Velocidade de fase

$$\psi(x,t) = \psi_0 \, \text{Sen} \, (\underbrace{Kx - \omega t + \varepsilon}_{\varphi})$$

$$\left| \left(\frac{\partial \varphi}{\partial t} \right)_x \right| = \omega$$

$$\left| \left(\frac{\partial \varphi}{\partial x} \right)_t \right| = K$$

$$v = \left(\frac{\partial x}{\partial t} \right)_\varphi = \left(\frac{\partial x}{\partial \varphi} \right)_t \cdot \left(\frac{\partial \varphi}{\partial t} \right)_x = \frac{\left(\frac{\partial \varphi}{\partial t} \right)_x}{\left(\frac{\partial \varphi}{\partial x} \right)_t} = \frac{\omega}{K}$$

$\boxed{v = \pm \frac{\omega}{K}}$ → Velocidade de fase da onda

x ——— x ———

Batimento de duas ondas

$$E_1 = E_{01} \, \text{Cos} \, (K_1 x - \omega_1 t) \qquad \varepsilon_1 = \varepsilon_2 = 0$$

$$E_2 = E_{02} \, \text{Cos} \, (K_2 x - \omega_2 t) \qquad E_{01} = E_{02}$$

$$K_1 > K_2 \quad e \quad \omega_1 > \omega_2$$

$$E = E_1 + E_2 = E_{01} \left[\text{Cos} \, (K_1 x - \omega_1 t) + \text{Cos} \, (K_2 x - \omega_2 t) \right]$$

$$\text{Cos} \, \alpha + \text{Cos} \, \beta = 2 \, \text{Cos} \left[\frac{\alpha + \beta}{2} \right] \cdot \text{Cos} \left[\frac{\alpha - \beta}{2} \right]$$

$$E = 2E_{01}\left[\cos\left\{\frac{(K_1x - \omega_1 t) + (K_2 x - \omega_2 t)}{2}\right\} \cdot \cos\left\{\frac{(K_1x - \omega_1 t) - (K_2 x - \omega_2 t)}{2}\right\}\right]$$

$$E = 2E_{01}\left[\cos\left\{\frac{(K_1 + K_2)x - (\omega_1 + \omega_2)t}{2}\right\} \cdot \cos\left\{\frac{(K_1 - K_2)x - (\omega_1 - \omega_2)t}{2}\right\}\right]$$

$\bar{K} = \frac{K_1 + K_2}{2}$ → número de onda médio

$\bar{\omega} = \frac{\omega_1 + \omega_2}{2}$ → frequência angular média

$K_m = \frac{K_1 - K_2}{2}$ → número de onda de modulação

$\omega_m = \frac{\omega_1 - \omega_2}{2}$ → frequência angular de modulação

$$\boxed{E = 2E_{01} \cos(\bar{K}x - \bar{\omega}t) \cdot \cos(K_m x - \omega_m t)}$$

ω_1, ω_2 ou $\bar{\omega}$ são valores altos

$\omega_1 \simeq \omega_2$ → valores próximos

$$\boxed{\bar{\omega} \gg \omega_m}$$

$$E = \underbrace{2E_0 \cos(K_m x - \omega_m t)}_{\text{amplitude}} \cdot \cos(\bar{K}x - \bar{\omega}t)$$

$$I \propto \left(2E_0 \cos(K_m x - \omega_m t)\right)^2 \qquad \omega_1, \omega_2$$

$$\boxed{I \propto 4E_{01}^2 \cos^2(K_m x - \omega_m t)}$$

$$I \propto 2E_{01}^2\left(1 + \cos(2K_m x - 2\omega_m t)\right)$$

$$2K_m = 2\cdot\left(\frac{K_1-K_2}{2}\right) = (K_1-K_2)$$

$$2\omega_m = \cancel{2}\left(\frac{\omega_1-\omega_2}{\cancel{2}}\right) = (\omega_1-\omega_2)$$

