Chapter 8 Solutions

8.1
$$E_x = E_0 \cos(kz - \omega t)$$
$$E_y = -E_0 \cos(kz - \omega t)$$
$$\tan\theta = \frac{E_y}{E_x} = \frac{-E_0 \cos(kz-\omega t)}{E_0 \cos(kz-\omega t)} = -1$$
$$\theta = 135°$$
$$\vec{\mathbf{E}} \cdot \vec{\mathbf{E}} = (E_{0x}^2 + E_{0y}^2)\cos^2(kz - \omega t)$$
$$E^2 = 2E_0^2 \cos^2(kz - \omega t)$$
$$E = \sqrt{2}E_0 \cos(kz - \omega t)$$

Thus the amplitude is $\sqrt{2}E_0$.

8.2
$$E_x = 3\cos(ky - \omega t)$$
$$E_z = 4\cos(ky - \omega t)$$
$$\tan\theta = \frac{E_x}{E_z} = \frac{3\cos(ky-\omega t)}{4\cos(ky-\omega t)} = \frac{3}{4}$$

$\theta = 37°$ up from the z-axis.

8.3 E_x lags E_z by $\pi/2$. This is right-handed circular polarization. The amplitude is $E_0 = 8$.

8.4 In each part the x and y components have the same amplitude E_0.

(a) $\vec{E} = (\hat{i} - \hat{j})E_0 \cos(kz - \omega t)$ is a P state at 135° or –45°.

(b) $\vec{E} = (\hat{i} - \hat{j})E_0 \sin(kz - \omega t)$ is also a P state at 135° or –45°.

(c) E_x leads E_y by $\pi/4$. Therefore it is an ε state and left-handed.

(d) E_y leads E_x by $\pi/2$. Therefore it is an R state.

8.5 E_x leads E_y by $\pi/2$. Therefore it is a left-handed circularly-polarized standing wave.

8.6
$$\vec{E}_R = \hat{i}E_0 \cos(kz - \omega t) + \hat{j}E_0 . \sin(kz - \omega t).$$
$$\vec{E}_L = \hat{i}E_0' \cos(kz - \omega t) - \hat{j}E_0' \sin(kz - \omega t).$$
$$\vec{E} = \vec{E}_R + \vec{E}_L = \hat{i}(E_0 + E_0')\cos(kz - \omega t) + \hat{j}(E_0 - E_0')\sin(kz - \omega t).$$

Let
$$E_0 + E_0' = E_{0x}'' \text{ and } E_0 - E_0' = E_{0y}'';$$
then
$$\vec{E} = \hat{i}E_{0x}'' \cos(kz - \omega t) + \hat{j}E_{0y}'' \sin(kz - \omega t).$$

From Eqs. (8.11) and (8.12) it is clear that we have an ellipse where $\varepsilon = -\pi/2$ and $\alpha = 0$.

8.7 $E_{0y} = E_0 \cos 25°; \quad E_{0z} = E_0 \cos 25°;$

$\vec{E}(x,t) = (0.91\hat{j} + 0.42\hat{k})E_0 \cos(kx - \omega t + \pi/2).$

8.8 $\vec{k} = k(\hat{i} + \hat{j})/\sqrt{2}.$

8.9 $\vec{E} = E_0[\hat{j}\sin(kx - \omega t) - \hat{k}\cos(kx - \omega t)].$

8.10 $\theta = 0°,\ I = I_1 \cos^2 0° = 200\ \text{W/m}^2.$

8.11 Half the energy is removed and half transmitted, hence $I = 150\ \text{W/m}^2$.

8.12 (8.24) $I(\theta) = I(0)\cos^2\theta$ so $I(\theta)/I(0) = \cos^2(60°) = (1/2)^2 = 0.25.$

