Chapter 10 Solutions

10.1 $(R+l)^2 = R^2 + a^2$; therefore $R = (a^2 - l^2)/2l \approx a^2/2l,\ lR = a^2/2,$ so for $\lambda \ll l,\ \ \lambda R \gg a^2/2.$ Therefore $R = (1\times 10^{-3})^2 10/2\lambda = 10$ m.

10.2 A "constant" phase shift is added due to the angle of the incident wave reaching the ends of the slit at different phase, so that (10.11) becomes $r = R - y(\sin\theta - \sin\theta_i) + \ldots.$ This constant carries through the integration, so that the definition of β in 10.18 (or 10.14) becomes $\beta = (kb/2)(\sin\theta - \sin\theta_i)$.

10.3 $d\sin\theta_m = m\lambda,\ \theta = N\delta/2 = \pi,\ \ 7\sin\theta = (1)(0.21),\ \delta = 2\pi/N = kd\sin\theta,\ \ \sin\theta = 0.03$ so $\theta = 1.7°$.
For $\sin\theta = 0.0009,\ \theta = 3\,\text{min}$.

10.4 Converging spherical wave in image space is diffracted by the exit pupil.

10.5 $I(\theta) = I(0)\left(\dfrac{\sin\beta}{\beta}\right)^2$

From figure 10.13, the first secondary maximum is at $\beta = 1.43\pi$:

$$I\left(\beta = 1.43\pi\right) = I(0)\left(\frac{\sin(1.43\pi)}{1.43\pi}\right)^2 = I(0)\left(\frac{0.9579}{4.492}\right)^2 = 0.0455I(0)$$

Then,

$$\frac{I(0)}{I(\beta = 1.43\pi)} = \frac{I(0)}{0.0455I(0)} = 21.98$$

10.6 $\beta = \pm\pi,\ \sin\theta = \pm\lambda/b \approx \theta,\ L\theta \approx \pm L\lambda/b,\ L\theta \approx \pm f_2\lambda/b.$

10.7 Is $R > b^2/\lambda$?, b = slit width.

$$b^2/\lambda = (1\times 10^{-4}\ \text{m})^2/(461.9\times 10^{-9}\ \text{m}) = 0.02\ \text{m} << 1\ \text{m}$$

Fraunhoffer:

(Half) angular width from the central maximum

$$\beta = \pi = (\text{kb}/2)\sin\theta_1,$$
$$\sin\theta_1 = 2\pi/kb = \lambda/b = (4.619\times 10^{-7}\ \text{m})/(1\times 10^{-4}\ \text{m});\ \theta = 0.26°.$$

To what order Young's fringe does θ_1 correspond?

$$\alpha = m'\pi = (ka/2)\sin\theta_1.$$
$$m' = (ka/2\pi)\sin\theta_1 = (a/\lambda)\sin\theta_1 = (2\times 10^{-4}\ \text{m})/(4.619\times 10^{-7}\ \text{m})\sin(0.26°) = 2 = a/b$$

So there are 4 "Young's Fringes" in the central maximum.

10.8 $b\sin\theta_m = m\lambda$, so,

$$b = m\lambda/\sin\theta_m = 10(1.1522\times 10^{-6}\,\text{m})/\sin(6.2°) = 1.07\times 10^{-4}\,\text{m}.$$

In water, $b\sin\theta_m = m\lambda,$ where $\lambda = n_{\text{air}}\lambda_o/n_{\text{water}}$.

$$\sin\theta_m = mn_{\text{air}}\lambda_o/(n_{\text{water}}b);$$
$$\theta_m = \sin^{-1}[10(1.00029)(1.1522\times 10^{-6}\,\text{m})/(1.33)(1.07\times 10^{-4}\,\text{m})]$$
$$= 4.65°.$$

10.9 $\lambda = (20\text{ cm})\sin 36.87° = 12\text{ cm}.$

10.10 Starting with,

$$y_m = \frac{s}{b} m\lambda$$

For plane waves, $s = f$,

$$y_m = \frac{f}{b} m\lambda$$

$$f = \frac{y_m b}{m\lambda} = \frac{(1.2\times10^{-3}\text{ m})(2.5\times10^{-4}\text{m})}{4(5.1836\times10^{-7}\text{m})} = 0.145\text{ m}$$

10.11 The first maximum is located at $\beta = 1.4303\pi$

$$\beta = 1.4303\pi = \left(\frac{kb}{2}\right)\sin\theta$$

Use $\lambda = \dfrac{2\pi}{\kappa}$:

For small angles:

$$1.4303 = \left(\frac{b}{\lambda}\right)\sin\theta \approx \frac{b}{\lambda}\theta$$

$$\frac{y}{f} = \tan\theta \approx \theta$$

Then,

$$1.4303\frac{\lambda f}{b} \approx y$$

10.12

$$b\sin\theta_m = m\lambda$$

For small angles,

$$\sin\theta \approx \frac{y}{f}$$

$$y_m = \frac{m\lambda f}{b}$$

The width of the central irradiance is the distance between the ±1 minumum = $2y_1$

$$= 2y = \frac{\lambda f}{b} = \frac{2(5.461\times10^{-7}\text{ m})(0.62\text{ m})}{1.5\times10^{-4}\text{ m}} = 0.00451\text{ m}$$

