2.1 Complementos a aula sobre movimento ondulatório

→ função de onda ⟹ descreve no (x, y, z, t) uma onda, podendo ser harmônico, ou pulso, etc

$$\psi = \psi_0 \operatorname{Sen}(Kx - \omega t + \phi)$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$ → Eq da onda, unidimensional

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Exercício: $\psi(y, t) = (y - 4t)^2$

a) $\psi(y,t)$ = Solução?

b) $v = ?$

c) direção, $+y$, $-y$,

$$\frac{\partial \psi}{\partial y} = 2(y - 4t)$$

$$\frac{\partial^2 \psi}{\partial y^2} = 2$$

$$\frac{\partial \psi}{\partial t} = 2(y - 4t) \cdot (-4)$$

$$= -8(y - 4t)$$

$$\frac{\partial^2 \psi}{\partial t^2} = (-8)(-4) = 32$$

$$\frac{\partial^2 \psi}{\partial y^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} \Rightarrow 2 = \frac{1}{v^2} \cdot 32 \qquad v^2 = 16 \qquad \boxed{v = \pm 4}$$

?

c) $\psi(y,t) = (\underbrace{y - 4t}_{})^2$

$t \rightarrow +$

$(-4t) \rightarrow$ cresce negativamente

$y \rightarrow$ cresce positivamente

$$v = \frac{\Delta y}{\Delta t} \Rightarrow \oplus \qquad \boxed{v = +4\hat{j}}$$

$$\psi(x,t) = \psi_0 \operatorname{Sen}(Kx - \omega t + \phi)$$

ϕ → fase inicial

argumento ⟹ φ ⟹ fase da onda

$$\varphi = Kx - \omega t + \phi$$

$$\left|\left(\frac{\partial \varphi}{\partial t}\right)_x\right| = \omega \quad \text{frequência temporal angular}$$

$$\boxed{\omega = \frac{2\pi}{T}}$$

$$\left|\left(\frac{\partial \varphi}{\partial x}\right)_t\right| = K \quad \text{número de onda}$$

frequência espacial (angular)

$$\boxed{K = \frac{2\pi}{\lambda}}$$

$$\frac{\partial \varphi}{\partial t} = -\omega$$

$$v = \left(\frac{\partial x}{\partial t}\right)_\varphi = \left(\frac{\partial x}{\partial \varphi}\right)\left(\frac{\partial \varphi}{\partial t}\right) = -\frac{\left(\frac{\partial \varphi}{\partial t}\right)}{\left(\frac{\partial \varphi}{\partial x}\right)} = \frac{-(-\omega)}{+K}$$

$$\frac{\partial \varphi}{\partial x} = +K$$

$$\psi = \psi_0 \operatorname{Sen}(Kx - \omega t + \phi)$$

$$v \to +\hat{i}$$

$$\boxed{v = \frac{\omega}{K}}$$

$$v = \frac{\cancel{2\pi} f}{\cancel{2\pi}/\lambda} \Rightarrow \boxed{v = \lambda f}$$

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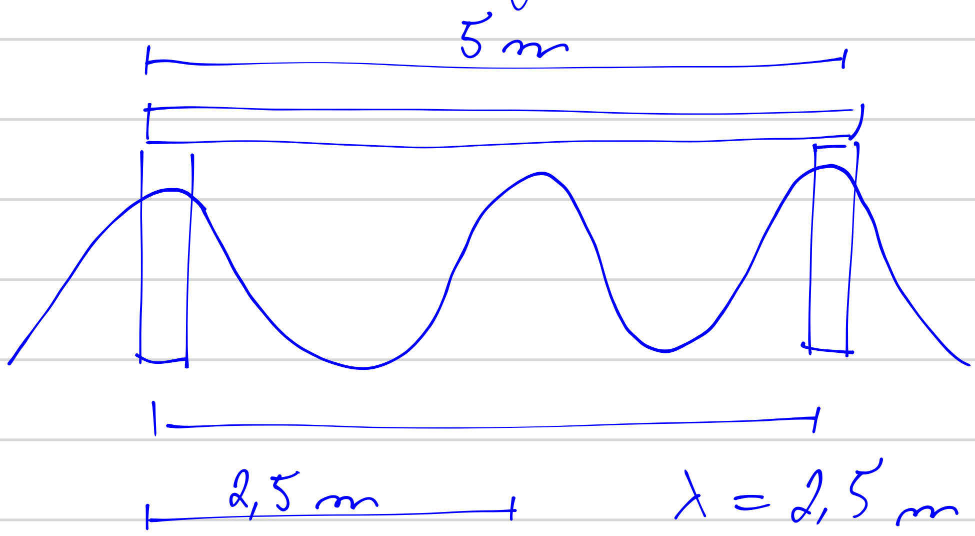
exercício 2.9 do Hecht