8.13 $T_t = (T_0 - T_{90})\cos^2\theta + T_{90}$

$T_t = T_0\cos^2\theta + T_{90} - T_{90}\cos^2\theta$

$T_t = T_0\cos^2\theta + T_{90}(1 - \cos^2\theta)$

$T_t = T_0\cos^2\theta + T_{90}\sin^2\theta$

8.14 Natural light is unpolarized, thus 50% if the incident irradiance is assumed to vibrate parallel to the transmission axis while the other 50% is absorbed. A perfect filter will thus transmit 50% of the incident irradiance (I_i). However a real filter will transmit a fraction of that: $(1/2)I_iT_0$. In the case of an HN-22 polarizer, $1/2\, T_0 = 22\%$, and $T_0 = 44\%$. Thus the transmitted irradiance is:

$$I_t = 1/2\, I_iT_0 = (0.5)(1000\ \text{W/m}^2)(0.44) = 220\ \text{W/m}^2$$

8.15 $HN-32$, so 32% of incident light is (ideally) transmitted. Since the transmitted light is now polarized in the same direction as the 2nd filter, 64% of light incident on it is transmitted.
$I_t = (0.32)(0.64)\ I_i = 0.2\, I_i.$

8.16 In natural light each filter passes 32% of the incident beam. Half of the incoming flux density is in the form of a *P*-state parallel to the extinction axis, and effectively none of this emerges. Thus, 64% of the light parallel to the transmission axis is transmitted. In the present problem $32\% I_i$ enters the second filter, and with a 30° angle,

$$\begin{aligned} I &= \frac{1}{2}T_0^2\cos^2\theta\ \ I_i \\ &= \frac{1}{2}(0.64)^2\cos^2 30 I_i \\ &= 0.15\ I_i \end{aligned}$$

8.17 In the case of an HN38S polarizer, $1/2\, T_0 = 38\%$, and $T_0 = 76\%$.

$$I_t = 1/2\,(T_0)^2 I_i = (0.5)(0.76)^2(1000\ \text{W/m}^2) = 289\ \text{W/m}^2$$

The resulting transmittance of the pair $= \dfrac{289\ \text{W/m}^2}{1000\ \text{W/m}^2} = 29\%$

8.18 $I = I_1\cos^2\theta = (200\ \text{W/m}^2)\cos^2 40° = 153\ \text{W/m}^2.$

8.19 In the case of an HN32S polarizer, $1/2\, T_0 = 32\%$, and $T_0 = 64\%$. Then for four polarizers;

$$I_t = 1/2(T_0)^4 I_i = (0.5)(0.64)^4 I_i = 0.084 I_i$$

8.20 In the case of an HN-32 polarizer, $1/2\,T_0 = 32\%$, and $T_0 = 64\%$

(a) $I_t = 1/2\,T_0 I_i = (0.5)(0.64)\,I_i = 0.32\,I_i$

(b) $I_2 = \frac{1}{2}T_0^2 \cos^2\theta\, I_i = \frac{1}{2}(0.64)^2 \cos^2 45°\, I_i = \frac{1}{2}(0.64)^2\left(\frac{1}{\sqrt{2}}\right)^2 I_i = 0.10 I_i$

8.21 $I_2 = \frac{1}{2}T_0^2 T_0 I_i = \frac{1}{2}T_0^3 I_i = \frac{1}{2}(0.64)^3 I_i = 0.13 I_i$

8.22 50% means all the light from the first polarizer, viz. $(1/2)I_i$, is passed by the second. Hence the angle is 90°.

8.23 $I = I_1 \cos^2\theta = (200\text{ W/m}^2)\cos^2 60° = (1/4)\,200\text{ W/m}^2 = 50\text{ W/m}^2$.

8.24 $I = I_1 \cos^2 45° = (200\text{ W/m}^2)0.50 = 100\text{ W/m}^2$.

8.25 The light from the first polarizer is at 10° and has an irradiance of $I_i \cos^2 30° = 0.75 I_i$; this makes an angle of 60° with the next filter, hence $I_2 = (0.75 I_i)\cos^2 50° = 0.31 I_i$.

8.26 I' = flux density through middle polarizer $= I_1 \cos^2\theta$ (8.24).

$$\begin{aligned} I' &= I_1 \cos^2(45°) = I_1/2 \\ I_2 &= I' \cos^2(45°) \\ &= (I_1/2)/2 = I_1/4 \end{aligned}$$

8.27 Without middle polarizer, $I_t = (I_i/2)\cos^2(50°) = 207\text{ W/m}^2$. With middle polarizer, $I_t = (I_i/2)\cos^2(25°)\cos^2(25°) = 337\text{ W/m}^2$.