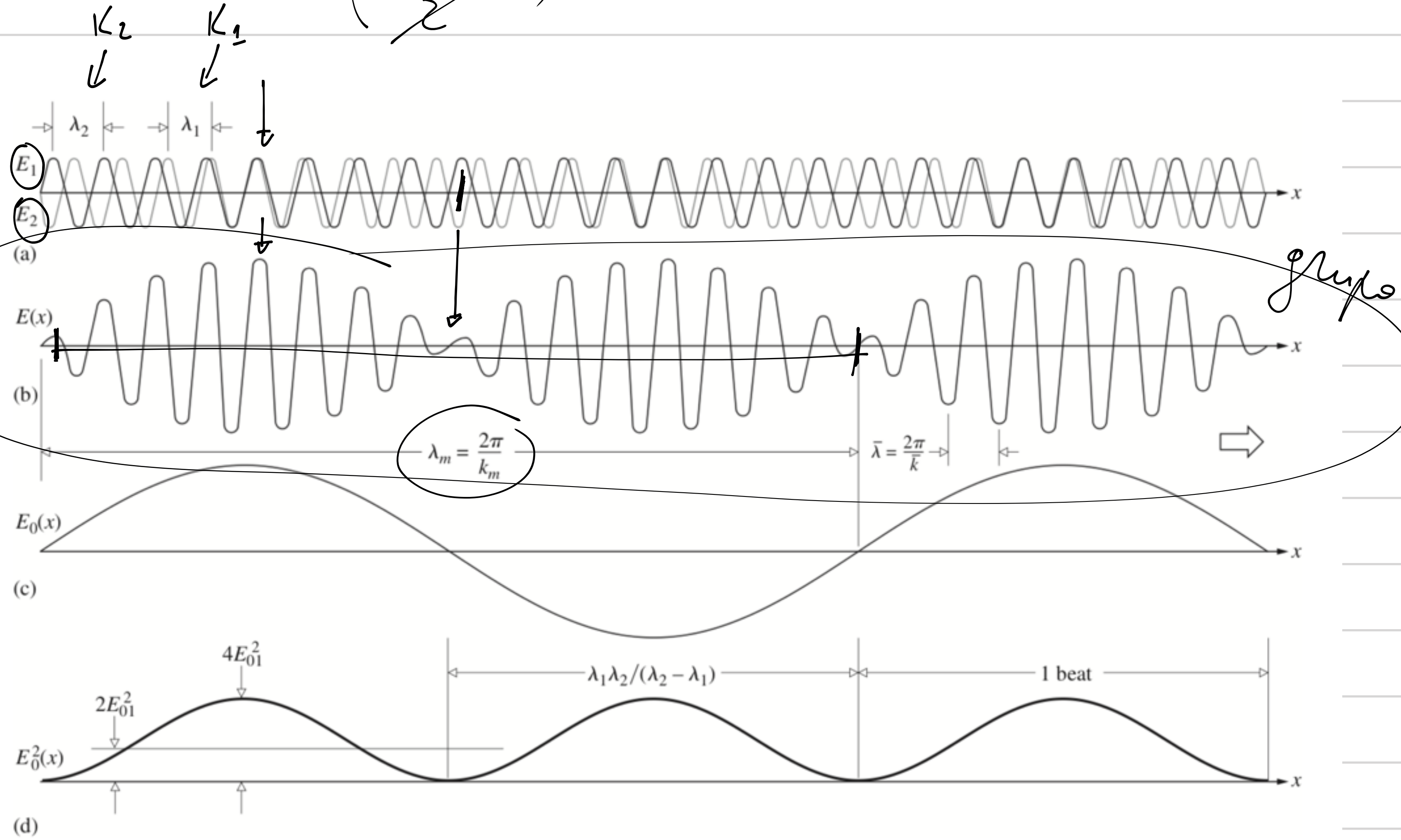


Figure 7.16 The superposition of two equal-amplitude harmonic waves of different frequency producing a beat pattern.

