Chapter 3 Solutions

3.1 Compare $E_y = 2\cos[2\pi \times 10^{14}(t - x/c) + \pi/2]$ to $E_y = A\cos[2\pi\nu(t - x/\upsilon) + \pi/2]$. (a) $\nu = 10^{14}$ Hz, $\upsilon = c$, and $\lambda = c/\nu = 3\times10^8/10^{14} = 3\times10^{-6}$ m, moves in positive x-direction, $A = 2$ V/m, $\varepsilon = \pi/2$ linearly polarized in the y-direction. (b) $B_x = 0, B_y = 0, B_z = E_y/c$.

3.2 $E_z = 0, E_y = E_x = E_0\sin(kz - \omega t)$ or cosine; $B_z = 0, B_y = -B_x = E_y/c$, or if you like,

$$\vec{E} = \frac{E_0}{\sqrt{2}}(\hat{i} + \hat{j})\sin(kz - \omega t), \qquad \vec{B} = \frac{E_0}{c\sqrt{2}}(\hat{j} - \hat{i})\sin(kz - \omega t).$$

3.3 First, by the right-hand rule, the directions of the vectors are right. Then $kE = \omega B$ and so $(2\pi/\lambda)E = \omega B = 2\pi\nu B$, hence $E = \lambda\nu B = cB$.

3.4 $\partial E/\partial x = -kE_0\sin(kx - \omega t); -\partial B/\partial t = -\omega B_0\sin(kx - \omega t)$; $-kE_0 = -\omega B_0; E_0 = (\omega/k)B_0$ and Eq. (2.33) $\omega/k = c$.

3.5 (a) The electric field oscillates along the line specified by the vector $-6\hat{i} + 3\sqrt{5}\hat{j}$. (b) To find E_0, dot $\vec{E}_0$ with itself and take the square root, thus $\sqrt{36+45}\ 10^4$ V/m $= 9\times10^4$ V/m. (c) From the exponential $\vec{k}\cdot\vec{r} = (\sqrt{5}x + 2y)(\pi/3)\times10^7$, hence $\vec{k} = (\sqrt{5}\hat{i} + 2\hat{j})(\pi/3)\times10^7$ and because the phase is $\vec{k}\cdot\vec{r} - \omega t$ rather than $\vec{k}\cdot\vec{r} + \omega t$ the wave moves in the direction of $\vec{k}$. (d) $\vec{k}\cdot\vec{k} = (\pi\times10^7)^2$, $k = \pi\times10^7\ \text{m}^{-1}$ and $\lambda = 2\pi/k = 200$ nm. (e) $\omega = 9.42\times10^{15}$ rad/s and $\nu = \omega/2\pi = 1.5\times10^{15}$ Hz. (f) $\upsilon = \nu\lambda = 3.00\times10^8$ m/s.

3.6 (a) The field is linearly polarized in the y-direction and varies sinusoidally from zero and $z = 0$ to zero at $z = z_0$. (b) Using the wave equation

$$\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = 0,$$

$$\left[-k^2 - \frac{\pi^2}{z_0^2} + \frac{\omega^2}{c^2}\right] E_0 \sin\frac{\pi z}{z_0}\cos(kx - \omega t) = 0$$

and since this is true for all x, z, and t each term must equal zero and so $k = (\omega/c)\sqrt{1 - (c\pi/\omega z_0)^2}$. (c) Moreover, $\upsilon = \omega/k = c/\sqrt{1 - (c\pi/\omega z_0)^2}$.

3.7 $\mathbf{\vec{B}} = \dfrac{(10\text{ v/m})}{c}(\cos 0.5\,\pi)\hat{\mathbf{j}}$

3.8 (a) $c = \nu\lambda$, so $\nu = c/\lambda = (3\times10^8\text{ m/s})/(550\times10^{-9}\text{ m}) = 5.45\times10^{14}$ Hz. (b) $\omega = 2\pi\nu = 2\pi(5.45\times10^{14}\text{ Hz}) = 3.42\times10^{15}$ rad/s; $k = 2\pi/\lambda = 2\pi/(550\times10^{-9}\text{ m}) = 1.14\times10^{-7}\text{ m}^{-1}$. (c) $E_0 = cE_0$, so $B_0 = E_0/c = (600\text{ V/m})/(3\times10^8\text{ m/s}) = 2\times10^{-6}\text{ V-s/m}^2 = 2\times10^{-6}$ T.

