# Chapter 4 Solutions

**4.1** $E_{os} \propto V E_{oi}/r = KV E_{oi}/r$; thus $V K/r$ must be unitless, and so $K$ has units of (length)$^{-2}$. The only quantity unaccounted for is $\lambda$ and so we conclude that $K = \lambda^{-2}$, and $I_i/I_s \propto K^2 \propto \lambda^{-4}$.

**4.2** The degree of Rayleigh scattering is proportional to $1/\lambda^4$. But $\lambda_y = 1.45\lambda_\upsilon$ and so $1/\lambda_y^4 = (1/1.45\lambda_\upsilon)^4$ hence violet is scattered $(1.45)^4 = 4.42$ times more intensely than yellow. The ratio of yellow to violet is 22.6%.

**4.3** The sinusoids represent the field, in this case the *E*-field of the disturbance. The wavefront is a surface of constant phase and it meets each sinusoid at the same point (same phase) in its development. The outward radial lines are rays and they are everywhere perpendicular to the wavefronts.

**4.4** (a) On the left-hand side are the inertial, drag force, and elastic force terms; on the right-hand side is the electric driving force. (b) $x_0(-\omega^2 + \omega_0^2 + i\gamma\omega) = (q_e E_0/m_e)\exp(i\alpha)$, forming the absolute square of both sides yields $x_0^2[(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2] = (q_e E_0/m_e)^2$ and $x_0$ follows by division and taking the square root. (c) As for $\alpha$, divide the imaginary parts of both sides of the first equation above, namely $x_0\gamma\omega = (q_e E_0/m_e)\sin\alpha$, by the real parts, $x_0(\omega_0^2 - \omega^2) = (q_e E_0/m_e)\cos\alpha$ to obtain $\alpha = \tan^{-1}[\gamma\omega/(\omega_0^2 - \omega^2)]$. $\alpha$ ranges continuously from 0 to $\pi/2$ to $\pi$.

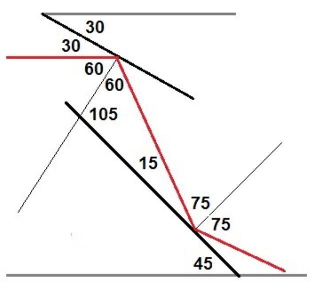
**4.5** (a) The phase angle is retarded by an amount $(n\Delta y 2\pi/\lambda) - \Delta y 2\pi/\lambda$ or $(n-1)\Delta y\omega/c$. Thus $E_p = E_0 \exp i\omega[t - (n-1)\Delta y/c - y/c]$ or $E_p = E_0 \exp[-i\omega(n-1)\Delta y/c]\exp i\omega(t - y/c)$. (b) Since $e^x \approx 1 + x$ for small $x$, if $n \approx 1$ or $\Delta y \ll 1$, $\exp[-i\omega(n-1)\Delta y/c] \approx 1 - i\omega(n-1)\Delta y/c$ and since $\exp(-i\pi/2) = -i$, $E_p = E_u + \omega(n-1)\Delta y(E_u/c)\exp(-i\pi/2)$.

**4.6** $\sin 58° = x/(5.0 \text{ m})$, $x = 4.2$ m.

**4.7** The statue is 16 m from the point of incidence, and since the ray-triangles are similar, 4 m : 16 m as 3 m : $Y$ and $Y = 12$ m.

**4.8** At the first mirror, $\theta_r = \theta_i$. For the second, $\theta_i' = 90 - \theta_r = 90 - \theta_i$ and $\theta_r' = \theta_i'$, so $\theta_r' = 90 - \theta_i$.

**4.9**




$\theta_n = 60$

$\theta_{r2} = 75$

**4.10** $\overline{OQ} = \dfrac{1/n_i}{\sin\theta_i} = \dfrac{1/n_t}{\sin\theta_t}$

$\sin\theta_i = \dfrac{1/n_i}{\overline{OQ}} \Rightarrow \dfrac{1}{\overline{OQ}} = n_i \sin\theta_i$

$\sin\theta_t = \dfrac{1/n_t}{\overline{OQ}} \Rightarrow \dfrac{1}{\overline{OQ}} = n_t \sin\theta_t$

$n_t \sin\theta_t = n_i \sin\theta_i$

**4.11** $n_i \sin\theta_i = n_t \sin\theta_t$, $\sin 30° = 1.52 \sin\theta_t$, $\theta_t = \sin^{-1}(1/3.04)$, so $\theta_t = 19°13'$.

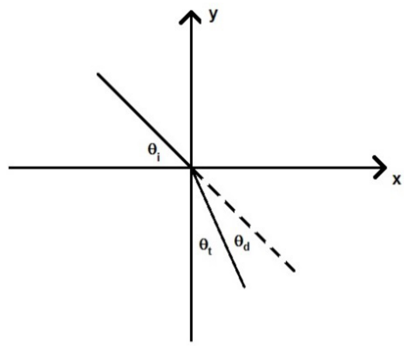
**4.12** $P_{\text{transverse}} = m\upsilon_i \sin\theta_i$
$= m\upsilon_f \sin\theta_t$

where "*m*" is the presumed mass. But $\upsilon_i = \dfrac{s_0}{t}$, $\upsilon_f = \dfrac{BP}{t}$. So

$$(s_0)\sin\theta_i = (BP)\sin\theta_t$$
$$\sin\theta_i = \frac{BP}{s_0}\sin\theta_t$$

The factor $\dfrac{BP}{s_0}$ corresponds to $n_{ti}$.

**4.13**




$n_t \sin\theta_t = n_i \sin\theta_i$

$\sin\theta_t = \dfrac{n_i}{n_t}\sin\theta_i = \dfrac{1}{1.5}\sin 30 = 0.333$

$\theta_t = 19.5°$

$\theta_d = \theta_i - \theta_t = 30.0° - 19.47° = 10.5°$

**4.14** The slope of the curve is $n_{it} = n_i/n_t$. Slope $\sim 0.75/1.00$, so that $n_t \simeq 1.33$. This suggests that the dense medium is water.

**4.15** $\theta_t = \sin^{-1}[(\sin 45°)/2.42] = 17°$, the angular deviation is $45° - 17° = 28°$.

**4.16** $\theta_t = \sin^{-1}[(n_w/n_g)\sin\theta_i] = \sin^{-1}[(8/9)\sin 45°] = 39°$. For a ray incident in the glass at this angle, $\theta_t = \sin^{-1}[(n_g/n_w)\sin 39°] = \sin^{-1}[(9/8)\sin 39°] = 45°$.

**4.17** (a) $n_{ti} = n_t/n_i = (c/\upsilon_t)/(c/\upsilon_i) = \upsilon_i/\upsilon_t = \nu\lambda_i/\nu\lambda_t = \lambda_i/\lambda_t$. Therefore $\lambda_t = \lambda_i 3/4 = 9$ cm. (b) $\sin\theta_i = n_{ti}\sin\theta_t$, $\theta_t = \sin^{-1}[(3/4)\sin 45°] = 32°$.