$$E(x,t) = 2E_{01}\underbrace{\cos(K_m x - \omega_m t)}\cdot\underbrace{\cos(\bar{K}x - \bar{\omega}t)}$$

$$v_g = \frac{\omega_m}{K_m} \qquad v = \frac{\bar{\omega}}{\bar{K}}$$

→ Velocidade de grupo

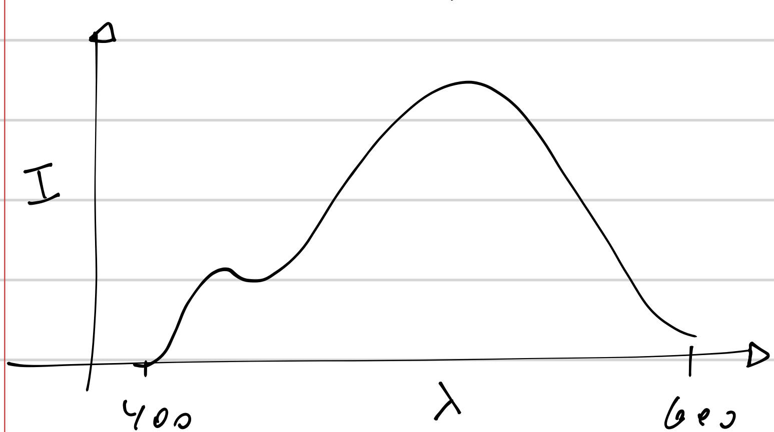
Meio Dispersivo

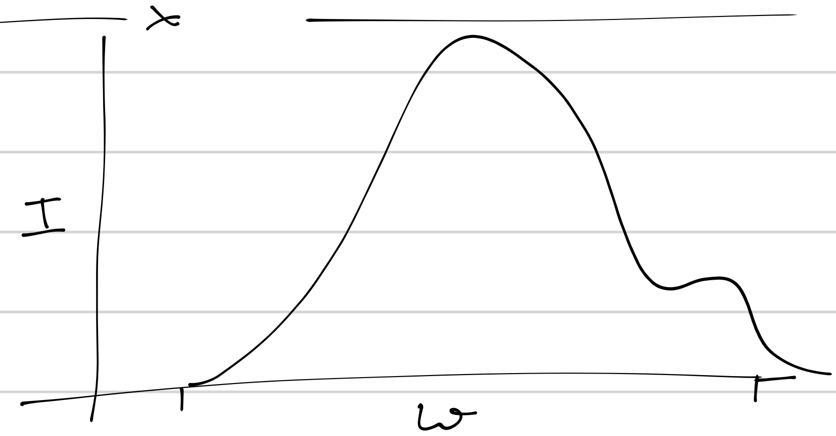
$\boxed{n = n(\omega)} \longrightarrow$ Dispersão