20 ondas em 10 segundos $\Rightarrow$ 2 ondas por Seg

$f = 2\ Hz$



$\lambda = 2{,}5\ m$

T = período temporal

f = frequência temporal

ω = " " angular

λ = período espacial → Comprimento de onda

K = frequência espacial angular

→ número de onda

$f = \frac{1}{T}$ $\quad T = \frac{1}{2} = 0{,}5\,s$ $\quad \mathbb{K} \equiv \frac{1}{\lambda}$ (não angular)

$\omega = \frac{2\pi}{T}$ $\quad \omega = \frac{2\pi}{0{,}5} = 4\pi \frac{rad}{s}$ $\quad K = \frac{2\pi}{\lambda}$ (angular)

$K = \frac{2\pi}{2{,}5} = \frac{4}{5}\pi \frac{rad}{m}$

$v = \lambda \cdot f = 2{,}5 \cdot 2 = 5\,m/s$

$\mathbb{K} = \frac{1}{2{,}5} = \frac{2}{5}\frac{1}{m} = \frac{2}{5}m^{-1}$

2.2º $\quad \psi = \psi(x,t) \Rightarrow E =$

amplitude $= 10^3\,V/m$

$T = 2{,}2 \times 10^{-15}\,s$

$v = 3\times10^8\,m/s \rightarrow$ onda eletromagnética

$-\hat{x}$ $\quad \psi(0,0) = 10^3\,V/m$

$$E(x,t) = E_0 \,Sen\left[Kx - \omega t + \phi\right] = E_0\,Sen\left[\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t + \phi\right]$$

$v = \lambda f$ $\quad \boxed{\lambda = v \cdot T}$ $\quad \omega = \frac{2\pi}{T} = \frac{2\pi}{2{,}2\times10^{-15}\,s}\frac{rad}{s}$

$E(0,0) = E_0 = E_0\,Sen(K\,0 - \omega \cdot 0 + \phi)$

$\phi = \pi/2$

$\lambda = (3 \cdot 10^8)(2{,}2 \times 10^{-15}\,s) =$

$$\boxed{E(x,t) = E_0\,Sen\left[\frac{2\pi}{vT}x - \frac{2\pi}{T}\cdot t + \frac{\pi}{2}\right]}$$

2.50 $\quad \vec{E} = \vec{E}_0 \, e^{i(3x - \sqrt{2}y - 9,9\times10^8 t)}$

$\vec{k} = ?$

$k = ?$

$\vec{v} = ?$

$\vec{E} = \vec{E}_0 \sin\left[3x - \sqrt{2}y - 9,9\times10^8 t\right] \qquad \phi = 0$

$\boxed{3x - \sqrt{2}y}$

$\omega = 9,9\times10^8 \, \frac{rad}{s}$

$\vec{k}\cdot\vec{r}$

$\vec{k} = k_x\hat{i} + k_y\hat{j} + \cancel{k_z\hat{k}}$

$\vec{r} = x\hat{i} + y\hat{j} + \cancel{z\hat{k}}$

$\vec{k}\cdot\vec{r}$

$k_x x + k_y y = 3x - \sqrt{2}y$

$$\begin{cases} k_x = 3 \\ k_y = -\sqrt{2} \\ k_z = 0 \end{cases}$$

$\vec{k} = 3\hat{i} - \sqrt{2}\hat{j}$

$k^2 = k_x^2 + k_y^2 + k_z^2 = 9 + 2$

$\boxed{k = \sqrt{11}}$

$v = \lambda f = \frac{2\pi}{k} f$

$v = \frac{\omega}{k} = \frac{9,9\cdot10^8 \ rad/s}{\sqrt{11} \ rad/m} \simeq 3\times10^8 \, m/s$

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passagem intermediária

$\frac{\partial\psi}{\partial x} = -\frac{1}{v}\frac{\partial\psi}{\partial t}$

$\boxed{\frac{\partial\psi}{\partial t} = [-v]\cdot\frac{\partial\psi}{\partial x}}$