8.28 $I_1 = (1/2)I_i;\ I_2 = I_1\cos^2 30°;\ I_3 = I_2 \cos^2 30°;\ I_4 = I_3 \cos^2 30°$ hence $I_4 = (1/2)I_i \cos^2 30° \cos^2 30° \cos^2 30° = 0.21\,I_i = 0.21(200\text{ W/m}^2) = 42\text{ W/m}^2$.

8.29 $I_2 = I_1 \cos^2\theta = 30\% I_i$ and $I_i = 2I_1;\ 30\% I_i = (1/2)I_i \cos^2\theta;\ 0.60 = \cos^2\theta, \theta = 39°$.

8.30 With $\theta = \omega t$, the emergent flux density is

$$\begin{aligned} I &= \tfrac{1}{2}E_{01}^2 \sin^2\theta\cos^2\theta = (E_{01}^2/8)(1-\cos 2\theta)(1+\cos 2\theta) \\ &= (E_{01}^2/8)(1-\cos^2 2\theta) = (E_{01}^2/16)(1-\cos 4\theta) = (I_1/8)(1-\cos 4\theta). \end{aligned}$$

8.31 No. The crystal performs as if it were two oppositely oriented specimens in series. Two similarly oriented crystals in series would behave like one thick specimen and thus separate the *o*- and *e*-rays even more.

8.32 The polarization of the light is lost in the specular reflection from the pencil dot.

8.33 Light scattered from the paper passes through the polaroids and becomes linearly polarized. Light from the upper left filter has its $\vec{E}$-field parallel to the principal section (which is diagonal across the second and fourth quadrants) and is therefore an *e*-ray. Notice how the letters P and T are shifted downward in an extraordinary fashion. The lower right filter passes an *o*-ray so that the letter C is undeviated. Note that the ordinary image is closer to the blunt corner.

8.34 (a) and (c) are two aspects of the previous problem. (b) shows double refraction because the polaroid's axis is at roughly 45° to the principal section of the crystal. Thus both an *o*- and an *e*-ray will exist.

8.35 When $\vec{E}$ is perpendicular to the CO_3 plane the polarization will be less than when it is parallel. In the former case, the field of each polarized oxygen atom tends to reduce the polarization of its neighbors. In other words, the induced field is down while $\vec{E}$ is up. When $\vec{E}$ is in the carbonate plane two dipoles reinforce the third and vice versa. A reduced polarizabillity leads to a lower dielectric constant, a lower refractive index, and a higher speed. Thus $\upsilon_{\parallel} > \upsilon_{\perp}$.

8.36 For calcite, $n_o > n_e$. The indices are computed in the usual way, using $n = \sin[(\alpha + \delta_m)/2]\sin(\alpha/2)$, where δ_m is the angle of minimum deviation of either beam.

8.37 $\sin\theta_c = n_{\text{balsam}}/n_o = 1.55/1.658 = 0.935;\ \theta_c \sim 69°$.

8.39 The light initially enters the prism normal to the surface. Thus both the e and o rays travel the same path. At the 45° interface, the optical axis of the material is rotated by 90°. Thus the e and rays are refracted due to the different indices of refraction.

Refraction of o-ray across interface between two quartz prisms:

$$n_{i1}\sin\theta_{i1} = n_{t2}\sin\theta_{t2}$$

$$(1.5534)\sin 45° = (1.5443)\sin\theta_{t2}$$

$$\theta_{t2} = \sin^{-1}\left[\frac{1.5534}{1.5443}(0.7071)\right] = 45.34°$$

Refraction of e-ray across interface between two quartz prisms:

$$n_{i1}\sin\theta_{i1} = n_{t2}\sin\theta_{t2}$$

$$(1.5443)\sin 45° = (1.5534)\sin\theta_{t2}$$

$$\theta_{t2} = \sin^{-1}\left[\frac{1.5443}{1.5534}(0.7071)\right] = 44.66°$$

Refraction of o-ray exiting Wollaston prism:

$$n_{i2}\sin\theta_{i2} = n_{t3}\sin\theta_{t3}$$

$$(1.5534)\sin 0.34° = (1)\sin\theta_{t3}$$

$$\theta_{t3} = \sin^{-1}[1.5534(0.0059)] = 0.53°$$

Retraction of e-ray exiting Wollaston prism:

$$n_{i2}\sin\theta_{i2} = n_{t3}\sin\theta_{t3}$$

$$(1.5443)\sin 0.66° = (1)\sin\theta_{t3}$$

$$\theta_{t3} = \sin^{-1}[1.5443(0.0058)] = 1.02°$$

The angle separating the two is: $\theta = 0.53° + 1.02° = 1.55°$

8.40 (c) Undesired energy in the form of one of the *P*-states can be disposed of without local heating problems. (d) The Rochon transmits an undeviated beam (the *o*-ray), which is therefore achromatic as well.