10.13 $y_m = f\sin\theta_m \approx \dfrac{m\lambda f}{b}$

$$y_1 \approx \frac{\lambda f}{b}$$

$$y_2 \approx \frac{2\lambda f}{b}$$

The distance between the 1st and 2nd zeroes is:

$$y_2 - y_1 = \frac{2\lambda f}{b} - \frac{\lambda f}{b} = \frac{\lambda f}{b} = \frac{(4.861\times 10^{-7}\text{ m})(0.60\text{ m})}{2.0\times 10^{-4}\text{ m}} = 0.00146\text{ m}$$

10.14 $\alpha = (ka/2)\sin\theta,\ \beta = (kb/2)\sin\theta.\ a = mb,\ \alpha = m\beta,\ \alpha = m2\pi,$

$N =$ number of fringes $= \alpha/\pi = m2\pi/\pi = 2m.$

10.15 Is $R > b^2/\lambda$?, $b =$ slit width.

$$b^2\lambda = (1\times 10^{-4}\text{ m})^2/(5\times 10^{-7}\text{ m}) = .02\text{ m} \ll 2.5\text{ m}.$$

Fraunhofer.

(Half) angular width of central maximum from

$$\beta = \pi = (kb/2)\sin\theta_1.$$
$$\sin\theta_1 = 2\pi/kb = \lambda/b = (5\times 10^{-7}\text{ m})/(1\times 10^{-4}\text{ m});\quad \theta_1 = 0.29°.$$

To what order Young's fringe does θ_1 correspond?

$$\alpha = m'\pi = (ka/2)\sin\theta_1.\quad m' = (ka/2\pi)\sin\theta_1 = (a/\lambda)\sin\theta_1$$
$$= (2\times 10^{-4}\text{ m})/(5\times 10^{-7}\text{ m})\sin(0.29°) = 2.$$

So there are 4 "Young's Fringes" in the central maximum.

10.16 For single slit diffraction:

$$b\sin\theta_{mD} = m_D\lambda$$

For two slit interference:

$$a\sin\theta_{mI} = m_I\lambda$$

Since there are 11 maxima, there must be 5 peaks on either side of the central peak ($m_I = \pm 5$). The outer edge of the central diffraction (θ_{1D}) then corresponds to the location of the edge of the 5th interference fringe (θ_{5I}).

$$\sin\theta_{5I} = \sin\theta_{1D}$$
$$\theta_{5I} = \theta_{1D}$$
$$\frac{5\lambda}{a} = \frac{\lambda}{b}$$
$$b = \frac{a}{5} = \frac{0.100\text{ mm}}{5} = 0.020\text{ mm}$$

10.17 $\alpha = 3\pi/2N = \pi/2,\ I(\theta) = I(0)[(\sin\beta)/\beta]^2/N^2$ and $I/I(0) \approx 1/9.$

10.18 For the principle maximum, the phasors line on a straight line ($\rightarrow\rightarrow\rightarrow$); the amplitude is $3E_0$. The phasors at the subsidiary maximum ($\rightarrow\leftarrow\rightarrow$) have an amplitude of E_0. The irradiance ratio is:

$$\left(\frac{E_0}{3E_0}\right)^2 = \frac{1}{9}$$

10.19 For 16 slits, the amplitude is 16 E_0, thus the irradiance ratio is:

$$\left(\frac{E_0}{16E_0}\right)^2 = \frac{1}{256}$$

For 8 slits, the amplitude is 8 E_0 and the irradiance ratio is:

$$\left(\frac{E_0}{8E_0}\right)^2 = \frac{1}{64}$$

The ratio between the two is 64/256 = 0.25. The 8-slit screen will have wider $m = 0$ peaks.

10.20 $I(\theta) = I(0)\left(\frac{\sin\beta}{\beta}\right)^2$

$$\beta = \frac{\pi}{\lambda} b\sin\theta = \frac{\pi}{\lambda}\frac{a}{4}\sin\theta = \frac{\alpha}{4}$$

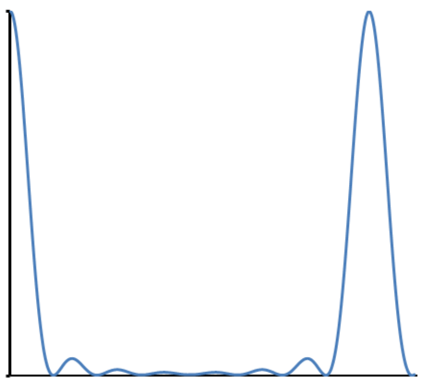
For the 2nd order maximum, $m = 2$, thus $\beta = \alpha/4 = \pi/2$.

$$I(\theta) = I(0)\left(\frac{\sin\pi/2}{\pi/2}\right)^2 = \left(\frac{1}{1.5707}\right)^2 = 0.405$$

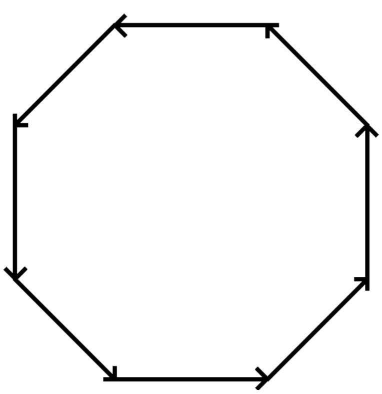
Where we set $I(0) = 1$. Thus:

$$\frac{I(\theta)}{I(0)} = 0.405$$

10.21 (a)

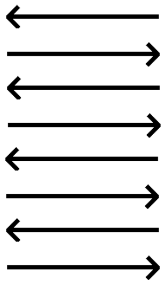


(b)

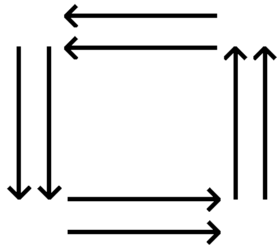


The phasors form an octagon.