**4.18** $\lambda_t = \lambda_i/n_{ti} = 600/1.5 = 400$ nm, color depends on frequency, which is the number of peaks that arrive per sec. This doesn't change when going into a new medium. Thus it is still orange.

**4.19** 1.00 sin 55° = $n$ sin 40°; $n$ = 1.27 or 1.3.

**4.20** 1.33 sin 35° = 1.00 sin $\theta_t$; $\theta_t$ = 50°.

**4.21** For $\theta_i$ = 0, 10, 20, 30, 40, 50, 60, 70, 80, 90 degrees, $\theta_t$ = 0, 6.7, 13.3, 19.6, 25.2, 30.7, 35.1, 38.6, 40.6, 41.8 degrees respectively.

**4.22** Consider one ray on each side of the beam, with a perpendicular separation $D$. The width of the beam on the interface is $D/\cos\theta_i$. Likewise, the width of the beam at the interface is $D'/\cos\theta_t$, where $D'$ is the perpendicular separation (width) of the rays in the glass, and $D/\cos\theta_i = D'/\cos\theta_t$. (4.4) $n_i \sin\theta_i = n_t \sin\theta_t$ so

$$\begin{aligned}\cos\theta_t &= (1-\sin^2\theta_t)^{1/2}\\ &= (1-\sin^2\theta_i/n_g^2)^{1/2}\end{aligned}$$

so

$$D' = \frac{D\left(1-\dfrac{\sin^2\theta_i}{n_t^2}\right)^{1/2}}{\cos\theta_i}$$

**4.23** (4.4) $n_i \sin\theta_i = n_t \sin\theta_t$ so sin(60.0°) = $n_t$ sin $\theta_\varepsilon$. Diameter of emerging beam ($D$) is related to the difference in horizontal displacement of red and violet light ($h$) by $D$ cos(60.0°) = $h$ (See Problem 4.19). Red:

$\sin\theta_{\text{red}} = \sin(60.0°)/n_{\text{red}} = (\sqrt{3}/2)/(1.505)$, $\theta_{\text{red}} = 35.1°$;

tan $\theta_{\text{red}} = h_{\text{red}}/10.0$ cm so $h_{\text{red}}$ = (10.0 cm) tan(35.1°) = 7.04 cm.

Violet:

$\sin\theta_{\text{violet}} = \sin(60.0°)/n_{\text{violet}} = (\sqrt{3}/2)/(1.545)$; $\theta_{\text{violet}} = 34.1°$;

$h_{\text{violet}} = (10.0\text{ cm})\tan(34.1°) = 6.77$ cm. $D = h/\cos(60.0°) =$

$(h_{\text{red}} - h_{\text{violet}})/\cos(60.0°) = (7.04 - 6.77)/(0.5) = 0.54$ cm .

**4.24** $$\frac{S_i}{S_o} = \frac{n_t}{n_i} = \frac{1}{1.47}$$

$$S_i = \frac{10\text{ cm}}{1.47} = 6.8\text{ cm}$$

**4.25** $$\frac{S_i}{S_o} = \frac{n_t}{n_i}$$

$$\frac{S_i}{3\text{ cm}} = \frac{1}{1.5} \Rightarrow S_i = 2\text{ cm}$$

8 cm + 2 cm = 10 cm

**4.26** 1.00 sin 35° = 1.50 sin $\theta_{t1}$; $\theta_{t1}$ = 22.48° and cos 22.48° = (2.00 cm)/$L$; $L$ = 2.16 cm or 2.2 cm.

**4.27** sin $\theta_i$ = $n$ sin $\theta_i$/2; since $\sin 2\alpha = 2\sin\alpha\cos\alpha$, $\sin\theta_i = 2\sin(\theta_i/2)\cos(\theta_i/2)$ and so setting these two expressions equal we get

$$1.70\sin(\theta_i/2) = 2\sin(\theta_i/2)\cos(\theta_i/2);\ \cos\theta_i/2 = 0.85;$$

$$31.79° = \theta_i/2;\ \theta_i = 63.6°.$$

**4.28** The glass will change the depth of the object from $d_R$ to $d_A$, where $d_A/d_R = 1.00/1.55$; but $d_R$ = 1.00 mm; hence, $d_A$ = 0.645 mm and the camera must be raised 1.00 mm − 0.645 mm = 0.355 mm.

**4.29** $d_{A1}/d_{R1} = 1.50/1.33$; $d_{R1}$ = 1.00 m; $d_{A1}$ = 1.1278 m; $d_{R2} = d_{A1}$ + 0.20 m; $d_{A2}/d_{R2} = 1.00/1.50$; $d_{A2}$ = 1.3278(1.00/1.50) = 0.885 m.

**4.30** The number of waves per unit length along $\overline{AC}$ on the interface equals $(\overline{BC}/\lambda_i)/(\overline{BC}\sin\theta_i) = (\overline{AD}/\lambda_t)/(\overline{AD}\sin\theta_t)$. Snell's Law follows on multiplying both sides by $c/\nu$.

**4.31** With the origin in the plane of incidence, $z = 0$; with the origin on the interface $y = 0$ so $(\vec{k}_i \cdot \vec{r}) \to k_{ix}x$

$$(\vec{k}_i \cdot \vec{r} + \varepsilon_r) \to k_{rx}x + \varepsilon_r$$
$$(\vec{k}_t \cdot \vec{t} + \varepsilon_t) \to k_{tx}x + \varepsilon_t$$

and as $\varepsilon_r = \varepsilon_t = 0$, Eq. (4.19) becomes $k_{ix} = k_{rx} = k_{tx}$ or $k_i\sin\theta_i = k_r\sin\theta_r = k_t\sin\theta_t$. Since $k = 2\pi/\lambda$

$$\frac{\sin\theta_i}{\lambda_i} = \frac{\sin\theta_t}{\lambda_t},$$

which is the condition derived in Problem (4.26) for wave front continuity.

**4.32** Let $\tau$ be the time for the wave to move along a ray from $b_1$ to $b_2$, from $a_1$ to $a_2$, and from $a_1$ to $a_3$. Thus $\overline{a_1a_2} = \overline{b_1b_2} = \upsilon_i\tau$ and $\overline{a_1a_3} = \upsilon_t\tau$. $\sin\theta_i = \overline{b_1b_2}/\overline{a_1b_2} = \upsilon_i\tau\,\overline{a_1b_2}$, $\sin\theta_t = \overline{a_1a_3}/\overline{a_1b_2} = \upsilon_t\tau\,\overline{a_1b_2}$, $\sin\theta_r = \overline{a_1a_2}/\overline{a_1b_2} = \upsilon_i\tau\,\overline{a_1b_2}$, $\sin\theta_i/\sin\theta_t = \upsilon_i/\upsilon_t = n_t/n_i = n_{ti}$ and $\theta_i = \theta_r$.