8.41 (8.25) $\tan\theta_p = n_t/n_i = 9.0/1.0,\ \theta_p = 83.7°$. The dipole is perpendicular to the plane of incidence.

8.42 $\tan\theta_p = n_t/n_i = 1.33/1.00,\ \theta_p = 53°$.

8.43 $\tan\theta_p = n_t/n_i = 1.65/1.33,\ \theta_p = 51.1°$.

8.44 $\sin\theta_c = \dfrac{n_t}{n_i}$

Use $\tan\theta_P = \dfrac{n_t}{n_i}$,

$$\sin\theta_c = \tan\theta_P$$
$$\sin(41°) = \tan\theta_P = 0.656$$
$$\theta_P = \tan^{-1} 0.656 = 33.3°$$

8.45 $\tan\theta_P = n_t/n_i$; $n_t = \tan 54.30° = 1.39$.

8.46 $\tan\theta_P = n_g/n_e = 1.65/1.36 = 1.21$ and $\theta_p = 50.5°$; $n_e \sin\theta_p = n_g \sin\theta_t$; $\sin\theta_t = (1.36/1.65)\sin 50.50° = 0.636$ and $\theta_t = 39.5°$.

8.47 (4.5) $\sin\theta_i/\sin\theta_t = n_{ti}$; $\sin\theta_t = \sin\theta_i/n_{ti} = \sin(40°)/1.5$; $\theta_t = 25.4°$.
(8.30) $R_\| = \tan^2(\theta_i - \theta_t)/\tan^2(\theta_i + \theta_t) = \tan^2(-14.6°)/\tan^2(65.4°) = 0.014$.
(8.31) $R_\perp = \sin^2(\theta_i - \theta_t)/\sin(\theta_i + \theta_t) = \sin^2(-14.6°)/\sin^2(65.4°) = 0.077$.
(8.32) $R = \frac{1}{2}(R_\| + R_\perp) = 0.0455$.
(8.33) $V = I_p/(I_p + I_n) = (R_\perp + R_\|)/(R_\perp + R_\| + R) = 67\%$.

8.48 If we take $I_{r\perp} > I_{r\|}$, then $I_P = I_{r\perp} - I_{r\|}$. we also have $I_r = I_{r\perp} + I_{r\|}$. The degree of polarization is:

$$V_r = \frac{I_{r\perp} - I_{r\|}}{I_{r\perp} + I_{r\|}}$$

In the case of natural light,

$$I_{i\|} = I_{i\perp} = \frac{1}{2} I_i$$
$$I_{r\|} = R_\| I_{i\|} = \frac{1}{2} R_\| I_i$$
$$I_{r\perp} = R_\perp I_{i\perp} = \frac{1}{2} R_\perp I_i$$
$$V_r = \frac{I_{r\perp} - I_{r\|}}{I_{r\perp} + I_{r\|}} = V_r = \frac{\frac{1}{2} R_\perp I_i - \frac{1}{2} R_\| I_i}{\frac{1}{2} R_\perp I_i + \frac{1}{2} R_\| I_i} = \frac{R_\perp - R_\|}{R_\perp + R_\|}$$

8.49 (4. 5) $\sin\theta_i/\sin\theta_t = n_{ti}$; $\sin\theta_t = \sin\theta_t/n_{ti} = \sin(70°)/1.5$; $\theta_t = 38.8°$.
(8.26) $R_\| = \tan^2(\theta_i - \theta_t)/\tan^2(\theta_i + \theta_t) = \tan^2(-31.2°)/\tan^2(108.8°) = 0.0425$.
(8.27) $R_\perp = \sin^2(\theta_i - \theta_t)/\sin^2(\theta_i + \theta_t) = \sin^2(-31.2°)/\sin^2(108.8°) = 0.299$.
(8.28) $R = \frac{1}{2}(R_\| + R_\perp) = 0.171$.

