Chapter 12 Solutions

12.1 $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$

$I_{\max}$ occurs at $\varphi = 0$:

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$$

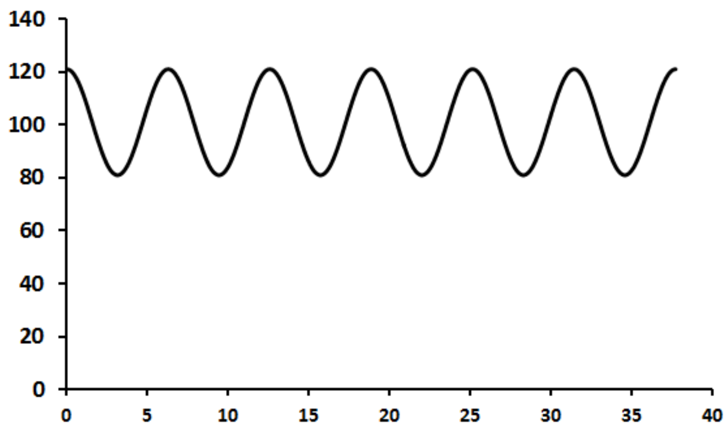
Since $I_1 = 100 I_2$:

$$I_{\max} = (10\sqrt{I_2} + \sqrt{I_2})^2 = 121\, I_2$$

$I_{\min}$ occurs at $\varphi = \pi$:

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = (\sqrt{I_1} - \sqrt{I_2})^2 = (10\sqrt{I_2} - \sqrt{I_2})^2 = 81 I_2$$

$$\upsilon = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{40\, I_2}{202\, I_2} = 0.198$$



12.2 $I = I_0 \cos^2\alpha + I_0 \cos^2(\alpha + \pi/2)$

Use $\cos(\alpha + \pi/2) = \cos\alpha\cos\pi/2 - \sin\alpha\sin\pi/2 = -\sin\alpha$

$$I = I_0\cos^2\alpha + I_0\sin^2\alpha = I_0$$

12.3 $A_c = \pi\left(\frac{d_c}{2}\right)^2 = \frac{\pi}{4}d_c^2 = \frac{\pi}{4}\left(\frac{\bar{\lambda}_0}{\theta_s}\right)^2 \sim \left(\frac{\bar{\lambda}_0}{\theta_s}\right)^2$

$A_s \sim d_s^2 = l^2\theta_s^2$

$\theta_s = \frac{d_s}{l}$

$A_c \sim \left(\frac{l\bar{\lambda}_0}{d_s}\right)^2 = \frac{(l\bar{\lambda}_0)^2}{A_s}$

12.4 $A_c \sim \left(\frac{l\bar{\lambda}_0}{A_s}\right)^2 = \frac{(2.0\text{ m})^2(5.00\times10^{-7}\text{ m})^2}{(1.00\times10^{-6}\text{ m}^2)}$

$A_c \sim 1.00\times10^{-6}\text{ m}^2$

12.5 Starting with the result from problem 12.3,

$$A_c \sim \left(\frac{l\bar{\lambda}_0}{d_s}\right)^2 = \frac{(l\bar{\lambda}_0)^2}{A_s}$$

Use, $A_s = l^2\Omega_s$:

$$A_c \sim \frac{(l\bar{\lambda}_0)^2}{A_s} = \frac{l^2\bar{\lambda}_0^2}{l^2\Omega_s}$$

$$A_c \sim \frac{\bar{\lambda}_0^2}{\Omega_s}$$

12.6 $A_c \sim \frac{\bar{\lambda}_0^2}{\Omega_s}$

$\Omega_s = \frac{A_s}{l^2} = \frac{\pi r^2}{l^2}$

Using $r = l\theta$,

$$\Omega_s = \frac{\pi r^2}{l^2} = \frac{\pi l^2\theta^2}{l^2} = \pi\theta^2$$

$$A_c \sim \frac{\bar{\lambda}_0^2}{\pi\theta^2} = A_c \sim \frac{\bar{\lambda}_0^2}{\Omega_s} = \frac{(5.50\times10^{-7}\text{ m})^2}{\pi\left(\frac{9.3\times10^{-3}\text{ rad}}{2}\right)^2} = 4.45\times10^{-9}\text{ m}^2 = 4.45\times10^{-3}\text{ mm}^2$$