(d) $E(y,t) = E_0 \sin(ky - \omega t + \varepsilon)$; $E(0, 0) = 0 = E_0 \sin(\varepsilon), \varepsilon = 0$;
$B(y, t) = B_0 \sin(ky - \omega t + \varepsilon)$; $B(0, 0) = 0 = B_0 \sin(\varepsilon), \varepsilon = 0$;
$E(y, t) = (600 \text{ V/m}) \sin((1.14 \times 10^{-7} \text{ m}^{-1})y - (3.42 \times 10^{15} \text{ rad/s})t)$;
$B(y, t)\ (2 \times 10^{6} T) \sin((1.14 \times 10^{-7} \text{ m}^{-1})y - (3.42 \times 10^{15} \text{ rad/s})t)$.

3.9 $\vec{\mathbf{B}} = (-\hat{\mathbf{i}} + \hat{\mathbf{j}})\dfrac{E}{c}\sin(kz - \omega t + \pi/6)$

$\vec{\mathbf{B}}(0,0) = (-\hat{\mathbf{i}} + \hat{\mathbf{j}})\dfrac{E}{c}\sin(\pi/6)$

3.10 $\vec{\mathbf{E}} = (\hat{\mathbf{i}} + \hat{\mathbf{j}})E_0 \sin(kz - \omega t + \pi/6)$
$\vec{\mathbf{E}}(\lambda/2, 0) = (\hat{\mathbf{i}} + \hat{\mathbf{j}})E_0 \sin(-\pi - 0 + \pi/6)$
$\vec{\mathbf{E}}(\lambda/2, 0) = (\hat{\mathbf{i}} + \hat{\mathbf{j}})E_0 \sin(-5\pi/6)$
$\vec{\mathbf{E}}(\lambda/2, 0) = (\hat{\mathbf{i}} + \hat{\mathbf{j}})E_0 (-0.5)$

3.11 At $y = 0$ and $t = 0$, $\vec{\mathbf{E}} = E_0\hat{\mathbf{i}}$. The wave travels in the $+y(\hat{\mathbf{j}})$ direction. As a result, $\vec{\mathbf{B}}$ must be in the $-\hat{\mathbf{k}}$ direction. Then,

$$\vec{\mathbf{B}}(x, y, z, t) = -\frac{1}{C}E_0\hat{\mathbf{k}}\exp[i(ky - \omega t)]$$

3.12 At $z = 0$ and $t = 0$, $\vec{\mathbf{B}} = B_0\hat{\mathbf{j}}$. The wave travels in the $+z\ (\hat{\mathbf{k}})$ direction. As a result, $\vec{\mathbf{E}}$ must be in the $-\hat{\mathbf{i}}$ direction. Then,

$$\vec{\mathbf{E}}(x, y, z, t) = -cB_0\hat{\mathbf{i}}\exp[i(kz - \omega t)]$$

3.13 By Gauss' law, $E = \sigma/\varepsilon_0$, where $\sigma = q/A$ is the surface charge density. Putting the average value of this electric field into $u_E = \varepsilon_0 E^2/2$ gives $u_E = \sigma^2/8\varepsilon_0$.

3.14 $u_B = B^2/2\mu_0$; $c = 1/\sqrt{\varepsilon_0\mu_0}$, so $c^2\varepsilon_0 = 1/\mu_0$. $u_B = c^2\varepsilon_0 B^2/2$; $E = cB$, so $u_B = \varepsilon_0(cB)^2/2 = \varepsilon_0 E^2/2 = u_E$.

3.15 $\left\langle \cos^2(\vec{k}\cdot\vec{r} - \omega t)\right\rangle = (1/T)\int_t^{t+T}\cos^2(\vec{k}\cdot\vec{r} - \omega t')\,dt'$. Let $\vec{k}\cdot\vec{r} - \omega t' = x$; then
$\left\langle \cos^2(\vec{k}\cdot\vec{r} - \omega t)\right\rangle = -(1/\omega T)\int\cos^2 x\,dx = -(1/2\,\omega T)\int(1 + \cos 2x)\,dx = -(1/2\omega T)[x + 0.5\sin 2x]_{\vec{k}\cdot\vec{r}-\omega t}^{\vec{k}\cdot\vec{r}-\omega(t+T)}$.
Similarly use $\left\langle \sin^2(\vec{k}\cdot\vec{r} - \omega t)\right\rangle = (1/2)\left\langle 1 - \cos 2(\vec{k}\cdot\vec{r} - \omega t)\right\rangle$ and
$\left\langle \sin(\vec{k}\cdot\vec{r} - \omega t)\cos(\vec{k}\cdot\vec{r} - \omega t)\right\rangle = (1/2)\left\langle \sin 2(\vec{k}\cdot\vec{r} - \omega t)\right\rangle$.

