Chapter 7 Solutions

7.1 $E_0^2 = 36 + 64 + 2(6)(8)\cos \pi/2 = 100$, $E_0 = 10$; $\tan \alpha = 8/6$, $\alpha = 53.1° = 0.93$ rad.
$E = 10\ \sin(120\pi t + 0.93)$.

7.2 $E_1 = E_{01}\cos(\omega t)$; $E_2 = E_{01}\cos(\omega t + \alpha_2)$.

$$\begin{aligned} E &= E_1 + E_2 = E_{01}\cos\omega t + E_{01}\cos(\omega t + \alpha_2) \\ &= E_{01}(2\cos\tfrac{1}{2}(\omega t + \omega t + \alpha_2)\cos\tfrac{1}{2}(\omega t - \omega t - \alpha_2)) \\ &= 2E_{01}\cos(\omega t + \alpha_2/2)\ \cos(-\alpha_2/2). \end{aligned}$$

Recall $\cos(-\theta) = \cos\theta$, so,

$$E = (2E_{01}\cos(\alpha_2/2))(\cos(\omega t + \alpha_2/2)) = E_0\cos(\omega t + \alpha).$$

To show that this follows from (7.9) and (7.10), recall that $\cos\theta = \sin(\theta + \pi/2)$ so that

$$\alpha_1 \to \alpha_1 + \pi/2 = \pi/2,\ \alpha_2 \to \alpha_2 + \pi/2.$$

7.3 In phase: $\alpha_1 = \alpha_2\ \cos(\alpha_2 - \alpha_1) = \cos(0) = 1$.

$$\begin{aligned} (7.9)\ E_0^2 &= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1) \\ &= E_{01}^2 + E_{02}^2 + 2E_{01}E_{02} = (E_{01} + E_{02})^2. \end{aligned}$$

Out of phase, $\alpha_2 - \alpha_1 = \pi$, $\cos(\alpha_2 - \alpha_1) = \cos\pi = -1$.

$$(7.9)\ E_0^2 = E_{01}^2 + E_{02}^2 - 2E_{01}E_{02} = (E_{01} - E_{02})^2.$$

7.4 $OPL = \Sigma_i n_i x_i = \Sigma(c/\upsilon_i)x_i = \Sigma_i ct_i$, where t_i is the time spent in medium *i*. But ct_i is also the distance the light would travel, in vacuum.

7.5 $1\text{ m}/500\text{ nm} = 0.2 \times 10^7 = 2{,}000{,}000$ waves. In the glass

$$0.05/\lambda_0/n = 0.05(1.5)/500\text{ nm} = 1.5 \times 10^5;$$

in air

$$0.95/\lambda_0 = 0.19 \times 10^7;$$

total 2,050,000 waves.

$$\begin{aligned} OPD &= [(1.5)(0.05) + (1)(0.95)] - (1)(1), \\ OPD &= 1.025 - 1.000 = 0.025\text{ m}, \\ \Lambda/\lambda_0 &= 0.025/500\text{ nm} = 5 \times 10^4\text{ waves}. \end{aligned}$$

7.6 $OPL_B = nx = (1.00)(100\text{ cm}) = 100\text{ cm} = 1.00\text{ m}$.

$$\begin{aligned} OPL_A &= \Sigma_i n_i x_i = (1.00)(89\text{ cm}) + 2(1.52)(0.5\text{ cm}) \\ &\quad + (1.33)(10\text{ cm}) = 103.82\text{ cm} = 1.0382\text{ m}. \end{aligned}$$

$$\Lambda = OPL_A - OPL_B = 1.0382 - 1.00 = .00382\text{ m}.$$

$$\begin{aligned} (7.16)\ \delta &= k_0\Lambda = (2\pi/\lambda_0)\Lambda = 2\pi(3.82 \times 10^{-3}\text{ m})/5.00 \times 10^{-7}\text{ m} \\ &= 1.28 \times 10^4\pi. \end{aligned}$$

An integer multiple of 2π, so waves are in phase.

7.7 $E_1 = E_{01}\sin[\omega t - k(x+\Delta x)]$, so $\alpha_1 = -k(x+\Delta x)$. $E_2 = E_{01}\sin[\omega t - kx]$,
So $\alpha_2 = -kx$.