(c)



(d)



(e) The angles for b) are 45°, c) are 180°, d) are 90°.

10.22 (10.17) $I(\theta) = I(0)(\sin\beta/\beta)^2$, where $\beta \equiv (kD/2)\sin\theta$. "Miniscule Area" corresponds to the limit $D \to 0$. As $D \to 0, \beta \to 0$, so

$$\lim_{D\to 0} I(\theta) = \lim_{\beta\to 0}(I(0)(\sin\beta/\beta)^2);$$

As $\beta \to 0$, $\text{sinc}(\beta) \to 1$, so $\lim_{D\to 0} I(\theta) = I(0)$, i.e., same in all directions.

10.23 The minima occur at m = 1.

$$Y = \frac{\lambda R}{b}$$
$$Z = \frac{\lambda R}{a}$$

Using $R \approx f$:

$$Y = \frac{\lambda f}{b} = \frac{(5.43\times 10^{-7}\text{ m})(1\text{ m})}{1\times 10^{-4}\text{ m}} = 5.43\text{ mm}$$
$$Z = \frac{\lambda f}{b} = \frac{(5.43\times 10^{-7}\text{ m})(1\text{ m})}{1.99\times 10^{-4}\text{ m}} = 2.73\text{ mm}$$

10.24
$$I(\theta) = I(0)\left(\frac{\sin\alpha'}{\alpha'}\right)^2\left(\frac{\sin\beta'}{\beta'}\right)^2$$

$$\alpha' = \frac{kaZ}{2R}, \quad a = 0.1\text{ mm}$$
$$\beta' = \frac{kbY}{2R}, \quad b = 0.2\text{ mm}$$

When $Y = 0$ and $Z = 1$ mm:

$$\beta' = 0,\ \ \alpha' = \frac{\pi(1\times10^{-4}\,\text{m})(1\times10^{-3}\,\text{m})}{(5.43\times10^{-7}\,\text{m})(10\ \text{m})} = 0.05786$$

$$I(\theta) = I(0)\left(\frac{\sin\alpha'}{\alpha'}\right)^2 = I(0)\left(\frac{\sin(0.05786)}{0.05786}\right)^2 = 0.999\,I(0)$$

When $Z = 0$ and $Y = 1$ mm:

$$\alpha' = 0,\ \ \beta' = \frac{\pi(2\times10^{-4}\,\text{m})(1\times10^{-3}\,\text{m})}{(5.43\times10^{-7}\,\text{m})(10\ \text{m})} = 0.1157$$

$$I(\theta) = I(0)\left(\frac{\sin\beta'}{\beta'}\right)^2 = I(0)\left(\frac{\sin(0.1157)}{0.1157}\right)^2 = 0.996\,I(0)$$

10.25 (from 10.41) $\tilde{E} \propto \iint e^{ik(Yy+Zz)/R}dS.$ and $I(Y,Z) \propto \langle \tilde{E}^2\rangle.$ If $\tilde{E}$ is an even function of $(Y,Z), \tilde{E}(-Y,-Z) = \tilde{E}(Y,Z).$ If $\tilde{E}$ is an odd function of $(Y,Z), \tilde{E}(-Y,-Z) = -\tilde{E}(Y,Z),$ but $I(-Y,-Z) = I(Y,Z).$

10.26 If the aperture is symmetrical about a line, the pattern will be symmetrical about a line parallel to it. Moreover, the pattern will be symmetrical about yet another line perpendicular to the aperture's symmetry axis. This follows from the fact that Fraunhofer patterns have a center of symmetry.

10.27 For the solution to this problem, please refer to the solutions at the end of the book.

10.28 Three parallel short slits, note the $N - 2 = 1$ subsidiary maximum between each peak.

10.29 Two parallel short slits.

10.30 An equilateral triangular hole.

10.31 A cross-shaped hole.

10.32 The *E*-field of a rectangular hole.

10.33 $q_1 = 1.22\dfrac{R\lambda}{2a}$

Use $R = f = 1$ m:

$$2q_1 = 1.22\frac{(1\ \text{m})(6.56\times10^{-7}\,\text{m})}{(1.2\times10^{-2}\,\text{m})} = 6.67\times10^{-5}\,\text{m}$$

10.34 $q_1 = 1.22\dfrac{f\lambda}{D} = 1.22\dfrac{(1.4\ \text{m})(5.40\times10^{-7}\ \text{m})}{(1.5\times10^{-2}\ \text{m})} = 6.15\times10^{-6}\ \text{m}$

$2q_1 = 1.23\times10^{-5}\,\text{m}$

If we doubled the lens diameter (*D*), then the disk would shrink to: $2q_1 = 0.62\times10^{-5}\,\text{m}$