8.50 The first step is to solve for $R_\perp$.

$$R_\perp = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)}$$

The problem give $\theta_i = 56°$, use Snell's law to obtain θ_t :

$$n_i \sin\theta_i = n_t \sin\theta_t$$
$$\sin 56° = 1.5 \sin\theta_t \Rightarrow \theta_t = 33.55°$$

$$R_\perp = \frac{\sin^2(56° - 33.55°)}{\sin^2(56° + 33.55°)} = \frac{\sin^2(22.45°)}{\sin^2(89.55°)} = \frac{0.3819^2}{0.9999^2} = 0.1458$$
$$R_\parallel = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} = \frac{\tan^2(22.45°)}{\tan^2(89.55°)} = \frac{0.41319^2}{127.32^2} = 1.0531 \times 10^{-5}$$
$$V_r = \frac{R_\perp - R_\parallel}{R_\perp + R_\parallel} = \frac{0.14579}{0.14581} = 0.9999$$

8.51 If the reflected light is totally polarized and the total reflectance is 10%:

$$R = 10\% = \frac{1}{2}(R_\parallel + R_\perp)$$

Set $R_\parallel = 0$:

$$R_\perp = 2(0.10) = 0.20$$
$$T_\perp = 1 - R_\perp = 0.80$$
$$T = \frac{1}{2}(T_\parallel + T_\perp) = \frac{1}{2}(1 + 0.80) = 0.90$$
$$T = 90\%$$

8.52 $n_o = 1.6584$, $n_e = 1.4864$. Using Snell's law, $\sin\theta_i = n_o \sin\theta_{to} = 0.766$, $\sin\theta_i = n_e \sin\theta_{te} = 0.766$, $\sin\theta_{to} \approx 0.462$, $\theta_{to} \approx 27°31'$; $\sin\theta_{te} \approx 0.515$, $\theta_{te} \approx 31°1'$; $\Delta\theta \approx 3°30'$.

8.53 (3.59) $n \equiv c/v = \lambda_o/\lambda_n$, so $\lambda_n = \lambda_o/n$.
Ordinary $\lambda_n = \lambda_o/n_o = 589.3\text{ nm}/1.5443 = 381.6\text{ nm}$.
Extraordinary $\lambda_n = \lambda_o/n_o = 589.3\text{ nm}/1.5533 = 379.4\text{ nm}$.
Same frequency $\nu = c/\lambda_o = (3 \times 10^8\text{ m/s})/5.893 \times 10^{-7}\text{ m} = 5.091 \times 10^{14}\text{ Hz}$.

8.54 E_x leads E_y by $\pi/2$. They were initially in phase and $E_x > E_y$. Therefore the wave is left-handed, elliptical, and horizontal.

8.55 After passing through the first polarizer, the beam is polarized linearly in the vertical direction. The first waveplate rotates the linear polarization from zero to $\pi/20$ rad. When the beam passes through the second waveplate, its fast axis is at $\pi/20$ radians with respect to the vertical. As a result, its fast axis is aligned with the polarization of the incident light and thus does nothing. When the beam passes through the next (third) polarizer, its fast axis is oriented at $3\pi/20$ rad with respect to the vertical and thus rotates the linear polarization from $\pi/20$ to $\pi/10$ rad. The beam's polarization is now aligned with the orientation of the fourth waveplate (and each even numbered waveplate thereafter) and thus does nothing to the polarization. Only the odd numbered waveplates rotate the beam, thus at the end of the sequence the polarization of the beam is rotated by a total of $\pi/4$ radians. The incident intensity is reduced by half as the light passes through the first polarizer (assuming no surface reflections). After passing through the waveplates, the resulting light has been rotated by $\pi/4$ radians and is thus now horizontally polarized. It is therefore transmitted by the 2nd polarizer, which is horizontal.

8.56 Placing the quarter wave plate first will have no effect on the irradiance. The irradiance will be affected with the quarter wave plate following the polarizer.