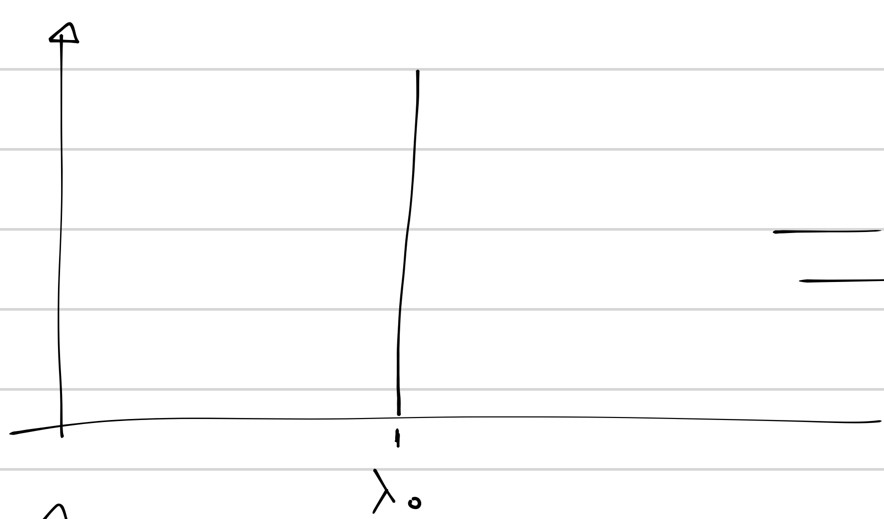
$\longrightarrow n \equiv \dfrac{c}{v}$

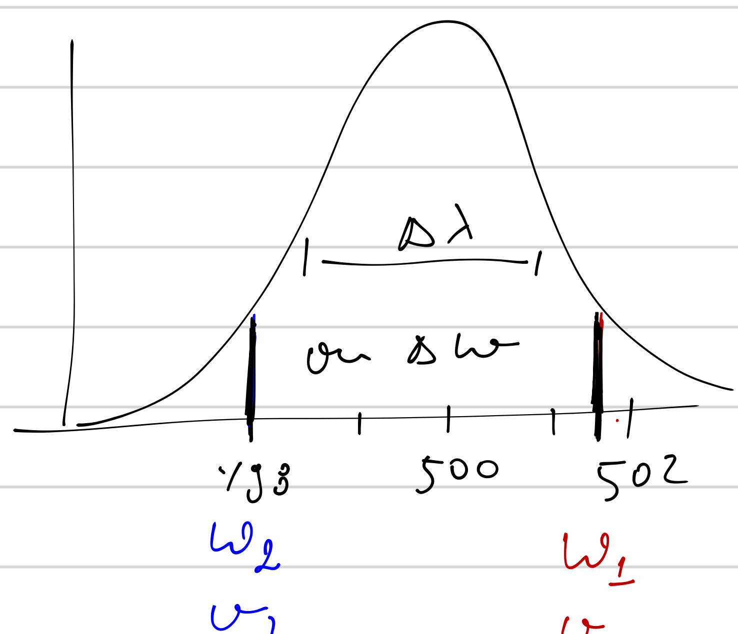
→ Vácuo → ambiente não dispersivo

→ Demais (todos) → São meios dispersivos

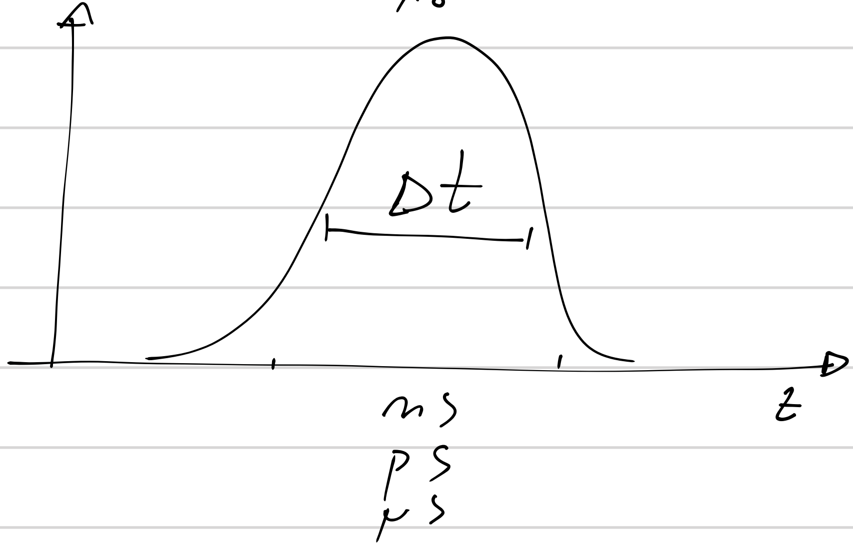


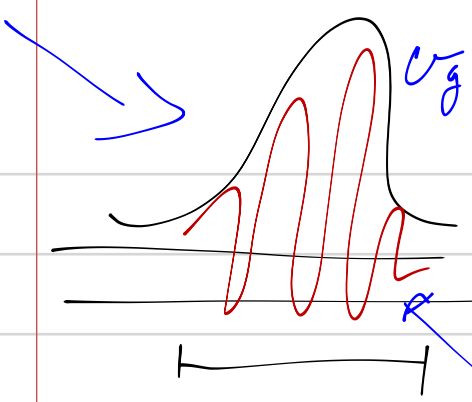






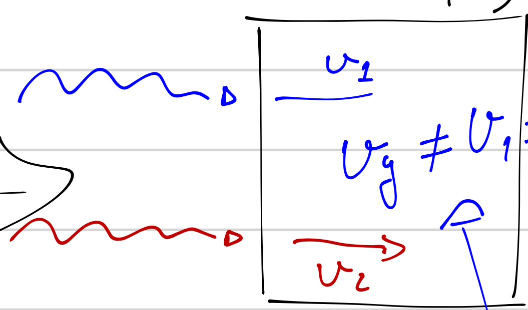
$\boxed{v_1 > v_2}$



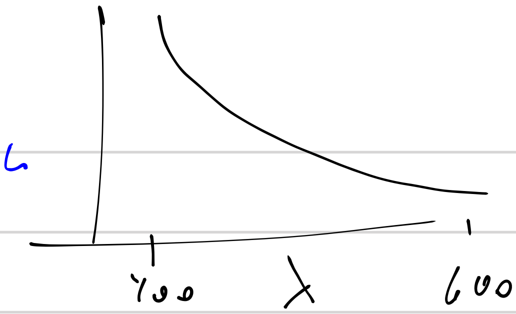


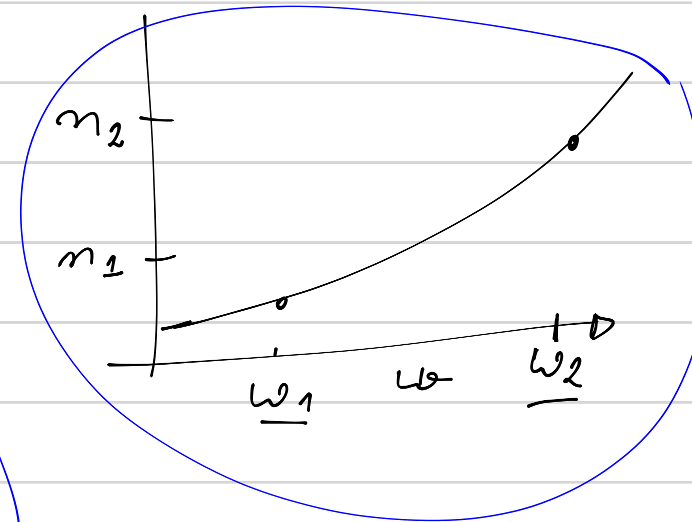