10.35 Using:

$$\lambda = \frac{\lambda_0}{n} = \frac{5.50\times10^{-7}\,\text{m}}{1.337} = 4.11\times10^{-7}\,\text{m}$$

$$q_1 = 1.22\frac{R\lambda}{2a} = 1.22\frac{(2.1\times10^{-2}\,\text{m})(4.11\times10^{-7}\,\text{m})}{(6\times10^{-3}\,\text{m})} = 1.755\times10^{-6}\,\text{m}$$

$$2q_1 = 3.51\ \mu\text{m}$$

10.36 From section 10.2.5, first "ring" (maximum) occurs for $u = kaq/R = 5.14$.

Interpolating from Table 10.1, $J_1(5.14) \simeq -0.33954$

From (10.55) $I/I(0) = \left[\dfrac{2J_1(u)}{u}\right]^2 = \left[\dfrac{2(-0.33954)}{5.14}\right]^2 = 0.0175$

10.37 $I(\theta) = I(0)\left[\dfrac{2J_1(u)}{u}\right]^2 = \dfrac{I(0)}{u^2}\left[\dfrac{2(\sin u - \cos u)}{\sqrt{\pi u}}\right]^2$

This is minimum when $\sin u = \cos u$,

$$u = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \cdots$$

$$= (4m-3)\frac{\pi}{4}, \qquad m = 1, 2, 3, \ldots$$

$$u = ka\sin\theta = (4m-3)\frac{\pi}{4}$$

$$\sin\theta = \frac{(4m-3)}{2\pi a}\frac{\pi}{4}\lambda = \frac{(4m-3)}{8a}\lambda$$

$$\frac{d}{dm}(\sin\theta) = (\cos\theta)\frac{d\theta}{dm} = \frac{4\lambda}{8a}$$

$$\Delta\theta(\cos\theta) = \frac{4\lambda}{8a}\Delta m$$

If $\Delta m = 1$,

$$\Delta\theta = \frac{\lambda}{2a\cos\theta}$$

10.38 From Eq. (10.58), $q_1 \approx 1.22(f/D)\lambda \approx \lambda$.

10.39 For the solution to this problem, please refer to the solutions in the back of the textbook.

10.40 (10.57) $q_1 = 1.22(R\lambda/2a)$

$$= 1.22[(3.76\times10^8\,\text{m})(6.328\times10^{-7}\,\text{m})]/2[1\times10^{-3}\,\text{m}]$$

$$= 1.45\times10^5\,\text{m}.$$

10.41 (10.59) $(\Delta\varphi)_{\min} = 1.22\lambda/D = [1.22(5.50\times10^{-7}\,\text{m})]/7.5\times10^{-4}\,\text{m}$

$$= 8.9\times10^{-4}\ \text{rad},$$

This is about 5 times worse resolution.

10.42 $(\Delta\theta)_{\min} = \dfrac{1.22\lambda}{D}$

$$D = \frac{1.22\lambda}{(\Delta\theta)_{\min}}$$

$$D = \frac{1.22(5.5\times10^{-7}\,\text{m})}{5\times10^{-6}\,\text{rad}} = 0.134\ \text{m}$$

10.43 $\lambda = \dfrac{\lambda_0}{n} = \dfrac{5.50\times10^{-7}\,\text{m}}{1.337} = 4.11\times10^{-7}\ \text{m}$

$$(\Delta\theta)_{\text{min}} = \frac{1.22\lambda}{D} = \frac{1.22(4.11\times10^{-7}\,\text{m})}{2\times10^{-3}\,\text{m}} = 2.51\times10^{-4}\ \text{rad} = 0.0144°$$

10.44 $(\Delta\theta)_{\text{min}} = 2.51\times10^{-4}\ \text{rad} = \dfrac{L}{D} = \dfrac{L}{2.5\times10^{-1}\,\text{m}}$

$$L = 6.28\times10^{-5}\,\text{m}$$

10.45 1 part in 1000. 3 yd ≈ 100 inches.

$$\Delta\theta = 2.51\times10^{-4}\,\text{rad} = \frac{L}{D} \quad L = 0.1 \text{ inch}$$

$$D = \frac{0.1}{2.51\times10^{-4}} = 398 \text{ inches}$$

10.46 (10.59) $\Delta\varphi_{\text{min}} = 1.22\lambda/D = [1.22(5.50\times10^{-7}\ \text{m})]/5.08\ \text{m}$
$= 1.32\times10^{-7}\ \text{rad};$

or $\Delta\varphi = 1.32\times10^{-7}\,\text{rad}(360°/2\pi\ \text{rad}) = 7.55\times10^{-6}\,°$; or

$\Delta\varphi = (7.55\times10^{-6}\,°)(3600\ \text{sec/degree}) = 2.72\times10^{-2}$ arc sec.

To be resolved, $s = r\Delta\varphi$ ($\Delta\varphi$ in radians).

$$s = (3.844\times10^{8}\ \text{m})(1.32\times10^{-7}) = 50.7\ \text{m}.$$

To be resolved by eyes,

$$s = r\Delta\varphi = r(1.22\lambda/D)$$
$$= (3.844\times10^{8}\ \text{m})[1.22(5.50\times10^{-7}\ \text{m})/(4\times10^{-3}\ \text{m})] = 6.44\times10^{4}\ \text{m}.$$

10.47 (a) $(\Delta\theta)_{\text{min}} = \dfrac{1.22\lambda}{D} = \dfrac{1.22(5.50\times10^{-7}\,\text{m})}{1\times10^{-1}\,\text{m}} = 6.71\times10^{-6}\,\text{rad}$

(b) $(\Delta\theta)_{\text{min}} = \dfrac{L}{D}$

$$L = D(\Delta\theta)_{\text{min}} = (1\times10^{6}\ \text{m})(6.71\times10^{-6}\ \text{rad}) = 6.71\ \text{m}$$

10.48 Resolution is proportional to wavelength, as a result, blue lasers are able to read smaller locations on the storage media. Thus blue lasers allow one to store more information on each disk (e.g., higher resolution "HD" movies may be recorded on an individual disk).