3.16 Using the identity $\cos^2\alpha = \dfrac{1}{2}(1 + \cos 2\alpha)$ we have

$$\left\langle \cos^2\omega t\right\rangle_T = \left\langle \frac{1}{2}[1 + \cos 2\omega t]\right\rangle_T = \frac{1}{2}\left[1 + \left\langle \cos 2\omega t\right\rangle_T\right] = \frac{1}{2}[1 + (\operatorname{sinc}\omega T)\cos 2\omega t].$$

3.17 Using the identity $\sin^2\alpha = \dfrac{1}{2}(1 - \cos 2\alpha)$ we have

$$\left\langle \sin^2\omega t\right\rangle_T = \left\langle \frac{1}{2}[1 - \cos 2\omega t]\right\rangle_T = \frac{1}{2}[1 - \left\langle \cos 2\omega t\right\rangle_T] = \frac{1}{2}[1 - (\operatorname{sinc}\omega T)\cos 2\omega t].$$

3.18 $I = \langle S \rangle_T = \left\langle c^2 \varepsilon_0 \left| \vec{E}_0 \times \vec{B}_0 \right| \cos^2(\vec{k} \cdot \vec{r} - \omega T) \right\rangle =$
$c^2 \varepsilon_0 \left| \vec{E}_0 \times \vec{B}_0 \right| \left\langle \cos^2(\vec{k} \cdot \vec{r} - \omega t) \right\rangle = c^2 \varepsilon_0 E_0 B_0 / 2;\ E_0 = cB_0, c = 1/\sqrt{\mu_0 \varepsilon_0},$ so
$\varepsilon_0 c = 1/\mu_0 c.\ I = E_0^2 / 2c\mu_0.$ If $E_0 = 15.0$ V/m,
$I = (15.0 \text{ V/m})^2 / 2(3 \times 10^8 \text{ m/s})(4\pi \times 10^{-7} \text{ m-kg/C}^2) = .9375/\pi \text{ W/m}^2 = .298 \text{ W/m}^2.$

3.19 Start with the following relationship for the irradiance:

$$I = \frac{c\varepsilon_0}{2} E_0^2$$

$$I = \frac{P}{A} = \frac{1 \times 10^{-3}\,\text{W}}{1 \times 10^{-4}\,\text{m}^2} = \frac{c\varepsilon_0}{2} E_0^2 = \frac{3 \times 10^8 \text{ m/s} \cdot 8.85 \times 10^{-12}\,\text{C} \cdot \text{V}^{-1}\ \text{m}^{-1}}{2} E_0^2$$

$$\frac{2 \times 10 \text{ W/m}^2}{2.7 \times 10^{-3}\,\text{C} \cdot \text{V}^{-1}\text{s}^{-1}} = E_0^2$$

Using the definition of a Volt (V = J/C):

$$E_0^2 = 7.4 \times 10^3 \text{ V}^2/\text{m}^2$$

$$E_0 = 86 \text{ V/m}$$

3.20 $A = \pi r^2 = \pi \left(\dfrac{1}{\sqrt{\pi}} \right)^2 = 1.0 \text{ cm}^2$

$40\text{J/s} \times 60 \text{ s/min} = 2.4 \times 10^3 \text{ J/min}$

3.21 (a) Since $\vec{\mathbf{E}} = \upsilon \vec{\mathbf{B}}$, Then use $\upsilon = \dfrac{\omega}{k} = \dfrac{1.80 \times 10^{15}}{1.20 \times 10^7} = 1.50 \times 10^8$ m/s, to obtain:

$$\vec{\mathbf{B}} = (1.50 \times 10^8 \text{ m/s})(-100 \text{ V/m})\ \hat{\mathbf{i}} \exp[i(kz - \omega t)]$$

