

## Chapter 6 Solutions

- 6.1** (6.7)  $M_T = (-s_{i1}/s_{o1})(-s_{i2}/s_{o2}) = -s_i/s_o$ . Let  $s_o = \infty$  so that  $s_o = s_{o1}$ ,  $s_{i1} = f_1$ ,  $s_{o2} = -(s_{i1} - d)$ ,  $s_i = f$ . Substituting into (6.7),

$$(-f_1/s_o)(-s_{i2}/-(s_{i1} - d)) = -f/s_o.$$

$$f = f_1(s_{i2}/(-s_{o2})) = f_1(s_{i2}/(s_{i1} - d)).$$

From

$$1/s_{o2} + 1/s_{i2} = 1/f_2; \quad 1/s_{i2} = 1/f_2 - 1/s_{o2}, \quad s_{i2} = s_{o2}f_2/(s_{o2} - f_2).$$

$$f = (-f_1/s_o)(s_{o2}f_2/(s_{o2} - f_2))$$

$$= -f_1f_2/(s_{o2} - f_2) = f_1f_2/(s_{i1} - d + f_2).$$

$$1/f = (s_{i1} - d + f_2)/f_1f_2 = 1/f_1 + (s_{i1} - d)/f_1f_2.$$

But  $s_{i1} = f_1$ , so,  $1/f = 1/f_1 + 1/f_2 - d/f_1f_2$ .

- 6.2** From Eq. (6.8),  $1/f = 1/f' + 1/f' - d/f'f' = 2/f' - 2/3f'$ ,  $f' = 3f'/4$ .

From Eq. (6.9),  $\overline{H_{11}H_1} = (3f'/4)(2f'/3)/f' = f'/2$ . From Eq. (6.10),

$$\overline{H_{22}H_2} = -(3f'/4)(2f'/3)/f' = -f'/2.$$

- 6.3** From Eq. (6.2),  $1/f = 0$  when  $-(1/R_1 - 1/R_2) = (n_l - 1)d/n_lR_1R_2$ . Thus  $d = n_l(R_1 - R_2)/(n_l - 1)$ .

**6.4** 
$$\frac{1}{f} = (n_2 - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{(n_2 - 1)d}{n_2R_1R_2} \right)$$

Taking  $d = 1$  cm,  $R_1 = 10$  cm and  $R_2 = -9$  cm,

$$\frac{1}{f} = 0.5 \left[ \frac{1}{10 \text{ cm}} - \frac{1}{9 \text{ cm}} + \frac{0.5(1 \text{ cm})}{1.5(10 \text{ cm})(9 \text{ cm})} \right]$$

$$\frac{1}{f} = 0.0037 \text{ cm}^{-1}$$

$$f = -270 \text{ cm}$$

- 6.5**  $1/f = 0.5[1/6 - 1/10 + 0.5(3)/1.5(6)(10)]$ ,  $f = +24$ ;  
 $h_1 = -24(0.5)(3)/10(1.5) = -2.4$ ,  $h_2 = -24(0.5)(3)/6(1.5) = -4$ .

- 6.6** Since  $|h_1| = |h_2|$  it follows from Eqs. (6.3) and (6.4) that  $-f(n_l - 1)d_l/R_2|n_l = -f(n_l - 1)d_l/R_1|n_l$  and  $|R_2| = |R_1|$  which means the lens is a sphere.

- 6.7**  $f = (1/2)nR/(n - 1)$ ;  $h_1 = +R$ ,  $h_2 = -R$ .

- 6.8** This is a thick lens with  $-R_2 = R_1 = R = 10$  cm and  $d = 2R = 20$  cm.

$$\begin{aligned} (6.2) \quad 1/f &= (n_l - 1)[1/R_1 - 1/R_2 + (n_l - 1)d/n_lR_1R_2] \\ &= (1.33 - 1)[1/10 - 1/(-10) + (1.33 - 1)(20)/1.33(10)(-10)]; \\ f &= 20.2 \text{ cm.} \end{aligned}$$

**6.9** From Problem (6.6) or (6.7), (6.2) becomes  $1/f = ((n_l - 1)/n_l)(2/R)$ , with  $R = +10$  cm.

$1/f = ((1.4 - 1)/1.4)(2/10) = 0.057 \text{ cm}^{-1}$ ,  $f = 17.5$  cm. (6.1)  $1/f = 1/s_o + 1/s_i$ , where  $s_o$  and  $s_i$  are measured from the principal planes.  $1/s_i = 1/f - 1/s_o = 1/(17.5) \text{ cm} - 1/(400 - 10) \text{ cm}$ ;  $s_i = 18.3$  cm.  
(6.7)  $M_T = -s_i/s_o = -18.3/390 = -.047$ . Image is real, inverted, and 0.047 times the size of the object.