10.49 $(\Delta\theta)_{\text{min}} = \dfrac{1.22\lambda}{D}$

$$D = \frac{1.22\lambda}{(\Delta\theta)_{\text{min}}}$$

Use $(\Delta\theta)_{\text{min}} = L/r$:

$$D = \frac{1.22\lambda r}{L} = \frac{1.22(5.50\times10^{-7}\ \text{m})(1.61\times10^{5}\ \text{m})}{5\times10^{-2}\ \text{m}} = 2.16\ \text{m}$$

10.50 $(\Delta\theta)_{\min} = \dfrac{1.22\lambda}{D} = \dfrac{1.22(5.50\times10^{-7}\ \text{m})}{2.4\ \text{m}} = 2.976\times10^{-7}\ \text{rad}$

$$L = D(\Delta\theta)_{\min} = (6\times10^{5}\,\text{m})(2.976\times10^{-7}\ \text{rad}) = 0.168\ \text{m}$$

10.51 (10.32) $a\sin\theta_m = m\lambda;\ \sin\theta_1 \simeq y/R$, so

$a(Y/R) = \lambda;\ Y = R\lambda/a = (2.0\ \text{m})(6.943\times10^{-7}\ \text{m})/(3.0\times10^{-6}\ \text{m}) = 0.46\ \text{m}.$

10.52 (10.32) $a\sin\theta_m = m\lambda; \sin\theta_3 = 3\lambda/a = 3(5.00\times10^{-7}\ \text{m})/(6.0\times10^{-6}\ \text{m});$
$\theta_3 = 16°.$

10.53 (10.32) $a\sin\theta_m = m\lambda.$

$a = 2\lambda/\sin\theta_2 = 2(5.50\times10^{-7}\,\text{m})/\sin(25°) = 2.6\times10^{-6}\,\text{m}.$

10.54 $a\sin\theta = m\lambda$

$$a = \frac{m\lambda}{\sin\theta} = \frac{2(6.562816\times10^{-7}\,\text{m})}{\sin(42.00°)} = 1.9616\times10^{-6}\,\text{m}$$

$$\frac{1}{a} = 509{,}789\ \text{lines/m} = 5{,}098\ \text{lines/cm}$$

$$\sin\theta = \frac{m\lambda}{a} = \frac{2(4.861327\times10^{-7}\,\text{m})}{1.9616\times10^{-6}\,\text{m}} = 0.4956$$

$$\theta = 29.71°$$

10.55 From Eq. (10.32), where $a = 1/(1000\ \text{lines per cm}) = 0.001\ \text{cm per line}$

(center to center), $\sin\theta_m = 1(650\times10^{-9}\,\text{m})/(0.001\times10^{-2}\,\text{m}) = 6.5\times10^{-2}$ and $\theta_1 = 3.73°.$

10.56 $a\sin\theta_m = m\lambda.\ \sin\theta_m \simeq Y_m/R;\ Y_m = (m\lambda/a)R.$

10,000 lines/cm = 10^6 lines/m so $a = 10^{-6}$ m.

$Y_1(589.5923\ \text{nm}) = [1(5.895923\times10^{-7}\ \text{m})/10^{-6}\ \text{m}](1.00\ \text{m}) = 0.5895923\ \text{m}$

$Y_1'(588.9953\ \text{nm}) = 0.5889953\ \text{m}$

$$Y_1 - Y_1' = 5.97\times10^{-4}\ \text{m}$$

10.57 $N = \dfrac{\lambda}{m(\Delta\lambda)_{\min}} = \dfrac{589.2938\,\text{nm}}{(1)(0.597\ \text{nm})} = 987$

This is twice the number of grooves for $m = 2$.

10.58 $a\sin\theta = m\lambda$

$$a = \frac{1}{5.9\times10^{5}\,\text{groove/m}} = 1.695\times10^{-6}\,\text{m/groove}$$

For $\lambda = 400$ nm:

$$\sin\theta_{400} = \frac{m\lambda}{a} = \frac{1(4.00\times10^{-7}\ \text{m})}{1.695\times10^{-6}\ \text{m/groove}}$$

$$\theta_{400} = 13.65°$$

For $\lambda = 720$ nm:

$$\sin\theta_{720} = \frac{m\lambda}{a} = \frac{1(7.20\times10^{-7}\,\text{m})}{1.695\times10^{-6}\,\text{m/groove}}$$
$$\theta_{720} = 25.14°$$
$$\Delta\theta = \theta_{720} - \theta_{400} = 11.49°$$

10.59 (10.32) $a\sin\theta_m = m\lambda$, so $\sin\theta_m = m\lambda/a$; is $\theta_2(\text{red}) > \theta_3(\text{violet})$?

$$5000\text{ lines/cm} = 5\times10^5\text{ lines/m};\quad a = 2\times10^{-6}\text{ cm.}$$
$$\sin\theta_2(\text{red}) = 2(7.8\times10^{-7}\,\text{m})/(2\times10^{-6}\,\text{m});\quad \theta_2(\text{red}) = 51.3°.$$
$$\sin\theta_3(\text{violet}) = 3(3.90\times10^{-7}\,\text{m})/(2\times10^{-6}\,\text{m}) = 35.8°.$$

Spectra do not overlap. Note: Can see "by inspection" by comparing factor of 2 in wavelength to factor of 3/2 in m's.