(b) $\eta = \sqrt{k_E} = \dfrac{c}{\upsilon} = 1.5$

(c) $k_E = \dfrac{\varepsilon}{\varepsilon_0}$, then $\varepsilon = k_E \varepsilon_0 = (1.5)^2 \varepsilon_0 = 2.25 \varepsilon_0 = 1.99 \times 10^{-11}$ F/m

(d) $I = \dfrac{\varepsilon \upsilon}{2} E_0^2 = \dfrac{(1.99 \times 10^{-11} \text{ F/m})(1.50 \times 10^8 \text{ m/s})}{2} (-100 \text{ V/m})^2 = 14.9 \text{ W/m}^2$

3.22 Total Power = 20 W; Total Area at 1.0 m $= 4\pi(10.0 \text{ m})^2 = 4\pi \text{ m}^2$;
I = Power/Area $= (20 \text{ W})/(4\pi \text{ m}^2) = 5/\pi \text{ W/m}^2 = 1.6 \text{ W/m}^2$.

3.23 (a) $\tau = 1/\nu = 10^{-7}$ s, $\upsilon = c$, $\lambda = c/\nu = C\tau = 30$ m.
(b) $E_y = 0.08 \cos[(2\pi\nu(t - x/c)],\ B_z = E_y/c.$) (c) By Eq. (3.44),
$\langle S \rangle_T = c\varepsilon_0 E_0^2 / 2.$

3.24 Will find I, then E_0 using Eq. (3.44). Total Power $= L = 3.9 \times 10^{26} W$; Total area at
$1.5 \times 10^{11}\text{m} = 4\pi(1.5 \times 10^{11}\text{m})^2 = 9.0\pi \times 10^{22}\text{m}^2 = 2.8 \times 10^{23}\text{m}^2$.
I = Power/Area $= (3.9 \times 10^{26}\,\text{W}) / (2.8 \times 10^{23}\,\text{m}^2) = 1.4 \times 10^3\,\text{W/m}^2$.

From Eq. (3.44) $I = (c\varepsilon_0/2)E_0^2$, so $E_0 = \sqrt{2I/c\varepsilon_0}$;

$$E_0 = \sqrt{\frac{2(1.4\times10^3\ \text{W/m}^2)}{(3\times10^8\ \text{m/s})(8.85\times10^{-12}\,\text{s}^2\text{-C}^2/\text{m-kg})}}$$

$$E_0 = 1.0\times10^3\ \text{V/m}$$

3.25 $\vec{E}_0 = (E_0/\sqrt{2})(-\hat{i}+\hat{j}),\ \vec{k} = (2\pi/\sqrt{2}\lambda)(\hat{i}+\hat{j}),$ hence
$\vec{E} = (10/\sqrt{2})(-\hat{i}+\hat{j})\cos[(\sqrt{2}\pi/\lambda)(x+y)-\omega t]$ and
$I = c\varepsilon_0 E_0^2/2 = 0.13\ \text{W/m}^2$.

3.26 (a) $l = c\Delta t = (3.00\times10^8\ \text{m/s})(2.00\times10^{-9}\ \text{s}) = 0.600\ \text{m}$. (b) The volume of one pulse is $V = l\pi R^2 = 2.9\times10^{-6}\ \text{m}^3$; therefore $(6.0\ \text{J})/V = 2.0\times10^6\ \text{J/m}^3$.

3.27 From Eq. (3.44), $I = c\varepsilon_0 E_0^2/2$ and so
$E_0 = \sqrt{2I/c\varepsilon_0} = \sqrt{7.535\times10^{22}} = 2.7\times10^{11}\ \text{V/m}$.

3.28 $u = (\text{power})(t)/(\text{volume}) = (10^{-3}\,\text{W}(t)/[(\pi r^2)(ct)] = 10^{-3}\,\text{W}/[\pi(10^{-3})^2(3\times10^8)],\ u = 1.1\times10^{-6}\ \text{J/m}^3$.

3.29 $V = Al = A\upsilon t$ so that
$N/At = nV/At = n\upsilon = 100\ \text{m}^{-3}\,6\ \text{m/60s} = 10\ \text{m}^{-2}\text{s}^{-1}$.

3.30 $I/E = I/h\upsilon = (19.88\times10^{-2})/(6.63\times10^{-34})(100\times10^6) = 3\times10^{24}$ photons/m^{2}s.
$n = (1/c)(I/E) = 10^{16}$ photons/m^3.