**4.33** $n_i\sin\theta_i = n_t\sin\theta_t$, $n_i(\hat{k}_i \times \hat{u}_n) = n_t(\hat{k}_t \times \hat{u}_n)$, where $\hat{k}_i$, $\hat{k}_t$ are unit propagation vectors. Thus $n_t(\hat{k}_t \times \hat{u}_n) - n_i(\hat{k}_i \times \hat{u}_n) = 0$, $(n_t\hat{k}_t - n_i\hat{k}_i) \times \hat{u}_n = 0$. Let $n_t\hat{k}_t - n_i\hat{k}_i = \vec{\Gamma} = \Gamma\hat{u}_n$. $\Gamma$ is often referred to as the *astigmatic constant*; $\Gamma$ is the difference between the projections of $n_t\hat{k}_t$ and $n_i\hat{k}_i$ on $\hat{u}_n$; in other words, take the dot product $\vec{\Gamma}\cdot\hat{u}_n$: $\Gamma = n_t\cos\theta_t - n_i\cos\theta_i$.

**4.34** Since $\theta_i = \theta_r$, $\hat{k}_{ix} = \hat{k}_{rx}$ and $\hat{k}_{iy} = -\hat{k}_{ry}$, and since $(\hat{k}_i \cdot \hat{u}_n)\hat{u}_n = \hat{k}_{iy}$, $\hat{k}_i - \hat{k}_r = 2(\hat{k}_i \cdot \hat{u}_n)\hat{u}_n$.

**4.35** Since $\overline{SB'} > \overline{SB}$ and $\overline{B'P} > \overline{BP}$, the shortest path corresponds to $B'$ coincident with $B$ in the plane of incidence.

**4.36** (Refer to Figure 4.35.) Let $\overline{SP} = a$, distance along interface $(S \rightarrow B) = x$, distance of $S$ and $P$ from interface = $h$.

$$t = \frac{\overline{SB}}{\upsilon_i} + \frac{\overline{BP}}{\upsilon_i}$$
$$= \frac{(h^2 + x^2)^{1/2}}{\upsilon_i} + \frac{(h^2 + (a-x)^2)^{1/2}}{\upsilon_i}$$

Minimize $t(x)$ *w.r.t.* $x$.

$$\frac{dt}{dx} = \frac{x}{\upsilon_i (h^2 + x^2)^{1/2}} + \frac{-(a-x)}{\upsilon_i (h^2 + (a-x)^2)^{1/2}} = 0$$

$$\sin\theta_i = \sin\theta_r \ \text{ or } \ \theta_i = \theta_r$$

**4.37** The mirrors are set as two sides of the acute triangle. The front of the laser is placed along the third side. The inscribed triangle is found by adjusting the position and the angle of the laser beam until the incoming and reflected beams meet on the third side of the triangle. This follows from format's principle since the alignment relies on the law of reflection.

**4.38** $n_1 \sin\theta_i = n_2 \sin\theta_t$, $\theta_t = \theta_i'$, $n_2 \sin\theta_i' = n_1 \sin\theta_t'$, $n_1 \sin\theta_i = n_1 \sin\theta_t'$ and $\theta_i = \theta_t'$. $\cos\theta_t = d/\overline{AB}$, $\sin(\theta_i - \theta_t) = a/\overline{AB}$, $\sin(\theta_i - \theta_t) = (a/d)\cos\theta_t$, $d\sin(\theta_i - \theta_t)/\cos\theta_t = a$.

**4.39** The left and right beams will be parallel if $\theta_t$ (Left) = $\theta_t$ (Right) in the final medium ($a$). Since all interfaces are parallel, the transmitted angle into a medium equals the incident angle at the next medium.

At each interface (4.5) $\sin\theta_i = n_{ti} \sin\theta_t$.

Left: $\sin\theta_i = n_{1a} \sin\theta_{t1} = n_{1a}(n_{a1} \sin\theta_{ta}) = n_{1a} n_{a1}(n_{2a} \sin\theta_{t2})$
$= n_{1a} n_{a1} n_{2a}(n_{a2} \sin\theta_{ta}) = \sin\theta_{ta}$.

Right: $\sin\theta_i = n_{1a} \sin\theta_{t1} = n_{1a}(n_{21} \sin\theta_{t2}) = n_{1a}(n_{21} \sin\theta_{ta})$
$= n_{1a} n_{21}(n_{a2} \sin\theta_{ta}) = \sin\theta_{ta}$.

For each beam, $\theta_{ta} = \theta_i$.

**4.40** Rather than propagating from point $S$ to point $P$ in a straight line, the ray traverses a path that crosses the plate at a sharper angle. Although in so doing the path lengths in air are slightly increased, the decrease in time spent within the plate more than compensates. This being the case, we might expect the displacement $a$ to increase with $n_{21}$. As $n_{21}$ gets larger for a given $\theta_i$, $\theta_t$ decreases, $\theta_i - \theta_t$ increases, and from the results of Problem 4.34, $a$ clearly increases.

**4.41** (a) In the case of a single thick piece of plexiglass, when looking down, one sees the text from the sheet underneath. Along the vertical surfaces, one sees the text from the page that is reflected off of the plexiglass – air interface. (b) When pressed lightly together, there is a very thin airgap between the two pieces of plexiglass. As a result, looking down through each, one sees the text beneath. However, along the vertical surfaces of each of the two pieces of plexiglass, one sees the text from the page that is reflected off of each of the plexiglass – air interfaces. This image is from total internal reflection due to the shallow angle at which the light from the text is incident on the vertical plexiglass-air interface. (c) In the case of two pieces pressed lightly together with castor oil between the two pieces, one sees an image which is the same as in photgraph a (the case of a single, thick piece of plexiglass). This occurs because the index of refraction of castor oil is very nearly the same as that of plexiglass. As a result, the light from the text incident on the vertical plexiglass-castor oil interface does not undergo total internal reflection, but instead is transmitted through the interface (as if the interface wasn't even there).

**4.42** From Eq. (4.40), $r_{\|} = (1.52\cos 30° - \cos 19°13')/(\cos 19°13' + 1.52\cos 30°)$, where from Problem 4.8, $\theta_t = 19°13'$. Similarly, $t_{\|} = 2\cos 30°/(\cos 19°13' + 1.52\cos 30°)$, $r_{\|} = 0.165$, $t_{\|} = 0.766$.