10.60 $\sin\theta = \dfrac{m\lambda}{a} = \dfrac{2(7.00\times10^{-7}\,\text{m})}{1.6949\times10^{-6}\,\text{m}} = 0.826$

$$\theta = 55.69°$$
$$\beta = \frac{kb}{2}\sin\theta$$
$$\pi = \frac{\pi}{\lambda}b\sin\theta$$
$$b = \frac{\lambda}{\sin\theta} = \frac{7.00\times10^{-7}\,\text{m}}{0.826} = 8.47\times10^{-7}\,\text{m}$$

10.61 The largest value of m in Eq. (10.32) occurs when the sine function is equal to one, making the left side of the equation as large as possible, then $m = a/\lambda = (1/1\times10^6)/(3.0\times10^8\text{ m/s}\div4.0\times10^{14}\text{ Hz}) = 1.3$, and only the first-order spectrum is visible.

10.62 (10.32) $a\sin\theta_m = m\lambda$, where $\lambda = \lambda_o/n$.

$$\sin\theta_m = m\lambda/a;\quad \sin\theta_1(\text{vacuum})/\sin\theta_1(\text{Mongo})$$
$$= [(1)\lambda_o/a]/[(1)\lambda_o/na];\quad n = \sin(20.0°)/\sin(18.0°) = 1.11.$$

10.63 $\sin\theta_i = n\sin\theta_n$ Optical path length difference is $m\lambda$, $a\sin\theta_m - na\sin\theta_n = m\lambda$. $a(\sin\theta_m - \sin\theta_n) = m\lambda$.

10.64 $R = \dfrac{\lambda}{(\Delta\lambda)_{\min}} = mN = 2(60{,}000) = 120{,}000$

$$(\Delta\lambda)_{\min} = \frac{\lambda}{R} = \frac{5.40\times10^{-7}\,\text{m}}{120{,}000} = 0.0045\text{ nm}$$

10.65 $R = mN = 10^6$, $N = 78\times10^3$. Therefore $m = 10^6/78\times10^3$,
$\Delta\lambda_{fsr} = \lambda/m = 500\text{ nm}/(10^6/78\times10^3) = 39$ nm.

$R = Fm = F2b_f d/\lambda = 10^6\Delta\lambda_{fsr} = \lambda^2/wn_f d = 0.0125$ nm.

10.66 $R = \lambda/\Delta\lambda = 5892.9/5.9 = 999$, $N = R/m = 333$.

10.67 The diffraction pattern will be the same as for a single hole.

10.68 $y = L\lambda/d,\ d = 12\times10^{-6}/12\times10^{-2} = 10^{-4}$ m.

10.69 (From 10.75) $E_l = [-K_l\varepsilon_A\rho\lambda/(\rho+r)][\sin(\omega t - k\rho - kr)]_{r=r_{l-1}}^{r=r_l}$; $\sin(\omega t - k\rho - kr_l) - \sin(\omega t - k\rho - kr_{l-1})$

$$= \sin(\omega t - k\rho - k(r_o + l\lambda/2))$$
$$-\sin(\omega t - k\rho - k(r_o + (l-1)(\lambda/2)))$$
$$= \sin(\omega t - k(\rho + r_o) - 2\pi l\lambda/2\lambda) - \sin(\omega t - k(\rho + r_o) - (2\pi(l-1)\lambda/2\lambda)).$$

Recall $\sin(\alpha-\beta) = \sin\alpha\cos\beta^{+}\cos\alpha\sin\beta$ and $\sin\pi l = \sin(l-1)\pi = 0.$
$\cos(l-1)\pi = -\cos l\pi;\ \cos l\pi = (-1)^l\pi$, so

$$E_l = (-1)^{l+1}(2K_l\varepsilon_A\rho\lambda)/(\rho+r_o)\sin(\omega t - k(\rho+r_o)).$$

10.70 $A = 2\pi\rho^2\int_0^{\varphi}\sin\varphi d\varphi = 2\pi\rho^2(1-\cos\varphi)$,

$\cos\varphi = [\rho^2 + (\rho+r_0)^2 - r_l^2]/2\rho(\rho+r_0),\ r_l = r_0 + l\lambda/2.$

Area of first l zones $A = 2\pi\rho^2 - \pi\rho(2\rho^2 + 2\rho r_0 - l\lambda r_0 - l^2\lambda^2/r)/(\rho+r_0)$,

$A_l = A - A_{l-1} = \lambda\pi\rho(r_0 + (32l-1)\lambda/4)(\rho+r_0).$

10.71 (10.78) becomes

$E = |E_1|/2 + \left(|E_1|/2 - |E_2| + |E_3|/2\right) + \cdots + \left(|E_{m-1}|/2 - |E_m|\right),$

so that (10.80) becomes $E < |E_2|/2 - |E_m|/2$ and (10.82) becomes
$E > |E_1|/2 - |E_m|/2$ so that (10.84) $E \approx E < |E_1|/2 - |E_m|/2$.

10.72 $N_F = \dfrac{(\rho+r_0)}{\rho r_0\lambda}R^2 = \dfrac{\rho R^2}{\rho r_0\lambda} + \dfrac{r_0R^2}{\rho r_0\lambda} = \dfrac{R^2}{r_0\lambda} + \dfrac{R^2}{\rho\lambda}$

For a collimated light source, $\rho\to\infty, 1/\rho\to0$:

$$N_F = \frac{R^2}{r_0\lambda} = \frac{(3\times10^{-3}\text{ m})^2}{(6.00\text{ m})(5.00\times10^{-7}\text{ m})} = 3$$

This would appear as a bright spot. When $N_F \geq 1$, one obtains a Fresnel diffraction pattern.