3.31 $N/t = P/h\upsilon = P\lambda/hc = 2.8\times10^{20}\ \text{s}^{-1}$.

3.32 $P_e = iV = (0.25)(3.0) = 0.75$ W. This is the electrical power dissipated.
The power available as light is $P_l = (0.01)P_e = 75\times10^{-4}$ W. (a) The photon flux is $P_l/h\upsilon = P_l\lambda/hc = 2.1\times10^{16}$ photons/s. (b) There are 2.1×10^{16} in volume $(3\times10^8)(1s)(10^{-3})\ \text{m}^2$. Therefore $2.1\times10^{16}/3\times10^5 = 0.69\times10^{11}$ is the number of photons per cubic meter.
(c) $I = 75\times10^{-4}\ \text{W}/10\times10^{-4}\ \text{m}^2 = 7.5\ \text{W/m}^2$.

3.33 $I = P/4\pi r^2 = 100\text{w}/4\pi(1\text{m})^2 = 8\text{W/m}^2,\ E_0 = \sqrt{2I/\varepsilon_0 c} = 78\text{V/m},$ and $B_0 = E_0/c = 2.6\times10^{-7}\,\text{T}$.

3.34 Imagine two concentric cylinders of radii r_1 and r_2 surrounding the wave. The energy flowing per second through the first cylinder must pass through the second; that is, $\langle S_1\rangle 2\pi r_1 = \langle S_2\rangle 2\pi r_2$, and so $\langle S\rangle 2\pi r =$ constant and $\langle S\rangle$ varies inversely with r. Therefore, since $\langle S\rangle \propto E_0^2$, E_0 varies as $\sqrt{1/r}$.

3.35 $p = E/c = h\nu/c = 2.2\times10^{-23}\ \text{kg m s}^{-1}$

3.36 $\langle dp/dt\rangle = \langle dW/dt\rangle/c$, with area A, $\langle P\rangle = \langle dp/dt\rangle/A = \langle dW/dt\rangle/Ac = I/c$.

3.37 (a) $\vec{\mathbf{E}}(z,t) = \hat{\mathbf{i}}\left(6.0\dfrac{\text{V}}{\text{m}}\right)\cos k(z-ct)$

(b) Since $B_0 = \dfrac{E_0}{c}$, thus $\vec{\mathbf{B}}(z,t) = -\hat{\mathbf{j}}\dfrac{\vec{\mathbf{E}}}{c}\cos k(z-ct)$

(c) $\vec{\mathbf{P}}_\upsilon = \hat{\mathbf{k}}\dfrac{\varepsilon_0}{c}E_0^2\cos^2 k(z-ct)$

3.38 From Eq. (3.52) the force exerted by the beam of light, $AP = \Delta p/\Delta t$, where $p(\text{incident}) = \xi/c$.
For reflected light at normal incidence, Δp = twice the incident momentum $= 2(\xi/c)$

$$AP = 2(\xi/c)/\Delta t, \text{ but, } I = \frac{\xi}{Area \cdot time}, \text{ so } P = 2I/c.$$

At an angle θ with respect to the normal, only the component of momentum normal to the surface changes, so p (normal) $= p\cos\theta$, so, $P(\theta) = 2I\cos\theta/c$.

3.39 $E = P \cdot t = (300w)(100s) = 3\times10^4$ J, $p = E/c = 10^{-4}$ kg m/s.

3.40 (a) $\langle P = 2\langle S\rangle/c\rangle = 2(1.4\times10^3 \text{ W/m}^2)/(3\times10^8 \text{ m/s}) = 9\times10^{-6} \text{ N/m}^2$.
(b) S, and therefore P, drops off with the inverse square of the distance, and hence
$\langle S\rangle = [(0.7\times10^9 \text{ m})^{-2}/(1.5\times10^{11} \text{ m})^{-2}]\ (1.4\times10^3 \text{ W/m}^2) = 6.4\times10^7 \text{ W/m}^2$, and $\langle P\rangle = 0.21 \text{ N/m}^2$.

3.41 I (absorbed) $= \alpha I$ and I (scattered) $= (1-\alpha)\,I$; the pressure arises from both contributions, viz.
$P = \alpha I/c + 2(1-\alpha)\,I/c = (2-\alpha)\,I/c$.