(7.9) $$E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$$
$$= E_{01}^2 + E_{01}^2 + 2E_{01}^2\cos(-kx - (-k(x+\Delta x))) = 2E_{01}^2(1+\cos k\Delta x)$$
$$= 2E_{01}^2(\cos(0)+\cos(k\Delta x)) = 4E_{01}^2\cos^2(k\Delta x/2),$$

(see Problem 7.2),

$$E_0 = 2E_{01}\cos(k\Delta x/2).$$

(7.10) $$\tan\alpha = \frac{E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2}{E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2}$$
$$= \frac{E_{01}\sin(-k(x+\Delta x) + E_{01}\sin(-kx))}{E_{01}\cos(-k(x+\Delta x)) + E_{01}\cos(-kx)}$$
$$= \frac{2\sin\frac{1}{2}(-k(x+\Delta x)-kx)\ \cos\frac{1}{2}(-k(x+\Delta x)+kx)}{2\cos\frac{1}{2}(-k(x+\Delta x)-kx)\ \cos\frac{1}{2}(-k(x+\Delta x)+kx)}$$
$$= \tan(-kx-(k\Delta x/2)),\quad \alpha = -k(x+(\Delta x/2)).$$

7.8 $E = E_1 + E_2 = E_{01}\{\sin[\omega t - k(x+\Delta x)] + \sin(\omega t - kx)\}$. Since

$$\sin\beta + \sin\gamma = 2\sin(1/2)(\beta+\gamma)\cos(1/2)(\beta-\gamma),$$
$$E = 2E_{01}\cos(k\Delta x/2)\sin[\omega t - k(x+\Delta x/2)].$$

7.9 $E = E_0\,\mathrm{Re}\left[e^{i(kx+\omega t)} - e^{i(kx-\omega t)}\right] = E_0\,\mathrm{Re}\left[e^{ikx}2i\sin\omega t\right]$
$= E_0\,\mathrm{Re}[2i\cos kx\sin\omega t - 2\sin kx\sin\omega t] = -2E_0\sin kx\sin\omega t.$

Standing wave with node at $x = 0$.

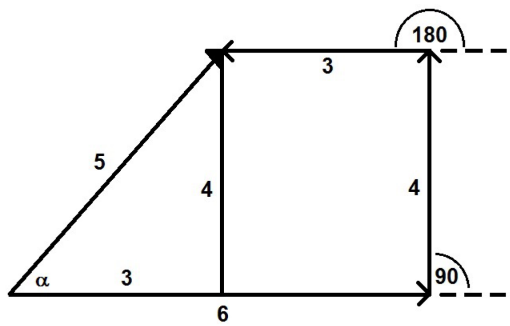
7.10 $E_i = 3\cos\omega t = 3\angle 0$, $(\alpha_1 = 0)$. $E_2 = 4\sin\omega t$, but $\sin\theta = \cos(\theta - \pi/2)$, so

$$E_2 = 4\cos(\omega t - \pi/2) = 4\angle -\pi/2. \quad E_3 = E_1 + E_2.$$
$$E_{3o}^2 = E_{01}^2 + E_{02}^2 + 2E_{01}E_{02}\cos(\alpha_2 - \alpha_1)$$
$$= 9 + 16 + 2(3)(4)\cos(-\pi/2),\ E_{3o} = 5.$$

(7.10) $\tan\alpha = (E_{01}\sin\alpha_1 + E_{02}\sin\alpha_2)/(E_{01}\cos\alpha_1 + E_{02}\cos\alpha_2)$
$= (3(0)+4(-1))/(3(1)+4(0)) = -4/3;\ \alpha = -53°;$

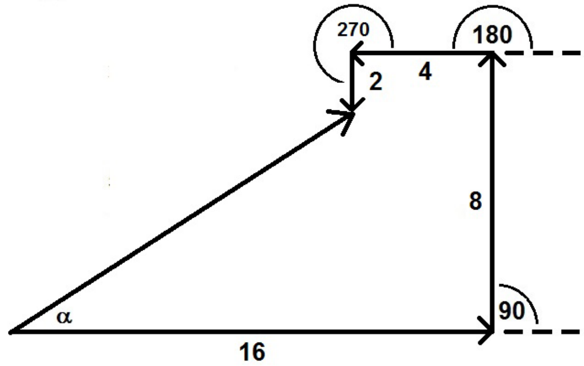
so $\varphi = 53° = 0.93$ rad. Note that $\alpha_1 < \varphi$, so E_1 leads E_3.

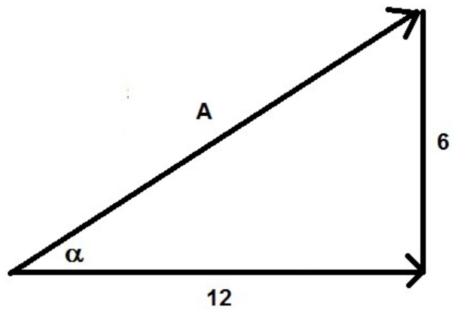
7.11



This is a 3 – 4 – 5 right triangle, thus A = 5,   = 53°.

7.12





$A = \sqrt{12^2 + 6^2} = 13.4$

$\alpha = \tan^{-1}\dfrac{6}{12} = 27°$

7.13 By Faraday's law, $\partial E/\partial x = -\partial B/\partial x$. Integrate to get

$$B(x,\,t) = -\int(\partial E/\partial x)\;dt = -2E_0 k\cos kx\int\cos\omega t\,dt$$
$$= -2E_0(k/\omega)\cos kx\sin\omega t.$$

But $E_0 k/\omega = E_0/c = B_0$; thus $B(x, t) = -2B_0 \cos kx \sin \omega t$.