**4.43** Starting with Eq. (4.34), divide top and bottom by $n_i$ and replace $n_{ti}$ with $\sin\theta_i / \sin\theta_t$ to get

$$r_{\perp} = \frac{\sin\theta_t \cos\theta_i - \sin\theta_i \cos\theta_t}{\sin\theta_t \cos\theta_i + \sin\theta_i \cos\theta_t},$$

which is equivalent to Eq. (4.42). Equation (4.44) follows in exactly the same way. To find $r_{\|}$ start the same way with Eq. (4.40) and get

$$r_{\|} = \frac{\sin\theta_i \cos\theta_i - \sin\theta_t \cos\theta_t}{\sin\theta_t \cos\theta_t + \sin\theta_i \cos\theta_i}.$$

There are several routes that can be taken now; one is to rewrite $r_{\|}$ as

$$r_{\|} = \frac{\sin\theta_i \cos\theta_t - \sin\theta_t \cos\theta_i}{\sin\theta_i \cos\theta_t + \sin\theta_t \cos\theta_i} \frac{\cos\theta_i \cos\theta_t - \sin\theta_i \sin\theta_t}{\cos\theta_i \cos\theta_t + \sin\theta_i \sin\theta_t}$$

and so

$$r_{\|} = \frac{\sin(\theta_i - \theta_t)\cos(\theta_i + \theta_t)}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}.$$

We call find $t_{\|}$, which has the same denominator, in a similar way.

**4.44** From Snell's Law $\theta_t = 13.99°$; from Eq. (4.43),

$$r_{\|} = \tan 8.01°/\tan 35.99° = 0.194;$$

using Eq. (4.42),

$$r_{\perp} = -\sin 8.01°/\sin 35.99° = -0.237;$$
$$[E_{or}]_{\|} = r_{\|}[E_{oi}]_{\|} = 1.94 \text{ V/m};$$
$$[E_{or}]_{\perp} = r_{\perp}[E_{oi}]_{\perp} = -4.74 \text{ V/m}.$$

**4.45** For small angles Snell's Law becomes $1\theta_i = n\theta_t$: from Eq. (4.42) using the identity $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ and using $\cos\theta_i = \cos\theta_t = 1$

$$r_{\perp} = -\frac{(\theta_i - \theta_t)}{(\theta_i + \theta_t)} = -\frac{(\theta_i - \frac{1}{n}\theta_i)}{(\theta_i + \frac{1}{n}\theta_i)} = -\frac{(n-1)}{(n+1)}.$$

**4.46**

$$t_{\perp} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t}$$

$$[t_{\perp}]_{\theta_i = 0} = \frac{2n_i \cos 0}{n_i \cos 0 + n_t \cos 0} = \frac{2n_i}{n_i + n_t}$$

$$t_{\parallel} = \frac{2n_i \cos\theta_i}{n_t \cos\theta_i + n_i \cos\theta_t}$$

$$[t_{\parallel}]_{\theta_i=0} = \frac{2n_i \cos 0}{n_t \cos 0 + n_i \cos 0} = \frac{2n_i}{n_i + n_t}$$

Thus, $[t_{\perp}]_{\theta_i=0} = [t_{\parallel}]_{\theta_i=0}$.

**4.47** Starting in air $(n_i = 1)$ and going into glass $(n_t = 1.5)$:

$$t_{\perp} = \frac{2n_i}{n_i + n_t} = \frac{2.0}{2.5} = 0.8$$

Starting in glass $(n_i = 1.5)$ and going into air $(n_t = 1)$:

$$t_{\perp} = \frac{2n_i}{n_i + n_t} = \frac{3.0}{2.5} = 1.2$$

**4.48** $r_{\perp} = \dfrac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$

$$[r_{\perp}]_{\theta_i=0} = \frac{n_i \cos 0 - n_t \cos 0}{n_i \cos 0 + n_t \cos 0} = \frac{n_i - n_t}{n_i + n_t}$$

$$[t_{\perp}]_{\theta_i=0} = \frac{2n_i}{n_i + n_t}$$

Air-to-Glass:

$$[r_{\perp}]_{\theta_i=0} = -\frac{0.5}{2.5} = -0.2$$

$$[t_{\perp}]_{\theta_i=0} = \frac{2n_i}{n_i + n_t} = \frac{2}{2.5} = 0.8$$

$$t_{\perp} + (-r_{\perp}) = 0.8 + 0.2 = 1$$

Glass-to-Air:

$$[r_{\perp}]_{\theta_i=0} = \frac{0.5}{2.5} = 0.2$$

$$[t_{\perp}]_{\theta_i=0} = \frac{2n_i}{n_i + n_t} = \frac{3}{2.5} = 1.2$$

$$t_{\perp} + (-r_{\perp}) = 1.2 - 0.2 = 1$$

**4.49** From (4.47), $R = r^2 = (n - 1/n + 1)^2 = (1.522 - 1/1.522 + 1)^2 = 0.043$.

$T = 1 - R = 0.957$.

**4.50** $T = 1 - R = 1 - r^2 = 1 - (n - 1/n + 1)^2 = 1 - (1.33 - 1/1.33 + 1)^2 = 0.98$.

From (4.55), $I_t = TI_i = (0.98)(500\ \text{W/m}^2) = 490\ \text{W/m}^2$.

**4.51** $r_{\parallel} = \dfrac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} = \dfrac{n_{ti} \cos\theta_i - \cos\theta_t}{\cos\theta_t + n_{ti} \cos\theta_i}$

Using Snell's Law:

$$n_i \sin\theta_i = n_t \sin\theta_t$$
$$\sin^2\theta_i = n_{ti}^2 \sin^2\theta_t$$
$$\cos^2\theta_t = 1 - \sin^2\theta_t$$
$$n_{ti} \cos\theta_t = n_{ti}\sqrt{1 - \sin^2\theta_t}$$

Using $1 - \sin^2\theta_t = 1 - \dfrac{1}{n_{ti}^2}\sin^2\theta_i\,[0,1]$,

$$n_{ti}\cos\theta_t = \sqrt{n_{ti}^2 - \sin^2\theta_i}$$
$$r_{\parallel} = \frac{n_{ti}\cos\theta_i - \dfrac{1}{n_{ti}}\sqrt{n_{ti}^2 - \sin^2\theta_i}}{n_{ti}\cos\theta_i + \dfrac{1}{n_{ti}}\sqrt{n_{ti}^2 - \sin^2\theta_i}}$$
$$r_{\parallel} = \frac{n_{ti}^2\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_i}}{n_{ti}^2\cos\theta_i + \sqrt{n_{ti}^2 - \sin^2\theta_i}}$$

Now, looking at $r_{\perp}$:

$$r_{\perp} = \frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}$$
$$r_{\perp} = \frac{\cos\theta_i - n_{ti}\cos\theta_t}{\cos\theta_i + n_{ti}\cos\theta_t}$$
$$r_{\perp} = \frac{\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n_{ti}^2 - \sin^2\theta_i}}$$

**4.52** $r_{\parallel} = \dfrac{n_{ti}^2\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_i}}{n_{ti}^2\cos\theta_i + \sqrt{n_{ti}^2 - \sin^2\theta_i}}$

$$r_{\parallel} = \frac{1.6^2\cos 30° - \sqrt{1.6^2 - \sin^2 30°}}{1.6^2\cos 30° + \sqrt{1.6^2 - \sin^2 30°}}$$
$$r_{\parallel} = \frac{2.217 - \sqrt{2.31}}{2.217 + \sqrt{2.31}} = 0.187$$
$$r_{\perp} = \frac{\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n_{ti}^2 - \sin^2\theta_i}}$$
$$r_{\perp} = \frac{\cos 30° - \sqrt{1.6^2 - \sin^2 30°}}{\cos 30° + \sqrt{1.6^2 - \sin^2 30°}} = \frac{-0.654}{2.386} = -0.274$$