3.42 The reflected component has a momentum change, and thus a pressure, of twice the incident momentum, while the absorbed component has a momentum change of the incident momentum.

$$P \text{ (reflected)} = 2(70.0\%)I/c = 2(0.700)(2.00\times10^6 \text{ W/m}^2)/(3\times10^8 \text{ m/s})$$
$$= .93\times10^{-2} \text{ N/m}^2.$$
$$P \text{ (absorbed)} = (30.0\%)I/c = (0.300)(2.00\times10^6 \text{ W/m}^2)/(3\times10^8 \text{ m/s})$$
$$= .20\times10^{-2} \text{ N/m}^2.$$
$$P = P \text{ (reflected)} + P \text{ (absorbed)} = 1.13\times10^{-2} \text{ N/m}^2.$$

3.43 $\langle S\rangle = 1400 \text{ W/m}^2$, $\langle P\rangle = 2(1400 \text{ W/m}^2/3\times10^8 \text{ m/s}) = 9.3\times10^{-6} \text{ N/m}^2$.
$F = \langle P\rangle A = (9.3\times10^{-6}\text{N/m}^2)(2\times10^3\text{m}^2) = 1.86\times10^{-2}\text{N}$

3.44 $\langle S\rangle = (200\times10^3 \text{ W})(500\times2\times10^{-6}\text{s})/A(1s)$,
$\langle F\rangle = A\langle P\rangle = A\langle S\rangle/c = 6.7\times10^{-7}\text{N}$.

3.45 $\langle F\rangle = A\langle P\rangle = A\langle S\rangle/c = 10 \text{ W}/3\times10^8 = 3.3\times10^{-8}\text{N}$,
$a = 3.3\times10^{-8}/100 \text{ kg} = 3.3\times10^{-10} \text{ m/s}^2$,
$\upsilon = at = 3.3\times10^{-10}t = 10$ m/s. Therefore $t = 3\times10^{10}$s or $t = 940$ years.

3.46 $\vec{B}$ surrounds $\vec{\upsilon}$ in circles, and $\vec{E}$ is radial, hence $\vec{E}\times\vec{B}$ is tangent to the sphere, and no energy radiates outward from it.

3.47 (a) $\nu = 5\times10^{14}$ Hz
(b) $\lambda = \text{v}/\nu = 0.65\text{c}/\nu = 3.9\times10^{-7}$ m = 390 nm
(c) $n = c/\text{v} = 1.538$

3.48 $c/\upsilon = 2.42$; $\upsilon = 1.24\times10^8$ m/s.

3.49 $\lambda_0 = 540$ nm; $n = \nu\lambda_0/\nu\lambda$; $\lambda_0/n = \lambda = 406$ nm.

3.50 $n = c/\upsilon = 1/0.90 = 1.11 = 1.1$.

3.51 $n = c/\upsilon = (3\times10^8 \text{ m/s})/(1.245\times10^8 \text{ m/s}) = 2.410$

3.52 $l = \upsilon t = (c/n)t = (3.00\times10^8 \text{ m/s})(1.00 \text{ s})/1.333 = 2.25\times10^8$ m.

3.53 $\lambda = \lambda_0/n = (500\text{ nm})/1.60 = 312.5\text{ nm}$;
$(1.00\times10^{-2}\text{ m})/(312.5\times10^{-9}\text{ m}) = 3.2\times10^{4}$ waves.

3.54 $t_1 = (20.0\text{ m})/(c/1.47)$ and $t_2 = (20.0\text{ m})/(c/1.63)$, hence $t_2 - t_1 = 3.2/c = 1.07\times10^{-8}$ s.

3.55 The number of waves is vacuum is $\overline{AB}/\lambda_0$. With the glass in place, there are $(\overline{AB} - L)/\lambda_0$ waves in vacuum and an additional L/λ waves in glass for a total of $(\overline{AB}/\lambda_0) + L(1/\lambda - 1/\lambda_0)$. The difference in number is $L(1/\lambda - 1/\lambda_0)$, giving a phase shift of $\Delta\phi$ of 2π for each wave; hence,
$2\pi L(1/\lambda - 1/\lambda_0) = 2\pi L(n/\lambda_0 - 1/\lambda_0) = 2\pi L/2\lambda_0 = 2000\pi$.

3.56 Thermal agitation of the molecular dipoles causes a marked reduction in K_e but has little effect on n. At optical frequencies n is predominantly due to electronic polarization, rotations of the molecular dipoles having ceased to be effective at much lower frequencies.