**6.10** (6.2)  $1/f = (n_l - 1)[1/R_1 - 1/R_2 + (n_l - 1)d/n_l R_1 R_2]$ ;  
(1.5 - 1)[1/23 - 1/20 + (1.5 - 1)(9.0)/1.5(23)(20)] = 0,  
 $f = \infty$ .

Generally, if

$$1/f = 0, [1/R_1 - 1/R_2 + (n_l - 1)d/n_l R_1 R_2] = 0;$$

$$\frac{n_l R_2}{n_l R_1 R_2} - \frac{n_l R_1}{n_l R_1 R_2} + \frac{(n_l - 1)d}{n_l R_1 R_2} = 0$$

$$(n_l - 1)d = n_l(R_1 - R_2); (R_1 - R_2) = (n_l - 1)n_l d$$

for  $n_l = 1.5$ ,  $(R_1 - R_2) = ((1.5 - 1)/1.5)d = d/3$ .

**6.11**  $f = 29.6 + 4.0 = 33.6$  cm;  $s_o = 49.8 + 2 = 51.8$  cm;  $\frac{1}{51.8} + \frac{1}{s_i} = \frac{1}{33.6}$

$s_i = 95.6$  cm from  $H_2$  and 91.6 cm from the back face.

**6.13** From Eq. (6.2),

$$1/f = (1/2)[1/4.0 - 1/(-15) + (1/2)4.0/(3/2)(4.0)(-15)] = 0.147$$

and  $f = 6.8$  cm.  $h_1 = -(6.8)(1/2)(4.0)/(-15)(3/2) = +0.60$  cm, while  $h_2 = -2.3$ . To find the image  $1/(100.6) + 1/s_i = 1/(6.8)$ ;  $s_i = 7.3$  cm or 5 cm from the back face of the lens.

**6.14** For both,  $-R_2 = R = R_1$ , so (6.2) becomes:

$$\frac{1}{f} = (n - 1) \left[ \frac{2}{R} - (n - 1) \frac{d}{nR^2} \right]$$

$$\frac{1}{f} = (1.5 - 1) \left[ \frac{2}{50} - (1.5 - 1) \frac{5.0}{1.5(50)^2} \right]; f = 50.8 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = 26.7 \text{ cm}$$

**6.15** (6.8)  $1/f = 1/f_1 + 1/f_2 - d/f_1 f_2 = 1/(+20) + 1/(-20) - 10/(20)(-20)$ ;

$f = +40$  cm. The principal planes are found from (6.9) and (6.10).

$$(6.9) \overline{H_{11}H_1} = fd/f_2 = (+40)(10)/(-20) = -20 \text{ cm.}$$

$$(6.10) \overline{H_{22}H_2} = fd/f_1 = (+40)(10)/(20) = +20 \text{ cm.}$$

**6.16**  $R_1 = \begin{bmatrix} 1 & -D_1 \\ 0 & 1 \end{bmatrix}$  from (6.16) where

$$D_1 = (n - 1)/R_1 = (1.5 - 1)/2.5 \text{ cm} = 0.2 \text{ cm}^{-1}.$$

$$T_{21} = \begin{bmatrix} 1 & 0 \\ d_{21}/n & 1 \end{bmatrix} \quad (6.24)$$

$$= \begin{bmatrix} 1 & 0 \\ 1.2/1.5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.8 & 1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 1 & -D_2 \\ 0 & 1 \end{bmatrix}$$

but  $R_2 = \infty$ , so  $D_2 = 0$ .

$$(6.29) \quad A = R_2 T_{21} R_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.8 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.2 \\ 0.8 & 1.16 \end{bmatrix}$$

Check:  $|A| = 1(1.16) - 0.2(0.8) = 1$ .

$$\begin{aligned} 6.17 \quad R_1 &= \begin{bmatrix} 1 & -\left(\frac{n_{t1} - n_{i1}}{R_1}\right) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\left(\frac{1.81 - 1.00}{11.0}\right) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -0.0736 \\ 0 & 1 \end{bmatrix} \\ T_{21} &= \begin{bmatrix} 1 & 0 \\ \frac{d}{n} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3.00}{1.81} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1.657 & 1 \end{bmatrix} \\ R_2 &= \begin{bmatrix} 1 & -\left(\frac{n_{t2} - n_{i2}}{R_2}\right) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\left(\frac{1.00 - 1.81}{-120}\right) \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -6.75 \times 10^{-3} \\ 0 & 1 \end{bmatrix} \\ A &= R_2 T_{21} R_1 = \begin{bmatrix} 1 & -6.75 \times 10^{-3} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1.657 & 1 \end{bmatrix} \begin{bmatrix} 1 & -0.0736 \\ 0 & 1 \end{bmatrix} \\ A &= \begin{bmatrix} 0.989 & -6.75 \times 10^{-3} \\ 1.657 & 1 \end{bmatrix} \begin{bmatrix} 1 & -0.0736 \\ 1.657 & 1 \end{bmatrix} \\ A &= \begin{bmatrix} 0.989 & -0.080 \\ 1.657 & 0.876 \end{bmatrix} \end{aligned}$$