8.57 This is left circularly polarized light. E_y leads by $\pi + \pi/2 = 3\pi/2$.

8.58 For right circularly polarized light E_y leads by $\pi/2$. The quarter wave plate with a vertical fast axis introduces a shift of $\pi/2$. As a result, one has linearly polarized light in the 2nd and 4th quadrants, which means that the state shifted 1/4 of the way around the circle in figure 8.42.

8.59 For right circularly polarized light E_x leads by $3\pi/2$. The quarter wave plate with a horizontal fast axis introduces a shift of $\pi/2$. As a result, one has linearly polarized light in the 1st and 3rd quadrants.

8.60 The a half-wave plate with a fast vertical axis will introduce a shift of π, thus linear light in the1st and 3rd quadrants will emerge.

8.61 The half-wave plate with a fast vertical axis will introduce a shift of π, thus left circularly polarized light will emerge.

8.62 The quarter-wave plate with a fast horizontal axis will introduce a shift of $\pi/2$, thus left elliptically polarized light will emerge.

8.65 $d = \dfrac{(2m+1)\lambda_0/2}{|n_o - n_e|} =$

Using $m = 0$

$$d = \frac{\lambda_0/2}{|n_o - n_e|} = \frac{(5.9\times 10^{-7}\text{ m})/2}{|1.5443 - 1.5534|} = \frac{2.95\times 10^{-7}\text{ m}}{0.0091} = 32.4\ \mu\text{m}$$

8.66 Emerging wave is elliptically polarized with $\varepsilon = (\pi/2 - \pi/4) = \pi/4$.

8.67 The polarizers are aligned. The cellophane is a half wave plate, so is seen as "dark" (no beam passing through in this region).

8.68 The *R*-state incident on the glass screen drives the electrons in circular orbits, and they reradiate reflected circular light whose $\vec{E}$-field rotates in the same direction as that of the incoming beam. But the propagation direction has been reversed on reflection, so that although the incident light is in an *R*-state, the reflected light is left-handed. It will therefore be completely absorbed by the right-circular polarizer.

8.69 $\Delta\varphi = 2\pi d\Delta n/\lambda_0$ but $\Delta\varphi = (1/4)(2\pi)$ because of the fringe shift.

Therefore $\Delta\varphi = \pi/2$ and $d = \dfrac{\lambda_0}{4\Delta n} = \dfrac{5.89\times 10^{-7}}{4(5\times 10^{-3})} = 2.94\times 10^{-5}\,\text{m}$.

8.70 Yes. If the amplitudes of the *P*-states differ. The transmitted beam, in a pile-of-plates polarizer, especially for a small pile.

8.71 Concentration is $10\text{ g}/1000\text{ cm}^3 = .01\text{ g/cm}^3$, so rotatory power $= .01(+66.45°)/10\text{ cm} = 0.06645°/\text{cm}$. Light travels through $1\text{ m} = 100\text{ cm}$, so emerging light is at 6.645° from vertical (clockwise).

8.72 Place the photoelastic material between circular polarizers with both retarders facing it. Under circular illumination no orientation of the stress axes is preferred over any other, and they will thus all be indistinguishable. Only the birefringence will have an effect, and so the isochromatics will be visible. If the two polarizers are different, that is, one an *R*, the other an *L*, regions where Δn leads to $\Delta\varphi = \pi$ will appear bright. If they are the same, such regions appear dark.

8.73 From (8.40) $\Delta\varphi = (2\pi/\lambda_o)l(|n_o - n_e|)$ so $|n_o - n_e| = \lambda_o\Delta\varphi/2\pi l.$

(8.40) $\Delta n = \lambda_o KE^2 = \lambda_o K(V/d)^2$ so $\lambda_o\Delta\varphi/2\pi l = \lambda_o K(V/d)^2$; $\Delta\varphi = 2\pi l K(V/d)^2$.