7.14 Fringes are spaces $\lambda/2$ vertically.

$$\sin\theta = \text{(fringes/cm) vertical/(fringes/cm) on film;}$$

$$\text{(fringes/cm) on film} = (1/(\lambda/2))/\sin\theta$$
$$= (1/5.50\times10^{-7}\text{ cm})/\sin(1°) = 1.04\times10^{8}\text{ cm}^{-1}.$$

7.15 Nodes are spaced at $\lambda/2$ apart.

$$c = \nu\lambda, \qquad \lambda = c/\nu = (3\times10^8\,\text{m/s})/(10^{10}\text{ Hz}) = 0.03\text{ m}.$$

Node spacing is .015 m.

7.16 (7.30) E (standing wave) = $2E_{ol}\sin kx\cos\omega t$ from two wave,

$$E_I = E_{0I}\sin(kx+\omega t);\quad E_R = E_{0I}\sin(kx-\omega t),$$

so,

$$E_I = 50\sin(\tfrac{2}{3}\pi x + 5\pi t);\quad E_R = 50\sin(\tfrac{2}{3}\pi x - 5\pi t).$$

7.17 $E_I = E_0 \sin(kx \mp \omega t)$

$E_R = \rho E_0 \sin(kx \pm \omega t)$

$$
\begin{aligned}
E = E_I + E_R &= E_0 \sin(kx \mp \omega t) + \rho E_0 \sin(kx \pm \omega t)\\
&= E_0[\sin kx \cos \omega t \mp \cos kx \sin \omega t + \rho \sin kx \cos \omega t \pm \rho \cos kx \sin \omega t]\\
&= E_0[(1+\rho)\sin kx \cos \omega t \mp (1-\rho)\cos kx \sin \omega t]\\
&= E_0[(1+\rho)\sin kx \cos \omega t + (1-\rho)(\sin(kx \mp \omega t) - \sin kx \cos \omega t)]\\
&= E_0[(\cancel{1}+\rho)\sin kx \cos \omega t - \cancel{\sin kx \cos \omega t} + \rho \sin kx \cos \omega t \pm (1-\rho)\sin(kx \mp \omega t)]\\
&= 2E_0 \rho \sin kx \cos \omega t + (1-\rho)E_0 \sin(kx \mp \omega t)
\end{aligned}
$$

7.18 Hear beat frequency = $\nu_2 - \nu_1 = 2$ Hz.

7.19 One can see that the relative phase of the two waves varies, and that a maximum occurs (positive or negative), and that a zero occurs when the relative phase is $\pm n\pi$ (n odd). Also at the maxima, the relative phase between one wave and the net wave is zero. At those zeroes where the relative phase between one wave and the net wave is $\pi/2$, the "faster" wave "laps" the slower one, and the relative phase changes abruptly.

7.20 $E_1 = E_{01} \cos[(k_c + \Delta k)x - (\omega_c + \Delta\omega)t]$;
$E_2 = E_{01} \cos[(k_c - \Delta k)x - (\omega_c - \Delta\omega)t]$;

$$
\begin{aligned}
E = E_1 + E_2 = 2E_{01} &\cos\frac{1}{2}[(k_c + \Delta k)x - (\omega_c + \Delta\omega)t + (k_c - \Delta k)x - (\omega_c - \Delta\omega)t] \times\\
&\cos\frac{1}{2}[(k_c + \Delta k)x - (\omega_c + \Delta\omega)t - (k_c - \Delta k)x + (\omega_c - \Delta\omega)t]\\
= 2E_{01}&[\cos(k_c x - \omega_c t)\cos(\Delta k x - \Delta\omega t)]
\end{aligned}
$$

so that $k_c = \bar{k}, \omega_c = \bar{\omega}, \Delta k = k_m, \Delta\omega = \omega_m$. Wavelength of envelope $\lambda_m = 2\pi/k_m = 2\pi/\Delta k$. Period of envelope $T_m = 2\pi/\omega_m = 2\pi/\Delta\omega$. Speed of envelope $\lambda_m/T_m = (2\pi/\Delta k)/(2\pi/\Delta\omega)$.

7.21
$$
\begin{aligned}
E &= E_0 \cos\omega_c t + E_0 \alpha \cos\omega_m t \cos\omega_c t\\
&= E_0 \cos\omega_c t + (E_0\alpha/2)[\cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t].
\end{aligned}
$$

Audible range $\nu_m = 20$ Hz to 20×10^3 Hz. Maximum modulation frequency $\nu_m(\max) = 20 \times 10^3$ Hz. $\nu_c - \nu_m(\max) \le \nu \le \nu_c + \nu_m(\max)$,
$\Delta\nu = 2\nu_m(\max) = 40 \times 10^3$ Hz.