6.18 When the object and image are both in air,  $n_i = n_o = 1$ . Since the distance between the vertex and the object is negative, take  $d_o$  is taken to be a negative.

$$\begin{aligned} \begin{bmatrix} \alpha_I \\ y_I \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ d_I & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -d_o & 1 \end{bmatrix} \begin{bmatrix} \alpha_o \\ y_o \end{bmatrix} \\ \begin{bmatrix} \alpha_I \\ y_I \end{bmatrix} &= \begin{bmatrix} a_{11} & a_{12} \\ a_{11}d_I + a_{21} & a_{12}d_I + a_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -d_o & 1 \end{bmatrix} \begin{bmatrix} \alpha_o \\ y_o \end{bmatrix} \\ \begin{bmatrix} \alpha_I \\ y_I \end{bmatrix} &= \begin{bmatrix} a_{11} - d_o a_{12} & a_{12} \\ d_I(a_{11} - a_{12}d_o) + a_{21} - d_o a_{22} & a_{12}d_I + a_{22} \end{bmatrix} \begin{bmatrix} \alpha_o \\ y_o \end{bmatrix} \\ \begin{bmatrix} \alpha_I \\ y_I \end{bmatrix} &= \begin{bmatrix} \alpha_o(a_{11} - d_o a_{12}) + a_{12}y_o \\ \alpha_o[d_I(a_{11} - a_{12}d_o) + a_{21} - d_o a_{22}] + y_o(a_{12}d_I + a_{22}) \end{bmatrix} \\ y_I &= \alpha_o[d_I(a_{11} - a_{12}d_o) + a_{21} - d_o a_{22}] + y_o(a_{12}d_I + a_{22}) \end{aligned}$$

6.19 Equation (6.36) is:

$$d_{I2} = \frac{-a_{21} + a_{22}d_{10}}{a_{11} - a_{21}d_{10}}$$

Where the matrix  $A$  is given by equation (6.31):

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 - \frac{D_2 d_l}{n_l} & -D_1 - D_2 + \frac{D_1 D_2 d_l}{n_l} \\ \frac{d_l}{n_l} & 1 - \frac{D_1 d_l}{n_l} \end{bmatrix}$$

In the case of a thin lens,  $d_l \rightarrow 0$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & -D_1 - D_2 \\ 0 & 1 \end{bmatrix}$$

Then (6.36) becomes:

$$d_{l2} = \frac{0 + d_{lo}}{1 + (D_1 + D_2)d_{lo}}$$

Using  $D_1 = \frac{n_l - 1}{R_1}$  and  $D_2 = \frac{n_l - 1}{R_2}$ , then

$$(D_1 + D_2) = \frac{1}{f}$$

And,

$$\begin{aligned} d_{l2} &= \frac{d_{lo}}{1 + \frac{d_{lo}}{f}} \\ \frac{1}{S_o} + \frac{1}{S_l} &= \frac{1}{f} \\ 1 + \frac{S_o}{S_l} &= \frac{S_o}{f} \\ \frac{S_o}{S_l} &= \frac{S_o}{f} - 1 \\ S_l &= \frac{S_o}{\frac{S_o}{f} - 1} = \frac{-S_o}{1 - \frac{S_o}{f}} \end{aligned}$$

This is equivalent to  $d_{l2} = \frac{d_{lo}}{1 + \frac{d_{lo}}{f}}$ , when one keeps in mind the fact that when  $S_o > 0$ ,  $d_{lo} < 0$ .