**4.53** $R_{\parallel} = r_{\parallel}^2 = 0.03497$

$$R_{\perp} = r_{\perp}^2 = 0.0751$$
$$T_{\parallel} = 1 - R_{\parallel} = 0.965$$
$$T_{\perp} = 1 - R_{\perp} = 0.925$$
$$T = \frac{1}{2}\left(T_{\parallel} + T_{\perp}\right) = \frac{1}{2}\left(0.965 + 0.925\right) = 0.945$$
$$R = 1 - T = 0.055$$

**4.54** Half of the unpolarized light s its *E*-field perpendicular to the plane of incidence $(I_{\perp} = 500\ \text{W/m}^2)$, then:

$$I_{T\perp} = 0.8 \times 500\ \text{W/m}^2 = 400\ \text{W/m}^2$$
$$R_{\perp} = 1 - T_{\perp} = 0.2$$
$$I_{T\perp} = 0.2 \times 500\ \text{W/m}^2 = 100\ \text{W/m}^2$$

**4.55** Half of the unpolarized light s its E-field perpendicular to the plane of incidence $(I_{\perp} = 1000\ \text{W/m}^2)$, then:

$$I_{R\perp} = 300\ \text{W/m}^2, \text{ then } R_{\perp} = \frac{300}{1000} = 0.3$$

$$I_{R\parallel} = 200\ \text{W/m}^2, \text{ then } R_{\parallel} = \frac{200}{1000} = 0.2$$

$$T_{\perp} = 1 - R_{\perp} = 0.7$$
$$T_{\parallel} = 1 - R_{\parallel} = 0.8$$
$$T = \frac{1}{2}(T_{\perp} + T_{\parallel}) = 0.75$$

**4.56** $I_{inc} = 2000\ \text{W/m}^2 = I_{R\perp} + I_{R\parallel} + I_{T\perp} + I_{T\parallel}$

$= 300\ \text{W/m}^2 + 200\ \text{W/m}^2 + (0.7)(1000\ \text{W/m}^2) + (0.8)(1000\ \text{W/m}^2)$

$= 300\ \text{W/m}^2 + 200\ \text{W/m}^2 + 700\ \text{W/m}^2 + 800\ \text{W/m}^2 = 2000\ \text{W/m}^2$

**4.57** From (4.47),

$R = r^2 = (n_t - n_i / n_t + n_i)^2 = (1.376 - 1.33/1.376 + 1.33)^2 = 0.000289.$

$T = 1 - R = 0.999711.$

From (4.55),

$I_t = TI_i = (0.999711)(400\ \text{W/m}^2) = 399.884 \simeq 400\ \text{W/m}^2.$

**4.58** $r \simeq n_t - n_i / n_t + n_i$. Air-water: $r = \dfrac{4/3 - 1}{4/3 + 1} = 1/7 = 0.14$. Air-crown glass:

$r = \dfrac{3/2 - 1}{3/2 + 1} = 1/5 = 0.20$. More reflectance for glass. From (4.54) and (4.56)

$I_r / I_i = R = r^2$. Air-water: $R = (1/7)^2 = 0.02$. Air-crown glass:

$R = \left(1/5\right)^2 = 0.04$ .

**4.59** $\sin x = x - x^3/3! + x^5/5! - \cdots$ and so $\sin(\alpha \pm \beta) = (\alpha \pm \beta)[1-(\alpha \pm \beta)^2/6]$ using Snell's Law $\theta_t(1-\theta_t^2/6+\ldots) = (\theta_i/n)(1-\theta_t^2/6+\ldots)$. Use $1\theta_i = n\theta_t$ and the fact that when $x$ is very small $(1+x)^{-1} \approx 1-x$ we have $\theta_t = (\theta_i/n)\,(1-\theta_i^2/6)(1+\theta_i^2/6n^2)$ dropping terms higher than the third power in $\theta_i$ we get $\theta_t = (\theta_i/n)\,[1-(n^2-1)\theta_i^2/6n^2]$ and so

$$\theta_i \pm \theta_t = \theta_i\left[1 \pm \frac{1}{n}\left(1-\frac{n^2-1}{6n^2}\theta_i^2\right)\right].$$

Using Eq. (4.42) and the power series representation of the sine, where terms higher than the third power in $\theta_i$ are dropped,

$$-r_\perp = \frac{n-1+\dfrac{\theta_i^2}{6n^2}[n^2-1-(n-1)^3]}{n+1-\dfrac{\theta_i^2}{6n^2}[n^2-1+(n-1)^3]} = \left(\frac{n-1}{n+1}\right)\left(1+\frac{\theta_i^2}{n}\right)$$

**4.60** $\cos(\theta_i+\theta_t)/\cos(\theta_i-\theta_t) = 1-2\theta_i^2/n$ multiplying by the ratio of the sines from the previous problem, viz., $[(n-1)/(n+1)](1-\theta_i^2/n)$ and dropping higher order terms yields the desired equation.

**4.61** From Snell's Law $n\sin\theta_t = 1\sin 90° = 1$ and so with Eq. (4.42) in mind,

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

and

$$\sin(90° \pm \theta_t) = \sin 90°\cos\theta_t \pm \cos 90°\sin\theta_t;$$

then

$$\sin(90° \pm \theta_t) = 1\cos\theta_t,$$

using $\sin^2\theta + \cos^2\theta = 1$ and Snell's Law

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sin(90° \pm \theta_t) = \sqrt{1-(1/n)^2}$$

and so $r_\perp \to -1$ at glancing incidence.

**4.62** Compute $dr_\perp/d\theta_i$ at $\theta_i = 90°$; we'll use $d\theta_t/d\theta_i = 0$ and then prove it; taking the derivative of Eq. (4.42) we get

$$dr_\perp/d\theta_i = -\cos(\theta_i-\theta_t)/\sin(\theta_i+\theta_t)$$
$$+\sin(\theta_i-\theta_t)\cos(\theta_i+\theta_t)/\sin^2(\theta_i+\theta_t)$$

and for $\theta_i = 90°$ this becomes

$$dr_\perp/d\theta_i = -\sin\theta_t/\cos\theta_t - \sin\theta_t\cos\theta_t/\cos^2\theta_t = 2\tan\theta_t$$

and using Snell's Law, i.e., $\sin\theta_t = 1/n$ when $\theta_i \approx 90°$, and the fact that

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t},$$
$$dr_\perp/d\theta_i = 2\tan\theta_t = 2\sin\theta_t/\cos\theta_t = 2/n\cos\theta_t = 2/\sqrt{n^2-1}:$$

this is the rise over the run at the end of the curve where $\theta_i \approx 90°$. Thus if $\alpha_\perp$ is the angle made *i*th the vertical $\tan\alpha_\perp = \sqrt{n^2-1/2}$.