12.7 $\Omega_s = \frac{A_s}{l^2}$

Using Eq. (12.3):

$$A_c \sim \frac{(l\bar{\lambda}_0)^2}{A_s}$$

$$\Omega_s = \frac{A_s}{l^2} = \frac{(l\bar{\lambda}_0)^2}{l^2A_c} = \frac{\bar{\lambda}_0^2}{A_c}$$

12.8 At low pressures, the intensity emitted from the lamp is low, the bandwidth is narrow, and the coherence length is large. The fringes will initially display a high contrast, although they'll be fairly faint. As the pressure builds, the coherence length will decrease, the contrast will drop off, and the fringes might even vanish entirely.

12.9 Over a long time interval, $E_1 \times E_2$ averages to zero. So, $\langle(E_1+E_2)^2\rangle_T \approx \langle E_1^2\rangle_T + \langle E_2^2\rangle_T$.

12.10 The net irradiance becomes more uniform as more waves are added. There will be a less distinct pattern, which corresponds to a smaller coherence length. The irradiance will become constant as the bandwidth goes to infinity.

12.11 Each sine function in the signal produces a cosinusoidal autocorrelation function with its own wavelength and amplitude. All of these are in phase at the zero delay point corresponding to $\tau = 0$. Beyond that origin the cosines soon fall out of phase, producing a jumble where destructive interference is more likely. (The same sort of thing happens when, say, a square pulse is synthesized out of sinusoids—everywhere beyond the pulse all the contributions cancel.) As the number of components increases and the signal becomes more complex—resembling random noise—the autocorrelation narrows, ultimately becoming a δ-spike at $\tau = 0$.

12.12 (12.1) $\upsilon = (I_{\max} - I_{\min})/(I_{\max} + I_{\min}) = 2|\text{sinc}(a\pi w/s\lambda)|/2$ (from 12.8, 12.9), $= |\text{sinc}(5\times 10^{-4}\pi/1\times 10^{-3})| = \text{sinc}(\pi/2) = 0.64$.

12.13 The irradiance at Σ_0 arising from a point source is $4I_0 \cos^2(\delta/2) = 2I_0(1+\cos\delta)$. For a differential source element of width *dy* at point S', y from the axis, the OPD to *P* at *Y* via the two slits is $\Lambda = (\overline{S'S_1} + \overline{S_1P}) - (\overline{S'S_2} + \overline{S_2P} = (\overline{S'S_1} - \overline{S'S_2}) + (\overline{S_1P} - \overline{S_2P} = ay/l + aY/s$ from Section 9.3. The contribution to the irradiance from *dy* is then $dI \propto (1+\cos k\Lambda)\,dy$, $I \propto \int_{-b/2}^{b/2} (1+\cos k\Lambda)dy$,

$$I \propto b + \frac{d}{ka}\left[\sin\left(\frac{aY}{s} + \frac{ab}{2l}\right) - \sin\left(\frac{aY}{s} - \frac{ab}{2l}\right)\right] I \propto b$$

$$+(d/ka)\left[\sin(kaY/s)\cos(kab/2l) + \cos(kaY/s)\sin(kab/2l)\right.$$
$$\left. -\sin(kaY/s)\cos(kab/2l) + \cos(kaY/s)\sin(kab/2l)\right],$$

$$I \propto b + (2l/ka)\sin(kab/2l)\cos(kaY/s).$$

12.14 $\upsilon = (I_{\max} - I_{\min})/(I_{\max} + I_{\min})$, $\quad I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\tilde{\gamma}_{12}|$,
$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}\,|\tilde{\gamma}_{12}|$, $\quad \nu = 4\sqrt{I_1 I_2}\,|\tilde{\gamma}_{12}|/2(I_1 + I_2)$.

