pressure, mid-film temperature rise, and temperature rise distribution across the film. Source: [1]



6. Advanced Topics

6.2 Thermal Behaviours

- Transition from boundary and mixed to full-film lubrication. Source: [1]

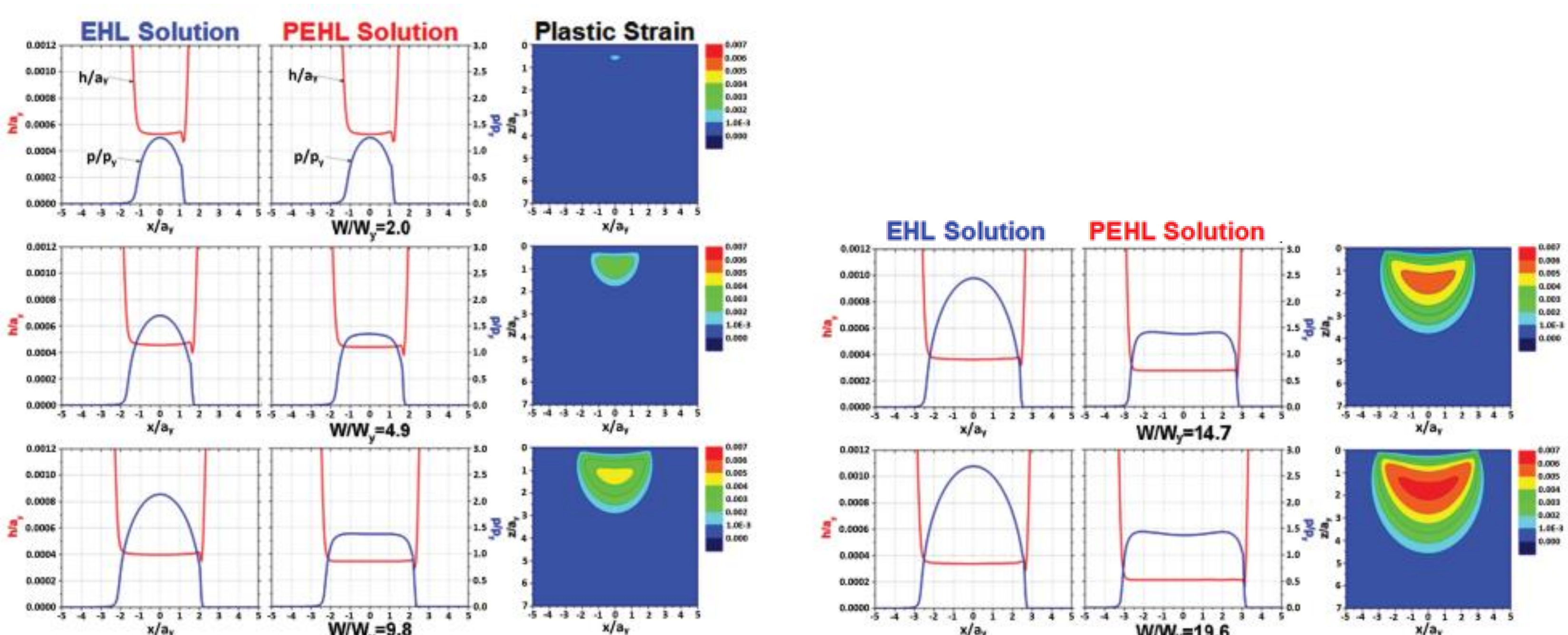


6. Advanced Topics

6.3 Plasto-Elastohydrodynamic Lubrication (PEHL)

- PEHL results under different loads in comparison with those from corresponding EHL solutions. Source: [1].

- Full coupling of Reynolds and the extension of the elastic half-space theory by incorporating plastic deformation.