7.22 $\upsilon = \omega/k = ak$, $\upsilon_g = d\omega/dk = 2ak = 2\upsilon$.

7.23 $\upsilon_g = \dfrac{d\omega}{dk} = \left(\dfrac{d\omega}{d\nu}\right)\left(\dfrac{d\nu}{d\lambda}\right)\left(\dfrac{d\lambda}{dk}\right)$

Use the fact that $\dfrac{d\omega}{d\nu} = 2\pi$, and $\dfrac{dk}{d\lambda} = \dfrac{-2\pi}{\lambda^2}$:

$$\upsilon_g = (2\pi)\left(\frac{d\nu}{d\lambda}\right)\left(\frac{\lambda^2}{-2\pi}\right) = -\lambda^2 \frac{d\nu}{d\lambda}$$

7.24 $\upsilon_g = \dfrac{d\omega}{dk} = \dfrac{d(2\pi\nu)}{d(2\pi/\lambda)} = \dfrac{d\nu}{d(1/\lambda)} = \dfrac{d}{d(1/\lambda)}\left(\dfrac{c}{n}\dfrac{1}{\lambda}\right)$

$$\upsilon_g = \frac{c}{n} + \frac{c}{\lambda}\frac{d(1/n)}{d(1/\lambda)}$$

7.25 $\upsilon_g = \dfrac{c}{n} + \dfrac{c}{\lambda}\dfrac{d(1/n)}{d(1/\lambda)} = \upsilon + \dfrac{c}{\lambda}\dfrac{d(n^{-1})}{d(1/\lambda)}$

$$\upsilon_g = \upsilon + (-1)(n^{-2})\frac{c}{\lambda}\frac{dn}{d(1/\lambda)}$$

$$\upsilon_g = \upsilon\left[1 - \frac{1}{n\lambda}\frac{dn}{d(1/\lambda)}\right]$$

$$\upsilon_g = \upsilon\left[1 - \frac{1}{n\lambda}\frac{d\nu}{d(1/\lambda)}\frac{dn}{d\nu}\right]$$

Insert $\upsilon_g = \dfrac{d\nu}{d(1/\lambda)}$:

$$\upsilon_g = \upsilon\left[1 - \frac{\upsilon_g}{n\lambda}\frac{dn}{d\nu}\right]$$

$$\upsilon = \upsilon_g + \frac{\upsilon_g \upsilon}{n\lambda}\frac{dn}{d\nu} = \upsilon_g\left[1 + \frac{\upsilon}{n\lambda}\frac{dn}{d\nu}\right]$$

$$\upsilon_g = \frac{\upsilon}{\left[1 + \frac{\nu}{n}\frac{dn}{d\nu}\right]}$$

7.26 $\upsilon = \dfrac{c}{n} = \dfrac{2.99\times10^8 \text{ m/s}}{1.449} = 2.063\times10^8 \text{ m/s}$

$$\upsilon_g = \frac{c}{n_g} = \frac{2.99\times10^8 \text{ m/s}}{1.462} = 2.045\times10^8 \text{ m/s}$$

$$\upsilon_g < \upsilon$$

7.27 $1/\upsilon_g = d(\nu/\upsilon)/d\nu$ and the rest follows.

7.28 From the previous problem $1/\upsilon_g = (n/c) - (\nu n^2/c^2)[d(c/n)]/d\nu$ and the rest follows.

7.29 $\upsilon = \sqrt{g/k + \dfrac{\gamma k}{\rho}}$ $\upsilon_g = \upsilon + k d\upsilon/dk$, where

$$k\frac{d\upsilon}{dk} = k\frac{\left[-\frac{g}{k^2} + \frac{\gamma}{\rho}\right]}{2\sqrt{g/k + \frac{\gamma k}{\rho}}}$$

$$= \frac{-\frac{g}{k} + \frac{\gamma k}{\rho}}{2\sqrt{g/k + \frac{\gamma k}{\rho}}} = \frac{\upsilon}{2}$$

7.30 We have $\lambda = 2\pi/k,\ d\lambda/dk = -2\pi/k^2 = -\lambda/k$ so that the term $kd\upsilon/dk = k(d\lambda/dk)(d\upsilon/d\lambda) = k(-\lambda/k)d\upsilon/d\lambda = -\lambda d\upsilon/d\lambda$ and the expression for υ_g follows.