12.15 When $S''S_1O' - S'S_1O' = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots,$ the irradiance due to S' is given by $I' = 4I_0 \cos^2(\delta'/2) = 2I_0(1+\cos\delta')$, while the irradiance due to S'' is $I'' = 4I_0\cos^2(\delta''/2) = 4I_0\cos^2(\delta'+\pi)/2 = 2I_0(1-\cos\delta')$. Hence $I' + I'' = 4I_0$.

12.16 Fringes disappear when $w = s\lambda/a$ so, $a = \lambda(s/w)$, from Figure 12.3,

$$l/b = s/w, \quad a = \lambda(l/b) = (5.893\times 10^{-7}\,\text{m})(1\,\text{m})/(1\times 10^{-4}\,\text{m})$$
$$= 5.893\times 10^{-3}\,\text{m}.$$

12.17 $d_c = \dfrac{1.22\bar{\lambda}_0}{\theta_s}$

For small angles, $\theta_s \approx 0.1\,\text{mm}/1\,\text{m} = 10^{-4}$ rad

$$d_c = \frac{1.22(5.00\times 10^{-7}\text{ m})}{10^{-4}\text{ rad}} = 6.1\times 10^{-3}\text{ m}$$

12.18 $I_1(t) = \Delta I_1(t) + \langle I_1 \rangle$; hence

$$\langle I_1(t+\tau) I_2(t) \rangle = \langle [\langle I_1 \rangle + \Delta I_1(t+\tau)][\langle I_2 \rangle + \Delta I_2(t)] \rangle$$

since $\langle I_1 \rangle$ is independent of time.

$$\langle I_1(t+\tau) I_2(t) \rangle = \langle I_1 \rangle \langle I_2 \rangle + \langle \Delta I_1(t+\tau) \Delta I_2(t) \rangle$$

if we recall that $\langle \Delta I_1(t) \rangle = 0$. Eq. (12.34) follows by comparison with Eq. (12.32).

12.20 From Eq. (12.22), $\upsilon = 2\sqrt{(10I)I}/(10I + I) = 2\sqrt{10}/11 = 0.57.$

12.21 Fringes disappear when $w = s\lambda/a$, so, $a = \lambda(s/w)$, from Figure 12.3,
$l/b = s/w$ where l = (mean) distance to sun; b = diameter of sun.
$a = \lambda(l/b) = [(5.50 \times 10^{-7}\text{ m})(1.50 \times 10^{11}\text{ m})]/2(6.96 \times 10^{8}\text{ m}) = 5.93 \times 10^{-5}\text{ m}.$

12.22 $\upsilon = \text{sinc}\left(\dfrac{a\pi b}{l\lambda}\right) = 0$

$$\frac{a\pi b}{l\lambda} = \pi$$

$$\frac{ab}{l\lambda} = 1$$

$$a = \frac{l\lambda}{b} = \frac{(1\text{ m})(5.00 \times 10^{-7}\text{ m})}{0.1 \times 10^{-3}\text{ m}} = 5\text{ mm}$$

12.23 $\upsilon = \left|\text{sinc}\left(\dfrac{a\pi b}{l\lambda}\right)\right| = 0.9$

Since $\text{sinc}(\pi/4) = 0.9$,

$$\frac{a\pi b}{l\lambda} = \frac{\pi}{4}$$

$$b = \frac{l\lambda}{4a} = \frac{(1\text{ m})(5.50 \times 10^{-7}\text{ m})}{4(2.0 \times 10^{-4}\text{ m})} = 6.875 \times 10^{-4}\text{ m} \approx 0.69\text{ mm}$$