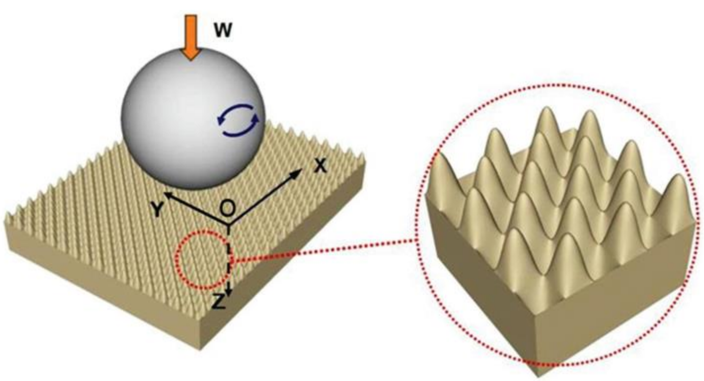


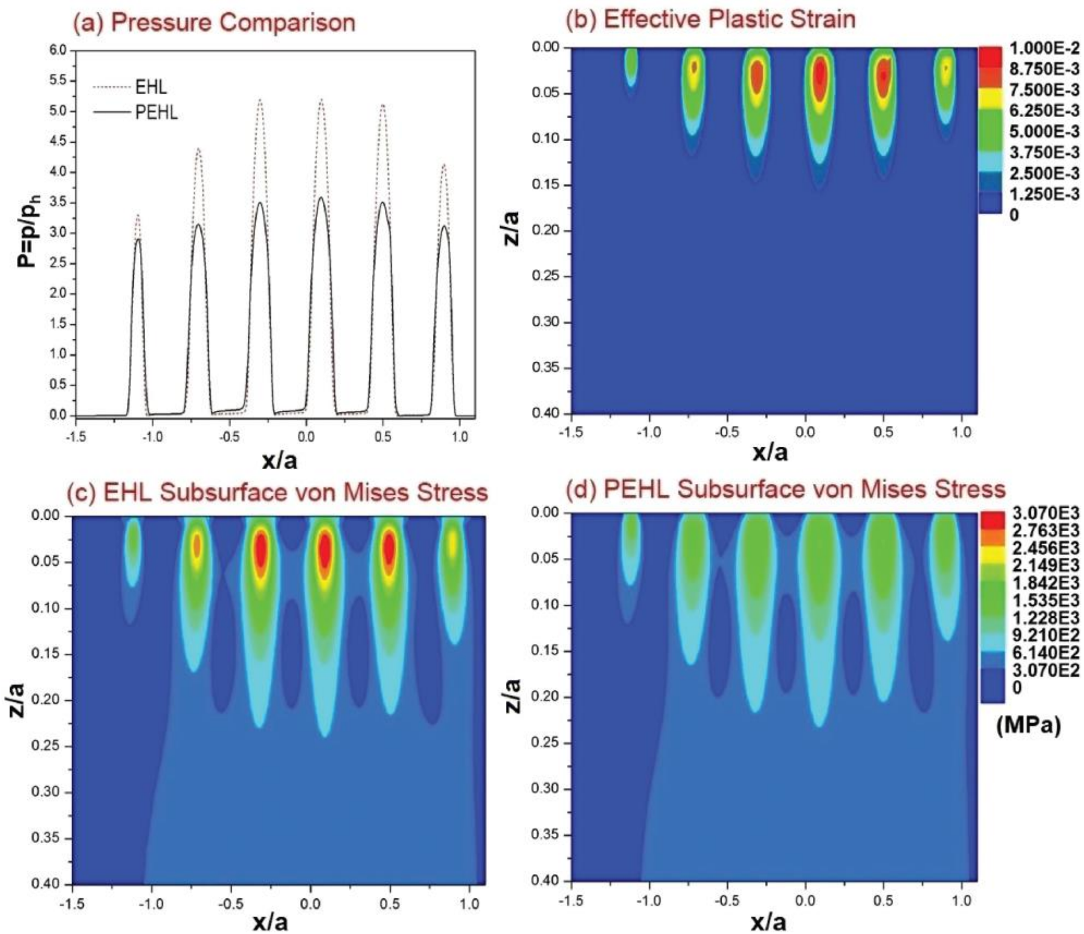
6. Advanced Topics

6.3 Plasto-Elastohydrodynamic Lubrication (PEHL)

- Sample PEHL solution with **sinusoidal surface roughness** and its comparison with corresponding EHL solution. Source: [1].