10.73 (10.91) $R_m^2 = mr_o\lambda$ so $R_m = (mr_o\lambda)^{1/2}$;

$R_1 = (1(1.00\text{ m})5.6819\times10^{-7}\text{ m})^{1/2} = 7.54\times10^{-4}$ m.

10.74 The full first zone has a radius $q_1 = 1.22R\lambda/2a$. Since area $= \pi q^2$, half the first zone corresponds to $q = q_1/\sqrt{2} = 1.22R\lambda/2\sqrt{2}a$; $I_o = \varepsilon_A^2A^2$, for a plane wave, so (10.55) becomes

$$I = \frac{I_o}{2R^2}\left[\frac{J_1(kaq/R)}{kaq/R}\right]^2 = \frac{I_o}{2R^2}\left[\frac{J_1(ka(1.22R\lambda/2\sqrt{2}a)/R)}{ka(1.22R\lambda/2\sqrt{2}a)/R}\right]^2$$

$$= \frac{I_o}{2R^2}\left[\frac{J_1(1.22\pi/\sqrt{2})}{1.22\pi/\sqrt{2}}\right]^2 \simeq \frac{I_o}{2R^2}(0.026) = \frac{I_o}{R^2}(0.013) \text{ (using Table 10.2)}$$

10.75 The primary wave at point P has the form given by equation (10.86):

$$E = \frac{\varepsilon_0}{(\rho + r_0)}\cos[\omega t - k(\rho + r_0)]$$

This follows from the fact that the first zone ($l = 1$) is positive ($l + 1 = 2$), thus $E_l = (-1)^{l+1} = 1$, and from $E_l - 2E$.

10.76 For 3 zones:

$$E = E_1 - E_2 + E_3 \approx E_1$$

This is twice the amplitude of the unobstructed wave: $E_u = \frac{1}{2}E$. Thus,

$$I_u = \frac{1}{4}I$$
$$I = 4I_u$$

10.77 $N_F = \dfrac{(\rho + r_0)}{\rho r_0 \lambda}R^2 = \dfrac{\cancel{\rho} R^2}{\cancel{\rho} r_0 \lambda} + \dfrac{\cancel{r_0} R^2}{\rho \cancel{r_0} \lambda} = \dfrac{R^2}{r_0 \lambda} + \dfrac{R^2}{\rho \lambda}$

For a collimated light source, $\rho \to \infty, 1/\rho \to 0$:

$$N_F = \frac{R^2}{r_0 \lambda}$$
$$r_0 = \frac{R^2}{N_F \lambda}$$

For the first bright spot, $N_F = 1$:

$$r_0 = \frac{(2.5\times 10^{-3}\text{ m})^2}{1(5.50\times 10^{-7}\text{ m})} = 11.4\text{ m}$$

The first dark spot, $N_F = 1$:

$$r_0 = 11.4\text{ m}/2 = 5.7\text{ m}$$

10.78 $N_F = \dfrac{(\rho + r_0)}{\rho r_0 \lambda}R^2 = \dfrac{(1\text{ m} + 1\text{ m})}{(5.00\times 10^{-7}\text{ m})}(1\times 10^{-3}\text{ m})^2 = 4$

The point P would be dark.

10.79 The disk obscures zones:

$$N_F = \frac{(\rho + r_0)}{\rho r_0 \lambda}R^2 = \frac{(1\text{ m} + 1\text{ m})}{(5.00\times 10^{-7}\text{m})}(.5\times 10^{-3}\text{ m})^2 = 1$$

The disk blocks the first zone, the annulus passes zones 2, 3 and 4.

$$E \approx -2E_u$$
$$\frac{I}{I_u} \approx 4$$

10.80 $R_m^2 = mr_0\lambda = mf\lambda$

$$f = \frac{R_m^2}{m\lambda} = \frac{(3.00\times10^{-3}\,\text{m})^2}{10(6.00\times10^{-7}\,\text{m})} = 1.5\text{ m}$$

10.81 $R_m^2 = mr_0\lambda = mf\lambda$

$$R_1 = \sqrt{1(2.00\text{ m})\,(6.47\times10^{-7}\text{ m})} = 1.14\text{ mm}$$

$$R_{30} = \sqrt{(30)(2.00\text{ m})(6.47\times10^{-7}\text{ m})} = 6.23\text{ mm}$$

10.82 $u = y\left[\dfrac{2(\rho_0 + r_0)}{\rho_0 r_0\lambda}\right]^{1/2}$

$$u = y\left[2\frac{\cancel{\rho}}{\cancel{\rho}r_0\lambda} + 2\frac{\cancel{r_0}}{\rho\cancel{r_0}\lambda}\right]^{1/2} = y\left[\frac{2}{r_0\lambda} + \frac{2}{\rho\lambda}\right]^{1/2}$$