$v_g = v = c$

$n = n(\omega)$



$v_g \neq v_1 \neq v_2 \neq c$





$n = \frac{c}{v}$

$n_1 = \frac{c}{v_1}$

$n_2 = \frac{c}{v_2}$

$v_1 = \frac{c}{n_1}$

$v_2 = \frac{c}{n_2}$

$\boxed{v_1 > v_2}$

$v_g = \frac{\omega_m}{K_m} = \frac{\omega_1 - \omega_2}{K_1 - K_2}$

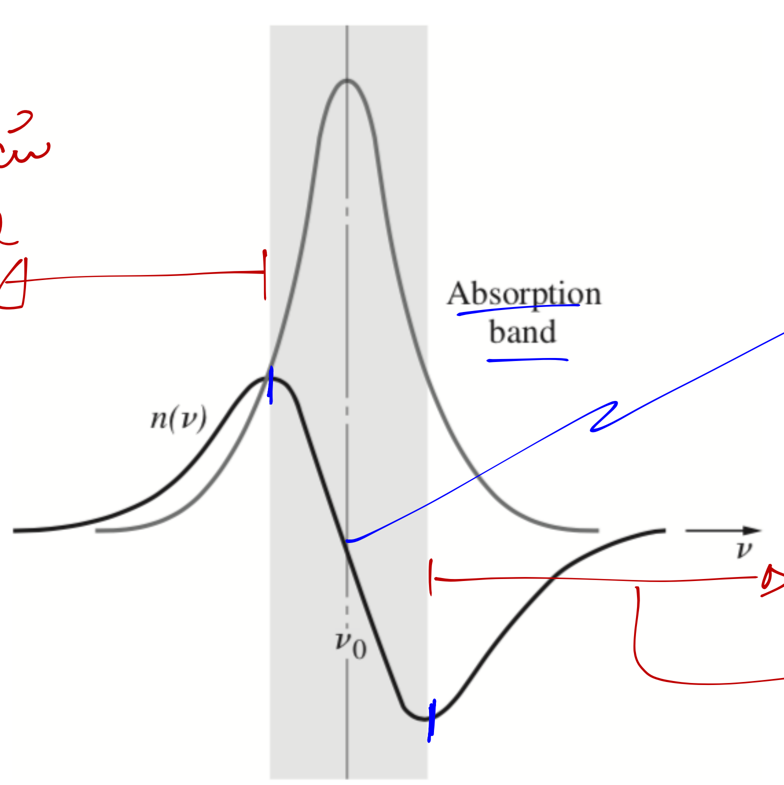
$\boxed{n_g \equiv \frac{c}{v_g}} \Longleftrightarrow \boxed{n \equiv \frac{c}{v}}$