- Full coupling of Reynolds and the extension of the elastic half-space theory by incorporating plastic deformation.



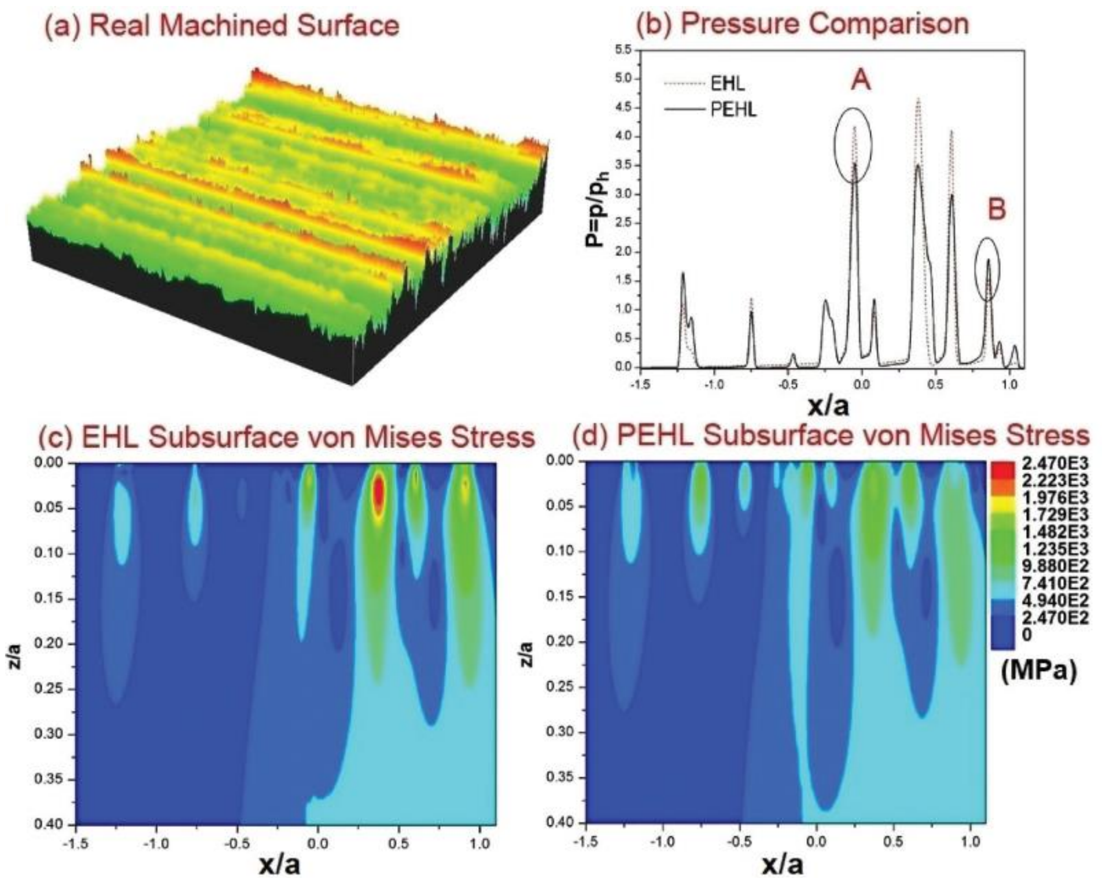


6. Advanced Topics

6.3 Plasto-Elastohydrodynamic Lubrication (PEHL)

- Sample PEHL solution with **real machined surface** and its comparison with corresponding EHL solution. Source: [1].

- Full coupling of Reynolds and the extension of the elastic half-space theory by incorporating plastic deformation.



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