7.31 $\upsilon_g = \upsilon + kd\upsilon/dk$ and $d\upsilon/dk = (d\upsilon/d\omega)(d\omega/dk) = \upsilon_g d\upsilon/d\omega.$ Since $\upsilon = c/n,$

$$\begin{aligned} d\upsilon/d\omega &= (d\upsilon/dn)(dn/d\omega) = -(c/n^2)\,dn/d\omega, \\ \upsilon_g &= \upsilon - (\upsilon_g ck/n^2)\,dn/d\omega = \upsilon/[1 + (ck/n^2)(dn/d\omega)] \\ &= c/[n + \omega(dn/d\omega)]. \end{aligned}$$

7.32 (7.40) $n_g \equiv c/\nu_g$. From Problem 7.31 $\nu_g = c/(n + 2(dn/d\omega)),$ so

$$n_g = n + \omega(dn/d\omega) = n(\nu) + 2\pi\nu(dn(\nu)/2\pi d\nu) = n(\nu) + \nu\left(\frac{dn(\nu)}{d\nu}\right).$$

7.33 Starting with:

$$\begin{aligned} n_g &= n(\nu) + \nu\frac{dn(\nu)}{d\nu} \\ &= n + \nu\frac{dn}{d\lambda}\frac{d\lambda}{d\nu} \end{aligned}$$

$$\frac{d\lambda}{d\nu} = \frac{d\lambda}{\left(\frac{c}{2\pi}\right)dk} = \frac{2\pi}{c}\left(-\frac{2\pi}{k^2}\right) = -\frac{4\pi^2}{ck^2}$$

using: $\nu = \dfrac{c}{2\pi}k$

$$\begin{aligned} n_g &= n - \frac{2\pi}{k}\frac{dn}{d\lambda} \\ n_g &= n - \lambda\frac{dn}{d\lambda} \end{aligned}$$

7.34 Using dimensional analysis:

$$\upsilon_g = \upsilon\left(1 + \frac{1}{n}\frac{dn}{dk}\right)$$

ν_g: m/s

n: dimensionless

k: m^{-1}

then we have the following dimensional relationship which cannot be true:

$$m/s = m/s\left(1 + \frac{1}{m^{-1}}\right) = m/s\,(1 + m)$$

7.35 For $\upsilon = a/\lambda,\ \upsilon_g = \upsilon - \lambda\,d\upsilon/d\lambda = a/\lambda + \lambda a/\lambda^2 = 2a/\lambda = 2\upsilon.$

7.36 (7.38) $$\begin{aligned} v_g &= v + k(dv/dk) = c/n - (kc/n^2)(dn/dk) \\ &= c/n - (kc/n^2)(dn/d\lambda)(d\lambda/dk) \\ &= c/n - (kc/n^2)(dn/d\lambda)(-2\pi/k^2) \\ &= c/n + (2\pi/k)(c/n^2)(dn/d\lambda) = c/n + (\lambda c/n^2)(dn/d\lambda) \end{aligned}$$

7.37 $$\upsilon_g = \frac{c}{n} + \frac{\lambda c}{n^2}\frac{dn}{d\lambda}$$

$$\upsilon_g = \frac{2.99\times 10^8\ \text{m/s}}{\overline{n}} + \frac{\overline{\lambda} c}{\overline{n}^2}\frac{\Delta n}{\Delta\lambda}$$

$$\overline{n} = \frac{n_1 + n_2}{2} = 1.3321$$

$$\overline{\lambda} = \frac{\lambda_1 + \lambda_2}{2} = 622.8\ \text{nm}$$

$$\upsilon_g = \frac{2.99\times 10^8\ \text{m/s}}{1.3321} + \frac{(622.8\ \text{nm})(2.99\times 10^8\ \text{m/s})}{(1.3321)^2}\left(-\frac{1.9\times 10^{-3}}{67\times 10^{-9}\ \text{m}}\right)$$

$$\upsilon_g = 2.22\times 10^8\ \text{m/s}$$

$$\overline{\upsilon} = \frac{c}{\overline{n}} = 2.25\times 10^8\ \text{m/s}$$

$$\overline{\upsilon} > \upsilon_g$$

7.38 $$\upsilon = \omega/k = \frac{\omega_0 \sin(kl/2)}{(kl/2)} = \omega_0\ \text{sinc}(kl/2);$$

$$\upsilon_g = d\omega/dk = \frac{\omega_0 l}{2}\sin(kl/2).$$

7.39 $\upsilon = \omega/k$ therefore $\omega^2 = \omega_p^2 + c^2(\omega/\upsilon)^2$ and

$$\upsilon = c/[1-(\omega_p/\omega)^2]^{1/2}; \quad \upsilon_g = d\omega/dk = c^2k/\omega = c[1-(\omega_p/\omega)^2]^{1/2}.$$

7.40 For $\omega^2 \gg \omega_i^2$, $n^2 = 1 - (Nq_e^2/\omega^2\varepsilon_0 m_e)\Sigma_i f_i = 1 - Nq_e^2/\omega^2\varepsilon_0 m_e$. Using the binomial expansion, we have $(1-x)^{1/2} \approx 1 - x/2$ for $x \ll 1$, so that

$$n = 1 - Nq_e^2/2\omega^2\ \varepsilon_0 m_e, \qquad dn/d\omega = -Nq_e^2/\varepsilon_0 m_e\omega^3.$$
$$\upsilon_g = c/[n + \omega(dn/d\omega)] = c/[1 + Nq_e^2/2\varepsilon_0 m_e\omega^2]$$

and $\upsilon_g < c, \upsilon = c/n = c/[1 - Nq_e^2/2\varepsilon_0 m_e\omega^2]$. By binomial expansion,

$$(1-x)^{-1} \approx 1 + x \text{ for } x \ll 1,\ \upsilon = c[1 + Nq_e^2/2\varepsilon_0 m_e\omega^2];\ \upsilon\upsilon_g = c^2.$$