## 6.20 Working in centimeters,

$$D_1 = (2.4 - 1.9)/R_1 = 0.1 \text{ cm}^{-1}, \quad D_2 = (1.9 - 2.4)/R_2 = -0.05 \text{ cm}^{-1}$$

therefore

$$\begin{aligned} A &= \begin{bmatrix} 1 & -D_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d/n_l & 1 \end{bmatrix} \begin{bmatrix} 1 & -D_1 \\ 0 & 1 \end{bmatrix} \\ A &= \begin{bmatrix} 1 & 0.05 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -0.1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1.02 & -0.052 \\ 0.4 & 0.96 \end{bmatrix} \end{aligned}$$

$$(1.02)(0.96) - (0.4)(-0.052) = 0.979 + 0.0208 = 1.$$

**6.21** We have

$$\det A = a_{11}a_{22} - a_{12}a_{21} = 1 - (D_1 + D_2)d_{21}/n_{t1} + D_1D_2d_{21}^2/n_{t1}^2 \\ + (D_1 + D_2)d_{21}/n_{t1} + D_1D_2d_{21}^2/n_{t1}^2 = 1.$$

**6.22**  $h_1 = n_{t1}(1 - a_{11})/(-a_{12}) = (D_2d_{21}/n_{t1})f = -(n_{t1} - 1)d_{21}f/R_2n_{t1}$ , from Eq. (5.70) where  $n_{t1} = n_t$ ;  $h_2 = n_{t2}(a_{22} - 1)/(-a_{12}) = -(D_1d_{21}/n_{t1})f$  from Eq. (5.71),  $h_2 = -(n_{t1} - 1)d_{21}f/R_1n_{t1}$ .

**6.23**  $A = R_2F_{21}R_1$ , but for the planar surface

$$R_2 = \begin{bmatrix} 1 & D_2 \\ 0 & 1 \end{bmatrix} \text{ and } D_2 = (n_{t1} - 1)/(-R_2) \text{ but } R_2 = \infty \quad R_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which is the unit matrix, hence  $A = F_{21}R_1$ .

**6.24**  $D_1 = (1.5 - 1)/0.5 = 1$  and  $D_2 = (1.5 - 1)/(-0.25) = 2$ ,  $A = \begin{bmatrix} 0.6 & -2.6 \\ 0.2 & 0.8 \end{bmatrix}$   
and  $|A| = 0.48 + 0.52 = 1$ .

**6.25** From the equation above (6.39),

$$-0.2 = -a_{12} = (n_{t1} - 1) \left\{ \frac{1}{R_1} + \frac{1}{R_2} \left[ \frac{(n_{t1} - d_{21})}{R_1n_{t1}} - 1 \right] \right\}.$$

Solving for the reciprocal of the second radius gives

$$\frac{1}{R_2} = - \left[ a_{12} + \frac{(n_{t1} - 1)}{R_1} \right] \frac{R_1n_{t1}}{(n_{t1} - 1)(n_{t1} - d_{21} - R_1n_{t1})} = 4 \text{ cm}^{-1}.$$

Then  $R_2 = 0.25 \text{ cm}$ .

**6.26** Starting with the three  $2 \times 2$  matrices from Eq. (6.33):

$$\begin{bmatrix} 1 & 0 \\ d_I & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -d_o & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{11}d_I + a_{21} & a_{12}d_I + a_{22} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -d_o & 1 \end{bmatrix} \\ = \begin{bmatrix} a_{11} - d_o a_{12} & a_{12} \\ d_I(a_{11} - a_{12}d_o) + a_{21} - d_o a_{22} & a_{12}d_I + a_{22} \end{bmatrix}$$

Then use Eq. (6.35):  $d_I(a_{11} - a_{12}d_o) + a_{21} - d_o a_{22} = 0$ , and Eq. (6.37)  $M_T = a_{12}d_I + a_{22}$ :

$$= \begin{bmatrix} a_{11} - d_o a_{12} & a_{12} \\ 0 & M_T \end{bmatrix}$$

Now take:

$$\det \begin{bmatrix} a_{11} - d_o a_{12} & a_{12} \\ 0 & M_T \end{bmatrix} = (a_{11} - d_o a_{12})M_T = 1 \\ M_T = \frac{1}{(a_{11} - d_o a_{12})}$$

**6.27**  $R_1 = \begin{bmatrix} 1 & -D_1 \\ 0 & 1 \end{bmatrix}$  from (6.16) where

$$D_1 = (n-1)/R_1 = (3/2-1)/-10.0 \text{ cm} = -0.050 \text{ cm}^{-1}.$$

$$T_{21} = \begin{bmatrix} 1 & 0 \\ d_{21}/n & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1.00/1.50 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.67 & 1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} 1 & -D_2 \\ 0 & 1 \end{bmatrix}$$

But  $R_2 = \infty$ , so  $D_2 = 0$ .

$$(6.29) \quad A = R_2 T_{21} R_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.67 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0.05 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.05 \\ 0.67 & 1.03 \end{bmatrix}$$

Check:  $|A| = 1(1.03) - (0.05)(0.67) = 1$ .