$n_1 = \frac{c}{v_1}$

$n_2 = \frac{c}{v_2}$

Dispersão normal

$\frac{dn}{d\omega} > 0$



$\frac{dn}{d\omega} < 0$

Dispersão anômala

$dn/d\omega > 0$

Dispersão normal

Figure 7.19 A typical representation of the frequency dependence of the index of refraction in the vicinity of an atomic resonance. Also shown is the absorption curve centered on the resonant frequency.

$$v_g = \frac{\omega_1 - \omega_2}{K_1 - K_2} = \frac{d\omega}{dK}$$

$$\boxed{\omega = K v}$$

$$\omega = K \cdot v$$

$$v_g = \frac{d}{dK}(K v) = \frac{dK}{dK} \cdot v + K \frac{dv}{dK} = v + K \frac{dv}{dK}$$

Vácuo $\frac{dv}{dK} = 0$ $\quad \boxed{v_g = v}$

Dispersivo $\frac{dv}{dK} \neq 0$ $\quad v = \frac{c}{n}$

$$dv = \frac{0 \cdot n - c\,dn}{n^2} = \frac{-c}{n^2}\,dn$$

$$v_g = v + K\left(-\frac{c}{n^2}\right)\frac{dn}{dK} = v - \frac{vK}{n}\frac{dn}{dK}$$

$$\boxed{v_g = v\left(1 - \frac{K}{n}\frac{dn}{dK}\right)}$$

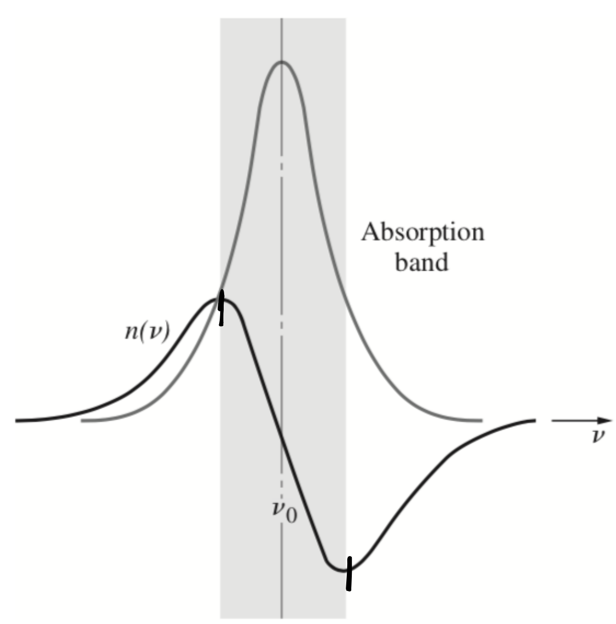


Figure 7.19 A typical representation of the frequency dependence of the index of refraction in the vicinity of an atomic resonance. Also shown is the absorption curve centered on the resonant frequency.

anômala → $\boxed{\frac{dn}{dK} < 0}$

$\boxed{v_g > v}$

normal $\boxed{\frac{dn}{dK} > 0}$

$\boxed{v_g < v}$

→ $n = n(\omega)$ → λ individual

→ n_g → grupo