7.41 $$E_1 = 2E_0\cos\omega t; \qquad E_2 = \frac{1}{2}E_0\sin 2\omega t. \qquad E = E_1 + E_2$$
$$= 2E_0\cos\omega t + \frac{1}{2}E_0\sin 2\omega t = E_0(2 + \sin\omega t)\cos\omega t.$$

Resultant is anharmonic, but periodic with period ω.

7.42 (a) Both sine and cosine terms are required because the function is neither symmetric nor anti-symmetric about the origin.

(b) Odd and even multiples are required because the waveform does not have half-wave or quarter-wave symmetry.

(c) 1/3

(d) DC Term = $A_0/2$, thus $A_0 = 2/3$

(e) $T = 2\pi/\omega$

7.43 $\int_0^\lambda akx \sin bkx\, dx = (1/2k)\left[\int_0^\lambda \cos[(a-b)kx]k\,dx - \int_0^\lambda \cos[(a+b)kx]kdx\right] = 0$

if $a \neq b$. Whereas if $a = b$,

$$\int_0^\lambda \sin^2 akx\, dx = (1/2k)\int_0^\lambda (1+\cos 2akx)k\, dx = \lambda/2.$$

The other integrals are similar.

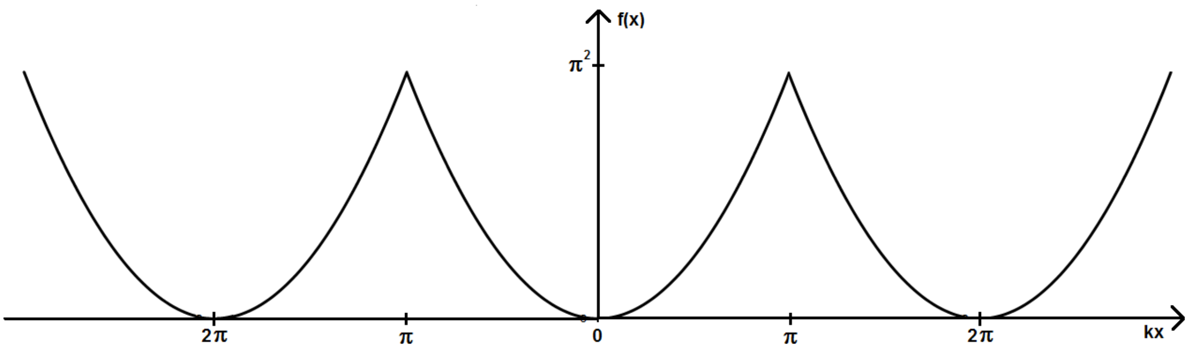
7.44 Even function, therefore $B_m = 0$.

$$A_0 = (2/\lambda)\int_{-\lambda/a}^{\lambda/a} dx = (2/\lambda)(2\lambda/a) = 4/a,$$
$$A_m = (2/\lambda)\int_{-\lambda/a}^{\lambda/a} (1)\cos mkx\, dx = (4/mk\lambda)\sin mkx\Big|_0^{\lambda/a}$$
$$= (2/m\pi)\sin 2m\pi/a.$$

7.45 $f(x) = 1 - \frac{8}{\pi^2}\left(\cos\left(\frac{\pi x}{2}\right) + \frac{1}{3^2}\cos\left(\frac{3+\pi x}{2}\right) + \frac{1}{5^2}\cos\left(\frac{5\pi x}{2}\right) + \cdots\right)$

7.46 $A_0 = 0$, $A_1 = A$, and all other $A_m = 0$ moreover $B_m = 0$ so $f(x) = A\cos(\pi x/L)$.

7.47



$$f(x) = \frac{\pi^2}{3} - 4\left(\frac{\cos(kx)}{1^2} - \frac{\cos(2kx)}{2^2} + \frac{\cos(3kx)}{3^2} - \cdots\right)$$

7.48 $A_m = 4/m^2$, $m \neq 0$; $A_0 = 8\pi^2/3$; $B_m = -4\pi/m$.

7.49 $A_m = -2(1+\cos m\pi)/\pi(m^2-1)$ where $m \neq 1$ and $A_1 = 0$.