**4.63** $[E_{or}]_\perp + [E_{oi}]_\perp = [E_{ot}]_\perp$; tangential field in incident medium equals that in transmitting medium, $[E_{ot}/E_{oi}]_\perp - [E_{or}/E_{oi}]_\perp = 1$, $t_\perp - r_\perp = 1$.
Alternatively, from Eqs. (4.42) and (4.44),

$$\frac{\sin(\theta_i - \theta_t) + 2\sin\theta_t\cos\theta_i}{\sin(\theta_i + \theta_t)} = 1$$

$$\frac{\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t + 2\sin\theta_t\cos\theta_i}{\sin\theta_i\cos\theta_t + \cos\theta_i\sin\theta_t} = 1.$$

**4.64** $n_i\sin\theta_i = n_t\sin\theta_t$ so, $\sin\theta_t = (n_i/n_t)\sin\theta_i = (1.00/1.52)\sin(30°) = 0.33$
$\theta_t = \sin^{-1}(0.33) = 19.2°$.
(Eq. 4.44) $t_\perp = 2\sin\theta_t\cos\theta_i/\sin(\theta_i + \theta_t) = 2\sin(19.2°)\cos(30°)/\sin(49.2°) = 0.75$.
(Eq. 4.42) $r_\perp = -\sin(\theta_i - \theta_t)/\sin(\theta_i + \theta_t) = -\sin(10.8°)/\sin(49.2°) =$
$-0.25$. $t_\perp + (-r_\perp) = 0.75 + (0.25) = 1.00$.

**4.65** Let $\theta_i = \theta_p = \pi/2 - \theta_t$. Reflected beam is polarized if $r_\perp$ or $r_\parallel$ equal zero.
(4.43)

$$r_\parallel = \tan(\theta_i - \theta_t)/\tan(\theta_i + \theta_t) = \tan(\pi/2 - \theta_t - \theta_t)/\tan(\pi/2 - \theta_t + \theta_t)$$
$$= \tan(\pi/2 - 2\theta_t)/\tan(\pi/2).$$

But $\tan(\pi/2)$ is infinite, so $r_\parallel = 0$.

**4.66** $\theta_i + \theta_t = 90°$ when $\theta_i = \theta_p$, $n_i\sin\theta_p = n_t\sin\theta_t = n_t\cos\theta_p$,
$\tan\theta_p = n_t/n_i = 1.52$, $\theta_p = 56°40'$.

**4.67** At $\theta_p$, $r_\parallel = 0$. So from (4.38) $(n_t/\mu_t)\cos\theta_i - (n_i/\mu_i)\cos\theta_t = 0$. Recall (4.4)
$n_i\sin\theta_i = n_t\sin\theta_t$. (3.59) $n = \sqrt{\varepsilon\mu/\varepsilon_0\mu_0}$ and $\cos^2\theta = 1 - \sin^2\theta$. Approach:
solve for $\tan\theta_p = \sin\theta_p/\cos\theta_p$ where $\theta_i = \theta_p$.

**4.68** $\tan\theta_p = n_t/n_i = n_2/n_1$, $\tan\theta'_p = n_1/n_2$, $\tan\theta_p = 1/\tan\theta'_p$.
$\sin\theta_p/\cos\theta_p = \cos\theta'_p/\sin\theta'_p$. Therefore $\sin\theta_p\sin\theta'_p - \cos\theta_p\cos\theta'_p = 0$,
$\cos(\theta_p + \theta'_p) = 0$, so $\theta_p + \theta'_p = 90°$.

**4.69** From Eq. (4.94), $\tan\gamma_r = r_\perp[E_{oi}]_\perp/r_\parallel[E_{oi}]_\parallel = (r_\perp/r_\parallel)\tan\gamma_i$ and from
Eqs. (4.42) and (4.43)

$$\tan\gamma_r = -\frac{\cos(\theta_i - \theta_t)}{\cos(\theta_i + \theta_r)}\tan\gamma_i.$$

**4.70** (4.56) $R = \left(\frac{E_{or}}{E_{oi}}\right)^2$ $E_{or}^2 = E_{or\parallel}^2 + E_{or\perp}^2$. $E_{oi}^2 = E_{oi\parallel}^2 + E_{oi\perp}^2$.

(4.34) $r_\perp \equiv \left(\frac{E_{or}}{E_{oi}}\right)_\perp$ · (4.38) $r_\parallel \equiv \left(\frac{E_{or}}{E_{oi}}\right)_\parallel$.

$$R = \frac{E_{or\perp}^2 + E_{or\|}^2}{E_{oi\perp}^2 + E_{oi\|}^2}$$

$$= \frac{(E_{or\perp}/E_{oi\perp})^2}{1+(E_{oi\|}/E_{oi\perp})^2}$$

$$+\frac{(E_{or\|}/E_{oi\|})^2}{(E_{oi\perp}/E_{oi\|})^2+1}$$

$$= \frac{r_\perp^2}{1+1/\tan^2\lambda_i} + \frac{r_\|^2}{\tan^2\gamma_i+1}$$

$$= R_\perp \sin^2\gamma_i + R_\| \cos^2\gamma_i$$

$$(4.57)\; T = \frac{n_t\cos\theta_t}{n_i\cos\theta_i}\left(\frac{E_{ot}}{E_{oi}}\right)^2$$

as above, $\left(\frac{E_{ot}}{E_{oi}}\right)^2 = t_\perp^2\sin^2\gamma_i + t_\|\cos^2\gamma_i$, and using (4.63, 4.64),

$$T_{\perp,\|} = \left(\frac{n_t\cos\theta_t}{n_i\cos\theta_i}\right)t_{\perp,\|}^2.$$

$T = T_\perp\sin^2\gamma_i + T_\|\cos^2\gamma_i.$

**4.71** Note that $\theta_e = 41.8°$. Note that $R_\perp$ increases steadily, while $R_\|$ has a minimum at $\theta_i \neq 0$.

**4.72** $T_\| = \dfrac{n_t\cos\theta_t\ t_\perp^2}{\cos\theta_i}$

from Eq. (4.45)

$$T_\| = \left(\frac{\sin\theta_i\cos\theta_t}{\sin\theta_t\cos\theta_i}\right)\frac{4\sin^2\theta_t\cos^2\theta_i}{\sin^2(\theta_i+\theta_t)\cos^2(\theta_i+\theta_t)}$$

$$T_\| = \frac{\sin 2\theta_i \sin 2\theta_t}{\sin^2(\theta_i+\theta_t)\cos^2(\theta_i+\theta_t)}$$

$T_\perp = n_t t_\perp^2\cos\theta_t/n_i\cos\theta_i$. From Eq. (4.44) and Snell's Law,

$$T_\perp = \left(\frac{\sin\theta_i\cos\theta_t}{\sin\theta_t\cos\theta_i}\right)\left(\frac{4\sin^2\theta_t\cos^2\theta_i}{\sin^2(\theta_i+\theta_t)}\right) = \frac{\sin 2\theta_i\sin 2\theta_t}{\sin^2(\theta_i+\theta_t)}.$$

**4.73** Use (4.62) and (4.43). $R_\| = r_\|^2 = \tan^2(\theta_i-\theta_t)/\tan^2(\theta_i+\theta_t) =$ $[\sin^2(\theta_i-\theta_t)/\cos^2(\theta_i-\theta_t)]\times[\cos^2(\theta_i+\theta_t)/\sin^2(\theta_i+\theta_t)]$. Note that $R_\|$ and $T_\|$ have now the same denominator.