8.74 $V_{\lambda/2} = \lambda_0/2n_0^3 r_{63} = 550\times 10^{-9}/2(1.58)^3 5.5\times 10^{-12} = 10^5/2(3.94) = 12.7\text{ kV}.$

8.75 $J = \begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix} = \begin{bmatrix}\cos 45°\\ \sin 45°\end{bmatrix} = \begin{bmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{bmatrix} = \dfrac{1}{\sqrt{2}}\begin{bmatrix}1\\ 1\end{bmatrix}$

8.76 $\vec{E}_1 \cdot \vec{E}_2^* = (1)(e_{21}^*) + (-2i)(e_{22})^* = 0, \quad \vec{E}_2 = (2, i)^T.$

8.77 (a) $E_1 = (1, 1, 0, 0)$ has relative irradiance of 1, and is horizontally polarized. $E_2 = (3, 0, 0, 3)$ has relative irradiance of 3, is right circularly polarized. For both, $V = 1$.
(b) $E = E_1 + E_2 = (4, 1, 0, 3)$, and has both a horizontal *P* component and an *R* component.
(c) (8.48) $V = (S_1^2 + S_2^2 + S_3^2)^{1/2} / S_o = (1^2 + 0^2 + 3^2)^{1/2} / 4 = 0.79.$
(d) $E = (1, 1, 0, 0) + (1, -1, 0, 0) = (2, 0, 0, 0)$ and is "natural" light (unpolarized).

8.78 (See Tables 8.5 and 8.6.)

$$S_t = \frac{1}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \text{Vertical } P$$

Relative irradiance = 1/2. (8.48) $V = (S_1^2 + S_2^2 + S_3^2)^{1/2} / S_o = 1.$

8.79 (See Tables 8.5 and 8.6.)

$$S_t = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = (45° P)$$

Relative irradiance = 1/2. (8.48) $V = (S_1^2 + S_2^2 + S_3^2)^{1/2} / S_o = 1.$

8.80 $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

8.81 $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Therefore light polarized at 45° is unchanged, as expected.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

A horizontal *P*-state is changed to an *R* state.

8.82 Fast axis $\alpha = +45°$, $\cos 90° = 0$, $\sin 90° = 1$. Quarter wave plate, $\Delta\varphi = \pi/2$; $\cos(\pi/2) = 0$, $\sin(\pi/2) = 1$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C^2 + S^2\cos\Delta\varphi & CS(1-\cos\Delta\varphi) & -S\sin\Delta\varphi \\ 0 & CS(1-\cos\Delta\varphi) & S^2 + C^2\cos\Delta\varphi & C\sin\Delta\varphi \\ 0 & S\sin\Delta\varphi & -C\sin\Delta\varphi & \cos\Delta\varphi \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0+1(0) & 0(1)(1-0) & -1(1) \\ 0 & 0(1)(1-0) & 1+0(0) & 0(1) \\ 0 & 1(1) & -(0)(1) & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

8.83 Quarter wave plate, $\Delta\varphi = \pi/2$; $\cos(\pi/2) = 0$, $\sin(\pi/2) = 1$. Vertical fast axis $\alpha = 0$, $\cos(0) = 1$, $\sin(0) = 0$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c^2 + s^2\cos\Delta\varphi & cs(1-\cos\Delta\varphi) & -s\sin\Delta\varphi \\ 0 & cs(1-\cos\Delta\varphi) & s^2 + c^2\cos\Delta\varphi & c\sin\Delta\varphi \\ 0 & s\sin\Delta\varphi & -c\sin\Delta\varphi & \cos\Delta\varphi \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1+0(0) & 1(0)(1-0) & -0(1) \\ 0 & 1(0)(1-0) & 0+1(0) & 1(1) \\ 0 & 0(1) & -1(1) & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

8.84 $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

8.85 (From Problem 8.83).

$$S_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = R\text{-state.}$$

E_y leads E_x by $\pi/2$ (see third part from right of Figure 8.7a).

$$S_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = L\text{-state.}$$

E_x leads E_y by $\pi/2$ (see third part from left of Figure 8.7a).

8.86
$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&-1\\0&0&1&0\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&-1\\0&0&1&0\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}$$

Half wave plate $\Delta\varphi=\pi$, $\cos(\pi)=1$, $\sin(\pi)=0$
fast vertaxis, $\alpha=0$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}=\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}=\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&-1&0\end{bmatrix}\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&-1&0\end{bmatrix}=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}$$