7.50 $f(x) = \frac{1}{\pi}\int_0^a E_0 L \frac{\sin kL/2}{kL/2}\cos kx\, dk$

$$= \frac{E_0 L}{2\pi}\int_0^a \frac{\sin(kL/2+kx)}{kL/2}\,dk + \frac{E_0 L}{2\pi}\int_0^a \frac{\sin(kL/2-kx)}{kL/2}\,dk.$$

Let $kL/2 = w$, $(L/2)\,dk = dw$, $kx = wx'$,

$$f(x) = \frac{E_0}{\pi}\int_0^b \frac{\sin(w+wx')}{w}\,dw + \frac{E_0}{\pi}\int_0^b \frac{\sin(w-wx')}{w}\,dw,$$

where $b = aL/2$. Let $w + wx' = t$, $dw/w = dt/t$, $0 \le w \le b$ and $0 \le t \le (x'+1)b$.

Let $w - wx' = -t$ in the other integral, $0 \le w \le b$ and $0 \le t \le (x'-1)b$.

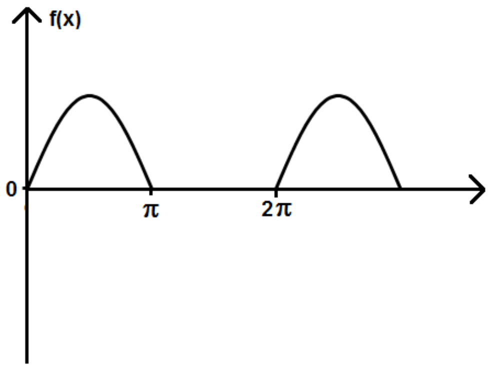
$$f(x) = \frac{E_0}{\pi}\int_0^{(x'+1)b} \frac{\sin t}{t}\,dt - \frac{E_0}{\pi}\int_0^{(x'-1)b} \frac{\sin t}{t}\,dt,$$

$$f(x) = \frac{E_0}{\pi}\,\mathrm{Si}\left[b(x'+1)\right] - \frac{E_0}{\pi}\,\mathrm{Si}\left[b(x'-1)\right],$$

with $x' = 2x/L$.

7.51 $E(t) = E_0\left(\frac{1}{\pi} + \frac{1}{2}\cos\omega t + \frac{2}{3\pi}\cos 2\omega t - \frac{2}{15\pi}\cos 4\omega t + \cdots\right)$

7.52 $f(x) = \frac{1}{\pi} + \frac{1}{2}\sin kx - \frac{2}{\pi}\left(\frac{\cos(2kx)}{1\cdot 3} + \frac{\cos(4kx)}{3\cdot 5} + \frac{\cos(6kx)}{5\cdot 7} + \cdots\right)$



7.53 (a) The envelopes of frequency spectra become more compressed as the wavelength increases. This is because the angular spatial frequency, k, is inversely proportional to the wavelength.

(b) The Fourier expansion of the square wave consists of only of odd harmonics of the fundamental frequency. Thus if one considers the number of frequency terms present in a range defined by a multiple of the square waves' fundamental harmonic, then there will be the same number of terms present.

(c) There are no even terms present because the expansion of a square wave consists only of odd harmonic terms.

7.54 By analogy with Eq. (7.61), $A(\omega) = (\Delta t/2)E_0\ \mathrm{sinc}\,(\omega_p - \omega)\Delta t/2$. From Table 1, sinc($\pi$/2) = 63.7%. Not quite 50% actually, sinc(π/1.65) = 49.8%.
$\left|(\omega_p - \omega)\Delta t/2\right| < \pi/2$ or $-\pi/\Delta t < \omega_p - \omega < \pi/\Delta t$; thus appreciable values of $A(\omega)$ lie in a range $\Delta\omega \sim 2\pi/\Delta t$ and $\Delta\nu\Delta t \sim 1$. Irradiance is proportional to $A^2(\omega)$, and $[\mathrm{sinc}\,(\pi/2)^2] = 40.6\%$.

7.55 $\Delta x_c = c\Delta t_c$, $\Delta x_c \sim c/\Delta\nu$. But $\Delta\omega/\Delta k_0 = \overline{\omega}/\overline{k}_0 = c$; thus $\left|\Delta\nu/\Delta\lambda_0\right| = \overline{\nu}/\overline{\lambda}_0$, $\Delta x_c \sim c\overline{\lambda}_0/\Delta\lambda_0\overline{\nu}$, $\Delta x_c \sim \overline{\lambda}_0^2/\Delta\lambda_0$. Or try using the uncertainty principle: $\Delta x \sim h/\Delta p$ where $p = h/\lambda$ and $\Delta\lambda_0 \ll \overline{\lambda}_0$.