Use (4.61) and (4.42). $R_\perp = r_\perp^2 = \sin^2(\theta_i-\theta)/\sin^2(\theta_i+\theta_t)$. Note that $R_\perp$ and $T_\perp$ have the same denominator.

**4.74** If $\Phi_i$ is the incident radiant flux or power and $T$ is the transmittance across the first air-glass boundary, the transmitted flux is then $T\Phi_i$. From Eq. (4.68) at normal incidence the transmittance from glass to air is also $T$. Thus a flux $T\Phi_i T$ emerges from the first slide, and $\Phi_i T^{2N}$ from the last one. Since $T = 1-R, T_t = (1-R)^{2N}$ from Eq. (4.67).

$$R = (0.5/2.5)^2 = 4\%, T = 96\%, T_t = (0.96)^6 \approx 78.3\%.$$

**4.75** $T = I(y)/I_0 = e^{-\alpha y}$, $T_1 = e^{-\alpha}$, $T = (T_1)^y$. $T_t = (1-R)^{2N}(T_1)^d$.

**4.76** At $\theta_i = 0, R = R_\parallel = R_\perp = [(n_t - n_i)/(n_t + n_i)]^2$. As $n_{ti} \to 1$, $n_t \to n_i$ and clearly $R \to 0$. At $\theta_i = 0$, $T = T_\parallel = T_\perp 4n_t n_i/(n_t + n_i)^2$ and since $n_t \to n_i$, $\lim_{n_{ti}\to 1} T = 4n_i^2/(2n_i)^2 = 1$. From Problem 4.61 and the fact that as $n_t \to n_i$ Snell's Law says that $\theta_t \to \theta_i$, we have

$$\lim_{n_{ti}\to 1} T_\parallel = \sin^2 2\theta_i / \sin^2 2\theta_i = 1, \lim_{n_{ti}\to 1} T_\perp = 1.$$

From Eq. (4.43) and the fact that $R_\parallel = r_\parallel^2$ and $\theta_t \to \theta_i$, $\lim_{n_{ti}\to 1} R_\parallel = 0$.

**4.77** (4.34) 
$$r_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$
$$= \frac{\cos\theta_i - n_{ti}\cos\theta_t}{\cos\theta_i + n_{ti}\cos\theta_t}$$
$$= \frac{\cos\theta_i - n_{ti}\sqrt{1-\sin^2\theta_t}}{\cos\theta_i + n_{ti}\sqrt{1-\sin^2\theta_t}}$$
$$= \frac{\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_i}}{\cos\theta_i + \sqrt{n_{ti}^2 + \sin^2\theta_i}}$$
$$r_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t}$$
$$= \frac{n_{ti}\cos\theta_i - \sqrt{1-\sin^2\theta_t}}{n_{ti}\cos\theta_i + \sqrt{1-\sin^2\theta_t}}$$
$$= \frac{n_{ti}^2\cos\theta_i - \sqrt{n_{ti}^2 - \sin^2\theta_t}}{n_{ti}^2\cos\theta_i + \sqrt{n_{ti}^2 - \sin^2\theta_t}}$$

**4.78** For $\theta_i > \theta_c$, Eq. (4.70) can be written

$$r_\perp = \frac{\cos\theta_i - i\left(\sin^2\theta_i - n_{ti}^2\right)^{1/2}}{\cos\theta_i + i\left(\sin^2\theta_i - n_{ti}^2\right)^{1/2}},$$
$$r_\perp r_\perp^* = \frac{\cos^2\theta_i + \sin^2\theta_i - n_{ti}^2}{\cos^2\theta_i + \sin^2\theta_i - n_{ti}^2} = 1.$$

Similarly $r_\parallel r_\parallel^* = 1$.

**4.79** $\theta_c = \sin^{-1}(1/1.5) = 42°$.

**4.80** Light incident from glass to air. $\theta_t$ increases as $\theta_i$ increases, if $n_i > n_t$, then $n_i/n_t > 1$, since $\sin\theta_t \le 1$, then there exists a $\theta_c$ such that $\sin\theta_i = n_t/n_i$ This max $\theta_i = \theta_c$.
(4.4) $n_i \sin\theta_i = n_t \sin\theta_t$ so, $\sin\theta_t = (n_i/n_t)\sin\theta_i$.
Maximum $\theta_t \le 90°$ as $\theta_t \to 90°$, $\sin\theta_t \to 1$ so, $\sin\theta_t = n_i/n_t = \sin\theta_c$.

**4.81** $1.00/2.417 = \sin\theta_c$; $\theta_c = 24°$ diamond refracts light back out and so looks brilliant.

**4.82** $\sin 48.0° = (1.00/n)$; $n = 1.35$.

**4.83** $\theta_i = 45° \rightarrow \theta_c$

$$\sin\theta_i = \frac{n_t}{n_i}, \text{ where } n_t = 1$$

$$n = \frac{1}{\sin 45°} = 1.41$$

**4.84** Light entering at glancing incidence is transmitted at the critical angle and those rays limit the cone of light reaching the fish; $\sin\theta_c = 1/1.333$; $\theta_c = 49°$ and the cone-angle is twice this or 98°.

**4.85** $\sin\theta_c = n_t/n_i$; $\theta_c = 59.1°$.

**4.86** From Eq. (4.73) we see that the exponential will be in the form $k(x - \upsilon t)$, provided that we factor out $k_t \sin\theta_i/n_{ti}$, leaving the second term as $\omega n_{ti} t/k_t \sin\theta_i$, which must be $\upsilon_t t$. Hence $\omega n_t/(2\pi/\lambda_t)n_i \sin\theta_i = \upsilon_t$, and so $\upsilon_t = c/n_i \sin\theta_i = \upsilon_i/\sin\theta_i$.

**4.87** From the defining equation, $\beta = k_t[(\sin^2\theta_i/n_{ti}^2) - 1]^{1/2} = 3.702\times10^6\ \text{m}^{-1}$, and since $y\beta = 1$, $y = 2.7\times10^{-7}$ cm.