8.87
$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&-1\\0&0&1&0\end{bmatrix}\frac{1}{2}\begin{bmatrix}1&0&1&0\\0&0&0&0\\1&0&1&0\\0&0&0&0\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1&0&1&0\\0&0&0&0\\0&0&0&0\\1&0&1&0\end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix}1&0&1&0\\0&0&0&0\\0&0&0&0\\1&0&1&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix}1&0&1&0\\0&0&0&0\\0&0&0&0\\1&0&1&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix}1&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&1\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix}1&0&1&0\\0&0&0&0\\0&0&0&0\\1&0&1&0\end{bmatrix}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}=\frac{1}{2}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix}1&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&1\end{bmatrix}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}=\frac{1}{2}\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

8.88 (8.42) $\Delta\varphi = 2\pi n_o^3 r_{63} V/\lambda_o$. At maximum transmission, $\Delta\varphi = \pi$. For $I_t = 0$, $\sin^2(\Delta\varphi/2) = 0$, so $(\Delta\varphi/2) = \pi$. $2\pi = 2\pi n_o^3 r_{63} V/\lambda_o$.

$$V = \lambda_o / n_o^3 r_{63} = (5.461\times10^{-7}\ \text{m})/((1.52)^3(8.5\times10^{-12}\ \text{m/V}) = 18.29\ \text{kV}.$$

If polarizers are parallel, I_t is a maximum at $V = 0$. Equivalent to $I_t = I_i \cos^2(\Delta\varphi/2)$, so

$$I_t/I_i = \cos^2(\pi V/2V_{\lambda/2})\ \text{(from 8.51)} = \cos^2(\pi/2) = 0.$$

8.89 $\begin{bmatrix} te^{i\varphi} & 0 \\ 0 & te^{i\varphi} \end{bmatrix}$, where a phase increment of φ is introduced into both components as a result of traversing the plate. vacuum: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, perfect absorber: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

8.90 $\begin{bmatrix} t^2 & 0 & 0 & 0 \\ 0 & t^2 & 0 & 0 \\ 0 & 0 & t^2 & 0 \\ 0 & 0 & 0 & t^2 \end{bmatrix}$ $\quad$ $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

8.91 $V = I_p/(I_p + I_u) = (S_1^2 + S_2^2 + S_3^2)^{1/2}/S_0$,

$I_p = (S_1^2 + S_2^2 + S_3^2)^{1/2}$; $I - I_p = I_u$.

$$S_o - (S_1^2 + S_2^2 + S_3^2)^{1/2} = I_u$$

$$\begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$5 - (0+0+1)^{1/2} = I_u.$$

8.92 **(a)** $$\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} = \begin{pmatrix} \cos\theta\cos\alpha + \sin\theta\sin\alpha \\ -\cos\theta\sin\alpha + \sin\theta\cos\alpha \end{pmatrix}$$
$$= \begin{pmatrix} \cos(\theta+\alpha) \\ \sin(\theta-\alpha) \end{pmatrix}$$

(b) $$\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \cos\alpha + i\sin\alpha \\ \cos\alpha - i\sin\alpha \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\alpha} \\ e^{-i\alpha} \end{pmatrix}$$

(c) $$\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \cos\alpha + i\sin\alpha \\ \cos\alpha + i\sin\alpha \end{pmatrix} = \frac{1}{\sqrt{2}}e^{i\alpha}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

8.93 $$\begin{pmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{pmatrix}\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$
$$= \begin{pmatrix} \cos\theta\cos^2\alpha + \sin\theta\cos\alpha\sin\alpha \\ \cos\theta\cos\alpha\sin\alpha + \sin\theta\sin^2\alpha \end{pmatrix}$$

This is a polarizer with axis given by α

8.94 **(a)** Quarter were plate with fast axis at +45°

(b) Quarter wave plate with fast axis at –45°

8.95 **(a)** $$\frac{1}{2\sqrt{2}}\begin{pmatrix}1+\sqrt{3} & -1+\sqrt{3}\\ 1-\sqrt{3} & 1+\sqrt{3}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\frac{1}{2\sqrt{2}}\begin{pmatrix}1+\sqrt{3}\\1-\sqrt{3}\end{pmatrix}$$

(b) $$\frac{1}{2\sqrt{2}}\begin{pmatrix}1+\sqrt{3} & -1+\sqrt{3}\\ 1-\sqrt{3} & 1+\sqrt{3}\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}=\frac{1}{2\sqrt{3}}\begin{pmatrix}-1+\sqrt{3}\\1+\sqrt{3}\end{pmatrix}$$