7.56 $\Delta\lambda_0 = 21\times 10^{-9}$ m

$\overline{\lambda}_0 = 446\times 10^{-9}$ m

$$\Delta l_c = \frac{c}{\Delta\nu} = \frac{c\overline{\lambda}_0}{\Delta\lambda_0 \nu} = \frac{\overline{\lambda}_0^2}{\Delta\lambda_0} = \frac{(446\times 10^{-9}\,\text{m})^2}{21\times 10^{-9}\,\text{m}} = 9.26\times 10^{-6}\,\text{m}$$

7.57 $\Delta x_c = c\Delta t_c = 3\times 10^8\,\text{m/s}\,10^{-8}\,\text{s} = 3\,\text{m}.$

$\Delta\lambda_0 \sim \lambda_0^2/\Delta x_c = (500\times 10^{-9}\,\text{m})^2/3\,\text{m},$

$\Delta\lambda_0 \sim 8.3\times 10^{-14}\ \text{m} = 8.3\times 10^{-5}\ \text{nm},$

$\Delta\lambda_0/\overline{\lambda}_0 = \Delta\nu/\overline{\nu} = 8.3\times 10^{-5}/500 = 1.6\times 10^{-7} \sim 1$ part in 10^7.

7.58 $\Delta\nu = 54\times 10^3$ Hz; $\quad \Delta\nu/\overline{\nu} = (54\times 10^3)(10{,}600\times 10^{-9}\ \text{m})/(3\times 10^8\ \text{m/s})$

$= 1.91\times 10^{-9}$. $\quad \Delta x_c = c\Delta t_c \sim c/\Delta\nu$, $\Delta x_c \sim (3\times 10^8)/(54\times 10^3)$

$= 5.55\times 10^3\,\text{m}.$

7.59 $\Delta\nu/\nu = 2/10^{10}$; $c = \nu\lambda$, so

$$\nu = c/\lambda = 3\times 10^8\ \text{m/s}/632.8\times 10^{-9}\ \text{m} = 4.74\times 10^{14}\ \text{Hz}.$$

(7.64) $\Delta l_c = c\Delta t_c$.

Frequency range is $\pm\, 2(4.74\times 10^4$ Hz) or 9.48×10^4 Hz, so

$\Delta t \simeq 1.05\times 10^{-5}$ s. $\qquad \Delta l_c = (3\times 10^8\ \text{m/s})(1.05\times 10^{-5}\ \text{s}) = 3.15\times 10^3\,\text{m}.$

7.60 $\Delta x_c = c\Delta t_c = 3\times 10^8 \times 10^{-10} = 3\times 10^{-2}$ m, $\quad \Delta\nu \sim 1/\Delta t_c = 10^{10}$ Hz,

$\Delta\lambda_0 \sim \overline{\lambda}_0^2/\Delta x_c$ (see Problem 7.35),

$\Delta\lambda_0 \sim (632.8\ \text{nm})^2/(3\times 10^{-2}\,\text{m}) = 0.013$ nm. $\Delta\nu = 10^{15}$ Hz,

$\Delta x_c = c\times 10^{-15} = 300$ nm, $\quad \Delta\lambda_0 \sim \overline{\lambda}_0^2/\Delta x_c = 133478$ nm.

7.61 $\Delta\nu/\nu = \Delta\lambda/\lambda$, (see Table 7.1)

$= (1\times 10^{-10}\,\text{m})/(600\times 10^{-9}\,\text{m}) = 1.67\times 10^{-4}.$

$c = \nu\lambda$, so $\nu = c/\lambda = (3\times 10^8\ \text{m/s})/(600\times 10^{-9}\ \text{m})$

$= 5.00\times 10^{14}$ Hz. $\quad \Delta\upsilon = (1.67\times 10^{-4})(5\times 10^{14}\ \text{Hz})$

$= 8.35\times 10^{14}$ Hz, so $\quad \Delta t \simeq 1.20\times 10^{-11}$ s.

(7.64) $\quad \Delta l_c = c\Delta t_c = (3\times 10^8\ \text{m/s})(1.20\times 10^{-11}\ \text{s}) = 3.60\times 10^{-3}$ m.

7.62 $\Delta l_c = 20\lambda_0$. (7.64) $\Delta l_c = c\Delta t_c$, so

$\Delta t_c = \Delta l_c/c = 20\lambda_0/c = 20(500\times 10^{-9}\ \text{m})/(3\times 10^8\ \text{m/s}) = 3.33\times 10^{-15}$ sec.

$\Delta\nu \simeq 1/\Delta t_c = 3\times 10^{14}$ Hz.

7.63 $\Delta\nu/\nu = \Delta\lambda/\lambda$, (see Table 7.1) = $(1.2 \times 10^{-9}$ m)/(500×10^{-9} m) = 0.0024.

$c = \nu\lambda$, so $\nu = c/\lambda = (3 \times 10^8 \text{ m/s})/(500 \times 10^{-9} \text{ m}) = 6.00 \times 10^{14}$ Hz.

$\Delta\nu$ = Frequency Bandwidth = $(0.0024)(6.00 \times 10^{14}$ Hz) = 1.44×10^{12} Hz.

$\Delta t_c \simeq 1/\Delta\nu = 6.94 \times 10^{13}$ s.

(7.64) $\Delta l_c = c\Delta t_c = (3 \times 10^8 \text{ m/s})(6.94 \times 10^{-13} \text{ s}) = 2.08 \times 10^{-4}$ m.