12.24 Using the van Cittert-Zernike theorem, we can find $\tilde{\gamma}_{12}(0)$ from the diffraction pattern over the apertures, and that will yield the visibility on the observation plane: $\upsilon = |\tilde{\gamma}_{12}(0)| = |\text{sinc}\beta|$.
From Table 1, $\sin u/u = 0.85$ when $u = 0.97$, hence $\pi by/l\lambda = 0.97$, and if $y = \overline{P_1P_2} = 0.50$ mm, then

$$b = 0.97(l\lambda/\pi y) = 0.97(1.5\text{ m})(500 \times 10^{-9}\text{ m})/\pi(0.50 \times 10^{-3}\text{ m})$$
$$= 0.46\text{ mm}.$$

12.25 (12.23) $\upsilon = |\tilde{\gamma}_{12}(\tau)|$.

(12.1) $\upsilon = (I_{max} - I_{min})/(I_{max} + I_{min}) = 2|\text{sinc}(a\pi w/s\lambda)|/2$ (from 12.8, 12.9),
$\upsilon = 0.90 = |\text{sinc}(a\pi w/s\lambda)| = |\text{sinc}(a\pi(1.0 \times 10^{-3}\text{ m})/(10.0\text{ m})(5.00 \times 10^{-7}\text{m}))| = |\text{sinc}(200\pi a)|$;
$\sin x \simeq x - x^2/3!$,
so $\text{sinc}(x) \simeq 1 - x^2/3!$; $0.90 = 1 - [(200\pi a)^2/6]$; $a = 1.23 \times 10^{-3}$ m.

12.26 $\upsilon = |\text{sinc}\,(a\pi b/l\lambda)|$; as shown in Figure 12.6, υ is a minimum when $(a\pi b/l\lambda) = m\pi$, $(m \neq 0)$. $b/l \simeq \sin(\alpha_2 - \alpha_1) \simeq (\alpha_2 - \alpha_1)$ for small angle, so minimum υ when $[a(\alpha_2 - \alpha_1)\pi/\lambda] = m\pi$; $a(\alpha_2 - \alpha_1) = m\lambda$.

12.27 From the van Cittert-Zernike theorem, the degree of coherence can be obtained from the Fourier transform of the source function, which itself is a series of δ-functions corresponding to a diffraction grating with spacing *a*, where $a \sin \theta_m = m\lambda$. The coherence function is therefore also a series of δ-functions. Hence the $\overline{P_1P_2}$, the slit separation *d*, must correspond to the location of the first-order diffraction fringe of the source if υ is to be maximum. $a\theta_1 \approx \lambda$, and so

$$d \approx l\theta_1 \approx \lambda l/a = (500\times10^{-9}\,\text{m})(2.0\text{ m})/(500\times10^{-6}\,\text{m}) = 2.0\text{ mm}.$$

12.28 $h = 0.32\dfrac{R\bar{\lambda}_0}{D} = 0.32\bar{\lambda}/\theta_s$

Use $\theta = 9.3\times10^{-3}$ rad, $\bar{\lambda} = 550$ nm

$$h = 0.32(5.5\times10^{-7}\,\text{m})/(9.3\times10^{-3})$$
$$= 1.89\times10^{-5}\,\text{m}$$

12.29 From example 12.1, $\theta_s = 0.0093$ rad, $\bar{\lambda}_0 = 5.00\times10^{-7}\,\text{m}$

$$h \approx 0.32\frac{\bar{\lambda}_0}{\theta_s} = 0.32\frac{5.00\times10^{-7}\,\text{m}}{0.0093\text{ rad}} = 1.7\times10^{-5}\,\text{m} = 0.017\text{ mm}$$

Compare this to the result in the example ~0.05 mm.

12.30 Two stars:

$$\theta_s = \left(m + \frac{1}{2}\right)\frac{\bar{\lambda}_0}{h}$$

From Eq. (12.29), one star:

$$\theta_s = 1.22\frac{\bar{\lambda}_0}{h}$$