**4.88** The penetration depth ($\delta$) is related to the attenuation coefficient ($\beta$) by $\delta = 1/\beta$. One first calculates the attenuation coefficient:

$$\beta = \frac{2\pi n_t}{\lambda_0}\left[\left(\frac{n_i}{n_t}\right)^2 \sin^2\theta_i - 1\right]^{1/2}$$

$$\beta = \frac{2\pi 1}{500\ \text{nm}}\left[\left(\frac{3}{2}\right)^2 \sin^2 60 - 1\right]^{1/2}$$

$$\beta = 1.257\times10^7[2.25\times0.75 - 1]^{1/2}$$

$$\beta = 1.257\times10^7\times0.8292 = 1.04\times10^6$$

$$\delta = 9.6\times10^{-7}\,\text{m}$$

**4.89** $\sin(\theta_c) = \dfrac{n_t}{n_i}$

$$\sin(\theta_c) = \frac{1.33}{2.42}$$

$$\theta_c = \sin^{-1}(0.55) = 33.3°$$

**4.90** $\sin(\theta_c) = \dfrac{n_t}{n_i}$

$$\sin(\theta_c) = \frac{1.46}{2.41}$$

$$\theta_c = \sin^{-1}(0.61) = 37.3°$$

**4.91** The beam scatters off the wet paper and is mostly transmitted until the critical angle is attained, at which point the light is reflected back toward the source. $\tan\theta_c = (R/2)/d$, and so $n_{ti} = 1/n_i = \sin[\tan^{-1}(R/2d)]$.

**4.92** 1.00029 sin 88.7° = $n$ sin 90°, $n$ = 1.00003.

**4.93** Can be used as a mixer to get various proportions of the two incident waves in the emitted beams. This could be done by adjusting the gaps. [For some further remarks, see H. A. Daw and J. R. Izatt, *J. Opt. Soc. Am.* **55**, 201 (1965).]

**4.94** Light traverses the base of the prism as an evanescent wave, which propagates along the adjustable coupling gap. Energy moves into the dielectric film when the evanescent wave meets certain requirements. The film acts like a waveguide, which will support characteristic vibration configurations or modes. Each mode has associated with it a given speed and polarization. The evanescent wave will couple into the film when it matches a mode configuration.

**4.95** From Fig. 4.62 the obvious choice is silver. Note that in the vicinity of 300 nm, $n_I \approx n_R \approx 0.6$, in which case Eq. (4.83) yields $R \approx 0.18$. Just above 300 nm, $n_I$ increases rapidly, while $n_R$ decreases quite strongly, with the result that $R \approx 1$ across the visible and then some.

**4.96**

| | | | | | | |
|---|---|---|---|---|---|---|
| RED | → | R | YELLOW | | | |
| CYAN | → | G | | G | → | G |
| | → | B | | | | |
| MAGENTA | → | R | | R | → | R |
| | → | B | | | | |
| YELLOW | → | R | | R | → | Y |
| | → | G | | G | → | |
| WHITE | → | R | | R | → | Y |
| | → | G | | G | → | |
| | → | B | | | | |

**4.97**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| RED | → | R | YELLOW | R | → | CYAN | | BLACK |
| CYAN | → | G | | G | → | | G | GREEN |
| | → | B | | | | | | |
| WHITE | → | R | | R | → | | | GREEN |
| | → | G | | G | → | | G | |
| | → | B | | | | | | |
| YELLOW | → | R | | R | → | | | GREEN |
| | → | G | | G | → | | G | |
| GREEN | → | G | | G | → | | G | GREEN |
| MAGENTA | → | R | | R | → | | | BLACK |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | $\rightarrow$ | B | | | | | | |

**4.98**

| Graph # | Color |
|---|---|
| 1 | Magenta |
| 2 | Orange |
| 3 | Light-Grey |
| 4 | Red |
| 5 | Plum-Red |
| 6 | Yellow-Green |
| 7 | Purple |

**4.99** $$t_{\parallel} = \frac{2\sin\theta_2 \cos\theta_1}{\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2)}$$

$$t'_{\parallel} = \frac{2\sin\theta_1 \cos\theta_2}{\sin(\theta_1 + \theta_2)\cos(\theta_2 - \theta_1)}$$

$$t_{\parallel}t'_{\parallel} = \frac{\sin 2\theta_1 \cos 2\theta_2}{\sin^2(\theta_1 + \theta_2)\cos^2(\theta_1 - \theta_2)} = T_{\parallel}$$

from Eq. (4.100). Similarly $t_{\perp}t'_{\perp} = T_{\perp}$.

$$r_{\parallel}^2 = \left[\frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}\right]^2 = \left[\frac{-\tan(\theta_2 - \theta_1)}{\tan(\theta_1 + \theta_2)}\right]^2$$

$$r_{\parallel}'^2 = \left[\frac{\tan(\theta_2 - \theta_1)}{\tan(\theta_1 + \theta_2)}\right]^2 = r_{\parallel}^2 = R_{\parallel}.$$

**4.100** (4.84) $E_{oi}t_{\parallel}(\theta_1)t'_{\parallel}(\theta_2) + E_{oi}r_{\parallel}(\theta_p)r_{\parallel}(\theta_p) = E_{oi}$.
(4.85) $E_{oi}r_{\parallel}(\theta_1)t_{\parallel}(\theta_1) + E_{oi}t_{\parallel}(\theta_1)r'_{\parallel}(\theta_2) = 0$ where $\theta_2 = \theta_i = \theta'_p$ and $r'_{\parallel}(\theta'_p) = 0$. From Problem (4.66), $\theta_p = \theta_1$. From (4.84), $t_{\parallel}(\theta_p)t'_{\parallel}(\theta'_p) + 0 = 1$; $t_{\parallel}(\theta_p)t'_{\parallel}(\theta'_p) = 1$. From (4.85), $r_{\parallel}(\theta_p)t_{\parallel}(\theta_p) + 0 = 0$ Since $t_{\parallel}(\theta_p) \neq 0$, $r_{\parallel}(\theta_p) = 0$. From (4.100), $T_{\parallel} = t_{\parallel}t'_{\parallel}$, when $T_{\parallel} = 1$, there is no reflected wave, as $T + R = 1$.

**4.101** From Eq. (4.45)

$$t'_{\parallel}(\theta'_p)t_{\parallel}(\theta_p) = \left[\frac{2\sin\theta_p \cos\theta'_p}{\sin(\theta_p + \theta'_p)\cos(\theta'_p - \theta_p)}\right]\left[\frac{2\sin\theta'_p \cos\theta_p}{\sin(\theta_p + \theta'_p)\cos(\theta'_p - \theta_p)}\right]$$

$$= \frac{\sin 2\theta'_p \sin 2\theta_p}{\cos^2(\theta_p - \theta'_p)} \text{ since } \theta_p + \theta'_p = 90°$$

$$= \frac{\sin^2 2\theta_p}{\cos^2(\theta_p - \theta'_p)} \text{ since } \sin 2\theta'_p = \sin 2\theta_p, \quad \frac{\sin^2 2\theta_p}{\cos^2(2\theta_p - 90°)} = 1.$$