3.57 From Eq. (3.70), for a single resonant frequency we have

$$n = \left[1 + \frac{Nq_e^2}{\varepsilon_0 m_e}\left(\frac{1}{\omega_0^2 - \omega^2}\right)\right]^{1/2};$$

since for low-density materials $n \approx 1$, the second term is $\ll 1$, and we need only retain the first two terms of the binomial expansion of n. Thus $\sqrt{1+x} \approx 1 + x/2$ and $n \approx 1 + Nq_e^2/[2\varepsilon_0 m_e(\omega_0^2 - \omega^2)]$.

3.58 (a) The polar molecule, water, in the liquid state, is relatively free to move in response to the incident radiation. In the solid state, the molecules are not free to move. (b) The radar (microwave) interacts strongly with the liquid water in the droplets.

3.59 The normal order of the spectrum for a glass prism is R, O, Y, G, B, V, with red (R) deviated the least and violet (V) deviated the most. For a fuchsin prism, there is an absorption band in the green, and so the indices for yellow and blue on either side (n_y and n_B) of it are extremes, that is, n_Y is the maximum, n_B the minimum, and $n_Y > n_O > n_R > n_V > n_B$. Thus the spectrum in order of increasing deviation is B, V, black band, R, O, Y.

3.60 Since $(Nq_e^2/\varepsilon_0 m_e)^{1/2}$ has dimensions of frequency, the right-hand side is dimensionless and the units check.

3.61 With ω in the visible, $\omega_0^2 - \omega^2$ is smaller for lead glass and larger for fused silica. Hence $n(\omega)$ is larger for the former and smaller for the latter.

3.62 Subtract 1 from each side of Eq. (3.70) and then invert both sides: $1/(n^2 - 1) = (\varepsilon_0 m_e/Nq_e^2)(\omega_0^2 - \omega^2)$; since $\omega = 2\pi c/\lambda$ the desired result follows.

3.63 C_1 is the value that n approaches as λ gets larger.

3.64 The horizontal values of $n\,(\omega)$ approached in each region between absorption bands increase as ω decreases.

$$n_1 = 1.54 \quad \lambda_1 = 400\text{nm}$$
$$n_2 = 1.50 \quad \lambda_2 = 800\text{nm}$$
$$1.54 = n_1 = C_1 + C_2/\lambda_1^2$$
$$1.50 = n_2 = C_1 + C_2/\lambda_2^2$$
$$\Delta n = .04 = C_2\left(\frac{1}{\lambda_1^2} - \frac{1}{\lambda_2^2}\right)$$
$$C_2 = \Delta n \lambda_1^2 \lambda_2^2 / \left(\lambda_2^2 - \lambda_1^2\right)$$
$$C_2 = 8.53\times 10^3\,\text{nm}^2$$
$$C_1 = n_1 - C_2/\lambda_1^2 = 1.45$$

3.65 Subtracting the two equations $1.557 = n_1 = C_1 + C_2/\lambda_1^2$ and $1.547 = n_2 = C_1 + C_2/\lambda_2^2$ gives $\Delta n = 0.01 = n_1 - n_2 = C_2(1/\lambda_1^2 - 1/\lambda_2^2)$ so that $C_2 = \Delta n \lambda_1^2 \lambda_2^2/(\lambda_2^2 - \lambda_1^2) = 3.78\times 10^3\,\text{nm}^2$. Then $C_1 = n_1 - C_2/\lambda_1^2 = 1.5345$ and n (610 nm) $= C_1 + C_2/\lambda_3^2 = 1.545$.

3.66 Binomially expanding $n^2 \approx 1 + A/(1-\lambda_0^2/\lambda^2)$ gives $n^2 \approx 1 + A(1+\lambda_0^2/\lambda^2)$ or $n^2 = (1+A)[1 + A\lambda_0^2/(1+A)\lambda^2]$. Taking the square root and expanding again gives $n \approx (1+A)^{1/2}[1 + A\lambda_0^2/2(1+A)\lambda^2]$. This has the Cauchy form with $C_1 = (1+A)^{1/2}$ and $C_2 = A\lambda_0^2/2(1+A)^{1/2}$.

3.67 $\upsilon = \dfrac{E}{h} = 2.66\times 10^{15}\,\text{Hz}$