$$\begin{bmatrix} n_i \alpha_i \\ y_i \end{bmatrix} = A \begin{bmatrix} n_i \alpha_i \\ y_i \end{bmatrix}$$

$$\alpha_i = 0, n_i = 1, y_i = y_i.$$

$$\begin{bmatrix} 0 \\ y_i \end{bmatrix} = A \begin{bmatrix} \alpha_i \\ y_i \end{bmatrix} = \begin{bmatrix} 1 & 0.05 \\ 0.67 & 1.03 \end{bmatrix} \begin{bmatrix} \alpha_i \\ y_i \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ y_i \end{bmatrix} = \begin{bmatrix} \alpha_i + (0.05)y_i \\ (0.67)\alpha_i + (1.03)y_i \end{bmatrix}$$

$$0 = \alpha_i + (0.05)y_i, \quad y_i = (0.67)\alpha_i + (1.03)y_i,$$

both yield  $\alpha_i = (-0.05)(2.0) = -0.10$  or  $0.10$  radians above the axis.

**6.28** (6.34)  $1/f = -a_{12} = -(D_1 + D_2 - D_1 D_2 d/n_l);$

$$D_1 = (n_l - 1)/R_1 = (1.5 - 1)/0.5 = 1.0;$$

$$D_2 = (n_l - 1)/R_2 = (1.5 - 1)/(-0.25) = -2.0.$$

$$1/f = -(1.0 - 2.0 - (1.0)(2.0)(0.3)/1.5), \quad f = 0.71.$$

$$\overline{V_1 H_1} = (1)(1 - a_{11})/ -a_{12},$$

$$(6.36) \quad a_{11} = 1 - D_2 d/n_l = 1 - (-2.0)(0.3)/1.5 = 1.4.$$

$$\overline{V_1 H_1} = (1 - 1.4)/1.4 = -0.29.$$

$$(6.37) \quad \overline{V_2 H_2} = (1)(a_{22} - 1)/ -a_{12};$$

$$a_{22} = 1 - (D_1 d)/n_l = 1 - (1.0)(0.3)/1.5 = 0.8;$$

$$\overline{V_2 H_2} = (0.8 - 1)/1.4 = -0.14.$$

**6.29** For two reflections

$$\begin{bmatrix} -1 & -2/(+r) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -d & 1 \end{bmatrix} \begin{bmatrix} -1 & -2/(-r) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ d & 1 \end{bmatrix}$$

and this yields the desired matrix. When  $d = r$  the matrix for two traversals becomes

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

and for four it is

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and since this is a unit matrix the light ray is back to where it started.

**6.30** See E. Slayter, *Optical Methods in Biology*.

$$\overline{PC}/\overline{CA} = (n_1/n_2)R/R = n_1/n_2,$$

while  $\overline{CA}/\overline{P'C} = n_1/n_2$ . Therefore triangles  $ACP$  and  $ACP'$  are similar; using the sine law

$$\sin \angle P'AC/\overline{PC} = \sin \angle APC/\overline{CA}$$

or  $n_2 \sin \angle P'AC = n_1 \sin \angle APC$ , but  $\theta_i = \angle P'AC$ , thus

$$\theta_i = \angle APC = \angle P'AC,$$

and the refracted ray appears to come from  $P'$ .

**6.31** From Eq. (5.6), let  $\cos \varphi = 1 - \varphi^2/2$ ; then

$$l_o = [R^2 + (s_o + R)^2 - 2R(s_o + R) + R(s_o + R)\varphi^2]^{1/2},$$

$$l_o^{-1} = [s_o^2 + R(s_o + R)\varphi^2]^{-1/2}, \quad l_i^{-1} = [s_i^2 - R(s_i - R)\varphi^2]^{-1/2},$$

where the first two terms of the binomial series are used,

$$l_o^{-1} \approx s_o^{-1} - (s_o + R)h^2 / 2s_o^3 R$$

where

$$\varphi \approx h/R, \quad l_i^{-1} \approx s_i^{-1} + (s_i - R)h^2 / 2s_i^3 R.$$

Substituting into Eq. (5.5) leads to Eq. (6.40).

**6.33** Because (a) is symmetrical and looks like a somewhat altered Airy pattern; this is spherical aberration. (b) This pattern is asymmetrical as if the Airy system were pulled off to the side, so it corresponds to a little coma. (c) This pattern is asymmetrical along two axes and must be due to astigmatism.

**6.34** Fig. P.6.34a is bi-axially asymmetric and therefore corresponds to astigmatism. (b) is elongated along one axis and is due to coma, and because the pattern isn't very complicated there isn't much of it.