For a collimated light source, $\rho \to \infty$, $1/\rho \to 0$:

$$u = y\left[\frac{2}{r_0\lambda}\right]^{1/2}$$

$$\upsilon = z\left[\frac{2}{r_0\lambda}\right]^{1/2}$$

$$u_1 = y_1\left[\frac{2}{r_0\lambda}\right]^{1/2} = (-1\times10^{-3}\,\text{m})\left[\frac{2}{(5\text{m})(5.00\times10^{-7}\,\text{m})}\right]^{1/2} = -0.8944$$

$$u_2 = y_2\left[\frac{2}{r_0\lambda}\right]^{1/2} = (1\times10^{-3}\,\text{m})\left[\frac{2}{(5\text{m})(5.00\times10^{-7}\,\text{m})}\right]^{1/2} = 0.8944$$

$$\upsilon_1 = z_1\left[\frac{2}{r_0\lambda}\right]^{1/2} = (-5\times10^{-4}\,\text{m})\left[\frac{2}{(5\text{m})(5.00\times10^{-7}\,\text{m})}\right]^{1/2} = -0.4472$$

$$\upsilon_2 = z_2\left[\frac{2}{r_0\lambda}\right]^{1/2} = (5\times10^{-4}\,\text{m})\left[\frac{2}{(5\text{m})(5.00\times10^{-7}\,\text{m})}\right]^{1/2} = 0.4472$$

$$I = \frac{I_u}{4}\{[2C(0.8944)]^2 + [2L(0.8944)]^2\{[2C(0.4472)]^2 + [2L(0.4472)]^2\}$$

$$I = \frac{I_u}{4}\{[2(0.761)]^2 + [2(0.331)]^2\}\{[2(0.445)]^2 + [2(0.049)]^2\}$$

$$I = \frac{I_u}{4}\{2.316 + 0.438\}\{0.792 + 0.0096\}$$

$$I = 0.55\ \ I_u = 16.6\text{ W}$$

10.83 (From 10.42 and 10.43), $I(0) \propto \frac{1}{2}(A\varepsilon_A/R)^2$, recall (3.46) $I = \varepsilon_o c\left\langle E^2\right\rangle_T$ so,
$I(0) = \frac{1}{2}\varepsilon_o c(A\varepsilon_A/R)^2$; I (incident) $= \frac{1}{2}\varepsilon_o c(A\varepsilon_A)^2$ = (flux) (area) =
10 W/m^2 $(5.0\times10^{-3}\text{ m})^2 = 2.5\times10^{-4}$ W; $I(0) = I(\text{incident})\,A/R^2$ =
$(2.5\times10^{-4}\text{ W})(5.0\times10^{-3}\text{ m})^2 = 6.25\times10^{-9}$ W.

10.84 For the solution to this problem, please refer to the textbook.

10.85 $I = (I_0/2)([1/2 - C(\upsilon_1)]^2 + [1/2 - S(\upsilon_1)]^2)$,
$I = (I_0/2)(1/\pi\upsilon_1)^2[\sin^2(\pi\upsilon_1^2/2) + \cos^2(\pi\upsilon_1^2/2)] = I_0/2(\pi\upsilon_1)^2$.

10.86 Fringes in both the clear and shadow region [see M. P. Givens and W. L. Goffe, *Am. J. Phys.,* **34**, 248 (1966)].

10.87 $u = y[2/\lambda r_0]^{1/2}$; $\Delta u = \Delta y \times 10^3 = 2.5$.

10.88 For the solution to this problem, please refer to the textbook.

10.89 We should see symmetry through the *x-y* plane in both patterns. The keyhole should bear some resemblance to the combined patterns of a circle and a rectanglar aperture. The image of the triangle should have nearly 3-fold symmetry.

10.90 As the slit widens, the pattern becomes more like that of a rectangular aperture (see Figure 10.49).

10.91 $\upsilon = z\left(\dfrac{2}{\lambda r_0}\right)^{1/2}$

$$\upsilon = z\left(\frac{2}{(5\times10^{-7}\text{ m})(2\text{ m})}\right)^{1/2}$$
$$= z(1414)$$
$$z = \pm\frac{1}{2}(1\times10^{-4}\,\omega)$$
$$V_1 = -0.1414 \qquad V_2 = 0.1414$$
$$\left|\tilde{B}_{12}\right| \approx 0.28$$
$$\frac{I_P}{I_u} = \frac{1}{2}\left|B_{12}\right|^2$$
$$= 0.0392$$

10.92 $\upsilon = z\left[\dfrac{2}{r_0\lambda}\right]^{1/2}$

Using $z_1 = 0$ and $z_2 = 0.70$ mm:

$$\upsilon_1 = 0,\ \upsilon_2 = (7.0\times10^{-4}\text{ m})\left[\frac{2}{(1\text{ m})\,(6.00\times10^{-7}\text{ m})}\right]^{1/2} = 1.278$$
$$B_{12} = 0.93$$
$$\frac{I_{P'}}{I_u} = \frac{1}{2}(0.93)^2 = 0.432$$
$$I_{P'} = 43\text{ W}$$

10.93 $\upsilon = z\left[\dfrac{2}{r_0\lambda}\right]^{1/2}$

Using $z_1 = 0$ and $z_2 = 0.70$ mm:

$$\upsilon_1 = 0,\ \ \upsilon_2 = (7.0\times10^{-4}\,\text{m})\left[\frac{2}{(1\text{m})\,(6.00\times10^{-7}\,\text{m})}\right]^{1/2} = 1.278$$

$$B_{2+} = -0.230$$

$$B_{-1} = 0.707$$

These are anti-parallel, 0.707 − 0.230 = 0.477.

$$\frac{I_P}{I_u} = \frac{1}{2}(0.477)^2 = 0.114$$