1ª passagem

$\frac{\partial^2\psi}{\partial x^2} = \frac{+1}{v^2}\frac{\partial^2\psi}{\partial t^2}$

$\boxed{\frac{\partial^2\psi}{\partial t^2} = [+v^2]\cdot\frac{\partial^2\psi}{\partial x^2}}$

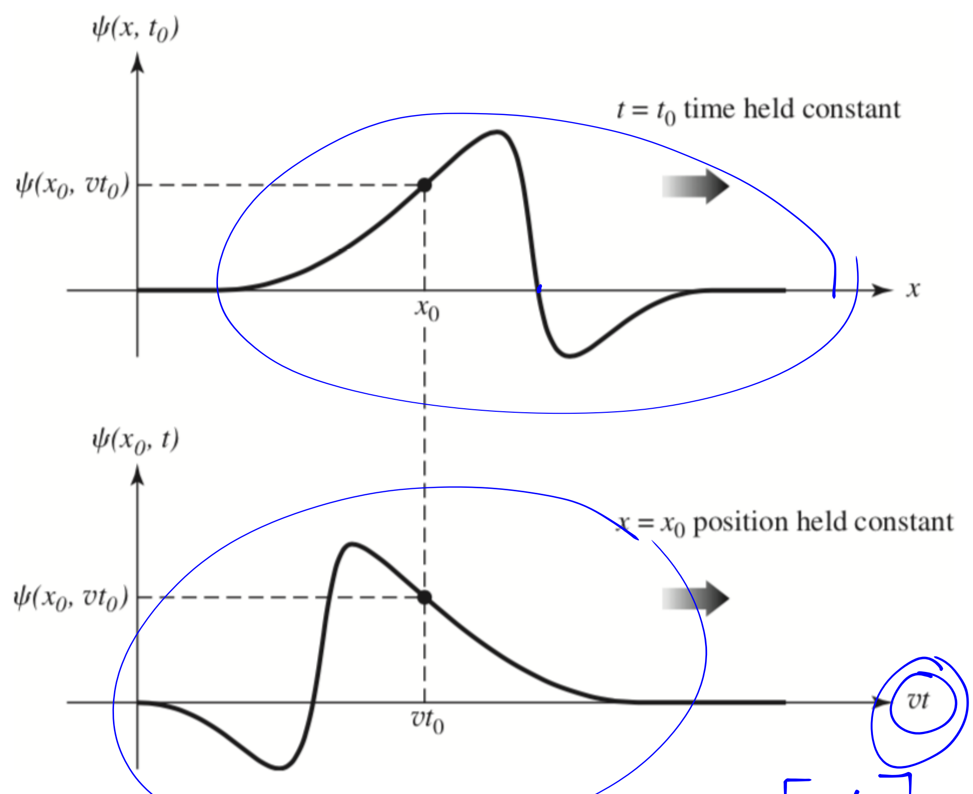


Figure 2.5 Variation of ψ with x and t.

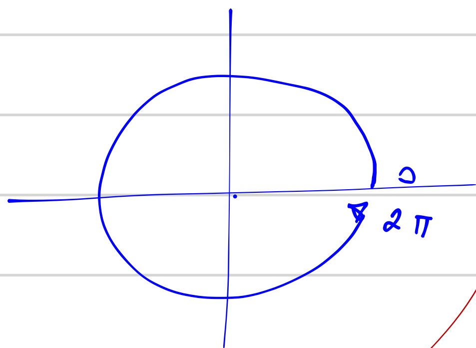
$[-v]$

Frequência espacial

$$\boxed{K = \frac{2\pi}{\lambda}} \quad \text{ou ainda} \quad \boxed{K \equiv \frac{1}{\lambda}} \quad (\text{ñ - angular})$$

$\Downarrow$

angular



$\lambda = 1\,m$

$x = 1\lambda$

$$\boxed{K = 2\pi \text{ rad}}$$

$\lambda = 0{,}5\,m$

$x = 1\,m = 2\lambda$

$$K = \frac{2\pi}{0{,}5} = 2\cdot(2\pi) = \boxed{4\pi}\ \frac{\text{rad}}{m}$$

$$K = \frac{\text{nº de onda}}{\lambda}$$

$\lambda = 1\,m$

$x \to 1\,m = 1\lambda$

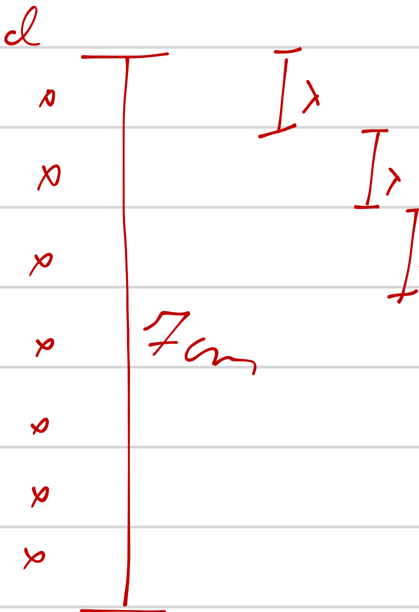
$$\boxed{K = 1\ m^{-1}}$$

$\lambda = 0{,}5\,m$

$x \to 1\,m \to 2\lambda$

$$K = \frac{2\lambda}{\lambda} = 2\ m^{-1}$$

$$K = \frac{1}{\lambda} = \left[\frac{1}{m}\right] \text{ Frequência espacial}$$



$\lambda = 1\,cm$

$$K_1 = \frac{7}{7\,cm} = \frac{1}{cm}$$

7 estruturas

$$\boxed{K = 1\ cm^{-1}}$$

$$K = \left(\frac{\text{nº de estruturas que se repetem}}{\text{comprimento}} \right)$$

$$K = \frac{2\pi}{\lambda} = \left[\frac{rad}{m}\right]$$

$x \to 1m$

$\lambda = 1m$

$$\frac{2\pi \cdot 1}{1m} = 2\pi$$

$x \to 1m$ $\quad 2 \cdot (2\pi) = 2 \cdot (2\pi)$

$\lambda = 0,5m$

$$\boxed{K = \frac{2\pi}{\lambda} = \frac{2\pi}{0,5}} \to \boxed{K = 4\pi \ \frac{rad}{m}}$$

λ

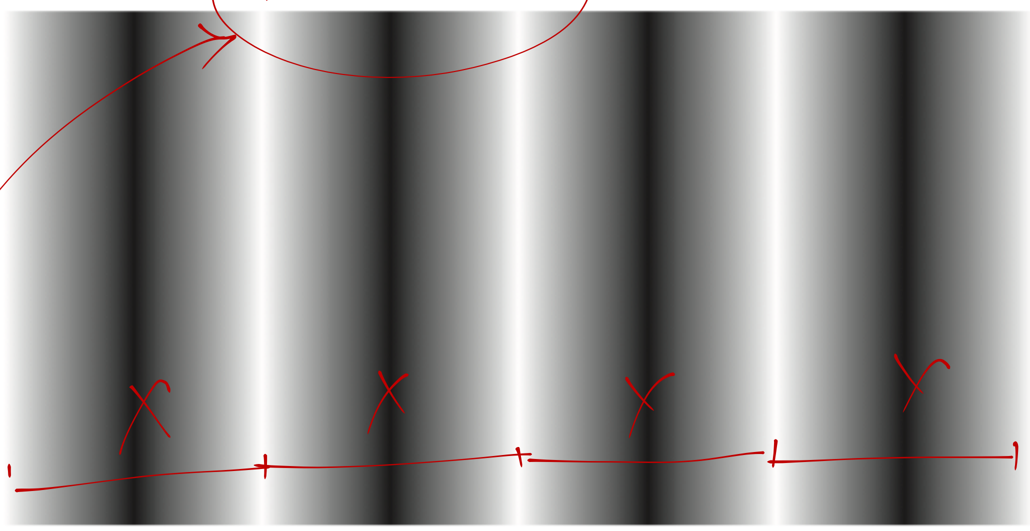


Figure 2.10 A sinusoidal brightness distribution of relatively low spatial frequency.

$x = 13\ cm$

4 repetições

$$K = \frac{4}{13\ cm} = 0,31\ cm$$

$$\boxed{K = 0,3\ cm^{-1}}$$

$\lambda = 3,2\ cm$

$$K = \frac{1}{\lambda} = \frac{1}{3,2\ cm}$$

$$\boxed{K = 0,31\ cm^{-1}}$$

$$\cancel{K = \frac{2\pi}{\lambda}}$$
